

Models for Heavy Baryons Based on the Symmetry between Heavy Quark and Heavy Diquark

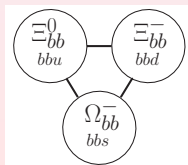
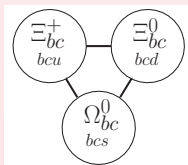
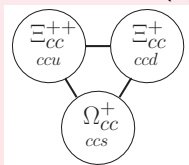
Alisa Shukhtina

P. G. Demidov Yaroslavl State University, Yaroslavl, Russia

The XXVII International Scientific Conference of Young Scientists and Specialists
(AYSS-2023), JINR, Dubna, Russia, 29 October — 3 November 2023

- Introduction
- Interpolating Currents
- Distribution Amplitudes
- Models of DAs
- Scale dependence
- Conclusions

- Hadrons are colorless systems of quarks and/or antiquarks
- According to spin, they are divided into two groups
 - 1 Conventional mesons are integer-spin hadrons composed of one quark and one antiquark
 - Light mesons (u -, d - and s -quarks)
 - **Heavy mesons** (one heavy quark — c - or b -quark)
 - Quarkonia (charmonia and bottomonia) and B_c -mesons
 - 2 Conventional baryons are systems having half-integer spins and containing three quarks
 - Light baryons (u -, d - and s -quarks)
 - Heavy baryons (one heavy quark — c - or b -quark)
 - **Doubly-heavy baryons** (two heavy quarks)
 - Triply-heavy baryons (all three quarks are heavy)
- Example: $SU(3)_F$ -triplets of doubly-heavy baryons



Experimental status of Ξ_{cc}^{++} (ccu) and Ξ_{cc}^{+} (ccd)

- **SELEX** Collab. (Fermilab) [PRL 89 (2002) 112001]
First double charmed baryon Ξ_{cc}^{+} was discovered in the decay channel $\Xi_{cc}^{+} \rightarrow \Lambda_c^{+} K^{-} \pi^{+}$ (Statistical significance 6.3σ)
- **SELEX** Collab. (Fermilab) [PLB 628 (2005) 18]
Confirmed in the decay channel $\Xi_{cc}^{+} \rightarrow p D^{+} K^{-}$ (Statistical significance 4.8σ)
- None of other experiments (**FOCUS**, **BaBar**, **Belle**, **LHCb**) confirm the existence of this resonance
- **LHCb** Collab. [PRL 119 (2017) 112001]
Have found its isospin partner Ξ_{cc}^{++} in the decay channel $\Xi_{cc}^{++} \rightarrow \Lambda_c^{+} K^{-} \pi^{+} \pi^{+}$ (Stat. significance $> 12\sigma$ at 13 TeV and $> 7\sigma$ at 8 TeV)
- Measurements by **SELEX** and **LHCb** show large mass difference $\Delta M = (103 \pm 2) \text{ MeV}$ between the states; this contradicts isospin symmetry

Experimental status of doubly-heavy baryons

- LHCb Collab. [PRL 121 (2018) 162002]

Ξ_{cc}^{++} was confirmed in the decay channel $\Xi_{cc}^{++} \rightarrow \Xi_c^+ \pi^+$
(Stat. significance 5.9σ)

- Mass & width by PDG [PTEP 2022 (2022) 083C01]

$$M_{\Xi_{cc}^{++}} = (3621.6 \pm 0.4) \text{ MeV}, \quad \Gamma_{\Xi_{cc}^{++}} = (2.56 \pm 0.27) \times 10^{-13} \text{ s}$$

