Investigating the conformal window in the Litim-Sannino model at 433 order

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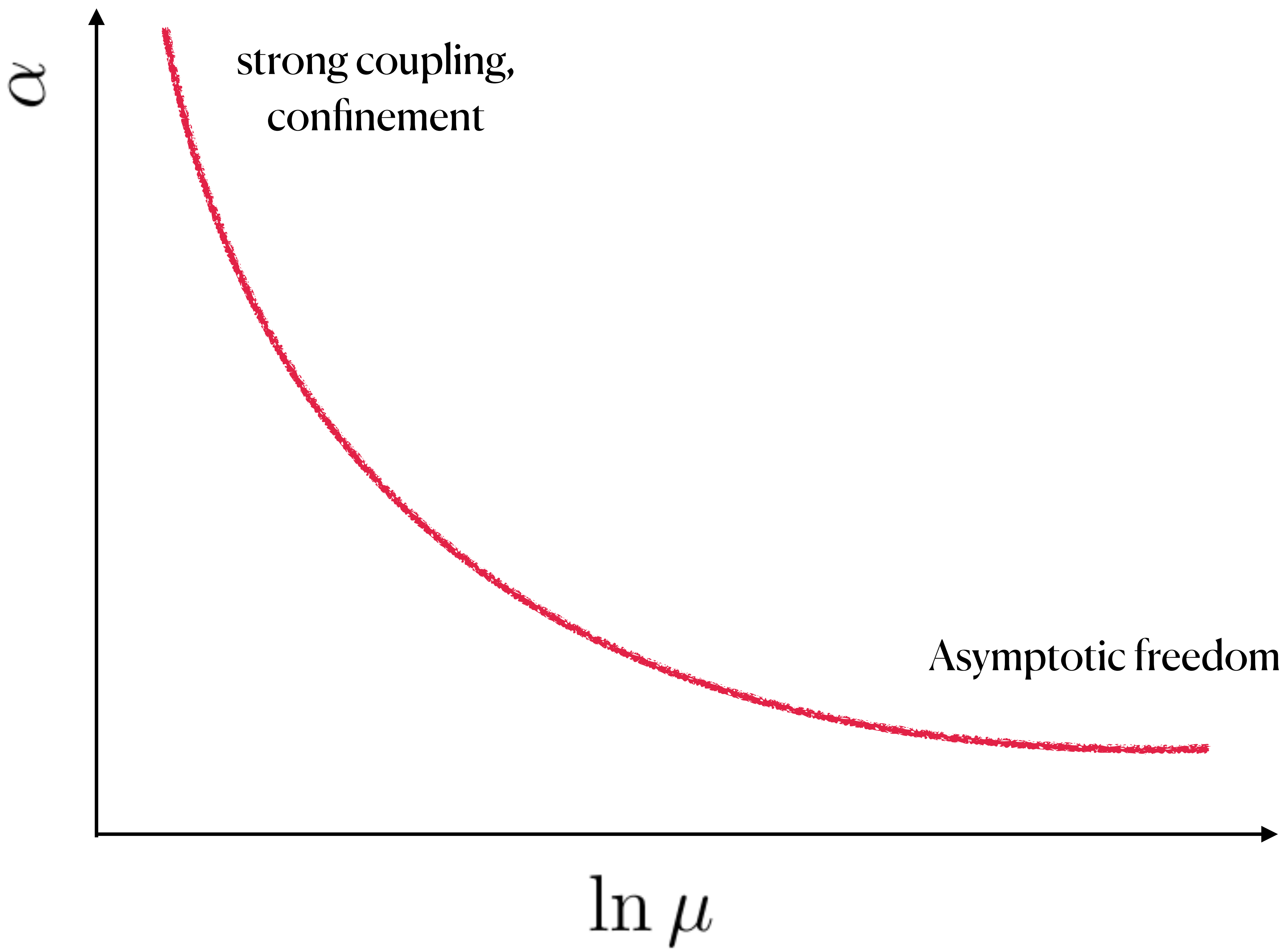
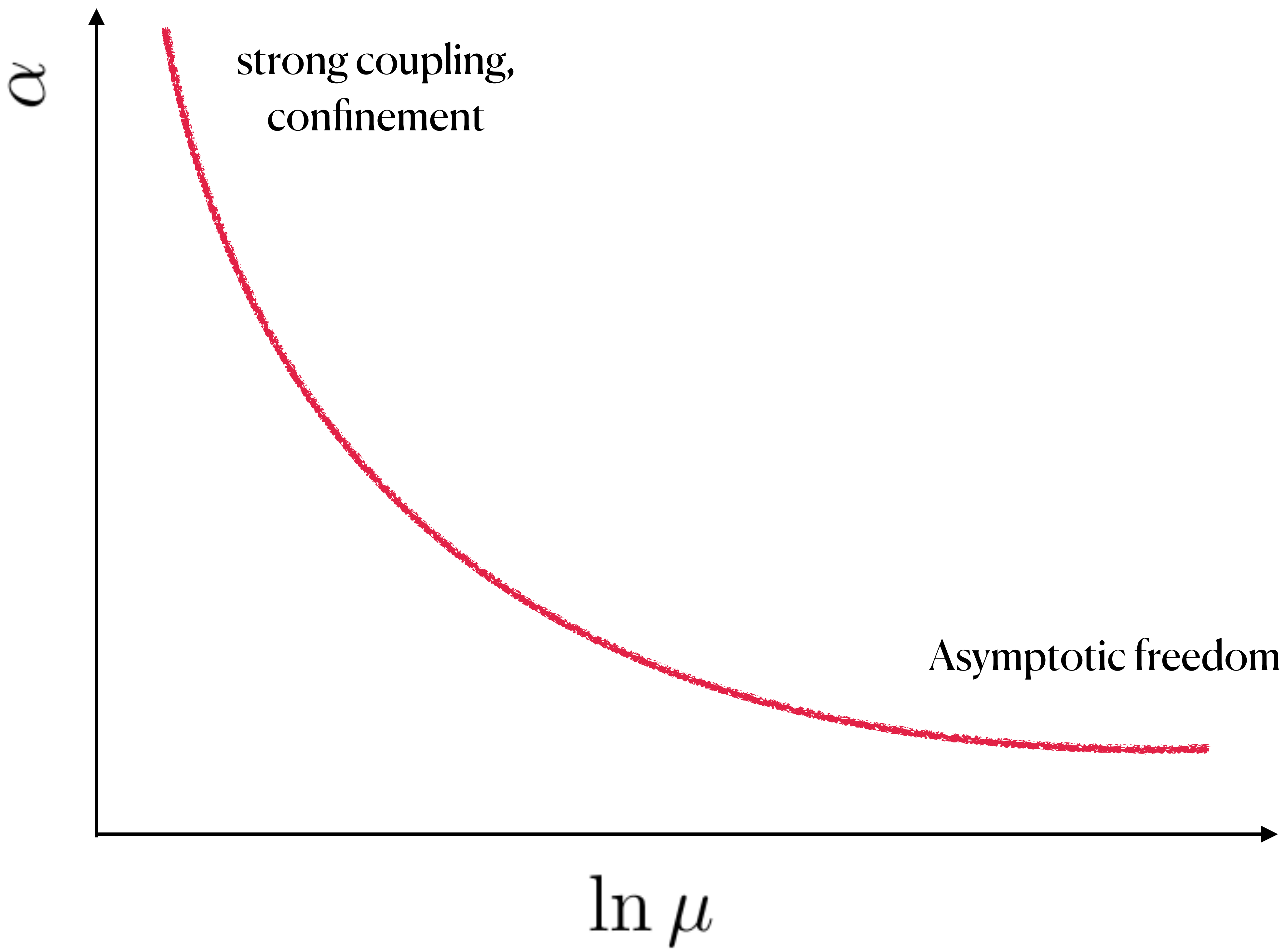
Motivation

