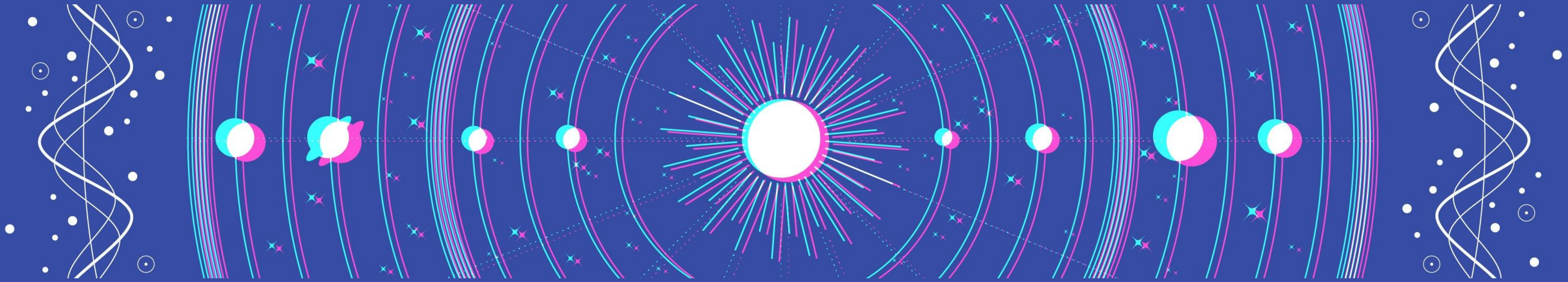
Black hole shadows as a source for testing extended theories of gravity



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1 Introduction

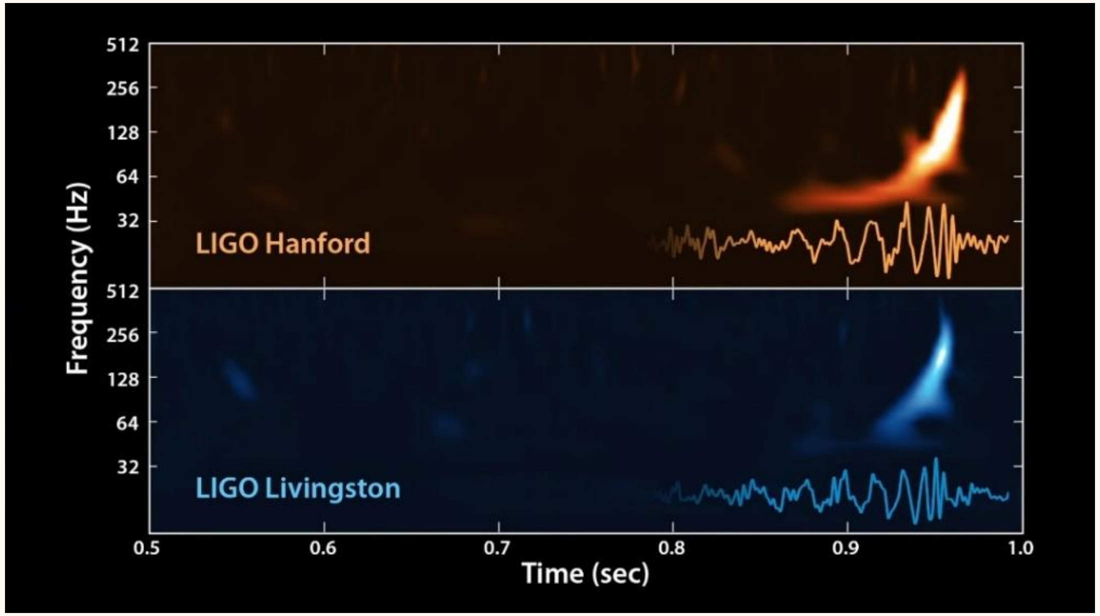
2 Shadow model at $A(r)=B^{-1}(r)$

3 Shadow model at $A(r) \neq B^{-1}(r)$

The existence of black holes was proven by:

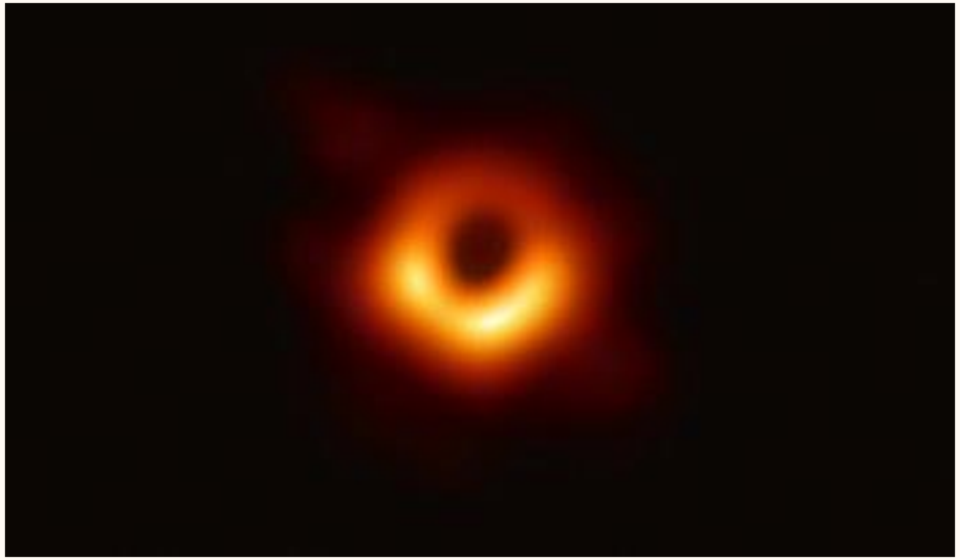
- results on binary systems dynamics
- gravitational wave astronomy
- direct imaging of black hole

