Relativistic description of asymmetric fully heavy tetraquarks

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Introduction

- “Ordinary” hadrons:
  - baryons qqq,
  - mesons $q\bar{q}$.
- Exotic hadrons:
  - tetraquarks $qq\bar{q}\bar{q}$,
  - pentaquarks $qqqq\bar{q}$, etc.
- Searches for the $X_{cc\bar{c}\bar{c}}$, $X_{bb\bar{b}\bar{b}}$ are conducted on the Large Hadron Collider (LHC) by the LHCb, ATLAS and CMS Collaborations.

Model description I

- $m_{\mathrm{c}} = 1.55$ GeV,
  $m_{\mathrm{b}} = 4.88$ GeV.
- Quark content:
  - symmetric:
    - $\mathrm{cc}\overline{\mathrm{cc}}$,
    - $\mathrm{cb}\bar{\mathrm{c}}\bar{\mathrm{b}}$,
    - $\mathrm{bb}\overline{\mathrm{bb}}$.
  - asymmetric:
    - $\mathrm{cc}\bar{\mathrm{c}}\bar{\mathrm{b}}$, $\mathrm{bc}\overline{\mathrm{cc}}$,
    - $\mathrm{cc}\overline{\mathrm{bb}}$, $\mathrm{bb}\overline{\mathrm{cc}}$,
    - $\mathrm{bb}\bar{\mathrm{b}}\bar{\mathrm{c}}$, $\mathrm{cb}\overline{\mathrm{bb}}$.

Model description II

- Diquark–antidiquark bound state: $\{(Q_1Q_2)-(\overline{Q}_3\overline{Q}_4)\}$.
- Ground state diquarks:
  - scalar (S) — $J=0$,
  - axialvector (A) — $J=1$.
- Diquark content:
  - only axialvector $(A\overline{A})$:
    - $cc\overline{cc}$, $bb\overline{bb}$,
    - $cc\overline{bb}$.
  - both axialvector and scalar $(A\overline{A}$, $A\overline{S}$, $S\overline{A})$:
    - $cb\overline{cb}$ $(+\ S\overline{S})$,
    - $cc\overline{cb}$, $bb\overline{bc}$.

Relativistic quark model I

- Relativistic Schrödinger-type quasipotential equation:

$$\left(\frac{b^2(M)}{2\mu_{\rm R}(M)}-\frac{\mathbf{p}^2}{2\mu_{\rm R}(M)}\right)\Psi_{\rm d,T}(\mathbf{p})=\int\frac{d^3q}{(2\pi)^3}\,V(\mathbf{p},\mathbf{q};M)\Psi_{\rm d,T}(\mathbf{q})$$

$$\mu_{\rm R}=\frac{E_1E_2}{E_1+E_2}=\frac{M^4-(m_1^2-m_2^2)^2}{4M^3}$$

$$b^2(M)=\frac{[M^2-(m_1+m_2)^2][M^2-(m_1-m_2)^2]}{4M^2}$$

Relativistic quark model II

- Diquark–antidiquark interaction quasipotential:

$$V(\mathbf{p},\mathbf{q};M)=\frac{\langle d(\mathcal{P})|\,J_\mu\,|d(\mathcal{Q})\rangle}{2\sqrt{E_{\rm d}}\sqrt{E_{\rm d}}}\,\frac{4}{3}\alpha_{\rm s}D^{\mu\nu}(\mathbf{k})\,\frac{\langle d'(\mathcal{P}')|\,J_\nu\,|d'(\mathcal{Q}')\rangle}{2\sqrt{E_{\rm d'}}\sqrt{E_{\rm d'}}}$$

$$+\Psi^*_{\rm d}(\mathcal{P})\Psi^*_{\rm d'}(\mathcal{P}')[J_{{\rm d};\mu}J^{\mu}_{\rm d'}\,V^{\rm V}_{\rm conf.}(\mathbf{k})+V^{\rm S}_{\rm conf.}(\mathbf{k})]\Psi_{\rm d}(\mathcal{Q})\Psi_{\rm d'}(\mathcal{Q}')$$

