The double Compton process in a strongly magnetized plasma

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Outline

- Introduction
- Photon dispersion in the magnetized medium
- Kinematic analysis
- Amplitude of double Compton process
- Photon production efficiency
- Conclusion

Introduction

Problem:

The influence of different quantum processes on the photon polarization states production in the magnetospheres of NS and its influence on the spectra formation.
(see, for example, Suleimanov V. et. al. A&A 2012)
The Compton scattering, $\gamma e \to \gamma e$, is a basic process that is considered when solving the radiation transfer problem.
But the number of photons does not change in this process.
In this talk we investigate the process $e\gamma \to e\gamma\gamma$ with taking into account the change of the photon dispersion properties.
As far as we know, previously, the process $e\gamma \to e\gamma\gamma$ in a plasma without a magnetic field was studied in the paper (A.P. Lightman, ApJ, 1981)
Multiple Compton Scattering in Magnetic Field — A. A. Mushtukov et. al. Phys. Rev. D 105, 103027 2022

Introduction

The conditions in a strongly magnetized NS (magnetars) are a very exotic.

The characteristics of outer crust of magnetar

$$B \sim 10^{14} - 10^{16} \text{ G.}, \; B \gg B_e,$$

$$B_e = m^2/e \simeq 4.41 \times 10^{13} \text{ G},$$

$$T \sim 10^8 - 10^9 \text{ K}, \; T \ll \mu - m,$$

$$\frac{p_F}{m} \simeq 0.34 \frac{B_e}{B} \frac{\rho}{\rho_6}, \quad \rho \gtrsim \rho_6 = 10^6 \text{g/cm}^3$$

In these conditions we will investigate the double Compton process and photon splitting process.

Some notations

p^μ and p'^μ is the momenta of the plasma electrons,
q^μ and q_N^μ are the momenta of initial and final photons,
The four-vectors with indices $\perp$ and $\parallel$ belong to the Euclidean $\{1, 2\}$-subspace and the Minkowski $\{0, 3\}$-subspace correspondingly in the frame were the magnetic field is directed along third axis;
$(ab)_\perp = (a\varphi\varphi b) = a_\alpha \varphi^\rho_\alpha \varphi_{\rho\beta} b_\beta$, $(ab)_\| = (a\tilde{\varphi}\tilde{\varphi} b) = a_\alpha \tilde{\varphi}^\rho_\alpha \tilde{\varphi}_{\rho\beta} b_\beta$.
The tensors $\varphi_{\alpha\beta} = F_{\alpha\beta}/B$ and $\tilde{\varphi}_{\alpha\beta} = \frac{1}{2}\varepsilon_{\alpha\beta\mu\nu}\varphi_{\mu\nu}$ are the dimensionless field tensor and dual field tensor correspondingly.

$$\Lambda_{\mu\nu} = (\varphi\varphi)_{\mu\nu}, \ \widetilde{\Lambda}_{\mu\nu} = (\tilde{\varphi}\tilde{\varphi})_{\mu\nu}$$

Photon dispersion in the magnetized medium

The polarization and dispersion properties of normal modes are connected with eigenvectors $\varepsilon_\alpha^{(\lambda)}(q)$ and eigenvalues of polarization operator $\varkappa^{(\lambda)}(q)$ correspondingly.

In the cold, quasidegenerate, moderately relativistic plasma:

$p_F/m \simeq v_F \ll 1$, (v_F is the Fermi velocity)

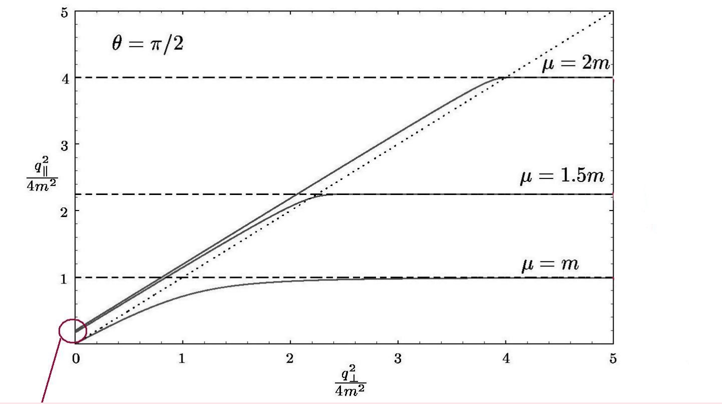
the physical polarization vectors of the photons

$$\varepsilon_\alpha^{(1)}(q) = -b_\alpha^{(1)} = \frac{(q\varphi)_\alpha}{\sqrt{q_\perp^2}}, \qquad \varepsilon_\alpha^{(2)}(q) = -b_\alpha^{(2)} = \frac{(q\tilde{\varphi})_\alpha}{\sqrt{q_\parallel^2}}$$

are just as in the pure magnetic field. The corresponding eigenvalues are

$$\varkappa^{(1)} \simeq -\frac{\alpha}{3\pi}\, q_\perp^2\,, \quad \varkappa^{(2)} \simeq -\frac{2\alpha}{\pi}\, eB \left[\mathcal{J}(q_\parallel) + H\left(\frac{q_\parallel^2}{4m^2}\right)\right]$$