- Isospin partner, $\Xi_{cc}^+(ccd)$, is under searches by LHCb

$$\Xi_{cc}^+ \rightarrow \Lambda_c^+ K^- \pi^+ \text{ [SCPMA 63 (2020) 221062]}$$

$$\Xi_{cc}^+ \rightarrow \Xi_c^+ \pi^+ \pi^- \text{ [JHEP 12 (2021) 107]}$$

- Strange partner, $\Omega_{cc}^+(ccs)$, is also under searches by LHCb

$$\Omega_{cc}^+ \rightarrow \Xi_c^+ K^- \pi^+ \text{ [SCPMA 64 (2021) 101062]}$$

- Searches of bottom-charmed baryons by LHCb

$$\Xi_{bc}^0 \rightarrow D^0 p K^- \text{ [JHEP 11 (2020) 095]}$$

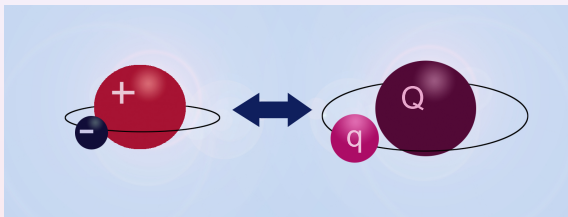
$$\Xi_{bc}^0 \rightarrow \Xi_c^+ \pi^- \text{ [Chin. Phys. C45 (2021) 093002]}$$

$$\Omega_{bc}^0 \rightarrow \Lambda_c^+ \pi^- \text{ [Chin. Phys. C45 (2021) 093002]}$$

$$\Xi_{bc}^+ \rightarrow J/\psi \Xi_c^+ \text{ [Chin. Phys. C47 (2023) 093001]}$$

Symmetry between heavy mesons and doubly-heavy baryons

- In HQET, heavy meson is the QCD analog of hydrogen atom



- Doubly-heavy baryon (DHB) contains heavy diquark — color state of two heavy quarks Q_1 и Q_2
- DHB is the QCD analog of deuterium
- Heavy quark fields are assumed to be static
- Being determined by light quark only, dynamics of heavy meson and DHB is similar

Local interpolating currents

- In HQET, B -meson-to-vacuum transition matrix element of axial-vector current (v^μ is the B -meson four-velocity)

$$\langle 0 | \bar{b}(0) \gamma^\mu \gamma_5 q(0) | B(v) \rangle = i f_B v^\mu$$

- $q(x)$ is a light quark field; $b(x)$ is the b -quark field
- $f_B \simeq 190 \text{ MeV}$ is the B -meson leptonic decay constant
- Local interpolating current of baryon

$$J(x) = \epsilon_{abc} [Q_1^a(x) C \Gamma Q_2^b(x)] \Gamma' q^c(x)$$

a, b, c are color indices, C is charged conjugation matrix
 $\Gamma^{(')} = \{I, \gamma_5, \gamma_\mu, \gamma_\mu \gamma_5, \sigma_{\mu\nu} = i[\gamma_\mu, \gamma_\nu]/2\}$

- Baryon-to-vacuum transition matrix element ($J^P = 1/2^+$)

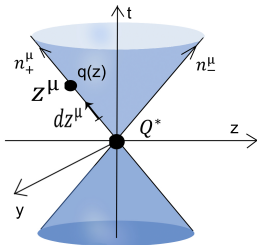
$$\langle 0 | J(0) | \Xi_{QQ'}(v) \rangle = i f_{\Xi_{QQ'}} u(v)$$

- For baryon with $J^P = 3/2^+$, the bispinor $u(v)$ is replaced by the vector-spinor $u^\mu(v)$

Wave-function of DHB in HQET

- To get a heavy-meson (B -meson) wave-function, local matrix element is replaced by non-local one
- In heavy meson, antiquark is assumed to be static, and massless light quark, being at a light-like separation z from it, determines the hadron dynamics
- In DHB, doubly-heavy diquark (DHD) is assumed to be static, and dynamics is also determined by massless light quark
- For bc -diquark, spin can be either $S = 0$ or $S = 1$ while cc - and bb -diquarks have spin $S = 1$
- Because of the heavy-quark symmetry (HQS), heavy-meson wave-function is characterized by two distribution amplitudes (DAs) [Grozin & Neubert, Phys. Rev. D55 (1997) 272]
- Transition matrix element from DHB state to the vacuum one is also determined by two DAs because of HQS
- Symmetry between heavy meson and DHB allows to generalize the heavy-meson theory developed to DHB wave-function