The first recorded gravitational waves from the BH merger*



* *B.P. Abbott et al. Phys. Rev. D, 93(12):122003, 2016.*

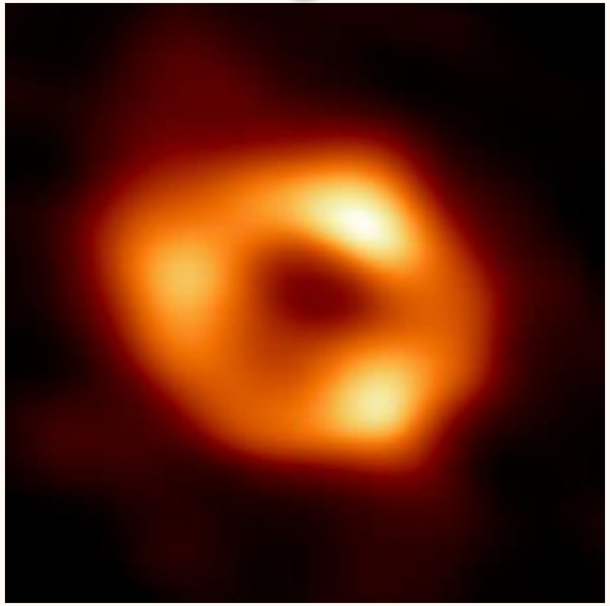
M 87*



BH shadow size from 4.3M to 6.1M

K. Akiyama, et al., Astrophys. J. 875 (1) L5 (2019).

Sgr A*



BH shadow size from 4.3M to 5.3M

The Event Horizon Telescope Collaboration, The Astrophysical Journal Letters 930 L17 (2022).

Quasi Schwarzschild metric*

$$ds^2 = -A(r)dt^2 + B(r)dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2),$$

Metric function**

$$A(r) = 1 - \frac{2M}{r} + \frac{Q}{r^2} + \frac{C_3}{r^3},$$

Q – tidal or electric charge

C_3 – third – order coefficient

$$\hat{r} = r/M,\ q = Q/M^2,\ c_3 = C_3/M^3.$$

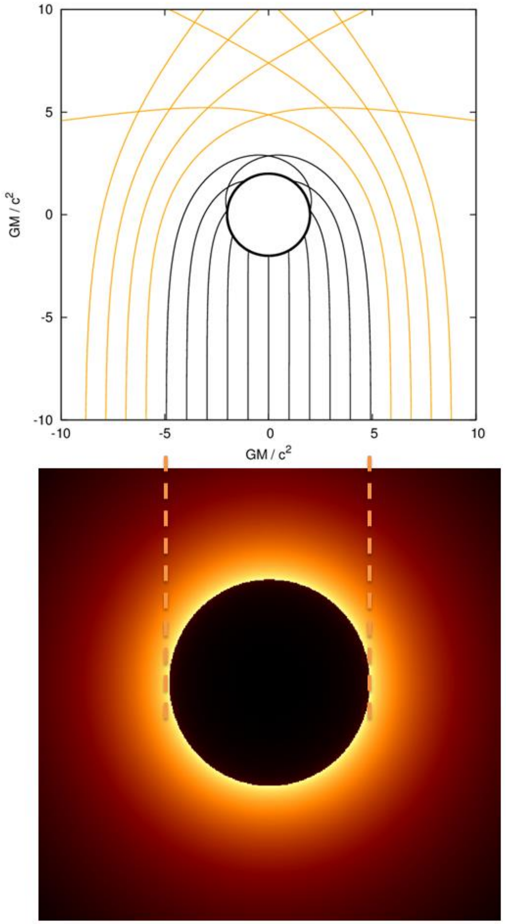
Sighting parameter

$D=27M^2$

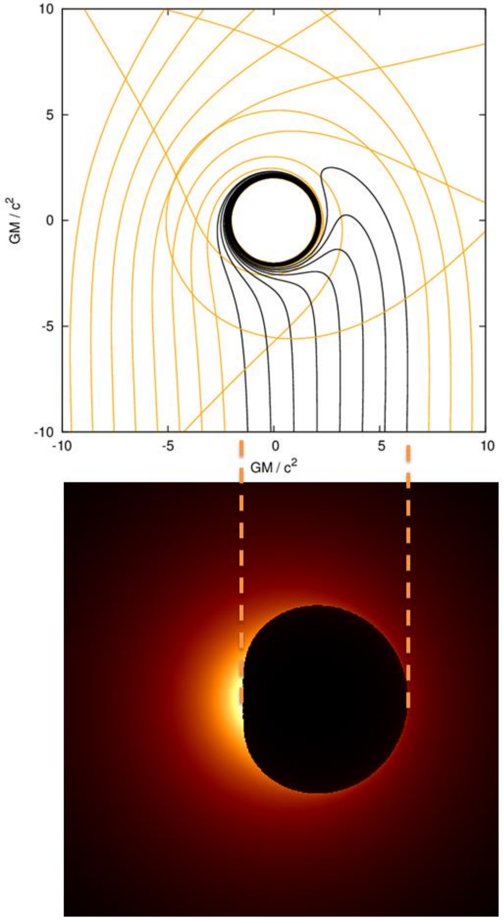
Mass - normalized metric function

$$A(\hat{r}) = 1 - \frac{2}{\hat{r}} + \frac{q}{\hat{r}^2} + \frac{c_3}{\hat{r}^3}.$$

Schwarzschild BH shadow***



Kerr BH shadow***



Cosimo Bambi. Phys. Rev. D, 87:107501, 2013.

