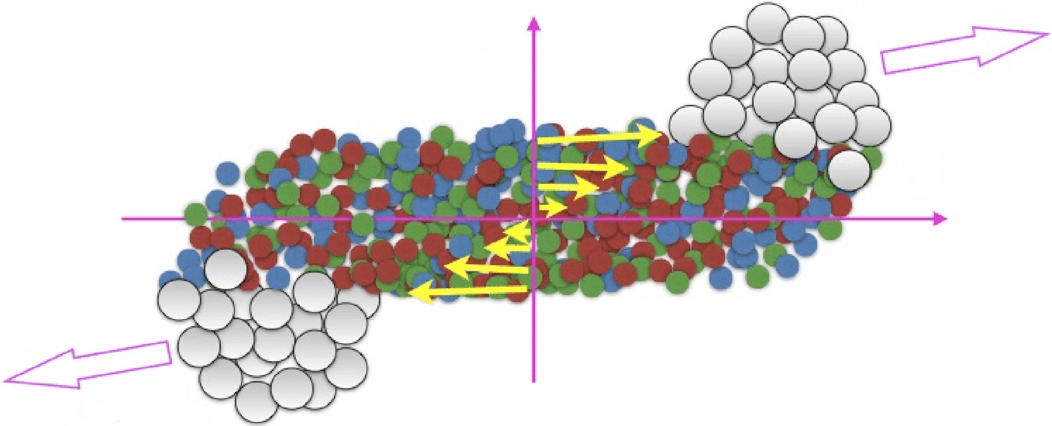
Negative moment of inertia and rotational instability of gluon plasma

D.A.Sychev
in collaboration with
V.V.Braguta, M.N.Chernodub, A.A.Roenko, I.E.Kudrov

BLTP JINR, MIPT

1 November 2023

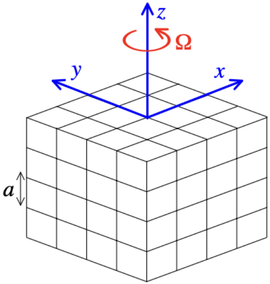
Motivation



Rigidity condition

$$v = \Omega \times r \tag{1}$$

$$\Omega = \frac{1}{2}\nabla \times v = \text{const} \tag{2}$$



Rotating coordinates

$$t = t_{\text{lab}}, \quad r = r_{\text{lab}}, \quad z = z_{\text{lab}}, \quad \varphi \sim (\varphi_{\text{lab}} - \Omega t) \tag{3}$$

$$g^{(\text{lab})}_{\mu\nu} = \eta_{\mu\nu} = \text{diag}(1, -1, -1, -1) \tag{4}$$

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = (1 - r^2\Omega^2)dt^2 - 2r^2\Omega dt d\varphi - dr^2 - r^2 d\varphi^2 - dz^2 \tag{5}$$

$$g_{\mu\nu} = \begin{pmatrix} 1 - r^2\Omega^2 & \Omega y & -\Omega x & 0 \\ \Omega y & -1 & 0 & 0 \\ -\Omega x & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{6}$$

Moment of inertia

$$E = E^{(\text{lab})} - J\Omega \tag{7}$$

$$dE^{(\text{lab})} = TdS + \boldsymbol{\Omega} d\boldsymbol{J} \tag{8}$$

$$dE = TdS - \boldsymbol{J} d\boldsymbol{\Omega} \tag{9}$$

$$F = E - TS \tag{10}$$

$$J = -\left(\frac{\partial F}{\partial \Omega}\right)_T \tag{11}$$

$$I = \frac{1}{\Omega} J \tag{12}$$

$$\mathrm{i}_2 = \frac{I}{V R_\perp^2}, \qquad R_\perp = \frac{L_s}{2} \tag{13}$$

$$K_2 = -\frac{I}{F_0 R_\perp^2} \tag{14}$$

Gluodynamics in rotating coordinates

$$S_G = \int d^4x \, \sqrt{\det g_{\alpha\beta}} \, \frac{1}{2g_{\mathrm{YM}}^2} \, g^{\mu\nu} g^{\rho\sigma} \, \mathrm{tr} F_{\mu\rho} F_{\nu\sigma} \tag{15}$$