Light-Cone Decomposition



$$n_{\pm}^{\mu} = \frac{1}{\sqrt{2}} (1, 0, 0, \mp 1)$$

$$n_+^2 = 0, \quad (n_+ n_-) = 1$$

$$dz'^{\mu} = dz'_- n_+^{\mu}$$

$$A_{\pm} = n_{\pm}^{\mu} A_{\mu}$$

$$A^\mu = A_+ n_-^\mu + A_- n_+^\mu + A_\perp^\mu$$

- Link between quarks is Wilson line

$$E(0, z) = \mathcal{P} \exp \left\{ -ig_{\text{st}} \int_0^{z_-} A_+^a(z'_-) \frac{\lambda^a}{2} dz'_- \right\}$$

- g_{st} is strong coupling constant, $A_{\mu}^a(z)$ is gluonic field
- Fock-Schwinger gauge is used

$$A_+(z) = 0 \quad \Rightarrow \quad E(0, z) = 1$$

- Heavy antiquark (DHD) is a static color source situated at the origin of the frame

Heavy-Meson Distribution Amplitudes

- Because spin of heavy antiquark is decoupled in HQS limit, $Q^*(0)$ in heavy meson is considered as a spinless particle
- Meson-to-vacuum transition at $z^2 = 0$ (here, $\hat{z} = z_\mu \gamma^\mu$)
[Grozin & Neubert, Phys. Rev. D55 (1997) 272]

$$\langle 0 | Q^*(0) E(0, z) q(z) | M(v) \rangle = f_M \left\{ \tilde{\varphi}_+(t) + [\tilde{\varphi}_-(t) - \tilde{\varphi}_+(t)] \frac{\hat{z}}{2t} \right\} U(v)$$

- f_M is a constant with dimension of a mass
- $t = (vz)$ is the time in heavy-meson rest frame
- $U(v)$ is non-relativistic wave-function of “spinor-like meson”

$$\hat{v} U(v) = U(v)$$

- Four-velocity of heavy meson at rest $v^\mu = (1, 0, 0, 0)$
- $\tilde{\varphi}_\pm(t)$ are distribution amplitudes (DAs) of heavy meson

Heavy-Meson Distribution Amplitudes

- DAs normalization $\tilde{\varphi}_{\pm}(0) = 1$
- Definition of decay constant f_M is reproduced at $z = 0$

$$\langle 0 | Q^*(0) q(0) | M(v) \rangle = f_M U(v)$$

- Fourier transforms of DAs are required in constructing matrix elements

$$\tilde{\varphi}_{\pm}(t) = \int_0^{\infty} d\omega e^{-i\omega t} \phi_{\pm}(\omega)$$

- Normalization accepted gives the probability meaning to DAs

$$\tilde{\varphi}_{\pm}(0) = \int_0^{\infty} d\omega \phi_{\pm}(\omega) = 1$$

- In HQS limit, transition matrix elements of DHB are parametrized in complete analogy with “spinor-like meson” due to decoupling of DHD spin

- Exponential model [Grozin & Neubert, PRD 55 (1997) 272]

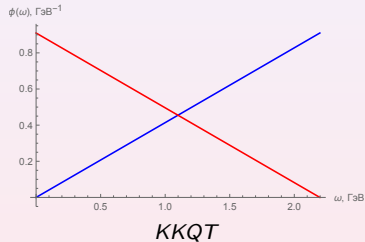
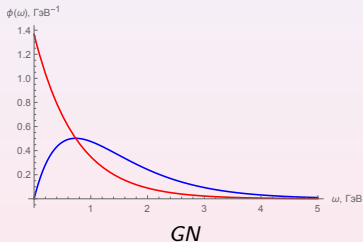
$$\phi_B^+(\omega) = \frac{\omega}{\omega_0^2} e^{-\omega/\omega_0}, \quad \phi_B^-(\omega) = \frac{1}{\omega_0} e^{-\omega/\omega_0}$$

- Naïve parton model (Linear model)
[Kawamura, Kodaira, Qiao & Tanaka, PLB 523 (2001) 111]

$$\phi_B^+(\omega) = \frac{\omega}{2\bar{\Lambda}^2} \theta(2\bar{\Lambda} - \omega), \quad \phi_B^-(\omega) = \frac{2\bar{\Lambda} - \omega}{2\bar{\Lambda}^2} \theta(2\bar{\Lambda} - \omega)$$