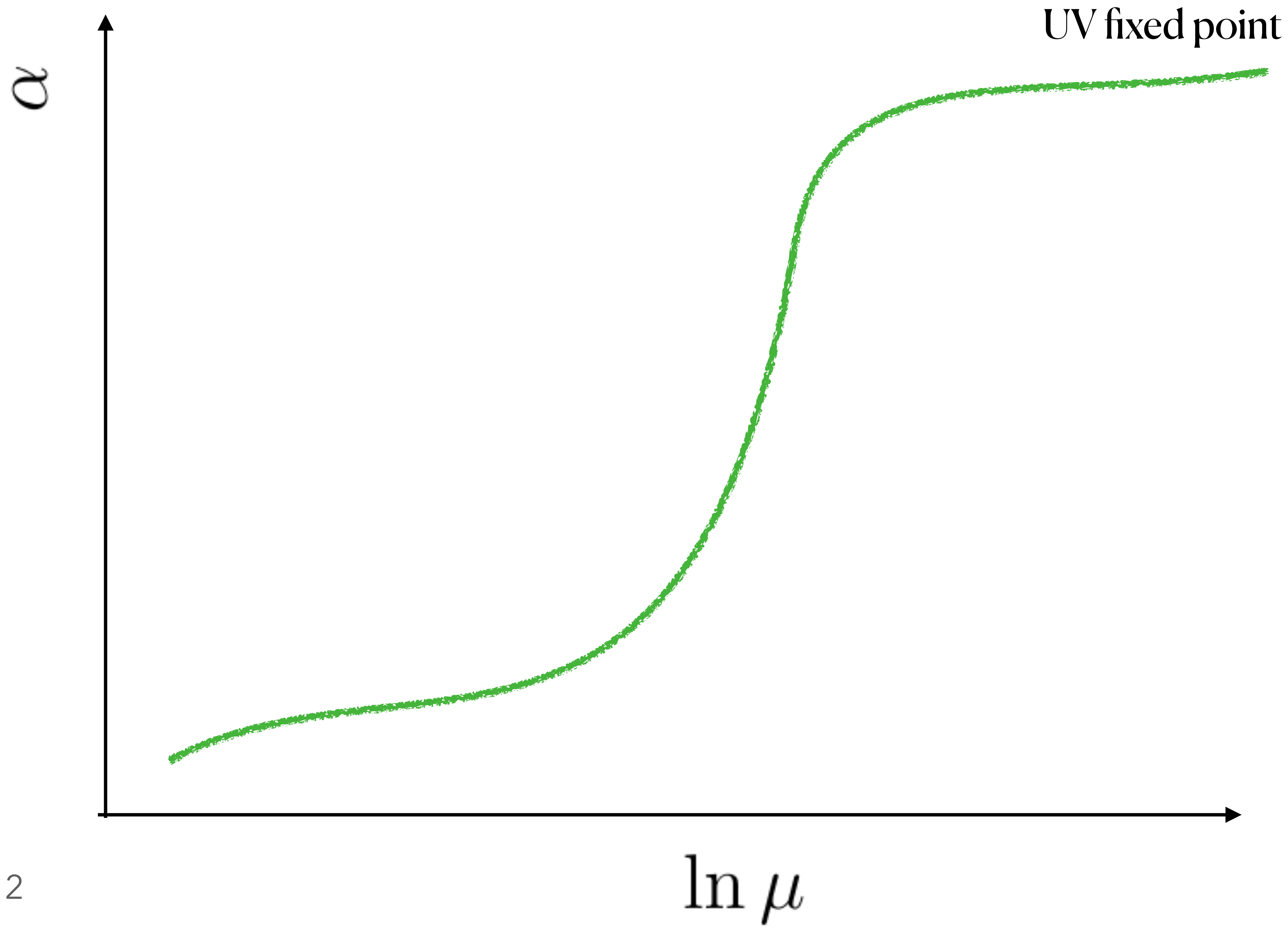
- QCD: Asymptotic Freedom [Gross, Wilczek, Politzer, (1971)]
  - UV complete
  - theory remains predictive



