Correlation functions in holographic RG flow of 3d supergravity

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based on a joint work with

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Outline

AdS/CFT-duality Maldacena'97; Witten'98; Gubser et.al,'98

AdS/CFT-correspondence relates $(d+1)$-dimensional gravity in AdS and d-dimentional conformal field theory in a flat spacetime, which is defined on a conformal boundary of AdS.
Then, each operator $\mathcal{O}(x)$ CFT with a conformal dimension Δ corresponds to a scalar field, $\tilde{\phi}$, "living" in the boundary of the dual gravity theory.
A source field $J(x)$ in a d-dimensional CFT and the partition function $Z(J(x))$:

$$\mathcal{S}' = \mathcal{S} - \int d^d x J(x)\mathcal{O}(x), \quad Z[J(x)] = e^{-W[J(x)]} = \langle e^{\int d^d x J(x)\mathcal{O}(x)} \rangle_{CFT}$$

The main statement of AdS/CFT-correspondence:

$$\boxed{\langle e^{\int d^d x J(x)\mathcal{O}(x)} \rangle_{CFT} = e^{-S_{AdS}(\tilde{\phi})}|_{\lim_{z\to 0}(\tilde{\phi}(x,z)z^{\Delta-d})=J(x)}}$$

where $J(x)$ is d-dimensional source field corresponds to the value on the boundary $(d+1)$-dimensional field $\tilde{\phi}$.

Holographic renormalization and RG flow Akhmedov'98; de Boer et.al'98

The supergravity action in $(d+1)$-dimensional spacetime with one scalar field:

$$S[g,\phi] = \int\limits_M d^{d+1}x\sqrt{g}\left(-\frac{1}{2\kappa^2}R + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + V(\phi)\right) - \frac{1}{2\kappa^2}\int\limits_{\partial M} d^dx\sqrt{\gamma}K,$$

where ∂M is a conformal boundary of asymptotic AdS.
The ansatz for the metric and the scalar field:

$$\begin{aligned} ds^2 = g_{\mu\nu}dx^\mu dx^\nu = dw^2 + \gamma_{ij}(w,x)dx^i dx^j, \\ \gamma_{ij}(w,x) = e^{2A(w)}\delta_{ij}, \quad \phi = \phi(w), \end{aligned} \tag{1}$$

then on-shell action takes the form

$$S_{on-shell} = -\frac{d-1}{d\kappa^2}\int\limits_{\Sigma_w} d^dx\sqrt{\gamma}K + \frac{1}{\kappa^2}\int\limits_{\Sigma_w} d^dx\xi(x),$$

where $K = 2\dot{A}$ and $\xi(x)$ is an arbitrary integration function of tranverse coordinates.

If there is an explicit expression for the superpotential $W(\phi)$, associated with the potential as

$$V(\phi) = \frac{1}{2}(W'^2 - 2W^2),$$

then the equations of motion are given by

$$\dot{A} = -W(\phi), \quad \dot{\phi} = W'(\phi),$$

and we get the on-shell action

$$S^{ren}_{on-shell} = \int\limits_{\Sigma_w} d^d x \sqrt{\gamma_B} W(\phi_B) + \frac{1}{\kappa^2} \int\limits_{\Sigma_w} d^d x \xi(x) + S_{ct}. \tag{2}$$

Using Dirichlet conditions, the domain wall solution holographically corresponds to deformations dual CFT either by relevant (marginal) operators or VEV of scalar operator. The main result I.Papadimitriou & K.Skenderis, JHEP **10** (2004) 075, I.Papadimitriou & K.Skenderis, IRMA Lect. Math. Theor. Phys. 8 (2005) 73-101

$$\boxed{\langle T_{ij} \rangle_{ren} = -\frac{1}{\kappa^2}(K_{(d)ij} - K_{(d)}\gamma_{ij}), \quad \langle O \rangle_{ren} = \frac{1}{\sqrt{\gamma}} \dot{\Phi}_{(\Delta)}.} \tag{3}$$

Outline

Holographic model

The action of a $3d$ truncated supergravity is Deger et.al, Nucl.Phys.B 573(2000)275:

$$S = \frac{1}{16\pi G_3}\int d^3x\sqrt{|g|}\left(R - \frac{1}{a^2}(\partial\phi)^2 - V(\phi)\right) + G.H.Y. \qquad (4)$$

The scalar field potential is

$$V(\phi) = 2\Lambda\cosh^2\phi\left((1-2a^2)\cosh^2\phi + 2a^2\right), \quad \Lambda<0. \qquad (5)$$