Relativistic quark model III

- ◇ Diquark–antidiquark interaction quasipotential in configuration space:

$$
\begin{aligned}
V(r) = {} & \Bigg[V_{\mathrm{Coul.}}(r) + V_{\mathrm{conf.}}(r) + \frac{1}{E_1 E_2}\Bigg\{ \mathbf{p}\Big[V_{\mathrm{Coul.}}(r) + V^{\mathrm{V}}_{\mathrm{conf.}}(r) \Big]\mathbf{p} - \frac{1}{4}\Delta V^{\mathrm{V}}_{\mathrm{conf.}}(r) + V'_{\mathrm{Coul.}}(r)\frac{\mathbf{L}^2}{2r} \Bigg\} \Bigg]_a \\
& + \Bigg[\Bigg\{ \frac{1}{2}\Bigg[\frac{1}{E_1(E_1+M_1)} + \frac{1}{E_2(E_2+M_2)} \Bigg] \frac{V'_{\mathrm{Coul.}}(r)}{r} - \frac{1}{2}\Bigg[\frac{1}{M_1(E_1+M_1)} + \frac{1}{M_2(E_2+M_2)} \Bigg] \frac{V'_{\mathrm{conf.}}(r)}{r} \\
& + \frac{\mu_{\mathrm{d}}}{4}\Bigg[\frac{1}{M_1^2} + \frac{1}{M_2^2} \Bigg] \frac{V'^{\mathrm{V}}_{\mathrm{conf.}}(r)}{r} + \frac{1}{E_1 E_2}\Bigg[V'_{\mathrm{Coul.}}(r) + \frac{\mu_{\mathrm{d}}}{4}\Bigg(\frac{E_1}{M_1} + \frac{E_2}{M_2} \Bigg) V'^{\mathrm{V}}_{\mathrm{conf.}}(r) \Bigg] \frac{1}{r} \Bigg\} \mathbf{L}(\mathbf{S}_1 + \mathbf{S}_2) \\
& + \Bigg\{ \frac{1}{2}\Bigg[\frac{1}{E_1(E_1+M_1)} - \frac{1}{E_2(E_2+M_2)} \Bigg] \frac{V'_{\mathrm{Coul.}}(r)}{r} - \frac{1}{2}\Bigg[\frac{1}{M_1(E_1+M_1)} - \frac{1}{M_2(E_2+M_2)} \Bigg] \frac{V'_{\mathrm{conf.}}(r)}{r} \\
& + \frac{\mu_{\mathrm{d}}}{4}\Bigg[\frac{1}{M_1^2} - \frac{1}{M_2^2} \Bigg] \frac{V'^{\mathrm{V}}_{\mathrm{conf.}}(r)}{r} + \frac{1}{E_1 E_2}\frac{\mu_{\mathrm{d}}}{4}\Bigg(\frac{E_1}{M_1} - \frac{E_2}{M_2} \Bigg) \frac{V'^{\mathrm{V}}_{\mathrm{conf.}}(r)}{r} \Bigg\} \mathbf{L}(\mathbf{S}_1 - \mathbf{S}_2) \Bigg]_b \\
& + \Bigg[\frac{1}{3E_1 E_2}\Bigg\{ \frac{1}{r}V'_{\mathrm{Coul.}}(r) - V''_{\mathrm{Coul.}}(r) + \frac{\mu_{\mathrm{d}}^2}{4}\frac{E_1 E_2}{M_1 M_2}\Bigg(\frac{1}{r}V'^{\mathrm{V}}_{\mathrm{conf.}}(r) - V''^{\mathrm{V}}_{\mathrm{conf.}}(r) \Bigg) \Bigg\} \times \Bigg[\frac{3}{r^2}\Big(\mathbf{S}_1\mathbf{r}\Big)\Big(\mathbf{S}_2\mathbf{r}\Big) - \mathbf{S}_1\mathbf{S}_2 \Bigg] \Bigg]_c \\
& + \Bigg[\frac{2}{3E_1 E_2}\Bigg\{ \Delta V_{\mathrm{Coul.}}(r) + \frac{\mu_{\mathrm{d}}^2}{4}\frac{E_1 E_2}{M_1 M_2}\Delta V^{\mathrm{V}}_{\mathrm{conf.}}(r) \Bigg\} \mathbf{S}_1\mathbf{S}_2 \Bigg]_d
\end{aligned}
$$