Motivation

- QCD: Asymptotic Freedom [Gross, Wilczek, Politzer, (1971)]
  - UV complete
  - theory remains predictive
- Generalisation: Asymptotic Safety [Weinberg (1980)]
  - known reliable examples away from d=4
  - Exception: d=4 Litim-Sannino model





Litim–Sannino Model

field	$SU(N_c)$	$U_L(N_f)$	$U_R(N_f)$
ψ_L	N_c	N_f	1
ψ_R	N_c	1	N_f
H	1	N_f	$\bar{N}_f$

gauge sector (QCD)

$$\mathcal{L} = -\frac{1}{4} F^{A\mu\nu} F^{A}_{\mu\nu} + \mathcal{L}_{gf} + \mathcal{L}_{gh} + \mathrm{Tr}(\bar{\psi} i \hat{D} \psi)$$
$$+ \mathrm{Tr}(\partial^\mu H^\dagger \partial_\mu H) - y\, \mathrm{Tr}[\bar{\psi}(H \mathcal{P}_R + H^\dagger \mathcal{P}_L)\psi]$$
$$- m^2\, \mathrm{Tr}(H^\dagger H) - u\, \mathrm{Tr}((H^\dagger H)^2) - v(\mathrm{Tr}(H^\dagger H))^2$$

Yukawa (chiral)

single trace

double trace

- interacting fixed points under perturbative control

Perturbative control

- Veneziano limit: $N_{f,c} \to \infty$ but $N_f/N_c = \text{const}$
- 't Hooft couplings:

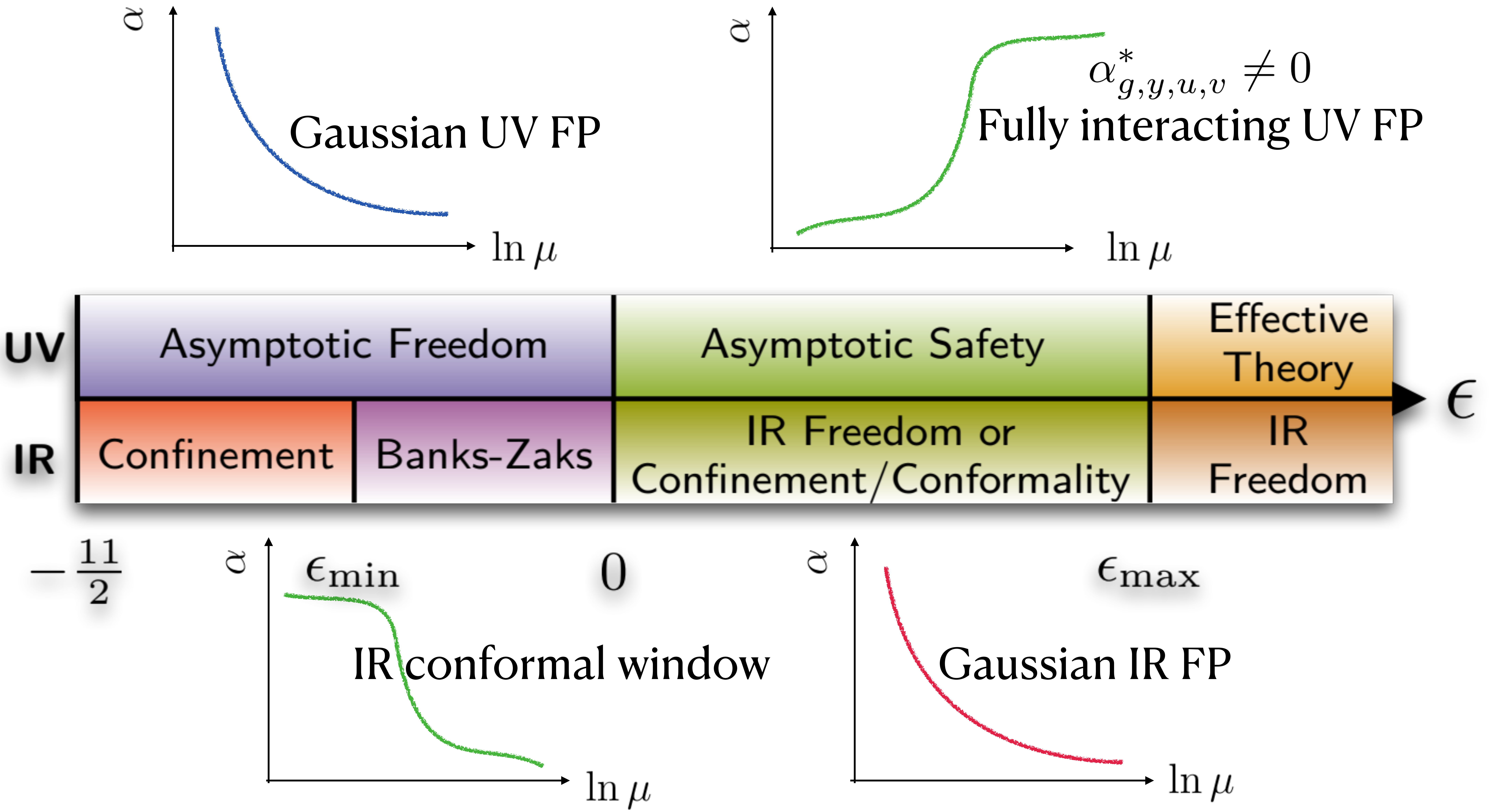
$$\alpha_g = \frac{N_c\, g^2}{(4\pi)^2} \qquad \alpha_y = \frac{N_c\, y^2}{(4\pi)^2} \qquad \alpha_u = \frac{N_f\, u}{(4\pi)^2} \qquad \alpha_v = \frac{N_f^2\, v}{(4\pi)^2}$$