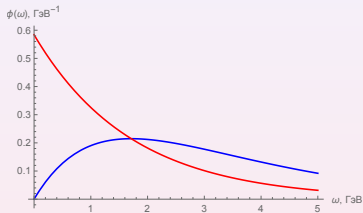
- $\bar{\Lambda} = m_B - m_b$ is effective mass of heavy meson ($\omega_0 = 2\bar{\Lambda}/3$)
- The value of $\bar{\Lambda}$ depends on the b -quark mass definition

- B -meson DAs at $\mu = 1$ GeV (blue line — ϕ^+ , red line — ϕ^-)

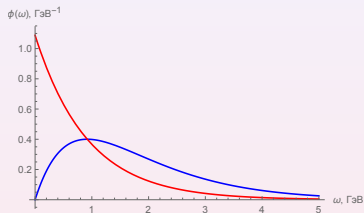


Models of doubly-heavy baryon DAs

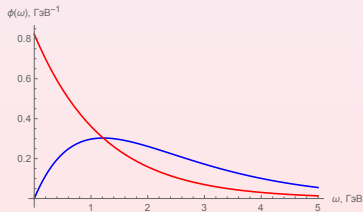
- $\bar{\Lambda} = M - m_{Q_1} - m_{Q_2}$ is effective mass of DHB
- DAs for Ξ_{bb^-} , Ξ_{cc^-} and Ξ_{bc^-} -baryons in the exponential model



Ξ_{bb}



Ξ_{cc}



Ξ_{bc}

Scale dependence of the ϕ_+ DA

- Evolution equation (B -meson) [Lange & Neubert (2003)]

$$\frac{d}{d \ln \mu} \phi_+^B(\omega, \mu) = - \int_0^\infty \gamma_+(\omega, \omega', \mu) \phi_+^B(\omega', \mu) d\omega'$$

- Evolution equation (Doubly-heavy baryon)

$$\frac{d}{d \ln \mu} \phi_+(\omega, \mu) = -2 \int_0^\infty \gamma_+(\omega, \omega', \mu) \phi_+(\omega', \mu) d\omega'$$

- B -meson DA [Lange & Neubert (2003)]

$$\phi_+^B(\omega, \mu) = \frac{1}{2\pi} \int_{-\infty}^\infty \varphi_0^B(t) f^B(\omega, \mu, \mu_0, it) dt,$$

- Doubly-heavy baryon DA

$$\phi_+(\omega, \mu) = \frac{1}{2\pi} \int_{-\infty}^\infty \varphi_0(t) f(\omega, \mu, \mu_0, it) dt,$$

- $\varphi_0(t)$ is the Fourier transform of $\phi_+(\omega, \mu_0)$ at fixed scale $\mu = \mu_0$ with respect to $\ln(\omega/\mu_0)$
- Depends on the choice of the DA model

Scale dependence of the ϕ_+ DA

- Solution for B -meson [Lange & Neubert (2003)]

$$f^B(\omega, \mu, \mu_0, it) \sim \left(\frac{\omega}{\mu_0}\right)^{it+g} \frac{\Gamma(1-it-g)\Gamma(1+it)}{\Gamma(1+it+g)\Gamma(1-it)}$$

- Solution for doubly-heavy baryon

$$f(\omega, \mu, \mu_0, it) \sim \left(\frac{\omega}{\mu_0}\right)^{it+2g} \left(\frac{\Gamma(1-it-g)\Gamma(1+it)}{\Gamma(1+it+g)\Gamma(1-it)}\right)^2$$

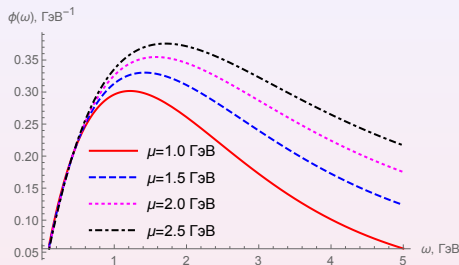
- Accounting evolution kernels between light quark and heavy quark
- Neglecting evolution kernel between heavy quarks; its impact is under study currently
- Quantities used

$$g(\mu, \mu_0) = \frac{8}{3\beta_0} \ln \frac{\alpha_s(\mu_0)}{\alpha_s(\mu)}, \quad \alpha_s(\mu) = \frac{4\pi}{2\beta_0 \ln(\frac{\mu}{\Lambda_{\text{QCD}}})}$$