Interactions I

◇ Interaction $V(r)$:

- $\Big[\ldots\Big]_a \equiv \mathrm{V}_{\mathrm{spin-ind.}}$,
- $\mathbf{L}(\mathbf{S}_{\mathrm{d}_1} \pm \mathbf{S}_{\mathrm{d}_2}) \equiv \mathrm{LS}_{\pm}$,
- $\dfrac{3}{r^2}\Big(\mathbf{S}_{\mathrm{d}_1}\mathbf{r}\Big)\Big(\mathbf{S}_{\mathrm{d}_2}\mathbf{r}\Big) - \mathbf{S}_{\mathrm{d}_1}\mathbf{S}_{\mathrm{d}_2} \equiv \mathrm{T}$,
- $\mathbf{S}_{\mathrm{d}_1}\mathbf{S}_{\mathrm{d}_2} \equiv \mathrm{SS}$.

Interactions II

- ◇ Symmetric compositions:
  - LS_{+} — diagonal,
  - $\mathrm{LS}_{-} \equiv 0$,
  - T — non-diagonal,
  - SS — diagonal.
- ◇ Non-diagonal elements arise only for a few states. They are very small numerically and can be ignored. Thus, effectively:
  - T — diagonal,

  and there is no mixing between any states.

Interactions III

◇ Asymmetric compositions:

- LS_+ — diagonal,
- LS_- — non-diagonal,
- T — non-diagonal,
- SS — diagonal.

◇ Significant mixing between the $n\,^{2S+1}\mathrm{L}_J$ and $n\,^{2S'+1}\mathrm{L}_J$ states arises.

Mixing I

◇ Notations:

- $M_{L=a,\ J=b} \equiv M_{a,b}$,
- $M_{L=a,\ J=b}(S=c,\ S'=d) \equiv M_{a,b}(c,\ d)$,
- $\Delta M_{a,b}(c,\ d) = \big[M_{a,b}(c,\ d)\big]_{\text{full}} - \big[M_{a,b}(c,\ d)\big]_{\text{spin-ind.}}$.

◇ P-wave:

- $J=1$: $\quad M_{1,1} = \lambda \begin{pmatrix} M_{1,1}(0,0) & \Delta M_{1,1}(0,1) & \Delta M_{1,1}(0,2) \\ \Delta M_{1,1}(1,0) & M_{1,1}(1,1) & \Delta M_{1,1}(1,2) \\ \Delta M_{1,1}(2,0) & \Delta M_{1,1}(2,1) & M_{1,1}(2,2) \end{pmatrix}$
- $J=2$: $\quad M_{1,2} = \lambda \begin{pmatrix} M_{1,2}(1,1) & \Delta M_{1,2}(1,2) \\ \Delta M_{1,2}(2,1) & M_{1,2}(2,2) \end{pmatrix}$

Mixing II

◇ D-wave:

- $J = 1$: $M_{2,1} = \lambda\begin{pmatrix} M_{2,1}(1,1) & \Delta M_{2,1}(1,2) \\ \Delta M_{2,1}(2,1) & M_{2,1}(2,2) \end{pmatrix}$
- $J = 2$: $M_{2,2} = \lambda\begin{pmatrix} M_{2,2}(0,0) & \Delta M_{2,2}(0,1) & \Delta M_{2,2}(0,2) \\ \Delta M_{2,2}(1,0) & M_{2,2}(1,1) & \Delta M_{2,2}(1,2) \\ \Delta M_{2,2}(2,0) & \Delta M_{2,2}(2,1) & M_{2,2}(2,2) \end{pmatrix}$
- $J = 3$: $M_{2,3} = \lambda\begin{pmatrix} M_{2,3}(1,1) & \Delta M_{2,3}(1,2) \\ \Delta M_{2,3}(2,1) & M_{2,3}(2,2) \end{pmatrix}$

Asymmetric compositions mass spectra I

◇ $A\bar{A}$-configuration:

Table 1: Masses of the ground states, radial and orbital excitations of the asymmetric ($cc\bar{c}\bar{b}$, $bc\bar{c}\bar{c}$, $cc\bar{b}\bar{b}$, $bb\bar{c}\bar{c}$, $bb\bar{b}\bar{c}$, $cb\bar{b}\bar{b}$) fully heavy tetraquarks in the $A\bar{A}$-configuration.