***S. Alexeyev, B. Latosh, V. Prokopov andE. Emtsova, J. Exp. Theor. Phys. 128 (5) 720(2019).*

****https://odysseyedu.wordpress.com/black-hole-shadow/*

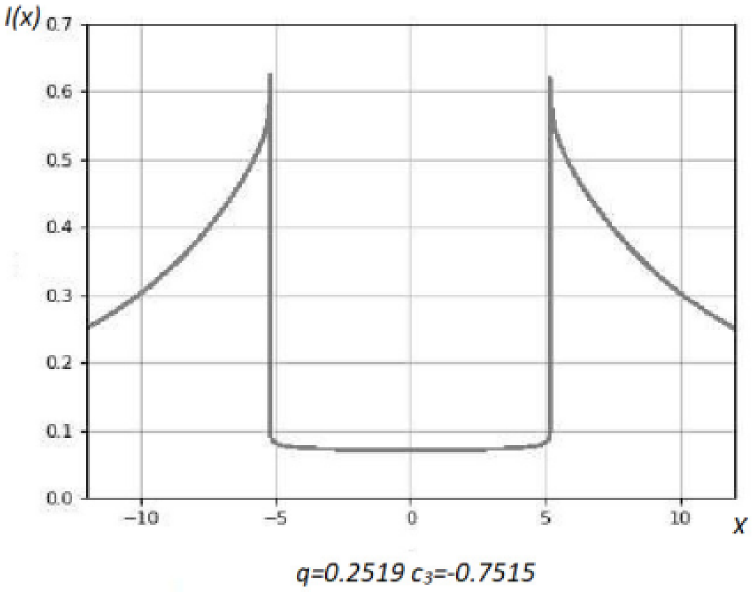


Fig. 1. The dependence of the intensity profile (I) in relative units for the shadow versus the distance from BH center on image plane X (in the units of M). The BH possesses by additional parameters q and c_3 and generates the shadow size the same as Schwarzschild BH of the equal size.

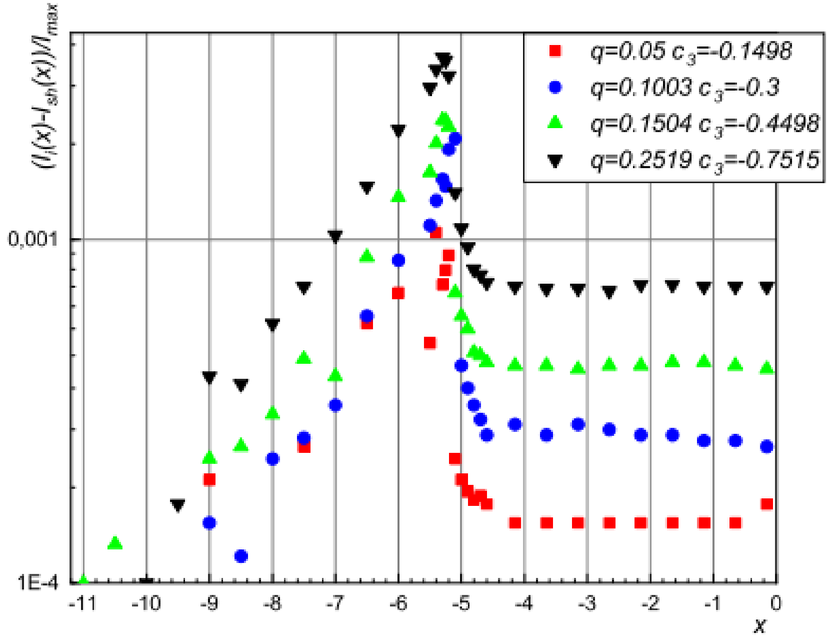


Fig. 2. The dependence of the intensity difference ($|I - I_{Sh}|/I_{max}$) between a BH with additional parameters q and c_3 and the corresponding Schwarzschild one versus the distance from BH center on image plane X in the units of M.

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Horndeski Theory

$$S = \int \sqrt{-g} d^4x (\mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5 + \mathcal{L}_4^{bH} + \mathcal{L}_5^{bH}),$$

$$\begin{aligned}
\mathcal{L}_2 &= G_2(X), \\
\mathcal{L}_3 &= -G_3(X)\Box\phi, \\
\mathcal{L}_4 &= G_4(X)R + G_{4X}[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2], \\
\mathcal{L}_5 &= G_5(X)G_{\mu\nu}\nabla^\mu\nabla^\nu - \frac{1}{6}G_{5X}[(\Box\phi)^3 - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3], \\
\mathcal{L}_4^{bH} &= F_4(X)\epsilon^{\mu\nu\rho\sigma}\epsilon_\sigma^{\alpha\beta\gamma}\nabla_{\mu\phi}\nabla_{\alpha\phi}\nabla_\nu\nabla_{\beta\phi}\nabla_\rho\nabla_{\gamma\phi}\nabla_\sigma\nabla_{\delta\phi}, \\
\mathcal{L}_5^{bH} &= F_5(X)\epsilon^{\mu\nu\rho\sigma}\epsilon^{\alpha\beta\gamma\delta}\nabla_{\mu\phi}\nabla_{\alpha\phi}\nabla_\nu\nabla_{\beta\phi}\nabla_\rho\nabla_{\gamma\phi}\nabla_\sigma\nabla_{\delta\phi}\nabla_\sigma\nabla_{\delta\phi}.
\end{aligned}$$

$$\begin{aligned}
A(r) &= 1 - \frac{2M}{r} - \frac{2C_7}{7r^7}, \\
B(r)^{-1} &= 1 - \frac{2M}{r} - \frac{C_7}{r^7},
\end{aligned}$$