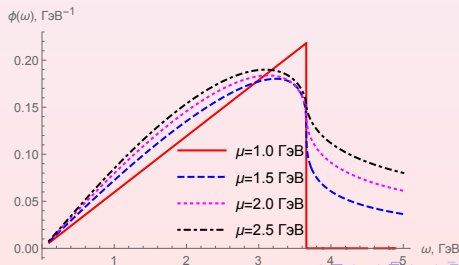
$$\beta_0 = 11 - \frac{2}{3}n_f, \quad n_f = 3, \quad \Lambda_{\text{QCD}} = 332 \text{ MeV}, \quad \mu_0 = 1 \text{ GeV}$$

Scale dependence of Ξ_{bb} -baryon DA, $\phi_+(\omega)$

- Exponential model



- Naïve parton model



- Theory describing heavy mesons on the light cone is generalized to doubly-heavy baryons
- For DAs of DHB, exponential and naïve parton models are proposed similar to heavy-meson DA models
- Scale dependence of the leading DA is obtained accounting for interaction of light quark with heavy ones only
- DAs are of interest in study of weak decays
 - In B -meson sector, semileptonic decays $B \rightarrow \pi \ell \nu_\ell$, $\ell = e, \mu$, are intensively studied both experimentally and theoretically
 - For DHBs, analogous decays are $\Xi_{bb} \rightarrow \Xi_b \ell^- \nu_\ell$
 - To calculate hadronic transition matrix elements, an information about wave-functions (DAs) of hadrons is needed

Backup Slides

- Braun, Ivanov and Korchemsky (BIK) model
[V. Braun, D. Ivanov & G. Korchemsky, PRD 69 (2004) 034014]

- B -meson DAs depend on the first reverse moments

$$\lambda_B^{-1}(\mu) = \int_0^\infty \frac{\phi_B^+(\omega, \mu)}{\omega} d\omega, \quad \frac{\sigma_B(\mu)}{\lambda_B(\mu)} = \int_0^\infty \frac{\phi_B^+(\omega, \mu)}{\omega} \ln \frac{\mu}{\omega} d\omega$$

- These moments require reevaluation for DHBs
- Lee and Neubert (LN) model
[S. J. Lee & M. Neubert, PRD 72 (2005) 094028]
 - Exponential model modified by “radiative tail”
 - “Radiative tail” needs to be recalculated
- Systematic parametrization of the leading B -meson DA
[T. Feldmann, P. Lüghausen and D. van Dyk, JHEP 10 (2022) 162]
 - Expansion of DA in terms of generalized Laguerre polynomials
 - Requires some analysis to be adopted to DHBs

- Baryons masses (MeV)

$$M_{\Xi_{cc}} = 3627 \pm 12 \text{ (KR)}, \quad 3651 \text{ (Ael)}, \quad 3480 \pm 50 \text{ (KL)}$$

$$M_{\Xi_{bc}} = 6914 \pm 13 \text{ (KR)}, \quad 6938 \text{ (Ael)}, \quad 6820 \pm 50 \text{ (KL)}$$

$$M_{\Xi_{bb}} = 10162 \pm 12 \text{ (KR)}, \quad 10235 \text{ (Ael)}, \quad 10090 \pm 50 \text{ (KL)}$$

- Lifetime (fs) [Karliner & Rosner, PRD (2014)]

Baryon	KR	Ael	KL
$\Xi_{cc}^{++} = ccu$	185	430 ± 100	450 ± 50
$\Xi_{cc}^{+} = ccd$	53	120 ± 100	160 ± 50
$\Xi_{bc}^{+} = bcu$	244	330 ± 80	300 ± 30
$\Xi_{bc}^0 = bcd$	93	280 ± 70	270 ± 30
$\Xi_{bb}^0 = bbu$	370	—	790 ± 20
$\Xi_{bb}^{-} = bbd$	370	—	800 ± 20