- Small expansion parameter: $\epsilon = \frac{N_f}{N_c} - \frac{11}{2}$ $\quad -\frac{11}{2} < \epsilon < \infty \quad |\epsilon| \ll 1$
- 1 loop gauge coefficient $\beta_g = \alpha_g^2 \left[\frac{4}{3}\epsilon + \mathcal{O}(\alpha^1)\right]$
- Conformal expansion: $\alpha^* = \epsilon\, a_{\text{LO}} + \epsilon^2\, a_{\text{NLO}} + \epsilon^3\, a_{\text{NNLO}} + \ldots$

211 Litim, Sannino (2014)	322 Litim, Bond, Medina, Steudtner (2017)	433 Litim, Riyaz, Stamou, Steudtner (2023)	433 Bednyakov, Mukhaeva (2023) soon

n loop gauge, *m* loop Yukawa, *l* loop scalar $\equiv (n, m, l)$

Conformal Window of weakly interacting FP



[Litim, Riyaz, Stamou, Steudtner (2023)]

Conformal Window

How to probe the UV conformal window

I. *Directly from beta functions* $\beta_{g,y,u,v}$ ϵ_{strict}

- fixed point values $\alpha^*_{g,y,u,v}(\epsilon)$ from $\beta_{g,y,u,v} = 0$)
  - coupling $0 < |\alpha^*| \lesssim 1$ [Weinberg, 1978]
  - vacuum stability $\alpha^*_u > 0$ and $\alpha^*_u + \alpha^*_v > 0$ [A.J. Paterson, 1980]
- critical exponents ϑ_i as eigenvalues of stability matrix $M_{xx'} = \left.\frac{\partial \beta_x}{\partial \alpha_{x'}}\right|_{\alpha=\alpha^*}$

$$\beta_i = \sum_j M_{ij}\left(\alpha_j - \alpha^*_j\right) + \text{subleading}$$

Relevant: $\vartheta_1 < 0$; Irelevant: $0 < \vartheta_{2,3,4}$

$$\vartheta_1 < 0 < \vartheta_{2,3,4}$$

II. ϵ*-expansion of* $\alpha^*_i(\epsilon)$ *and* $\vartheta_i(\epsilon)$ *(series is exact up to third order)* ϵ_{subl}

III. *Strong coupling constraints* $\alpha^*_x \gtrsim 1$

Investigating the UV conformal window

- Gauge coupling

$$\alpha_g^* = 0.456\epsilon + 0.781\epsilon^2 + 6.610\epsilon^3 + O(\epsilon^4),$$
$$\alpha_y^* = 0.211\epsilon + 0.508\epsilon^2 + 3.322\epsilon^3 + O(\epsilon^4),$$
$$\alpha_u^* = 0.200\epsilon + 0.440\epsilon^2 + 2.693\epsilon^3 + O(\epsilon^4),$$
$$\alpha_v^* = -0.137\epsilon - 0.632\epsilon^2 - 4.313\epsilon^3 + O(\epsilon^4)$$

