Introduction Gluodynamics under rotation Scale anomaly and magnetic gluon condensate OCOC Conclusions

Negative moment of inertia and rotational instability of gluon plasma

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Scale anomaly and magnetic gluon condensate

Negative Barnett effect

Motivation



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Rigidity condition

$$v = \Omega \times r \tag{1}$$

$$\Omega = \frac{1}{2} \nabla \times v = \text{const}$$
 (2)



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Rotating coordinates

$$t = t_{\mathsf{lab}}, \quad r = \eta_{\mathsf{ab}}, \quad z = z_{\mathsf{lab}}, \quad \varphi \sim (\varphi_{\mathsf{lab}} - \Omega t)$$
 (3)

$$g_{\mu\nu}^{(lab)} = \eta_{\mu\nu} = \text{diag}(1, -1, -1, -1)$$
 (4)

$$ds^{2} = g_{\mu\nu}dx^{\mu}dx^{\nu} = (1 - r^{2}\Omega^{2})dt^{2} - 2r^{2}\Omega dtd\varphi - dr^{2} - r^{2}d\varphi^{2} - dz^{2}$$
(5)

$$g_{\mu\nu} = \begin{pmatrix} 1 - r^2 \Omega^2 & \Omega y & -\Omega x & 0\\ \Omega y & -1 & 0 & 0\\ -\Omega x & 0 & -1 & 0\\ 0 & 0 & 0 & -1 \end{pmatrix}$$
(6)

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Moment of inertia

$$E = E^{(lab)} - J\Omega \tag{7}$$

$$dE^{(lab)} = TdS + \Omega dJ$$
(8)

$$dE = TdS - Jd\Omega$$
(9)

$$F = E - TS \tag{10}$$

$$J = -\left(\frac{\partial F}{\partial \Omega}\right)_{T} \tag{11}$$

$$I = \frac{1}{\Omega} J \tag{12}$$

$$i_{2} = \frac{I}{VR_{\perp}^{2}}, \qquad R_{\perp} = \frac{L_{s}}{2}$$

$$K_{2} = -\frac{I}{F_{0}R_{\perp}^{2}}$$
(13)
(14)

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Gluodynamics in rotating coordinates

$$S_G = \int d^4 x \, \sqrt{\det g_{\alpha\beta}} \, \frac{1}{2g_{\rm YM}^2} \, g^{\mu\nu} g^{\rho\sigma} \, {\rm tr} F_{\mu\rho} F_{\nu\sigma} \tag{15}$$

$$S_{G} = \frac{1}{g_{YM}^{2}} \int d^{4}x \operatorname{tr}[(1 - r^{2}\Omega^{2})F_{xy}F_{xy} + (1 - y^{2}\Omega^{2})F_{xz}F_{xz} + (1 - x^{2}\Omega^{2})F_{yz}F_{yz} + F_{x\tau}F_{x\tau} + F_{y\tau}F_{y\tau} + F_{y\tau}$$

$$+F_{z\tau}F_{z\tau}-2iy\Omega(F_{xy}F_{y\tau}+F_{xz}F_{z\tau})+2ix\Omega(F_{yx}F_{x\tau}+F_{yz}F_{z\tau})-2xy\Omega^2F_{xz}F_{zy}]$$
(16)

Simulations with a sign problem:

- Analytical continuation $\Omega_I = -i\Omega$
- Expansion coefficients at $\Omega=0$

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Temperature dependence of moment of inertia



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Condition of thermodynamic stability

$$\delta E - T \delta S - \mathbf{\Omega} \delta \mathbf{J} > 0 \tag{17}$$

$$g^{(W),\mu\nu} = -\frac{\partial^2 f(T, \mathbf{\Omega})}{\partial X_{\mu} \partial X_{\nu}}, \qquad X_{\mu} = (T, \Omega_i)$$
(18)

$$C_J = T\left(\frac{\partial S}{\partial T}\right)_J > 0 \tag{19}$$

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Decomposing moment of inertia

$$F = -T \ln \int DAe^{iS}$$
(21)

$$I^{\rm gl} = I^{\rm gl}_{\rm mech} + I^{\rm gl}_{\rm magn}$$
(22)