KR — [M. Karliner & J. L. Rosner, PRD 90 (2014) 094007]

Ael — [K. Anikeev et al., arXiv:hep-ph/0201071]

KL — [V. V. Kiselev, A. K. Likhoded, YΦH 172 (2002) 497]

- Braun, Ivanov and Korchemsky model [Braun, Ivanov & Korchemsky (2004)]

$$\phi_B^+(\omega, \mu) = \frac{4}{\pi \lambda_B} \frac{\omega \mu}{\omega^2 + \mu^2} \left[\frac{\mu^2}{\omega^2 + \mu^2} - \frac{2(\sigma_B - 1)}{\pi^2} \ln \frac{\omega}{\mu} \right],$$

$$\begin{aligned} \phi_B^-(\omega, \mu) = & -\frac{2}{\pi \lambda_B} \left(\frac{\omega \mu}{\omega^2 + \mu^2} + \operatorname{arctg} \frac{\omega}{\mu} - \frac{\pi}{2} + \right. \\ & \left. + \frac{4(\sigma_B - 1)}{\pi^2} \left\{ \operatorname{Im} \left[\operatorname{Li}_2 \left(\frac{i\omega}{\mu} \right) \right] - \operatorname{arctg} \frac{\omega}{\mu} \ln \frac{\omega}{\mu} \right\} \right) \end{aligned}$$

- Depend on the first reverse moments

$$\lambda_B^{-1}(\mu) = \int_0^\infty \frac{\phi_B^+(\omega, \mu)}{\omega} d\omega, \quad \frac{\sigma_B(\mu)}{\lambda_B(\mu)} = \int_0^\infty \frac{\phi_B^+(\omega, \mu)}{\omega} \ln \frac{\mu}{\omega} d\omega$$

- Lee and Neubert model [Lee & Neubert (2005)]

$$\phi_B^+(\omega, \mu) = \frac{N\omega}{\omega_0^2} e^{-\omega/\omega_0} + \\ + \theta(\omega - \omega_t) \frac{C_F \alpha_{\text{st}}(\mu)}{\pi\omega} \left[\left(\frac{1}{2} - \ln \frac{\omega}{\mu} \right) + \frac{4\bar{\Lambda}_{\text{DA}}}{3\omega} \left(2 - \ln \frac{\omega}{\mu} \right) \right]$$

- $\omega_0 = 2 [m_B - \bar{m}_b(\mu)] / 3$ like in GN model
- ω_t — граница, начиная с которой включается «радиационный хвост», обусловленный взаимодействием кварка с глюонным полем
- N — normalization coefficient:

$$\int_0^\infty \phi_B^+(\omega, \mu) d\omega = 1$$

- Lee and Neubert model [Lee & Neubert (2005)]

- Effective mass

$$\bar{\Lambda}_{\text{DA}}(\mu_f, \mu) = \bar{\Lambda}_{\text{SF}}(\mu_*, \mu_*) \left[1 + \frac{C_F \alpha_{\text{st}}(\mu)}{4\pi} \left(6 \ln \frac{\mu_f}{\mu} - \frac{7}{4} \right) \right] -$$

$$- \mu_f \frac{C_F \alpha_{\text{st}}(\mu)}{4\pi} \left(3 \ln \frac{\mu_f}{\mu} - \frac{9}{2} + \frac{4\mu_*}{\mu_f} \right)$$

- $\bar{\Lambda}_{\text{SF}}(\mu_*, \mu_*) = (0.65 \pm 0.06) \text{ ГэВ}$ на масштабе $\mu_* = 1.5 \text{ ГэВ}$
- DA of the non-leading twist