The potential V is related to the superpotential W by the expression:

$$V(\phi) = \frac{a^2}{4}\left(\frac{\partial W}{\partial\phi}\right)^2 - \frac{1}{2}W^2, \quad W = 2\sqrt{-\Lambda}\cosh^2\phi. \qquad (6)$$

We focus on the domain wall ansatz for the metric and the scalar field:

$$ds^2 = e^{2A(w)}\eta_{ij}dx^i dx^j + dw^2, \quad \phi = \phi(w). \qquad (7)$$

A special case of solutions (supersymmetric) can be found by the equations (BPS):

$$\dot{\phi} = a^2\frac{\partial W}{\partial\phi}, \quad \dot{A} = -W. \qquad (8)$$

The exact solution for the model

The solution of the eq. (8) for the scale factor Deger, JHEP 11 (2002) 025:

$$A = \frac{1}{4a^2}\ln\left(e^{8ma^2w} - 1\right), \quad m^2 = -\frac{\Lambda}{4}. \tag{9}$$

The solution for the scalar field:

$$\phi = \frac{1}{2}\ln\left(\frac{1+e^{-4ma^2w}}{1-e^{-4ma^2w}}\right), \quad 0 \le w < \infty. \tag{10}$$

The metric is represented as:

$$ds^2 = (e^{8ma^2w} - 1)^{\frac{1}{2a^2}}(-dt^2 + dx^2) + dw^2. \tag{11}$$

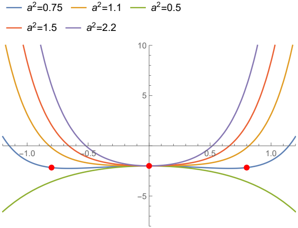

Figure: The dependence of $V(\phi)$ on different a^2

- *For* $a^2 \in \left(0; \frac{1}{2}\right) \cup (1; +\infty)$ *we have one extremum at* $\phi_1 = 0$, *(BPS. i.e. SUSY)*
- *For* $a^2 \in \left(\frac{1}{2}; 1\right)$ *we have three extrema at* $\phi_1 = 0$, $\phi_{2,3} = \frac{1}{2}\ln\left(\frac{1 \pm |a|\sqrt{1-a^2}}{2a^2 - 1}\right)$ *(Non-BPS).*

Scalar field in AdS and conformal dimension

The equation of motion for a scalar field AdS_3 AG & M. Usova, EPJ Plus 138 (2023) 3, 260:

$$\partial_w^2\phi + 2\sqrt{-\Lambda}\partial_w\phi - M^2\phi = 0. \tag{12}$$

We look for a general solution as $\phi \sim e^{-\Delta(w-w_0)}$ with Δ is a conformal dimension, then we find

$$\Delta_\pm = \sqrt{-\Lambda} \pm \sqrt{M^2 - \Lambda}. \tag{13}$$

The BF bound (assuming that $\Lambda = -1$) is given by

$$M^2 \geq -1. \tag{14}$$

The Dirichlet boundary conditions are $A(w) \sim w$ at $w \to \infty$, and the scalar field is dual to the operator $\mathcal{O}$ with Δ as $\phi(w,x) \sim e^{-(2-\Delta)}\phi_{(0)}(x)$. Then the expansion of $V(\phi)$ (5) at $\phi = 0$ up to the quadratic order:

$$V(\phi) = 2\Lambda a^2 + (4 - 4a^2)a^2\Lambda\phi^2 + \dots. \tag{15}$$

Then we have for the mass of ϕ :

$$M^2 = 4a^2(a^2 - 1), \quad \Rightarrow \quad \Delta = \Delta_+ = 1 + |1 - 2a^2|. \tag{16}$$

RG flow (10)-(11) AG & M. Usova'23, EPJ Plus 138 (2023) 3, 260:

1. for $a^2 \in \left(0; \frac{1}{2}\right)$ caused by a relevant operators;
2. for $a^2 = 1/2$ caused by a non-zero vacuum expectation value of a scalar operator;
3. for $a^2 \in [1; +\infty]$ caused by a non-zero VEV of a scalar operators;
4. for $a^2 \in (1/2; 1)$ the RG flow can't be described by (10)-(11). However, in AG & M. Usova'23 an asymptotic solution was constructed. This RG flow is triggered by a non-zero VEV of a scalar operator.

The solution for the scalar field near $\phi = 0$

The solution for the scalar field with the potential (15) near $\phi = 0$ which is valid for $\Delta > 1$, i.e. a $a^2 \in \left(0; \frac{1}{2}\right) \cup \left(\frac{1}{2}; 1\right)$ and $a^2 \in (1; +\infty)$

$$\phi(w,x) = e^{-(2-\Delta)w}\left(\phi_{(0)} + \dots\right) + e^{-\Delta w}\left(\phi_{(2\Delta-2)}(x) + \dots\right), \tag{17}$$

where $\phi_{(0)}$ *corresponds to the source*, *and* $\phi_{(2\Delta-2)}$ *corresponds to the VEV*.