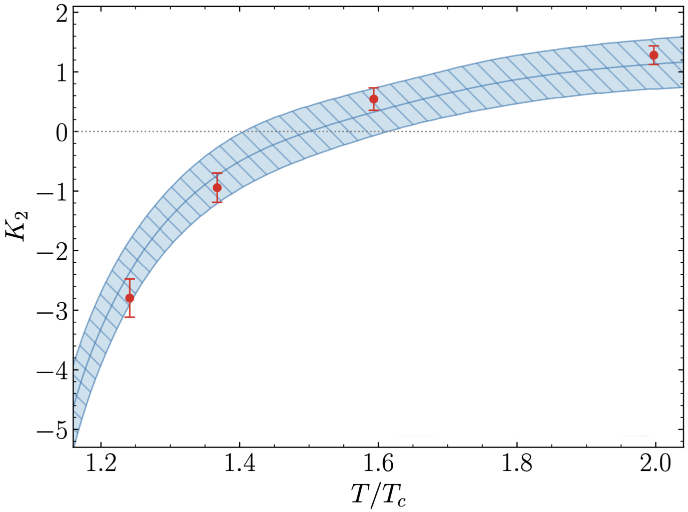
$$S_G = \frac{1}{g_{\mathrm{YM}}^2} \int d^4x \, \mathrm{tr}[(1-r^2\Omega^2) F_{xy} F_{xy} + (1-y^2\Omega^2) F_{xz} F_{xz} + (1-x^2\Omega^2) F_{yz} F_{yz} + F_{x\tau} F_{x\tau} + F_{y\tau} F_{y\tau} +$$

$$+ F_{z\tau} F_{z\tau} - 2iy\Omega(F_{xy} F_{y\tau} + F_{xz} F_{z\tau}) + 2ix\Omega(F_{yx} F_{x\tau} + F_{yz} F_{z\tau}) - 2xy\Omega^2 F_{xz} F_{zy}] \tag{16}$$

Simulations with a sign problem:

- Analytical continuation $\Omega_I = -i\Omega$
- Expansion coefficients at $\Omega = 0$

Temperature dependence of moment of inertia



Condition of thermodynamic stability

$$\delta E - T\delta S - \boldsymbol{\Omega}\delta \boldsymbol{J} > 0 \tag{17}$$

$$g^{(W),\mu\nu} = -\frac{\partial^2 f(T,\boldsymbol{\Omega})}{\partial X_\mu \partial X_\nu}, \qquad X_\mu = (T, \Omega_i) \tag{18}$$

$$C_J = T\left(\frac{\partial S}{\partial T}\right)_J > 0 \tag{19}$$

$$I > 0 \tag{20}$$

Decomposing moment of inertia

$$F = -T \ln \int DA e^{iS} \tag{21}$$

$$I^{\rm gl} = I^{\rm gl}_{\rm mech} + I^{\rm gl}_{\rm magn} \tag{22}$$

$$I^{\rm gl}_{\rm mech} = \frac{1}{T} \langle\!\langle \left(\boldsymbol{n} \cdot \boldsymbol{J}^{\rm gl} \right)^2 \rangle\!\rangle_T \tag{23}$$

$$J^{\rm gl}_i = \frac{1}{2} \int_V d^3x \, \epsilon_{ijk} M^{jk}_{\rm gl}(\boldsymbol{x}) \,, \qquad i,j = 1,2,3 \tag{24}$$

$$M^{ij}_{\rm gl}(\boldsymbol{x}) = x^i T^{j0}_{\rm gl}(\boldsymbol{x}) - x^j T^{i0}_{\rm gl}(\boldsymbol{x}) \tag{25}$$

$$T^{\mu\nu}_{\rm gl} = G^{a,\mu\alpha} G^{a,\nu}{}_{\alpha} - (1/4)\eta^{\mu\nu} G^{a,\alpha\beta} G^a_{\alpha\beta} \tag{26}$$

$$\langle\!\langle \mathcal{O} \rangle\!\rangle_T = \langle \mathcal{O} \rangle_T - \langle \mathcal{O} \rangle_{T=0} \tag{27}$$

Moment of inertia and magnetic gluon condensate

$$I^{\rm gl}_{\rm magn} = \int_V d^3x \Big[\langle\!\langle (\boldsymbol{B}^a \cdot \boldsymbol{x}_\perp)^2 \rangle\!\rangle_T + \langle\!\langle (\boldsymbol{B}^a \cdot \boldsymbol{n})^2 \rangle\!\rangle_T \boldsymbol{x}_\perp^2 \Big] \tag{28}$$

$$B^a_i = \frac{1}{2} \epsilon^{ijk} G^a_{jk} \tag{29}$$

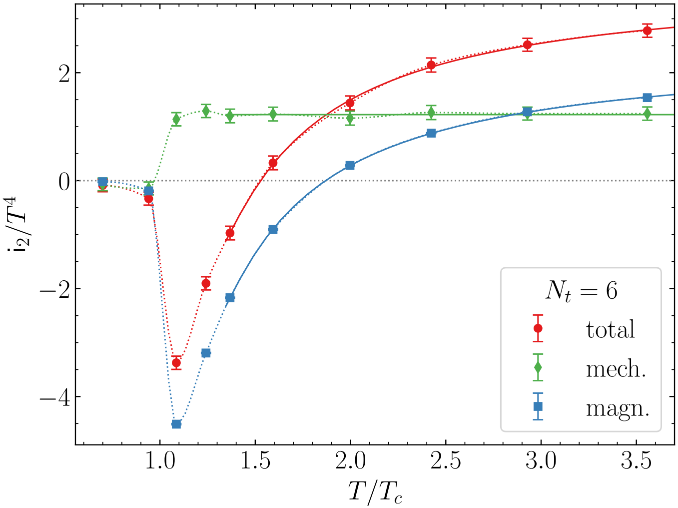
$$\langle\!\langle B^a_i B^a_j \rangle\!\rangle_T = \frac{1}{3} \delta_{ij} \langle\!\langle (\boldsymbol{B}^a)^2 \rangle\!\rangle_T \tag{30}$$