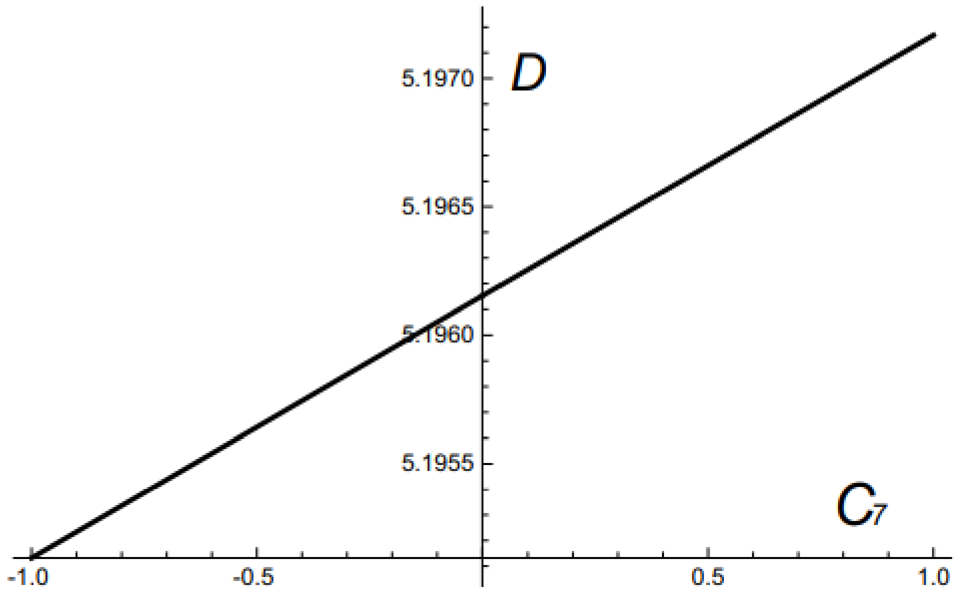

Fig. 3. The dependence of shadow size (D) versus the combination of model constants C_7 for Horndesky theory coupled with Gauss-Bonnet invariant (in the units of M, $M = 1$).

[12] Eugeny Babichev, Christos Charmousis, and Antoine Lehébel. Asymptotically flat black holes in Horndeski theory and beyond. JCAP, 04:027, 2017.

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5 Loop Quantum Gravity

$$A(r) = (1 - \frac{2Mr^2}{r^3 + 2Ml^2})(1 - \frac{\alpha\beta M}{\alpha r^3 + \beta M})$$

$$B(r)^{-1} = 1 - \frac{2Mr^2}{r^3 + 2Ml^2}$$

where l encodes the central energy density $3/8\pi l^2$,

[17] Jian-Ping Hu, Li-Li Shi, Yu Zhang, and Peng-Fei Duan. Analytical time-like geodesics in modified hayward black hole space-time. Astrophysics and Space Science, 363(10), 2018.

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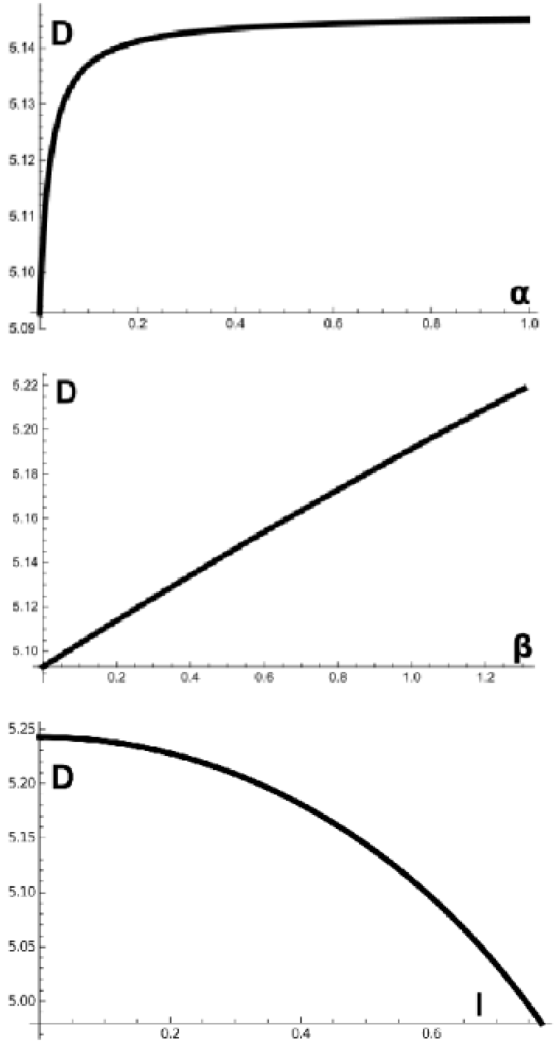

Fig. 4. The dependence of shadow size D upon the time delay α when $l = 0.5M$ and $\beta = 0.5$ (top image), upon the 1-loop quantum corrections β when $l = 0.5M$, $\alpha = 0.5$ (central image), upon the central energy density l when $\alpha = 0.5$, $\beta = 0.5$ (bottom image) for BH in modified Hayward metric in the units of M, $M = 1$.