The dispersion laws of the mode-2 photon for $B \simeq 10^{16}$ G and $T \simeq 50$ keV.



ω_{pl}^2

Kinematic analysis

The analyze show a feature that is connected with the appearance of the plasma frequency in the presence of real electrons which can be defined from the equation

$$\omega_{pl}^2 - \varkappa^{(2)}(\omega_{pl}, \mathrm{k} \to 0) = 0.$$

In our conditions $\omega_{pl}^2 = (2\alpha eB/\pi)v_F$.

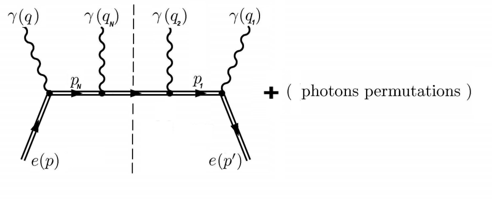
These facts lead to new polarization selection rules for competitive process of the photon splitting: in the region $q^2 > 0$ the splitting channels $\gamma_2 \to \gamma_2\gamma_2$, $\gamma_1 \to \gamma_2\gamma_2$ and $\gamma_1 \to \gamma_1\gamma_2$ are forbidden. Only the channel $\gamma_2 \to \gamma_1\gamma_1$ is kinematically allowed.

In this case the leading channel of mode 2 photons productions is

$$e\gamma_2 \to e\gamma_2\gamma_2.$$

Amplitude of double Compton process

In course of the work it was found that the approach used to obtain the amplitude the double Compton process can be easely extend to the cease multiple Compton scattering process.
Feynman diagrams for the process $e\gamma \rightarrow e(N\gamma)$ is



S-martix element has form

$$S^{s,s'}_{p,p'} = (ie)^{N+1} \int d^4 Z_1 ... d^4 Z_{N+1} \bar{\Psi}^{s'}_{\ell',p'}(Z_1) \hat{A}^*_{q_1}(Z_1) \hat{S}_{p_1}(Z_1, Z_2) \hat{A}^*_{q_2}(Z_2) ... \times$$

$$\hat{S}_{p_N}(Z_{N-1}, Z_N) \hat{A}_q(Z_{N+1}) \Psi^s_{\ell,p}(Z_{N+1}) + (\text{photon permutations}) \quad (1)$$

Where Z_k – 4-coordinate of k-th vortex, $\hat{S}_{p_k}(Z_k, Z_{k-1})$ – propagator with momenta p_k.

Amplitude of multiple Compton process

The amplitude of the process at an arbitrary Landau level is

$$M^{s,s'}_{\ell,\ell'} = \sum_{n_1,\dots,n_N} \frac{\exp\left[-\frac{i}{2\beta}\left\{q_x(p_N+p)_y\right\}\right]}{\sqrt{4M_\ell M_{\ell'}(m+M_\ell)(m+M_{\ell'})}} \left[\frac{q_y - iq_x}{\sqrt{q_\perp^2}}\right]^{n_N-\ell} \times$$
$$\sum_{s_1,\dots,s_N=\pm1} T^{s,s_k}_{n_N,\ell}(q,p_N,p) \times$$
$$\prod_{k=1}^{N} \frac{T^{s_{k-1},s_k}_{n_{k-1},n_k}(q_k,p_{k-1},p_k)}{[(p_k)^2_\parallel - M^2_{n_k}]} \left[\frac{(q_k)_y + i(q_k)_x}{\sqrt{(q_k)^2_\perp}}\right]^{n_{k-1}-n_k} \frac{\exp\left[\frac{i}{2\beta}\left\{(q_k)_x(p_{k-1}+p_k)_y\right\}\right]}{2M_{n_k}(m+M_{n_k})} \tag{2}$$

Here $p_0 = p'$, $n_0 = \ell'$, $s_0 = s'$, n_N – Landau level of electron from N-th propagator, $M_\ell = \sqrt{m^2 + 2eB\ell}$ – effective electron mass in the magnetic field, $q_{i\alpha} = (\omega_i, \mathrm{k}_i)$.