$$I_{\rm mech}^{\rm gl} = \frac{1}{T} \langle\!\langle \left(\boldsymbol{n} \cdot \boldsymbol{J}^{\rm gl}\right)^2 \rangle\!\rangle_T$$
(23)

$$J_{i}^{\rm gl} = \frac{1}{2} \int_{V} d^{3}x \,\epsilon_{ijk} M_{\rm gl}^{jk}(\boldsymbol{x}), \qquad i, j = 1, 2, 3$$
(24)

$$M_{\rm gl}^{ij}(\boldsymbol{x}) = x^i \, T_{\rm gl}^{j0}(\boldsymbol{x}) - x^j \, T_{\rm gl}^{i0}(\boldsymbol{x}) \tag{25}$$

$$T^{\mu\nu}_{\rm gl} = G^{a,\mu\alpha} G^{a,\nu}_{\ \alpha} - (1/4) \eta^{\mu\nu} G^{a,\alpha\beta} G^a_{\alpha\beta}$$
(26)

$$\langle\!\langle \mathcal{O} \rangle\!\rangle_{\mathcal{T}} = \langle \mathcal{O} \rangle_{\mathcal{T}} - \langle \mathcal{O} \rangle_{\mathcal{T}=0}$$
(27)

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Moment of inertia and magnetic gluon condensate

$$I_{\text{magn}}^{\text{gl}} = \int_{V} d^{3}x \Big[\langle\!\langle (\boldsymbol{B}^{a} \cdot \boldsymbol{x}_{\perp})^{2} \rangle\!\rangle_{T} + \langle\!\langle (\boldsymbol{B}^{a} \cdot \boldsymbol{n})^{2} \rangle\!\rangle_{T} \boldsymbol{x}_{\perp}^{2} \Big]$$
(28)
$$B_{i}^{a} = \frac{1}{2} \epsilon^{ijk} G_{jk}^{a}$$
(29)

$$\langle\!\langle B_i^a B_j^a \rangle\!\rangle_T = \frac{1}{3} \delta_{ij} \langle\!\langle (\boldsymbol{B}^a)^2 \rangle\!\rangle_T$$
(30)

$$I_{\text{magn}}^{\text{gl}} = \frac{2}{3} \int_{V} d^{3}x \, x_{\perp}^{2} \left\langle\!\left\langle \left(\boldsymbol{B}^{a}\right)^{2}\right\rangle\!\right\rangle_{T}$$
(31)

$$I_{\rm class} = \int_{V} d^{3}x \,\rho(\mathbf{x}) \,\mathbf{x}_{\perp}^{2}$$
(32)

$$\rho(T) \to \frac{2}{3} \langle\!\langle (\boldsymbol{B}^{a})^{2} \rangle\!\rangle_{T}$$
(33)

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Scale anomaly and gluon condensate

$$\langle\!\langle G^2 \rangle\!\rangle_T = \langle\!\langle \mathcal{B}^2 \rangle\!\rangle_T + \langle\!\langle \mathcal{E}^2 \rangle\!\rangle_T \tag{34}$$

$$\varepsilon - 3p = \langle T^{\mu}_{\mu} \rangle = \frac{\beta(\alpha_s)}{4\pi} \langle \langle (G^a_{\mu\nu})^2 \rangle \rangle_T \equiv - \langle \langle G^2 \rangle \rangle_T$$
(35)

$$\beta(\alpha_s(\mu)) = \frac{\mathrm{d}\alpha_s(\mu)}{\mathrm{d}\ln\mu} < 0 \tag{36}$$

$$\langle\!\langle \mathcal{O} \rangle\!\rangle_{\mathcal{T}} = \langle \mathcal{O} \rangle_{\mathcal{T}} - \langle \mathcal{O} \rangle_{\mathcal{T}=0}$$
(37)

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Conclusion:

Components of gluon condensate



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Barnett effect





- Lattice method for studying the dependence of the EoS of gluodynamics on the rotation was introduced.
- Below the "supervortical temperature" $T_s = 1.50(10) T_c$ negative moment of inertia suggests an instability of rigidly rotating gluon plasma.
- We expect qualitatively the same results for QCD with dynamical quarks.
- The rotational instability is related to the scale anomaly and the magnetic gluon condensate.

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Thanks for attention!