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Conformal Gravity

$$S=\frac{1}{16\pi G}\int d^4x\sqrt{-g}[R-\alpha(\phi^2R+6\partial_\mu\phi\partial^\mu\phi]-\frac{1}{2m_2^2}C^{\mu\nu\rho\sigma}C_{\mu\nu\rho\sigma}],$$

$$A(r) = 1-\frac{2M}{r}+\frac{Q_s^2}{r^2}+\frac{Q_s^2(-M^2+Q_s^2+\frac{6}{m_2^2})}{3r^4}+\ldots$$

$$B(r)^{-1} = 1-\frac{2M}{r}+\frac{Q_s^2}{r^2}+\frac{2Q_s^2(-M^2+Q_s^2+\frac{6}{m_2^2})}{3r^4}+\ldots,$$

if m_2=2, Q_s<0,9

[14] Yun Soo Myung and De-Cheng Zou. Black holes in new massive conformal gravity. Physical Review D, 100(6), 2019.

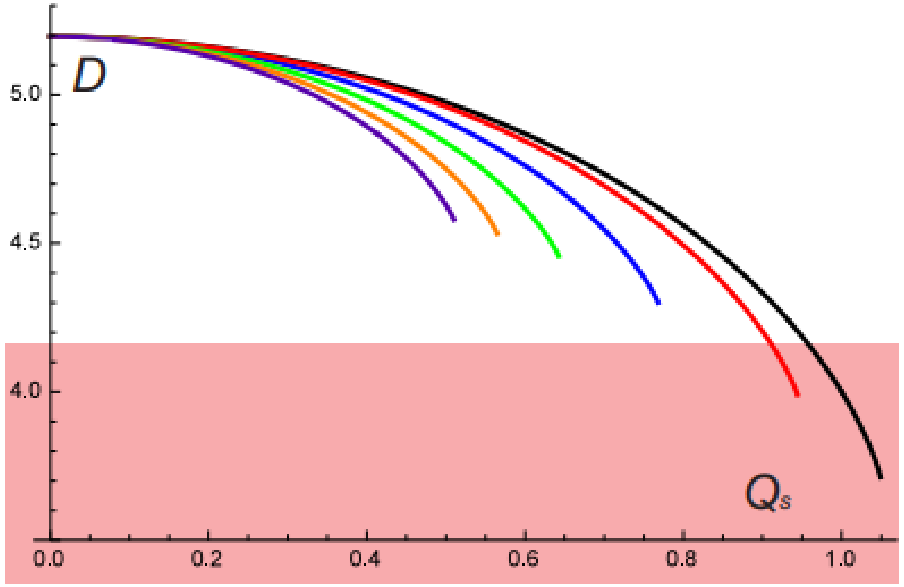

Fig. 5. The dependence of the shadow size D against the scalar charge Q_s for in new massive conformal gravity with different values of massive spin-2 mode m_2 (in the units of M, $M=1$). Black line corresponds to $m_2\to\infty$, red one corresponds to $m_2=2$, blue one corresponds to $m_2=1$, green one corresponds to $m_2=0.707$, orange one corresponds to $m_2=0.577$, purple one corresponds to $m_2=0.5$.

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Bumblebee mode

$$S_B = \int d^4x \mathcal{L}_B = \int d^4x (\mathcal{L}_g + \mathcal{L}_{gB} + \mathcal{L}_K + \mathcal{L}_V + \mathcal{L}_M),$$

$$\begin{aligned} A(r) &= (1+l)(1-\frac{2M}{r}), \\ B(r)^{-1} &= 1-\frac{2M}{r}. \end{aligned}$$

$-0{,}05 < l < 0{,}45$

[26] R. Casana, A. Cavalcante, F. P. Poulis, and E. B. Santos. Exact schwarzschild-like solution in a bumblebee gravity model. Physical Review D, 97(10), 2018.

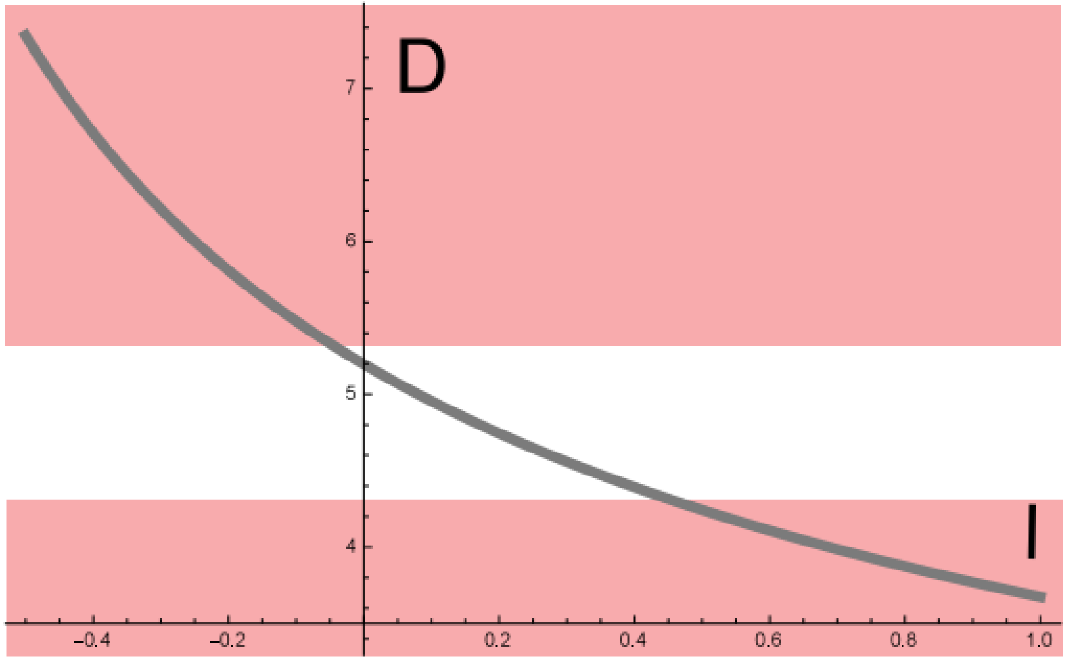

Рис. 1. The dependence of the shadow size D upon parameter l in alternative bumblebee generalization with Schwarzschild approximation (in the units of M, $M = 1$).