$$I^{\rm gl}_{\rm magn} = \frac{2}{3} \int_V d^3x \, x_\perp^2 \langle\!\langle (\boldsymbol{B}^a)^2 \rangle\!\rangle_T \tag{31}$$

$$I_{\rm class} = \int_V d^3x \, \rho(\boldsymbol{x}) \, \boldsymbol{x}_\perp^2 \tag{32}$$

$$\rho(T) \to \frac{2}{3} \langle\!\langle (\boldsymbol{B}^a)^2 \rangle\!\rangle_T \tag{33}$$

Moment of inertia and magnetic gluon condensate



Scale anomaly and gluon condensate

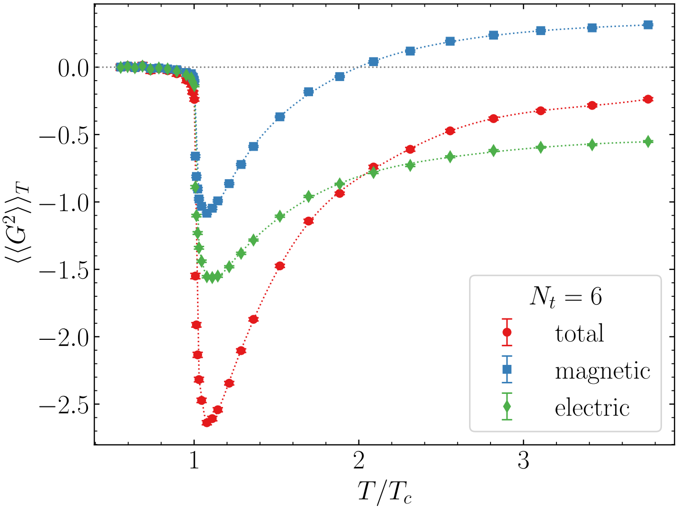
$$\langle\!\langle G^2 \rangle\!\rangle_T = \langle\!\langle \mathcal{B}^2 \rangle\!\rangle_T + \langle\!\langle \mathcal{E}^2 \rangle\!\rangle_T \tag{34}$$

$$\varepsilon - 3p = \left\langle T^{\mu}_{\ \mu} \right\rangle = \frac{\beta(\alpha_s)}{4\pi} \langle\!\langle (G^a_{\mu\nu})^2 \rangle\!\rangle_T \equiv -\langle\!\langle G^2 \rangle\!\rangle_T \tag{35}$$

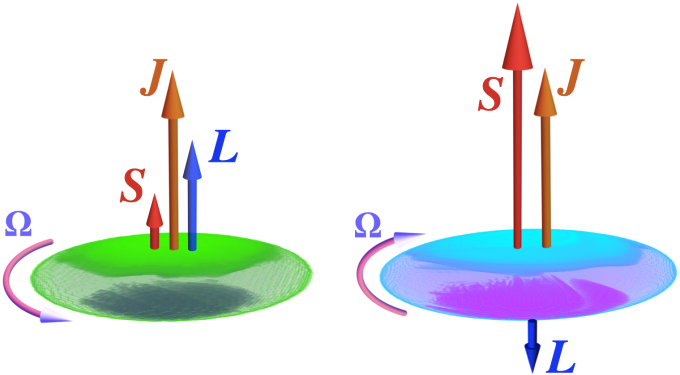
$$\beta\big(\alpha_s(\mu)\big) = \frac{\mathrm{d}\alpha_s(\mu)}{\mathrm{d}\ln\mu} < 0 \tag{36}$$

$$\langle\!\langle \mathcal{O} \rangle\!\rangle_T = \langle \mathcal{O} \rangle_T - \langle \mathcal{O} \rangle_{T=0} \tag{37}$$

Components of gluon condensate



Barnett effect



Summary

- Lattice method for studying the dependence of the EoS of gluodynamics on the rotation was introduced.
- Below the "supervortical temperature" $T_s = 1.50(10)\,T_c$ negative moment of inertia suggests an instability of rigidly rotating gluon plasma.
- We expect qualitatively the same results for QCD with dynamical quarks.
- The rotational instability is related to the scale anomaly and the magnetic gluon condensate.

Summary

Thanks for attention!