НЦФМ
НАЦИОНАЛЬНЫЙ ЦЕНТР ФИЗИКИ И МАТЕМАТИКИ

Analysis of $1/N_c$ corrections in the quark model for the pion transition form factor

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Usually, quark models are considered at the mean field level, since taking into account $1/N_c$ corrections significantly complicates calculations.

We perform the analysis of diagrams for calculating $1/N_c$ corrections applied to the transition form factor of a neutral pion.

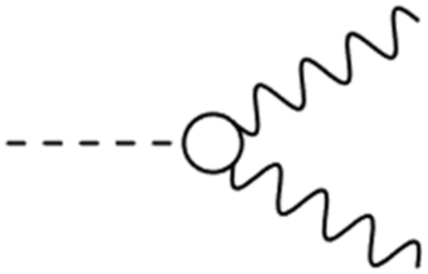


Fig.1. Transition of π^0-meson into two photons

From QGRAF to FORM via Python

- To generate diagrams with different N_c-factors, the QGRAF program is used

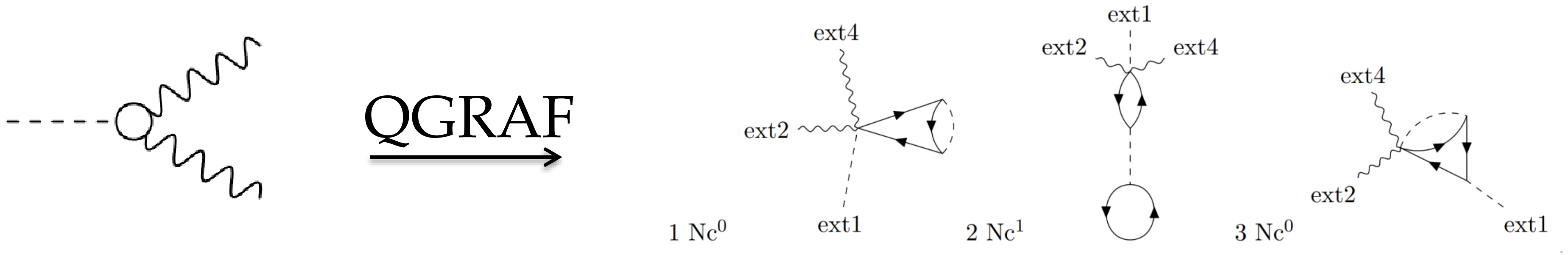


Fig.2. Examples of diagrams generated by QGRAF

- For theoretical calculations, it is necessary to input the necessary data in a format compatible with FORM.

Feynman diagrams

The diagrams consist of:

- external lines corresponding to free particles in the initial and final states;
- internal lines - propagators, which end at the vertices and describe the distribution of particles;
- vertices at which three or more lines meet, indicating local interactions of particles.