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f (Q) Gravity

$$S[g,\Gamma] = \int d^4x \sqrt{-g}\mathbb{Q}.$$

$$\begin{aligned} A(r) &= 1 - \frac{2M_{ren}}{r} - \alpha\frac{32}{r^2} \\ B(r)^{-1} &= 1 - \frac{2M_{ren}}{r} - \alpha\frac{96}{r^2} \\ 2M_{ren} &= 2M - \alpha\left(\frac{32}{3M} + c_1\right), \end{aligned}$$

-0,025<α<0,005

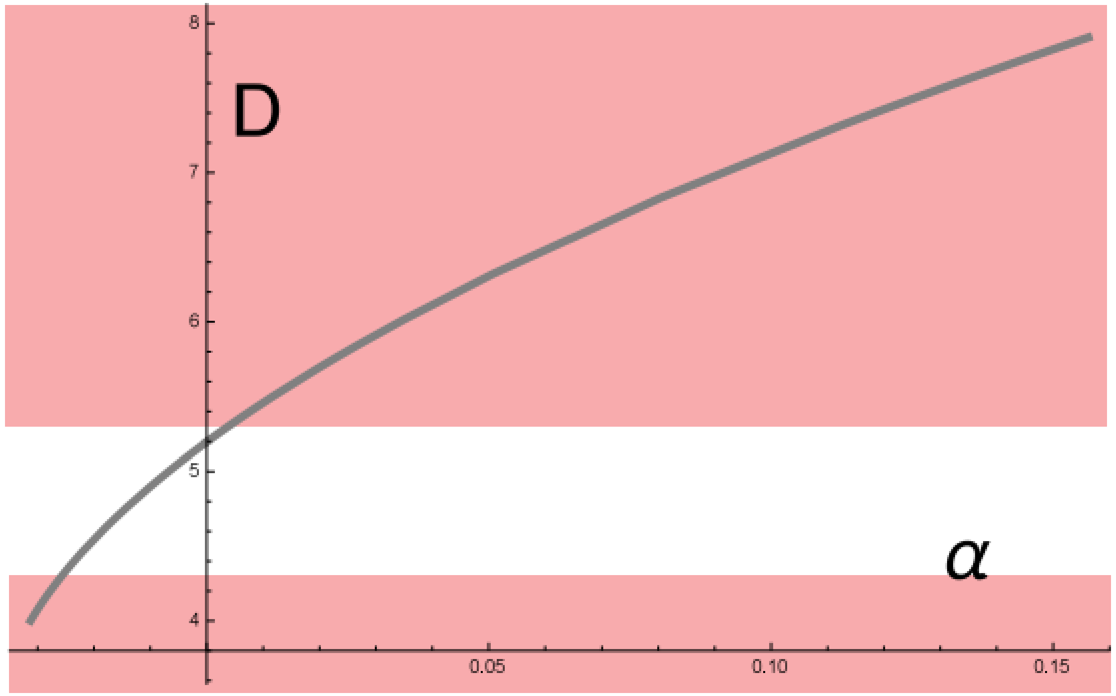

Рис. 2. The dependence of the shadow size D upon parameter α in $f(Q)$ gravity in M_{ren} units.

[8] Fabio D'Ambrosio, Shaun D. B. Fell, Lavinia Heisenberg, and Simon Kuhn. Black holes in $f(Q)$ Gravity. Physical Review D, 105(2), 2022.

Prokopov V. A., Alexeyev S. O., Zenin O. I. Black hole shadows constrain extended gravity 2: Sgr a* // *JETP*. — 2022.

Scalar Gauss-Bonnet gravity

$$A = -f(r)[1 + \frac{\zeta}{3r^3 f(r)} h(r)], \quad (34)$$

$$B = \frac{1}{f(r)}[1 - \frac{\zeta}{r^3 f(r)} k(r)], \quad (35)$$

where

$$h(r) := 1 + \frac{26}{r} + \frac{66}{5r^2} + \frac{96}{5r^3} - \frac{80}{r^4}, \quad (36)$$

$$k(r) := 1 + \frac{1}{r} + \frac{52}{3r^2} + \frac{2}{r^3} + \frac{16}{5r^4} - \frac{368}{3r^5}, \quad (37)$$

$$f(r) := 1 - \frac{2}{r}, \quad (38)$$

where ζ is the coupling parameter.

[18] Nicolás Yunes and Leo C. Stein. Nonspinning black holes in alternative theories of gravity. Phys. Rev. D, 83:104002, 2011.

Dependences of the radius of the photonic sphere and the radius of BH shadowon the coupling parameter in the first order:

$$r_{ph}^{sGB} = 3[1 - \frac{961}{2430}\zeta],$$

$$b_c^{sGB} = \sqrt{27}[1 - \frac{4397\zeta}{21870}].$$

[50] Adam Bauer, Alejandro Cárdenas-Avendaño, Charles F. Gammie, and Nicolás Yunes. Spherical accretion in alternative theories of gravity, 2021.