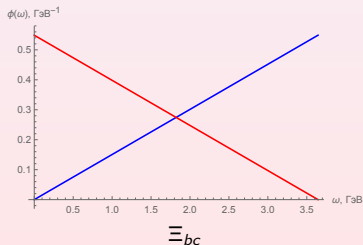
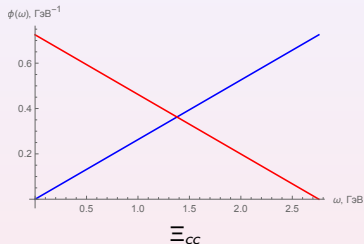
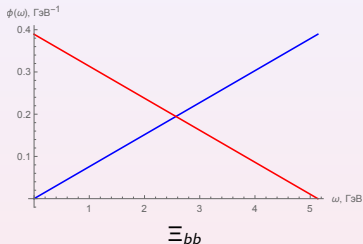
$$\phi_B^-(\omega, \mu) = \frac{N}{\omega_0} e^{-\omega/\omega_0} +$$

$$+ \theta(\omega - \omega_t) \frac{C_F \alpha_{\text{st}}}{\pi \mu} \left(-\frac{\mu}{2\omega} \left[1 + 2 \ln \frac{\omega}{\mu} \right] + \frac{\bar{\Lambda}_{\text{DA}} \mu}{3\omega^2} \left[3 - 2 \ln \frac{\omega}{\mu} \right] \right) +$$

$$+ \theta(\omega_t - \omega) \frac{C_F \alpha_{\text{st}}}{\pi \mu} \left(-\frac{\mu}{2\omega_t} \left[1 + 2 \ln \frac{\omega_t}{\mu} \right] + \frac{\bar{\Lambda}_{\text{DA}} \mu}{3\omega_t^2} \left[3 - 2 \ln \frac{\omega_t}{\mu} \right] \right).$$

Models of doubly-heavy baryon DAs

- DA for Ξ_{bb} , Ξ_{cc} и Ξ_{bc} baryons in linear model



QCD sum rules for DHB Distribution Amplitudes

- The construction of QCD sum rules for a matrix element $\langle 0 | \tilde{O}_{\Xi_{bc}}(t) J_{\Xi_{bc}}(-x) | \Xi_{bc}(\nu) \rangle$ in the Heavy Quark Effective theory of a is similar to the case of a heavy meson
- Similarly [Grozin & Neubert (1997)] we can get the QCD sum rules for the leading twist DA.

$$f_{\Xi_{bc}}^2 \varphi_+(\omega) = \frac{3\omega}{8\pi^2\tau} e^{(\bar{\Lambda}-\omega/2)\tau} \left[1 - e^{-(\varepsilon_c-\omega/2)\tau} \right] - \frac{\langle \bar{q}q \rangle}{8\tau} \tilde{f}_S\left(\frac{\omega}{2\tau}\right) e^{(\bar{\Lambda}-\omega/2)\tau}$$

- $\langle \bar{q}q \rangle$ — local condensate of light quarks
- τ — Borel parameter
- ε_c —
- Function $\tilde{f}_S(\nu)$ characterizes the energy distribution in a nonlocal quark condensate

QCD sum rules for DHB Distribution Amplitudes

- Distribution function [Braun et al. (1995)]

$$\tilde{f}_S(\nu) = \frac{\lambda^{p-2}}{\Gamma(p-2)} \nu^{1-p} e^{-\lambda/\nu}, \quad p = 3 + \frac{4\lambda}{m_0^2}$$

- $m_0^2 = \langle \bar{q} \sigma_{\mu\nu} G^{\mu\nu} q \rangle / \langle \bar{q} q \rangle$ — the ratio of quark-gluon condensate to quark condensate; λ — free parameter
- The first inverse moment of the leading twist DA

$$\lambda_{\Xi_{bc}}^{-1} = \int_0^\infty \frac{d\omega}{\omega} \varphi_+(\omega)$$

- Calculated similarly to the inverse moment of a heavy meson [Khodjamirian, Mandal & Mannel (2020)]
- Analytical form of sum rules for the first inverse moment of the Ξ_{bc} baryon (excluding Radiative Correction)

$$\lambda_{\Xi_{bc}}^{-1} = \frac{e^{\bar{\Lambda}\tau}}{4\pi^2 f_{\Xi_{bc}}^2} \left[1 - \frac{\pi^2 \langle \bar{q} q \rangle}{\tau \Gamma(p-2)} \lambda^{(p-3)/2} K_{p-1}(2\sqrt{\lambda}) \right]$$