$d\bar{d}'$	nL	n_r	L	S	J	J^P	$M_{cc\bar{c}\bar{b},\, bc\bar{c}\bar{c}}$	$M_{cc\bar{b}\bar{b},\, bb\bar{c}\bar{c}}$	$M_{bb\bar{b}\bar{c},\, cb\bar{b}\bar{b}}$
$A\bar{A}$	1S	0	0	0	0	0^+	9,606	12,848	16,102
				1	1	1^+	9,611	12,852	16,104
				2	2	2^+	9,620	12,859	16,108
	1P	0	1	1	0	0^-	9,875	13,106	16,326
				0	1	1^-	9,871	13,103	16,325
				1			9,877	13,108	16,326
				2			9,881	13,111	16,329
				1	2	2^-	9,875	13,106	16,327
				2			9,882	13,112	16,329
				2	3	3^-	9,881	13,110	16,330
	2S	1	0	0	0	0^+	10,063	13,282	16,481
				1	1	1^+	10,064	13,282	16,481
				2	2	2^+	10,064	13,283	16,481
	1D	0	2	2	0	0^+	10,113	13,330	16,513
				1	1	1^+	10,111	13,328	16,513
				2			10,114	13,331	16,514
				0	2	2^+	10,108	13,324	16,513
				1			10,113	13,330	16,514
				2			10,117	13,334	16,515
				1	3	3^+	10,111	13,327	16,515
				2			10,116	13,332	16,516
				2	4	4^+	10,114	13,329	16,516
	2P	1	1	1	0	0^-	10,265	13,468	16,631
				0	1	1^-	10,258	13,461	16,629
				1			10,264	13,468	16,630
				2			10,270	13,472	16,633
				1	2	2^-	10,260	13,463	16,630
				2			10,268	13,470	16,632
				2	3	3^-	10,263	13,466	16,631
	3S	2	0	0	0	0^+	10,442	13,629	16,765
				1	1	1^+	10,442	13,629	16,765
				2	2	2^+	10,440	13,628	16,764

Asymmetric compositions mass spectra II

◇ $A\bar{S}$-, $S\bar{A}$-configuration:

Table 2: Masses of the ground states, radial and orbital excitations of the asymmetric ($cc\bar{c}\bar{b}$, $bc\bar{c}\bar{c}$, $bb\bar{b}\bar{c}$, $cb\bar{b}\bar{b}$) fully heavy tetraquarks in the $A\bar{S}$-, $S\bar{A}$-configuration.

$\mathbf{d\bar{d}'}$	nL	$\mathbf{n_r}$	L	S	J	$\mathbf{J^P}$	$\mathbf{M_{cc\bar{c}\bar{b},bc\bar{c}\bar{c}}}$	$\mathbf{M_{bb\bar{b}\bar{c},cb\bar{b}\bar{b}}}$
$A\bar{S}$, $S\bar{A}$	1S	0	0	1	1	1^+	9,608	16,099
	1P	0	1		0	0^-	9,873	16,320
					1	1^-	9,872	16,321
					2	2^-	9,871	16,322
	2S	1	0		1	1^+	10,057	16,474
	1D	0	2		1	1^+	10,108	16,507
					2	2^+	10,107	16,508
					3	3^+	10,105	16,509
	2P	1	1		0	0^-	10,262	16,624
					1	1^-	10,260	16,624
					2	2^-	10,254	16,624
	3S	2	0		1	1^+	10,434	16,758

Threshold analysis: general I

- ◇ If energetically possible, the tetraquark will fall-apart into a meson pair through the quark rearrangement.

$$\Delta = M_{\mathrm{QQ'\overline{QQ}'}} - M^{\mathrm{lowest}}_{\mathrm{threshold}}$$

- ◇ If $\Delta < 0$, state is stable against fall-apart strong decays.
- ◇ The smaller $\Delta > 0$, the narrower is the state.