$T^{s,s'}_{n,m}(q,p,p')$ is an invariant expression and is written through Covariates and invariants K_i that look like:

$$K_{1\alpha} = \sqrt{\frac{2}{(p\tilde{\Lambda}p') + M_n M_m}} \left\{ M_n(\tilde{\Lambda}p')_\alpha + M_m(\tilde{\Lambda}p)_\alpha \right\} \qquad (3)$$

$$K_{2\alpha} = \sqrt{\frac{2}{(p\tilde{\Lambda}p') + M_n M_m}} \left\{ M_n(\tilde{\varphi}p')_\alpha + M_m(\tilde{\varphi}p)_\alpha \right\} \qquad (4)$$

$$K_3 = \sqrt{2\left[(p\tilde{\Lambda}p') + M_n M_m\right]} \qquad (5)$$

$$K_4 = -\sqrt{\frac{2}{(p\tilde{\Lambda}p') + M_n M_m}} (p\tilde{\varphi}p') \qquad (6)$$

Amplitude of double Compton process

The amplitude of the process $e\gamma \to e\gamma\gamma$ at an arbitrary Landau level is

$$M^{s,s'}_{\ell,\ell'} = \sum_{n_1,n_2} \frac{1}{[(p_1)^2_{\parallel} - M^2_{n_1}][(p_2)^2_{\parallel} - M^2_{n_2}]} \frac{1}{\sqrt{4M_\ell M_{\ell'}(m+M_\ell)(m+M_{\ell'})}} \times$$
$$\frac{\exp\left[\frac{i}{2\beta}\left\{q_{1x}(p'_y + (p_1)_y) + (q_2)_x((p_1)_y + (p_2)_y) - q_x((p_2)_y + p_y)\right\}\right]}{4M_{n_1}M_{n_2}(m+M_{n_1})(m+M_{n_2})} \times$$
$$\left[\frac{(q_1)_y + i(q_1)_x}{\sqrt{(q_1)^2_\perp}}\right]^{\ell'-n_1} \left[\frac{(q_2)_y + i(q_2)_x}{\sqrt{(q_2)^2_\perp}}\right]^{n_1-n_2} \left[\frac{q_y - iq_x}{\sqrt{q^2_\perp}}\right]^{n_2-\ell} \times$$
$$\sum_{s_1,s_2=\pm 1} T^{s,s_2}_{\ell,n_2}(q,p,p_2)\, T^{s_2,s_1}_{n_2,n_1}(q_2,p_2,p_1)\, T^{s_1,s'}_{n_1,\ell'}(q_1,p_1,p') \qquad (7)$$

Amplitude on ground Landau level

In the magnetar cease, the electrons are mostly occupy the ground Landau level.
In the simplest case, when all electrons are at the ground Landau level the amplitude is

$$\mathcal{M}^{tot}_{0,0} \simeq \exp\left[-\frac{(q_1)^2_\perp + (q_2)^2_\perp + (q^2_\perp)}{4eB}\right] \times$$
$$\frac{(K_1(p', p_1)\varepsilon_{q_1})(K_1(p_1, p_2)\varepsilon_{q_2})(K_1(p_2, p)\varepsilon_q)}{[(p_1)^2_\| + m][(p_2)^2_\| + m]} + \text{(photon permutations)} \quad (8)$$

Photon production efficiency

To analyze the efficiency of the process, $e\gamma \to e\gamma\gamma$ under consideration and to compare it with other competitive reactions we calculate the photon absorption rates which can be defined in the following way (M. Chistyakov and D.R. IJMPA 2009):

$$W_{e\lambda\to e\lambda_1\lambda_2} = \frac{eB}{64(2\pi)^7\omega_\lambda}\int |\,\mathcal{M}_{\lambda\to\lambda_1\lambda_2}\,|^2\, Z_\lambda Z_{\lambda_1} Z_{\lambda_2}(1-f_{E'})\times$$

$$f_E(1+f_{\omega_1})(1+f_{\omega_2})\delta(\omega+E-\omega_1-\omega_2-E')\frac{dp_z\, d^3q_1\, d^3q_2}{EE'\omega'\omega''}$$

$f_E = [e^{(E-\mu)/T}-1]^{-1}$ – equilibrium electron distribution function. The eigenvalue of the polarization operator $\varkappa^{(2)}$ becomes large near the electron-positron pair production threshold. This suggests that the renormalization of the wave function for a photon of this polarization should be taken into account:

$$\varepsilon_\alpha^{(2)}(q)\to\varepsilon_\alpha^{(2)}(q)\sqrt{Z_2}, \quad Z_2^{-1} = 1-\frac{\partial\varkappa^{(2)}(q)}{\partial\omega^2}\simeq 1.$$