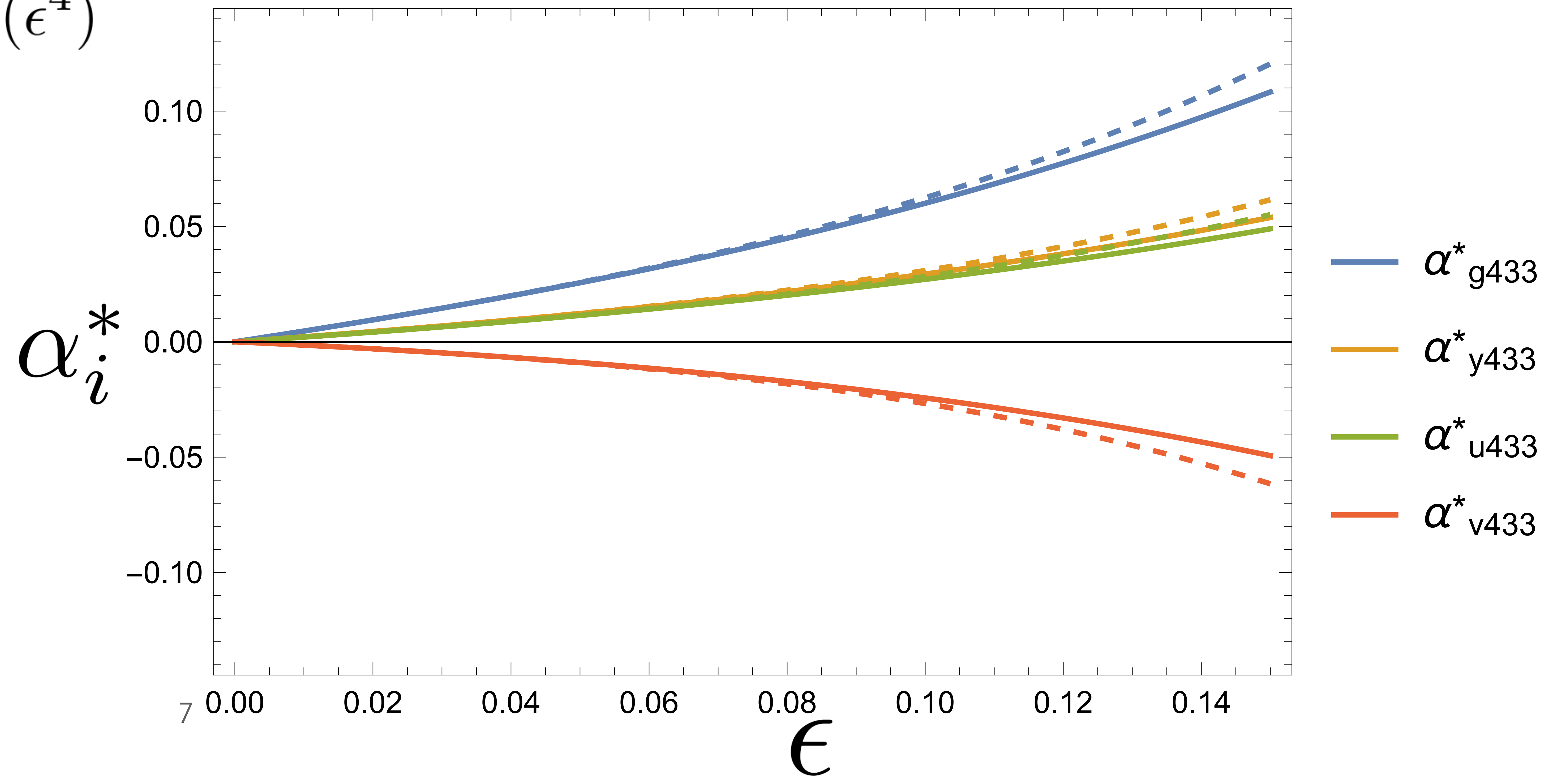
$$\epsilon_{strict} \sim (0.117 - 0.457)$$
$$\epsilon_{subl} \sim (0.117 - 0.363)$$

Competition of fluctuations

$$\beta_g^{(4)}|_{322} = +4.52\epsilon^5$$
$$\beta_y^{(3)}|_{322} = -0.14\epsilon^4$$
$$\beta_u^{(3)}|_{322} = +0.57\epsilon^4$$
$$\beta_v^{(3)}|_{322} = +2.02\epsilon^4$$

