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Multichannel scattering for the Schrödinger equation on a line with different thresholds at both infinities

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The study of the scattering process associated with the one-dimensional matrix Schrödinger equation is a classical pursuit within the realm of quantum theory. A comprehensive examination of the overarching characteristics of such scattering processes along a semi-axis, in particular the rigorous verification of unitarity in the context of the S-matrix with closed scattering channels, can be found in the the book ([1], Chap. 17). The inverse scattering problem for the one-dimensional Schrödinger equation on the whole axis in the case of single-channel scattering was solved by Fadeev [2-4]. In the context of multichannel scattering, which occurs over the entire real line, substantial research has been carried out, in particular in relation to the inverse scattering problem and the derivation of exact solutions for hierarchies of integrable nonlinear partial differential equations [5]. To the best of our knowledge, there is an unaddressed gap in the existing literature regarding the elucidation of the properties associated with the S-matrix, the Jost solutions and the existence of bound states in the general scenario involving multichannel scattering on a line with different threshold conditions existing on both the left and right limits of infinity.

Within the framework of this particular scattering problem, we embark on an analysis of the analytical structure characterising the Jost solutions and the transition matrix relating the Jost solutions as functions of the spectral parameter [6]. In addition, we provide the proof of unitarity for the scattering matrix under the more general circumstances where certain scattering channels may be closed and there are disparities in the threshold parameters at the left and right limits of the infinite line. In addition, we establish a necessary and sufficient condition that specifies the exact locations of bound states. While the formal expression of this condition is well documented in the context of multichannel scattering, as outlined in previous literature [5, 7, 8, 9], we demonstrate its applicability to the particular scenario of a scattering problem characterised by distinct thresholds at both the left and right extremes of infinity.

The results of the thesis are applicable in the electrodynamics of continuous media, in the theory of sound wave propagation, in the description of the passage of electrons through heterostructures, in quantum chemistry, in hydrodynamics and plasma physics, etc. In particular, when describing photon scattering through metamaterials with large spatial dispersion, the problem of proving the unitarity of the S-matrix in open channels arises.

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