Photon production efficiency

The analysis provides an estimate of the number of mode 2 photons produced in the process $e\gamma \to e\gamma\gamma$ in the magnetosphere of strongly magnetized NS:

$$\frac{dN}{dVdt} \simeq 1.6 \cdot 10^{17} \left(\frac{1}{\text{cm}^3\text{s}}\right) \tag{9}$$

estimation of the number of mode 2 photons produced in the process $e\gamma \to e\gamma\gamma$ in isotropic plasma without magnetic field for $T = 5$ KeV and $n_e = 3 \cdot 10^{13}$ cm^{-3}:

$$\frac{dN_{vac}}{dVdt} \simeq 2.8 \cdot 10^{20} \left(\frac{1}{\text{cm}^3\text{s}}\right) \tag{10}$$

Conclusion

- We have considered the double Compton process, $e\gamma \to e(N\gamma)$, in the presence of a strongly magnetized, charge asymmetric, cold plasma.
- As an application of the obtained results the double Compton process is considered. The amplitudes of the processes are obtained: $e\gamma \to e(N\gamma)$, $e\gamma \to e\gamma\gamma$. The amplitude of Double Compton process is obtained both in general form and in special ceases impotent for magnetars.

Conclusion

- The efficiency of photon production in the double Compton process in a magnetic field is considered. It is shown that it is suppressed in comparison with the vacuum one, but still is. However, the considered reaction can serve as a rather efficient mechanism for the production of polarized photons in the presence of a strongly magnetized plasma.
- Changes of dispersion properties of the photon in the magnetized medium are investigated.

Thank you!!!

Dispersion of a photon in a magnetized plasma

Photon dispersion features in cold $(T \ll \mu - m)$ magnetized plasma

- The figure shows, cold plasma the threshold of the pair birth is shifting to

$$4m^2 \to 4\mu^2$$

under conditions $k_z = 0$ $(\theta = \pi/2)$. In general, the shift of the pair birth is

$$4m^2 \to 2\left(\mu^2 - p_F|k_z| + \mu\sqrt{(p_F - |k_z|)^2 + m^2}\right)$$

in case $|q_z| < 2p_F$.
This result is consistent with a simple kinematic analysis of the process $\gamma_2 \to e^+e^-$ in degenerate plasma.

Propagator

We use fermion propagator in the following form (A. Kuznetsov and A. Okrugin 2011)

$$\hat{S}(X, X') = \sum_{n=0}^{\infty} \sum_{s=\pm 1} \int \frac{\mathrm{d}p_0 \mathrm{d}p_y \mathrm{d}p_z}{(2\pi)^3} \times$$

$$\times \frac{\mathrm{e}^{-\mathrm{i}\,(p\,(X-X'))_{\parallel} + \mathrm{i} p_y\,(X_2 - X_2')}}{p_{\parallel}^2 - M_n^2 - \mathcal{R}_{\Sigma}^s(p) + \mathrm{i}\,\mathcal{I}_{\Sigma}^s(p)} \phi_{p,n}^s(X_1) \bar{\phi}_{p,n}^s(X_1') \,.$$

$$\phi_{p,\ell}^s(X_1) = \frac{U_\ell^s[\xi(X_1)]}{\sqrt{2M_\ell(E_\ell + M_\ell)(M_\ell + m_f)}}$$

$$\mathcal{I}_{\Sigma}^s(p) = -\frac{1}{2}\, p_0\, \Gamma_n^s \quad \text{(V. C. Zhukovsky et al. 1994)}$$

Γ_n^s total fermion absorption width for polarization state s.

$eB = \beta$, $T^{s,s'}_{n,m}(q,p,p')$, has form:

$$T^{-,-}_{n,m}(q,p,p') = g_Z[2\beta\sqrt{nm}(K_1(q,p,p')\varepsilon_q)\mathcal{I}_{m-1,n-1}+$$
$$(m+M_n)(m+M_m)(K_1(q,p,p')\varepsilon_q)\mathcal{I}_{m,n}-$$
$$\sqrt{2\beta m}(m+M_n)K_3\frac{(j_z\Lambda q)-i(j_z\varphi q)}{\sqrt{q_{\|}^2}}\mathcal{I}_{m-1,n}- \quad (11)$$
$$\sqrt{2\beta n}(m+M_m)K_3\frac{(j_z\Lambda q)+i(j_z\varphi q)}{\sqrt{q_{\|}^2}}\mathcal{I}_{m,n-1}]$$