$\Delta\beta < 0$ stabilize the FP

$\Delta\beta > 0$ destabilize the FP



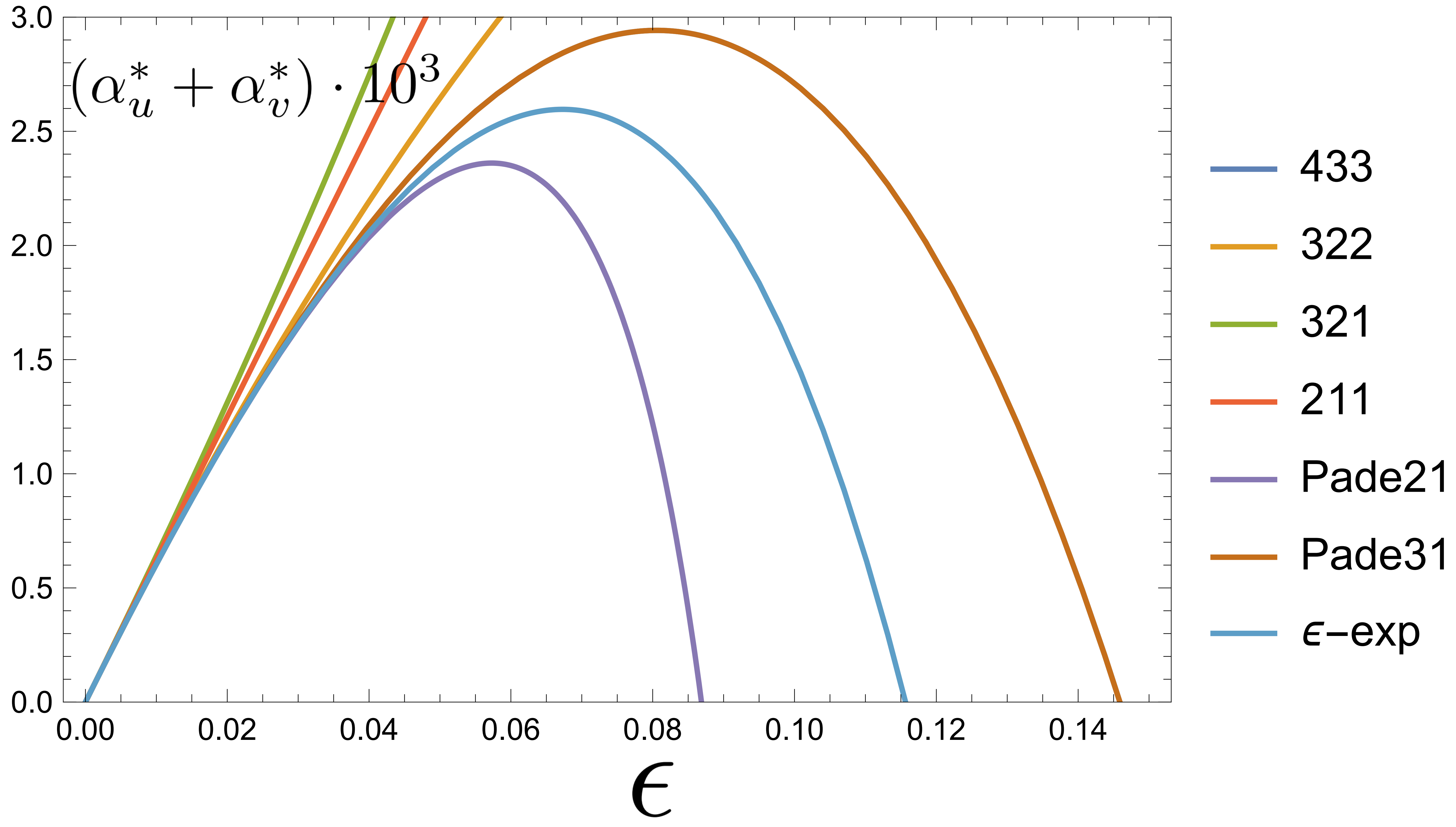
Investigating the UV conformal window

- Vacuum instability

$$\alpha_u^* + \alpha_v^* \approx +0.0625\,\epsilon - 0.192\,\epsilon^2 - 1.62\,\epsilon^3 + \dots$$