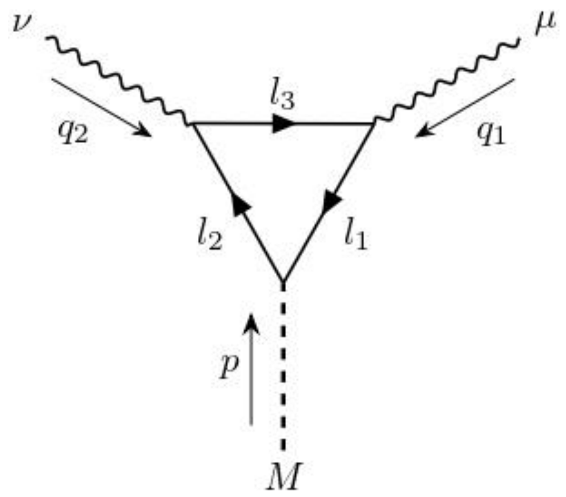
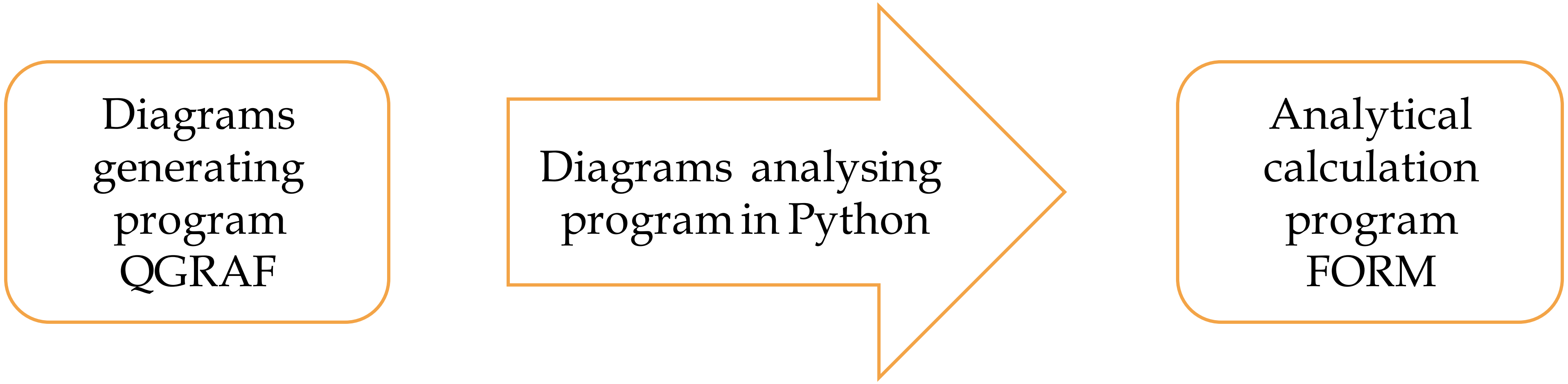


Fig. 3 Feynman diagram

The program should:

- Read the data from the diagram generator
- Analyze these diagrams
- Output the data in a format compatible with the program for analytical calculations



QCD

In the low-energy region, the strong coupling constant is not a small parameter, the use of methods beyond perturbation theory, for example, effective quark models, is required.

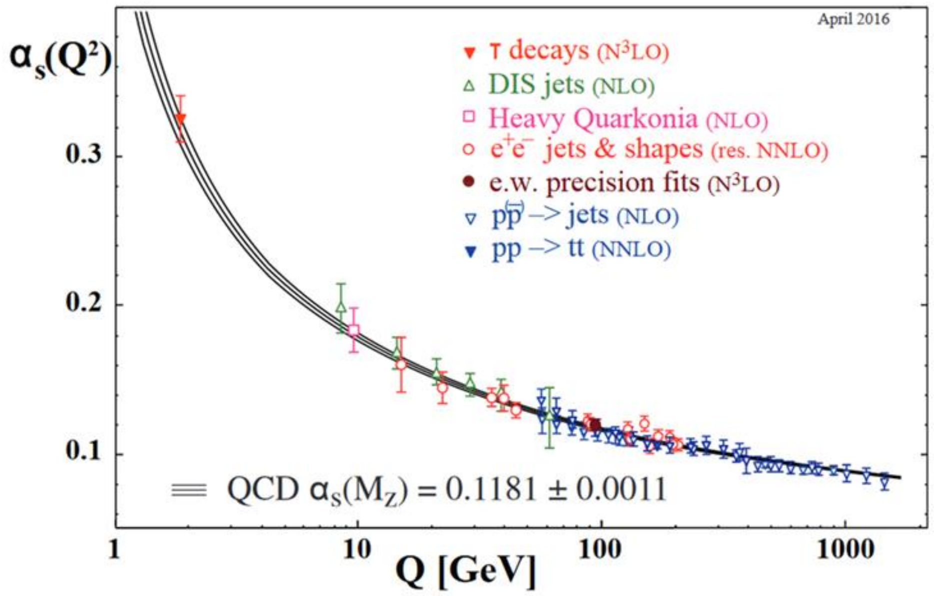


Fig. 4 Strong coupling constant

Nonlocal quark model

Lagrangian of the nonlocal model:

$$\mathcal{L} = \mathcal{L}_{free} + \mathcal{L}_{P,S} + \mathcal{L}_{V,A}$$

$$\mathcal{L}_{free} = \bar{q}(x)\left(i\hat{\partial} - m_c\right)q(x)$$

$$\mathcal{L}_{P,S} = \frac{G_1}{2}\left([J_S^a(x)]^2 + [J_P^a(x)]^2\right)$$

$$\mathcal{L}_{V,A} = \frac{G_2}{2}\left(\left[J_V^{a,\mu}(x)\right]^2 + \left[J_A^{a,\mu}(x)\right]^2\right)$$

m_c – quark current masses matrix, G_1 и G_2 –coupling constants,
$J_{\mathrm{M}}^{a,\{\mu\}}$ – nonlocal quark currents.