Threshold analysis: general II

- ◇ Many masses lie well above thresholds with $\Delta > 100$ MeV.
- ◇ Few masses lie in the $[-40 < \Delta < 100]$ MeV interval.
- ◇ Such behavior is seen for all quark compositions and all excitations.
- ◇ It is consistent with the lack of significant advances in experimental searches.

Threshold analysis: asymmetric

- ◇ The most promising to be stable states:

Table 3: Ground and excited states of the asymmetric ($cc\bar{c}\bar{b}$, $bc\overline{cc}$, $cc\overline{bb}$, $bb\overline{cc}$, $bb\bar{b}\bar{c}$, $cb\overline{bb}$) fully heavy tetraquarks, which lie slightly above or below the meson–meson fall-apart strong decay thresholds.

$QQ'\overline{QQ}'$	$\mathbf{d\bar{d}'}$	**nL**	**S**	$\mathbf{J^P}$	$\mathbf{M}_{QQ'\overline{QQ}'}$	$\mathbf{M}_{\text{thr.}}$	$\mathbf{\Delta}_{\text{max}}$	Meson pair
$cc\bar{c}\bar{b}$, $bc\overline{cc}$	$A\bar{A}$	1P	2	3^-	9,881	9,858	23	$J/\psi(1S)\ B_c^{\pm}(1\,^3P_2)$
		1D	1	3^+	10,111	10,013	98	$\eta_c(1S)\ B_c^{\pm}(1\,^3D_3)$
			2		10,116		103	
			2	4^+	10,114	10,126	-12	$J/\psi(1S)\ B_c^{\pm}(1\,^3D_3)$
	$A\bar{S}$, $S\bar{A}$	1D	1	3^+	10,105	10,013	92	$\eta_c(1S)\ B_c^{\pm}(1\,^3D_3)$
$cc\overline{bb}$, $bb\overline{cc}$	$A\bar{A}$	1P	0	1^-	13,103	13,017	86	$B_c^{\pm}(1\,^1S_0)\ B_c^{\pm}(1\,P_1)$
			1		13,108		91	
			2		13,111		94	
			1	2^-	13,106	13,035	71	$B_c^{\pm}(1\,^1S_0)\ B_c^{\pm}(1\,^3P_2)$
			2		13,112		77	
			2	3^-	13,110	13,094	16	$B_c^{\pm}(1\,^3S_1)\ B_c^{\pm}(1\,^3P_2)$
		1D	1	3^+	13,327	13,303	24	$B_c^{\pm}(1\,^1S_0)\ B_c^{\pm}(1\,^3D_3)$
			2		13,332		29	
			2	4^+	13,329	13,362	-33	$B_c^{\pm}(1\,^3S_1)\ B_c^{\pm}(1\,^3D_3)$
$bb\bar{b}\bar{c}$, $cb\overline{bb}$	$A\bar{A}$	1D	1	3^+	16,515	16,428	87	$\eta_b(1S)\ B_c^{\pm}(1\,^3D_3)$
			2		16,516		88	
			2	4^+	16,516	16,489	27	$\Upsilon(1S)\ B_c^{\pm}(1\,^3D_3)$
	$A\bar{S}$, $S\bar{A}$	1D	1	3^+	16,509	16,428	81	$\eta_b(1S)\ B_c^{\pm}(1\,^3D_3)$

Experimental data I

- In 2020 the LHCb Collaboration announced the discovery of the narrow resonance X(6900).
- Several other broad structures peaking at about 6.4 and 7.2 GeV were reported.
- In 2022 ATLAS and CMS Collaborations confirmed X(6900) and hinted on a few more states, including structures at 6.4 and 7.2 GeV.

Experimental data II

◇ Current observation status and our predictions:

Table 4: Exotic X states observed and hinted by the LHCb, ATLAS and CMS Collaborations in di-J/ψ and $J/\psi\,\psi(2S)$ invariant mass spectra and our candidates. All masses M and total widths Γ are given in MeV.