$$T^{-,+}_{n,m}(q,p,p') = ig_Z[\sqrt{2\beta m}(m+M_n)(K_2(q,p,p')\varepsilon_q)\mathcal{I}_{m-1,n-1}-$$
$$\sqrt{2\beta n}(m+M_m)(K_2(q,p,p')\varepsilon_q)\mathcal{I}_{m,n}+$$
$$2\beta\sqrt{nm}K_4\frac{(j_z\Lambda q)-i(j_z\varphi q)}{\sqrt{q_{\|}^2}}\mathcal{I}_{m-1,n}-$$
$$(m+M_n)(m+M_m)K_4\frac{(j_z\Lambda q)+i(j_z\varphi q)}{\sqrt{q_{\|}^2}}\mathcal{I}_{m,n-1}] \quad (12)$$

$$T^{+,-}_{n,m}(q,p,p') = -ig_Z[\sqrt{2\beta n}(m+M_m)(K_2(q,p,p')\varepsilon_q)\mathcal{I}_{m-1,n-1} -$$
$$\sqrt{2\beta m}(m+M_n)(K_2(q,p,p')\varepsilon_q)\mathcal{I}_{m,n} +$$
$$(m+M_n)(m+M_m)K_4\frac{(j_z\Lambda q) - i(j_z\varphi q)}{\sqrt{q^2_{\parallel}}}\mathcal{I}_{m-1,n} -$$
$$2\beta\sqrt{nm}K_4\frac{(j_z\Lambda q) + i(j_z\varphi q)}{\sqrt{q^2_{\parallel}}}\mathcal{I}_{m,n-1}] \tag{13}$$

$$T^{+,+}_{n,m}(q,p,p') = g_Z[2\beta\sqrt{nm}(K_1(q,p,p')\varepsilon_q)\mathcal{I}_{m-1,n-1} +$$
$$(m+M_n)(m+M_m)(K_1(q,p,p')\varepsilon_q)\mathcal{I}_{m,n} -$$
$$\sqrt{2\beta n}(m+M_m)K_3\frac{(j_z\Lambda q) - i(j_z\varphi q)}{\sqrt{q^2_{\parallel}}}\mathcal{I}_{m-1,n} - \tag{14}$$
$$\sqrt{2\beta m}(m+M_n)K_3\frac{(j_z\Lambda q) + i(j_z\varphi q)}{\sqrt{q^2_{\parallel}}}\mathcal{I}_{m,n-1}]$$

Cross section $e\gamma \longrightarrow e\gamma\gamma$

In our conditions ($T \ll \mu - m$) absorption rate can be expressed in terms of cross section $W_{e\lambda \to e\lambda_1\lambda_2} = n_e \sigma_{\lambda \to \lambda_1\lambda_2}$. For the leading channel

$$\sigma_{2\to 22} \simeq \frac{1}{2^6 (2\pi)^5 m^2 \omega} \int d\Omega_1 \int d\Omega_2 \int\limits_{\omega_{pl}/2}^{\omega-\omega_{pl}/2} d\omega'' \omega_1 (\omega - \omega_1) \mid \mathcal{M}_{2\to 22} \mid^2 ,$$

where

$$\mathcal{M}_{2\to 22} \simeq -2 \frac{(4\pi\alpha)^{3/2}}{m} \sin\theta \sin\theta_2 \sin\theta_1 \times \left[\left(2 - \frac{\omega_1}{\omega}\right) \cos\theta_2 + \left(1 + \frac{\omega_1}{\omega}\right) \cos\theta_1 \right]$$

θ, θ' and θ'' angles between photon momenta and direction of magnetic field.

Cross section $e\gamma \longrightarrow e\gamma\gamma$

After integration we get an expression for differential cross section, which is convenient for solving the problem of radiation transfer

$$\frac{d\sigma_{2\to 22}}{d\Omega_1 d\Omega_2} \simeq \frac{\alpha^3}{240\pi^2 m^4} (\omega - \omega_{pl})\Theta(\omega - \omega_{pl}) \times$$
$$\sin^2\theta \sin^2\theta_1 \sin^2\theta_2 \Big[(\omega + \omega_{pl})(23\cos^2\theta_2 + 44\cos\theta_2\cos\theta_1 +$$
$$23\cos^2\theta_1) - \frac{\omega_{pl}^2}{2\omega}(29\cos^2\theta_2 + 32\cos\theta_2\cos\theta_1 + 29\cos^2\theta_1) +$$
$$\frac{3\omega_{pl}^3}{4\omega^3}(4\omega - \omega_{pl})(\cos\theta_2 - \cos\theta_1)^2 \Big]$$

$\Theta(x)$ - theta function.