$$\epsilon_{strict} \sim (0.087 - 0.146)$$

$$\epsilon_{subl} \sim (0.087 - 0.116)$$

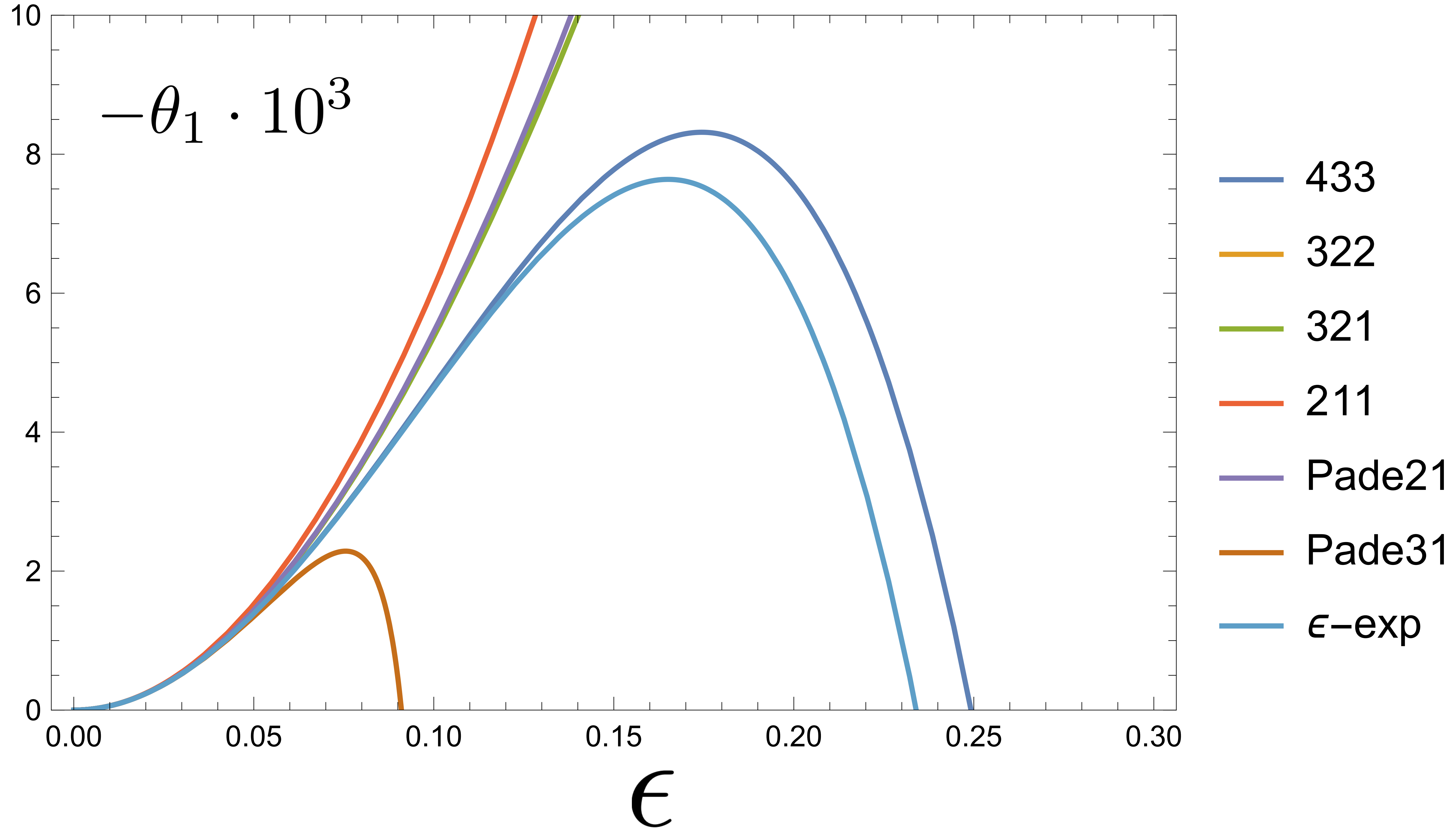


Investigating the UV conformal window

- Scaling exponents

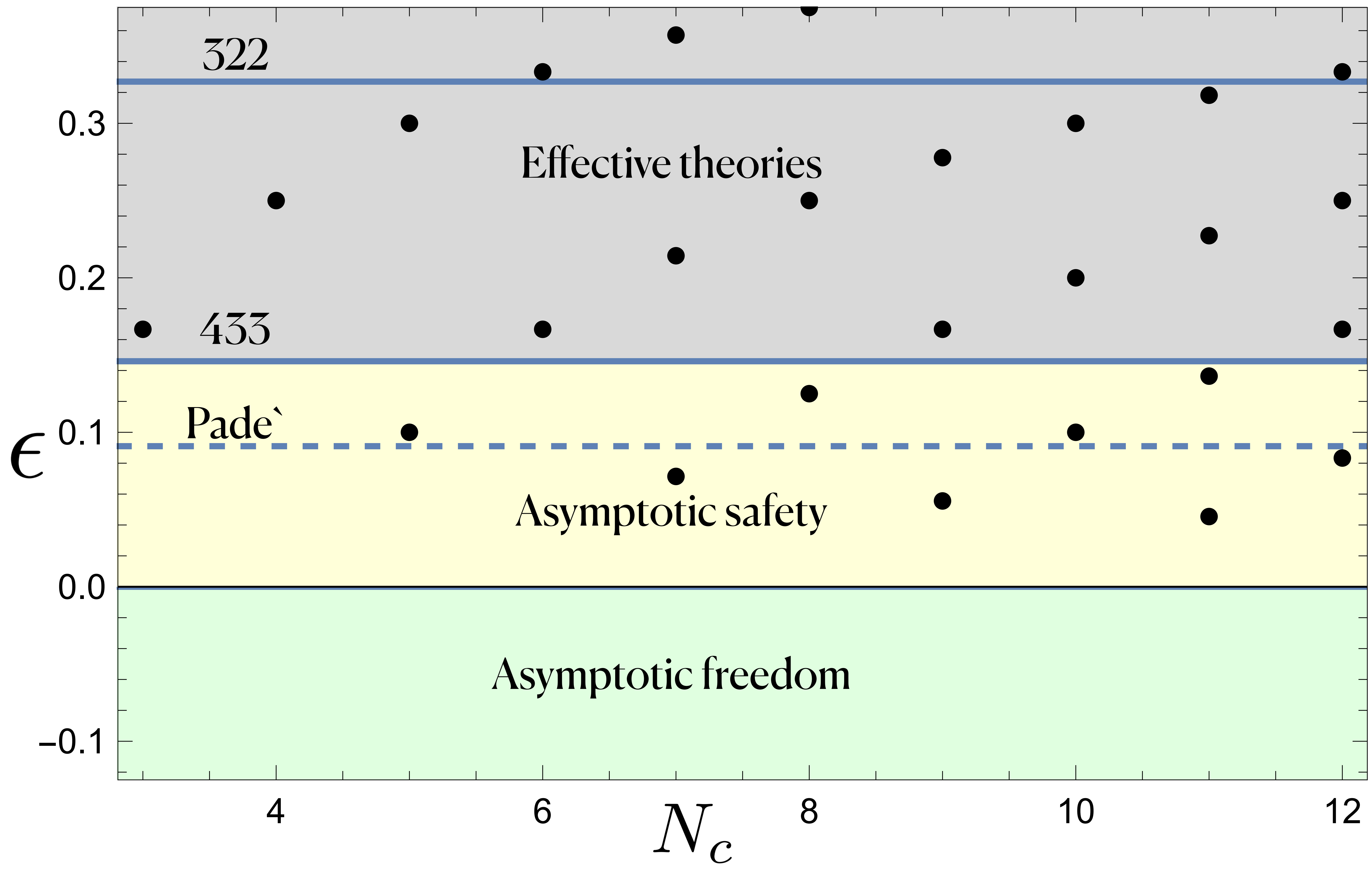
$$\theta_1 = -0.608\epsilon^2 + 0.707\epsilon^3 + 6.947\epsilon^4 + O(\epsilon^5)$$
$$\theta_2 = 2.737\epsilon + 6.676\epsilon^2 + 22.120\epsilon^3 + O(\epsilon^4)$$
$$\theta_3 = 2.941\epsilon + 1.041\epsilon^2 + 5.137\epsilon^3 + O(\epsilon^4)$$
$$\theta_4 = 4.039\epsilon + 9.107\epsilon^2 + 38.646\epsilon^3 + O(\epsilon^4)$$

$$\epsilon_{strict} \sim (0.091 - 0.249)$$
$$\epsilon_{subl} \sim (0.091 - 0.234)$$