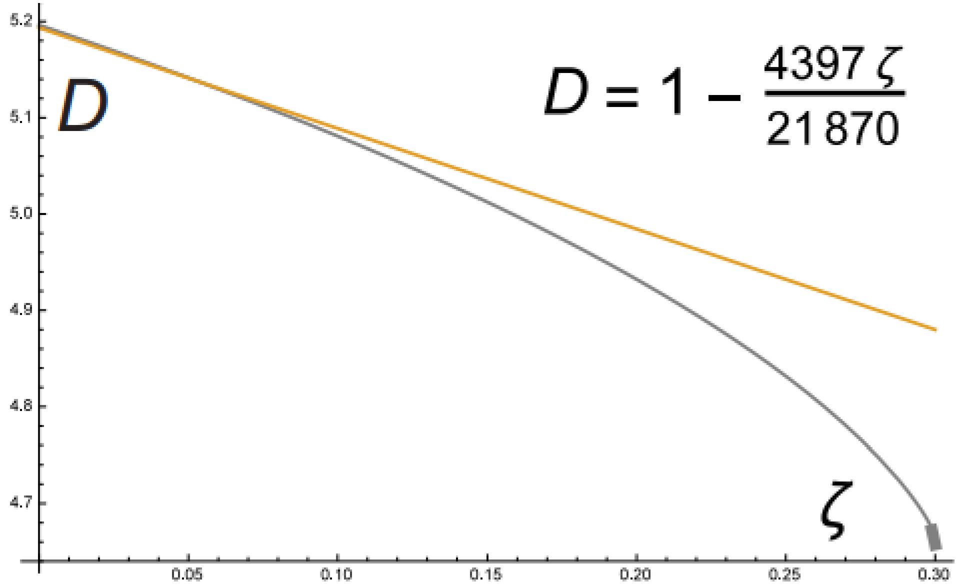

Fig. 8. The lower curve is the dependence of the shadow size D upon parameter ζ in scalar Gauss-Bonnet gravity (in the units of M, $M = 1$). The top line is the first order approximation.

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R+R^2 gravity

$$ds^2 = -f_t dt^2 + f_r dr^2 + r^2 d\Omega^2,$$

$$f_r \simeq \left(1 - \frac{2G_n M}{r}\right)^{-1} - \frac{\hat{\beta}\hbar G_n^2 M}{r^3} + O(G_n^3),$$
$$f_t \simeq \left(1 - \frac{2G_n M}{r}\right) - \frac{\hat{\alpha}\hbar G_n^2 M}{r^3} + O(G_n^3).$$

Hamilton-Jacobi Equation:

$$\lambda = \frac{\omega + a^2}{a} - \frac{2\omega'}{a}\frac{(f_r^{-1}r^2 + a^2)}{(f_r^{-1}r^2)'},$$
$$\eta = \frac{4(f_r^{-1}r^2 + a^2)}{(f_r^{-1}r^2)'^2}\omega'^2 - \frac{1}{a^2}\left[\omega - \frac{2(f_r^{-1}r^2 + a^2)}{(f_r^{-1}r^2)'}\omega'\right]^2,$$
$$x' = -\frac{\lambda}{\sin\theta_0},$$
$$y' = \pm\sqrt{\eta + a^2\cos^2\theta_0 - \frac{\lambda^2}{\tan^2\theta_0}},$$

The rotating metric obtained using the Newman-Janis algorithm by us:

$$ds^2 = -\frac{(f_r^{-1}r^2 + a^2\cos^2\theta)}{(\omega + a^2\cos^2\theta)^2}\rho^2 dt^2 - 2a\sin^2\theta\left[\frac{\omega - f_r^{-1}r^2}{(\omega + a^2\cos^2\theta)^2}\right]\rho^2 dt d\phi$$
$$+ \frac{\rho^2}{f_r^{-1}r^2 + a^2}dr^2 + \rho^2 d\theta^2$$
$$+ \rho^2\sin^2\theta\left[1 + a^2\sin^2\theta\frac{2\omega - f_r^{-1}r^2 + a^2\cos^2\theta}{(\omega + a^2\cos^2\theta)^2}\right]d\phi^2.$$

$$\omega = r^2\left(1 + \frac{(\hat{\alpha} + \hat{\beta})f_{ex}}{2}\right) \qquad f_{ex} = \frac{\hbar G_n^2 M}{r^3}$$

[1] Xavier Calmet, Roberto Casadio, Folkert / Quantum gravitational corrections to a star metric and the black hole limit // Phys. Rev. D 100, 086010 – Published 14 October 2019.