1. At $1<\Delta<2$ the parameter a^2 take the values $a^2 \in \left(0; \frac{1}{2}\right) \cup \left(\frac{1}{2}; 1\right)$ and *components of the solution are both decreasing as* $w \to \infty$.
2. At $\Delta>2$ the parameter $a^2 \in (1; +\infty)$ and *the first term diverges with* $w \to \infty$, which is associated with the VEV of the irrelevant operator.

• The special case $\Delta = 1$ (saturates the BF bound), i.e. when $a^2 = \frac{1}{2}$, the scalar field is given by:

$$\phi(w,x) = e^{-w}\left(-2w(\phi_{(0)}(x) + \dots) + \tilde{\phi}_{(0)}(x) + \dots\right). \tag{18}$$

For this case *the first term in brackets corresponds to the source, the second - VEV*.
• The special case, $\Delta = 2$ ($a^2 = 0$, $a^2 = 1$), for which the scalar field is given by:

$$\phi(w,x) = \left(\phi_{(0)}(x) + \dots\right) + e^{-2w}\left(\phi_{(2)}(x) + \dots\right). \tag{19}$$

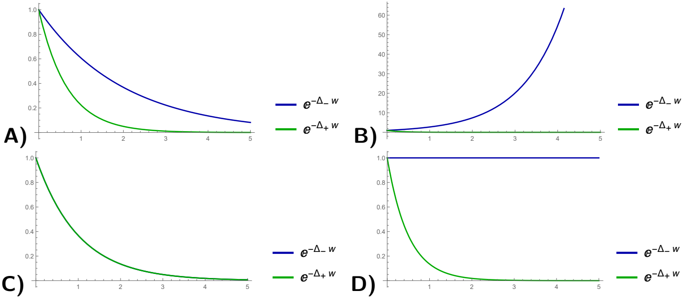

Figure: The dependence of $\phi(w,x)$ (17) on different a^2 : **A)** for $a^2 = \frac{3}{4}$; **B)** for $a^2 = \frac{3}{2}$; **C)** for $a^2 = \frac{1}{2}$; **D)** for $a^2 = 1$

Outline

Vacuum expected value (VEV) $\langle O_{ij}\rangle$ and $\langle T_{ij}\rangle$

The potential has an extreme point at $\phi=0$, the relation for $V(\phi)$ and $W(\phi)$ (6), (8), and the requirement that W also has an extremum at $\phi=0$, lead to two types of the expansion for W near $\phi=0$ Papadimitriou & Skenderies, JHEP 10(2004)075:

$$\begin{cases} W_-(\phi) = -1 - \frac{1}{2}(2-\Delta)\phi^2 + \ldots, \quad \Delta \in (1,2), \\ W_+(\phi) = -1 - \frac{1}{2}\Delta\phi^2 + \ldots, \quad \Delta > 2. \end{cases} \tag{20}$$

- $\Delta \in (1;2)$, i.e. $a^2 \in (0;1/2)\cup(1/2;1)$. Accordingly, we set $\xi=0$, then $S^{ren}_{on-shell}$ is

$$S^{ren}_{on-shell} = -\frac{1}{2}\int d^2x\sqrt{\gamma}K + \int d^2x\sqrt{\gamma}W_-, \tag{21}$$

where

$$S_{ct} = \int d^2x\sqrt{\gamma}W_-(\phi), \quad \text{valid for} \quad \Delta \neq 1 \quad \& \quad \Delta \neq 2.$$

The extrinsic curvature can be calculated as:

$$K = \partial_w \log e^{2A} = 2\dot{A}, \quad \dot{A} = -W_- \tag{22}$$

Taking into account (22) $S^{ren}_{on-shell}$ (21) has the form:

$$S^{ren}_{on-shell} = -\int d^2x\sqrt{\gamma}W_- + \int d^2x\sqrt{\gamma}W_- = 0. \tag{23}$$

The vacuum expectation value of the scalar operator is related to the canonical momentum as:

$$\langle \mathcal{O} \rangle_{ren} = \frac{\pi_{\phi(\Delta)}}{\sqrt{\gamma}} = \dot{\phi}, \quad \dot{\phi} = \frac{\partial W}{\partial \phi}. \tag{24}$$