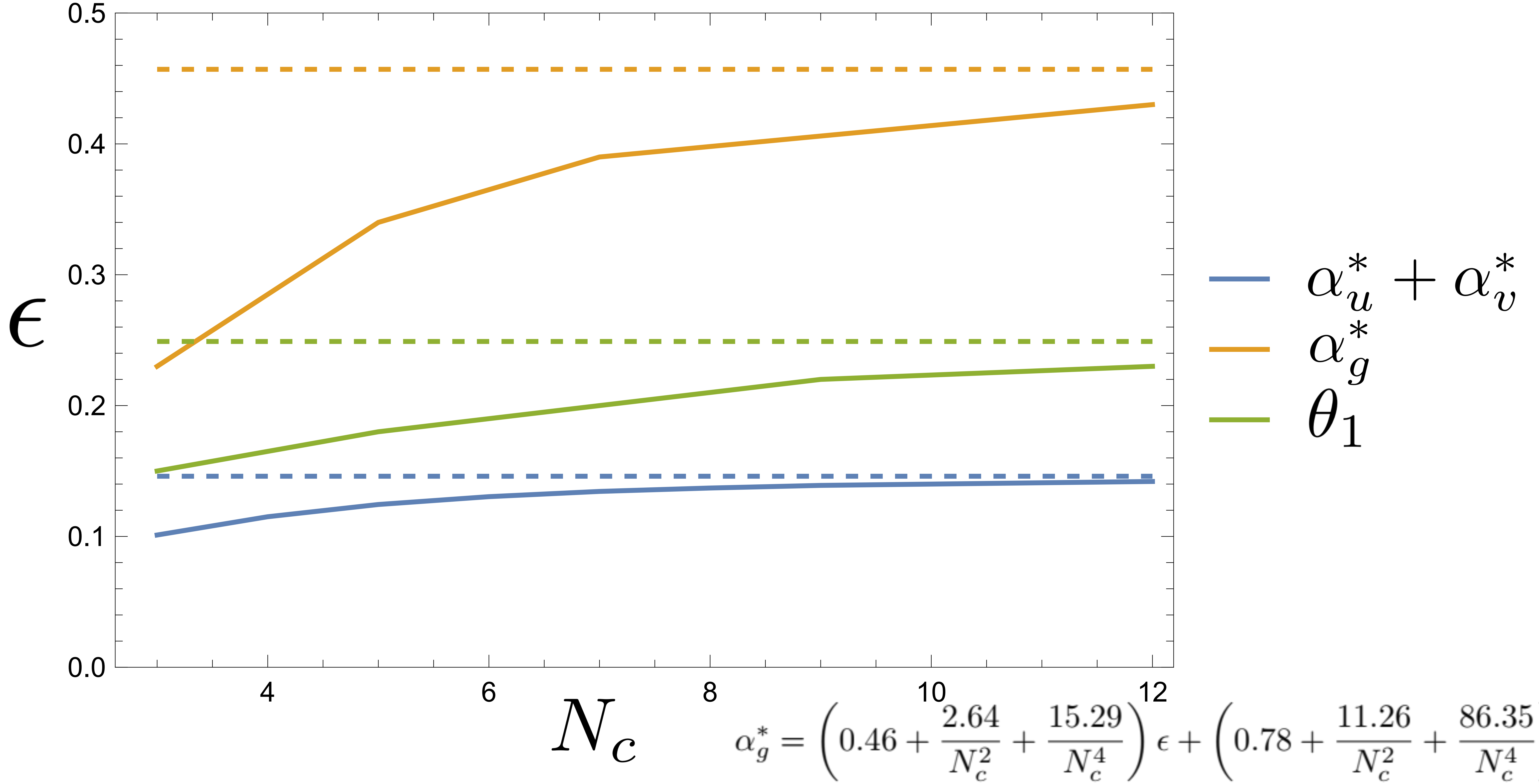
$$N_c = 2n \qquad N_f = 11n + j \qquad \epsilon = \frac{j}{N_c}$$

$$N_c = 2m+1 \quad N_f = 11m + 5 + j \quad \epsilon = \frac{j-\frac{1}{2}}{N_c}$$



Safe QFTs: (Nc,Nf) =(5,28), (7,39), (8,45), (9,50), (10,56), (11,61), (11,62), (12,67),....

Bounds from finite Nc



$$\alpha_g^* = \left(0.46 + \frac{2.64}{N_c^2} + \frac{15.29}{N_c^4}\right)\epsilon + \left(0.78 + \frac{11.26}{N_c^2} + \frac{86.35}{N_c^4}\right)\epsilon^2 + \left(6.61 + \frac{154.84}{N_c^2} + \frac{2206.62}{N_c^4}\right)\epsilon^3 + O(\epsilon^4)$$

$$\alpha_y^* = \left(0.21 + \frac{1.01}{N_c^2} + \frac{5.84}{N_c^4}\right)\epsilon + \left(0.51 + \frac{5.95}{N_c^2} + \frac{55.46}{N_c^4}\right)\epsilon^2 + \left(3.22 + \frac{70.04}{N_c^2} + \frac{967.31}{N_c^4}\right)\epsilon^3 + O(\epsilon^4)$$

$$\alpha_u^* = \left(0.20 + \frac{0.96}{N_c^2} + \frac{5.57}{N_c^4}\right)\epsilon + \left(0.44 + \frac{5.17}{N_c^2} + \frac{48.47}{N_c^4}\right)\epsilon^2 + \left(2.69 + \frac{60.76}{N_c^2} + \frac{853.03}{N_c^4}\right)\epsilon^3 + O(\epsilon^4)$$

$$\alpha_v^* = \left(-0.14 - \frac{0.67}{N_c^2} - \frac{3.85}{N_c^4}\right)\epsilon + \left(-0.63 - \frac{6.71}{N_c^2} - \frac{59.14}{N_c^4}\right)\epsilon^2 + \left(-4.31 - \frac{82.47}{N_c^2} - \frac{1055.82}{N_c^4}\right)\epsilon^3 + O(\epsilon^4)$$

Summary

- confirmed [Litim, Riyaz, Stamou, Steudtner (2023)] results
- found analytic expressions for fixed points for finite Nc
- estimated conformal window for (in)finite Nc
- set $\boxed{\epsilon_{\rm max} \approx 0.09 \pm 0.01}$

Thanks for attention!