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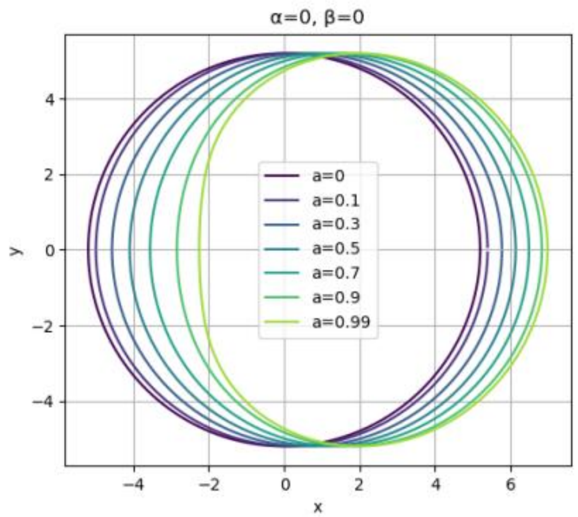
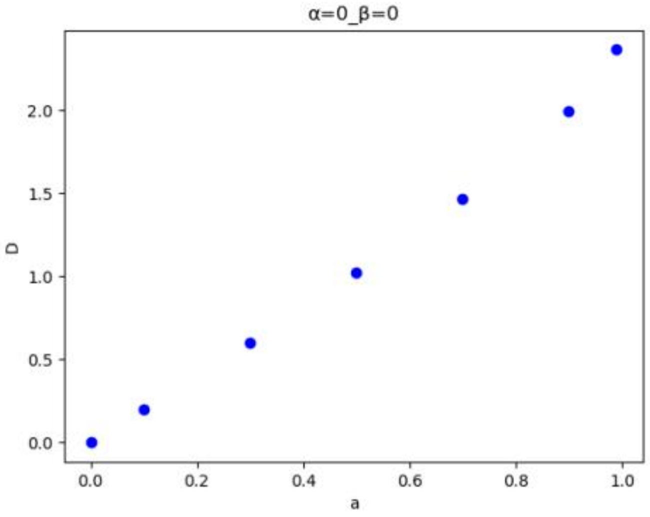
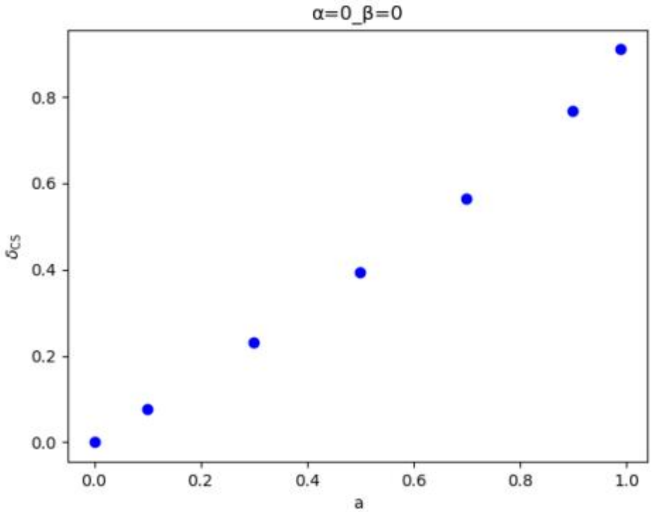
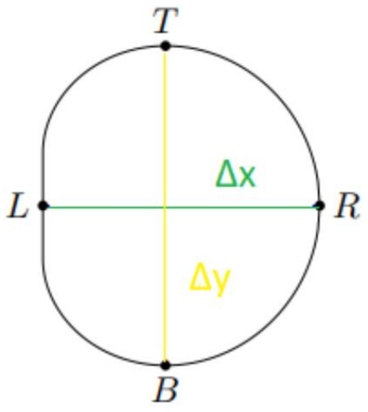
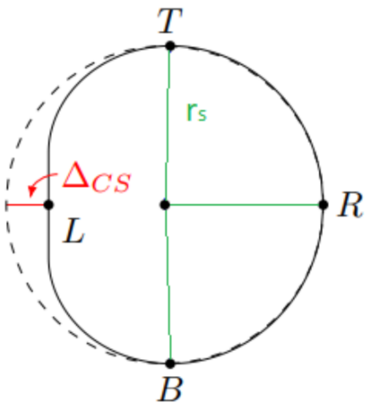
Rotating black hole simulation











$$D = \frac{x_{min} + x_{max}}{2},$$

$$\delta_{cs} = \Delta_{cs}/r_s,$$

Field	α	β	r_s
Scalar	0.0318	0.0318	5.244
Fermion	0.0849	-0.1273	5.22
Vector	0.1698	-0.2546	5.244
Graviton	4.52	-1.846	6.075

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Discussion and conclusions

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- In **conformal gravity**, large values of m_2 and Q_s should be excluded (for example, if m_2**=2**, then Q_s**<0.9**). In STEGR $f(Q)$ gravity observations **M87*** and **Sgr A*** limit the values of **α** as follows: **-0.025<α<0.005**. For an alternative generalization of the bumbelby metric with the Schwarzschild approximation: **-0.05 <** l **< 0.45**. These results demonstrate the maximum that can be achieved without taking into account the rotation of the black hole.

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- At the moment, using the Newman-Janis algorithm, a rotating **R+R²** metric has been obtained. Currently, black holes are being simulated with fixed quantum corrections. It is also planned to use the results of gravitational lensing of galactic clusters to test extended theories of gravity.

Пояснение по выбору альтернативной метрики бамбелби

Сферически-симметричное решение принимает вид:

$$A(r) = (1 - \frac{2M}{r}),$$
$$B(r) = \frac{1+l}{1 - \frac{2M}{r}}. \tag{58}$$

Расчёты показывают, что размер тени не зависит от параметра l. На самом деле, положение фотонной сферы не зависит от метрической функции $B(r)$, если на рассматриваемом масштабе $B(r) > 0$ и нет других особых точек (регулярна над горизонтом). Рассмотрим функцию:

$$\hat{u}(r) = u(r)B(r) = \frac{r^4}{D^2 A(r)} - r^2. \tag{59}$$

Она не зависит от метрической функции $B(r)$. Можно показать, что при выполнении условия существования фотонной сферы (60), выполняется и условие:

$$\hat{u}(r) = 0, \quad \frac{d\hat{u}(r)}{dr} = 0, \quad \frac{d^2\hat{u}(r)}{d^2r} > 0. \tag{60}$$

При $B(r) > 0$ автоматически выполняется первое условие системы (16), так как на фотонной сфере $u(r) = 0$.

$$\hat{u}'(r) = u'(r)B(r) + u(r)B'(r). \tag{61}$$

Но на фотонной сфере $u(r) = 0$ и $u'(r) = 0$. В рассматриваемом случае $B(r) > 0$ нет других особых точек. Следовательно, выполняется и второе условие. Аналогично и с третьим. Следовательно, и положение фотонной сферы также не зависит от метрической функции $B(r)$. При условии, что на масштабах фотонной сферы $B(r) > 0$ и нет других особых точек, для вычисления радиуса тени вместо системы (16) можно использовать (60) для упрощения вычислений.