Then the VEV of $\mathcal{O}$ can be calculated as follows:

$$\langle O \rangle_{ren} = W'_{-} - W'_{-} = 0. \tag{25}$$

• $\Delta > 2$, i.e. $a^2 \in (1; +\infty)$. This case corresponds to W_{+}, then we have for $S^{ren}_{on-shell}$:

$$S^{ren}_{on-shell} = -\frac{1}{2} \int d^2x \sqrt{\gamma} K + \int d^2x \sqrt{\gamma} W_{-} \tag{26}$$

We get that

$$S^{ren}_{on-shell} = -\int d^2x \sqrt{\gamma} W_{+} + \int d^2x \sqrt{\gamma} W_{-} = \int d^2x \, (\Delta - 1) \, \phi^2, \tag{27}$$

where

$$\phi \sim e^{-\Delta w}, \tag{28}$$

ϕ is rapidly decreasing as $w \to \infty$.
For this case, the expectation value of the scalar operator $\mathcal{O}$ is realized as:

$$\langle \mathcal{O} \rangle_{ren} = \frac{\pi_{\phi(\Delta)}}{\sqrt{\gamma}} = \dot{\phi} = W'_{+}, \tag{29}$$

and subtract the counterterm $S_{ct} = W_{-}$ from (29) :

$$\langle \mathcal{O} \rangle_{ren} = W'_{+} - W'_{-} = 2(1 - \Delta)\phi, \tag{30}$$

where

$$\phi \sim e^{-\Delta w}, \quad \text{at} \quad w \to \infty. \tag{31}$$

• If $\Delta = 1$ ($a^2 = 1/2$), W_+ and W_- coincide. The part of the divergent on-shell action is:

$$U(\phi) = -1 + \frac{1}{2}\left(\frac{1}{w} - 1\right)\phi^2 + \dots . \tag{32}$$

Then VEV of the scalar field is calculated using the following formula:

$$\langle \mathcal{O} \rangle_{ren} = 2\phi, \quad \phi \sim e^{-w}. \tag{33}$$

• If $\Delta = 2$ ($a^2 = 0$, $a^2 = 1$) the counterterm S_{ct} is a constant, and coincides with the first term in $W_\pm$:

$$U(\phi) = -1 \tag{34}$$

In this case, two common solutions can be obtained

$$W_- = -1, \quad W_+ = -\cosh\left(\sqrt{2}(\phi - \phi_0)\right). \tag{35}$$

- If W_- is realized in the background, then the expectation value is calculated as

$$\langle \mathcal{O} \rangle = W_-' - U' = 0. \tag{36}$$

- If W_+ is realized in the background, then we expand W_+ near ϕ_0 and get

$$W_+ = -1 - (\phi - \phi_0)^2. \tag{37}$$

Then the vacuum expactation value is calculated as

$$\langle \mathcal{O} \rangle = W_+' - U' = -2\phi. \tag{38}$$

The vacuum expectation value of the stress-energy tensor components $\langle T_{ij} \rangle$ Papadimitriou & Skenderies, JHEP 10 (2004) 075 defined as

$$\langle T_{ij} \rangle_{ren} = -\frac{1}{\kappa^2}(K_{(d)ij} - K_{(d)}\gamma_{ij}), \quad K_{ij} = \frac{1}{2}\partial_w \gamma_{ij}. \tag{39}$$

Compute (39) for the domain wall metric ($\kappa^2 = 1$) components

$$\langle T_{00} \rangle_{ren} = -\langle T_{11} \rangle_{ren} = -\dot{A}e^{2A}$$

These calculations make sense only for non-supersymmetric solutions ($a^2 > \frac{1}{2}$), where A (AG&Usova'EPJPlus'23) is specified as

$$A = a^2 \sqrt{-\frac{\Lambda}{2a^2 - 1}}(w - w_0).$$

Setting $\Lambda = -1$, we get

$$\langle T_{00} \rangle_{ren} = -\langle T_{11} \rangle_{ren} = -a^2 \sqrt{\frac{1}{2a^2 - 1}} e^{2a^2 \sqrt{\frac{1}{2a^2 - 1}}(w - w_0)}.$$

Outline

Conclusions

1. We have studied RG flows in a 3-dimensional supergravity model with a scalar field and non-trivial potential within the holographic duality.
2. By imposing Dirichlet conditions on the behavior of the gravitational model fields near the AdS boundary, holographic RG flows have been analyzed.
3. General solutions have been found for a scalar field near the extreme point of the potential and their behavior has been investigated.
4. Deformations of the dual conformal theory described by holographic RG flows have classified. These deformations are triggered by the relevant operator or by the VEV of the scalar operator.

Thank you for attention!