Collaboration		**Resonance**	**M**	**Γ**	**Our candidates**			
					nL	**S**	$\mathbf{J^{PC}}$	**M**
LHCb		X(6600)	$6400 \div 6600$		1S	2	2^{++}	6367
ATLAS	m_0, model A		$6410 \pm 80^{+80}_{-30}$	$590 \pm 350^{+120}_{-200}$				
	m_0, model B		$6650 \pm 20^{+30}_{-20}$	$440 \pm 50^{+60}_{-50}$				
	m_1, model A		$6630 \pm 50^{+80}_{-10}$	$350 \pm 110^{+110}_{-40}$				
CMS	BW_1, no interference		$6552 \pm 10 \pm 12$	$124^{+32}_{-26} \pm 33$	2S	0	0^{++}	6782
	BW_1, interference		6638^{+43+16}_{-38-31}	$440^{+230+110}_{-200-240}$				
LHCb	NRSPS, no interference	X(6900)	$6905 \pm 11 \pm 7$	$80 \pm 19 \pm 33$				
	NRSPS, interference		$6886 \pm 11 \pm 11$	$168 \pm 33 \pm 69$	2S	2	2^{++}	6868
ATLAS	m_2, model A		$6860 \pm 30^{+10}_{-20}$	$110 \pm 50^{+20}_{-10}$	1D	0	2^{++}	6921
	m_2, model B		$6910 \pm 10 \pm 10$	$150 \pm 30 \pm 10$		2	0^{++}	6899
	m_3, model β		$6960 \pm 50 \pm 30$	$510 \pm 170^{+110}_{-100}$		2	1^{++}	6904
CMS	BW_2, no interference		$6927 \pm 9 \pm 4$	$122^{+24}_{-21} \pm 18$		2	2^{++}	6915
	BW_2, interference		6847^{+44+48}_{-28-20}	191^{+66+25}_{-49-17}				
LHCb		X(7200)	$7200 \div 7400$		3S	0	0^{++}	7259
ATLAS	m_3, model α		$7220 \pm 30^{+10}_{-30}$	$90 \pm 60^{+60}_{-30}$				
CMS	BW_3, no interference		$7287^{+20}_{-18} \pm 5$	$95^{+59}_{-40} \pm 19$		2	2^{++}	7333
	BW_3, interference		7134^{+48+41}_{-25-15}	97^{+40+29}_{-29-26}				

Conclusion I

◇ Masses of ground and excited states of the fully heavy tetraquarks were calculated.

◇ The finite size of a diquark was taken into account.

Conclusion II

- ◇ Calculations for the asymmetric flavor compositions were carried out.
- ◇ Mixing between the states with the same $n\mathrm{L}_J$, but different S via the LS_-- and T-interactions was taken into account.

Conclusion III

◇ Asymmetric tetraquark states which are the most convenient for the experimental detection were identified.

◇ Masses of resonances in the di-J/ψ production detected at the LHCb, ATLAS and CMS agree with our predictions for the ground and excited $\mathrm{X}_{\mathrm{cc}\overline{\mathrm{cc}}}$ states.

Publications

◇ This talk is based on the following publications:

- Masses of the $QQ\overline{Q}\overline{Q}$ tetraquarks in the relativistic diquark–antidiquark picture, Physical Review D, 2020, vol. 102, №11, p. 114030;
- Heavy Tetraquarks in the Relativistic Quark Model, Universe, 2021, vol. 7, №4, p. 94;
- Fully heavy tetraquark spectroscopy in relativistic quark model, Memoirs of the Faculty of Physics, 2022, №4, p. 2241512;
- Fully Heavy Tetraquark Spectroscopy in the Relativistic Quark Model, Symmetry, 2022, vol. 14, №12, p. 2504;
- Relativistic description of the mass spectra of fully heavy tetraquarks, Memoirs of the Faculty of Physics, 2023, №4, p. 2341504;
- Relativistic description of asymmetric fully heavy tetraquarks in the diquark–antidiquark model, 2023, arXiv: 2310.20247.

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