Effective Lagrangian

$$\mathcal{L}_{eff} = \mathcal{L}_{free} - \frac{\left(P^a(x)\right)^2 + \left(\tilde{S}^a(x)\right)^2}{2G} + \sum \Phi_i J_i(x)$$

Here $P^a(x)$ and $\tilde{S}^a(x)$ are the observed meson fields (pseudoscalar and scalar, respectively), and Φ_i is the auxiliary field.

The field $\tilde{S}^a(x)$ has a non-zero vacuum mean

$$\langle \tilde{S}^a \rangle_0 \neq 0$$

In order to obtain a physical scalar field with zero vacuum average, it is necessary to shift the scalar field

$$\tilde{S}^a = S^a - \sigma_0$$

Dynamic mass

Shifting the scalar to obtain a physical scalar field with zero vacuum expectation leads to the appearance of a dynamic quark mass:

$$m(p) = m_c + m_{dyn} f^2(p)$$

Thus, the mass of current quarks becomes constituent:

$$m_{dyn} = G N_c \cdot 8 \int \frac{d_E^4 k}{(2\pi)^4} f^2(k) \frac{m(k)}{k^2 + m^2(k)}$$

$m_c = 5$ MeV – mass of current quarks

$m(0) \approx 300$ MeV – constituent mass

The coupling constant

To be consistent with QCD, the quark mass should not depend on Nc. This means that the interaction constant must have an inverse relationship with Nc

$$m_{dyn} = GN_c \cdot 8 \int \frac{d_E^4 k}{(2\pi)^4} f^2(k) \frac{m(k)}{k^2 + m^2(k)}$$

$$G \to \frac{1}{N_c}$$

Suppressed meson propagator

Since, in turn, the polarization operator also linearly depends on Nc:

$$\Pi(p^2) = i\frac{N_c}{(2\pi)^4}\int d^4k f^2(k^2) Sp[S(k_-)\Gamma S(k_+)\Gamma]$$

the meson propagator is suppressed by Nc:

$$D = \frac{1}{-G^{-1} + \Pi}$$

$$D_p^M \to \frac{1}{N_c}$$

Suppressed diagrams

Thus, all diagrams that contain a meson propagator inside the region under consideration, all other things being equal, have a contribution three times smaller than those that do not.

Fig. 5 & 6. Diagram with meson propagator (left) is suppressed by $1/N_c$ factor in comparison with diagram without one (right)

Quark loop

- The fermionic lines must not be interrupted.
- Only diagrams with bosons in the initial or final states are considered, so all fermion lines will form loops.
- A quark loop contributes N_c

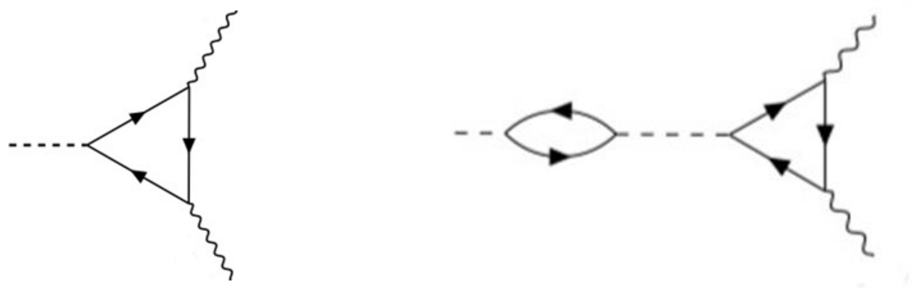


Fig. 7 & 8. Diagram without quark loop (left) is suppressed by diagram with one (right)

Calculation of the (N_c)-factor of the diagram

- Quark loop contributes N_c
- Meson propagator – $1/N_c$

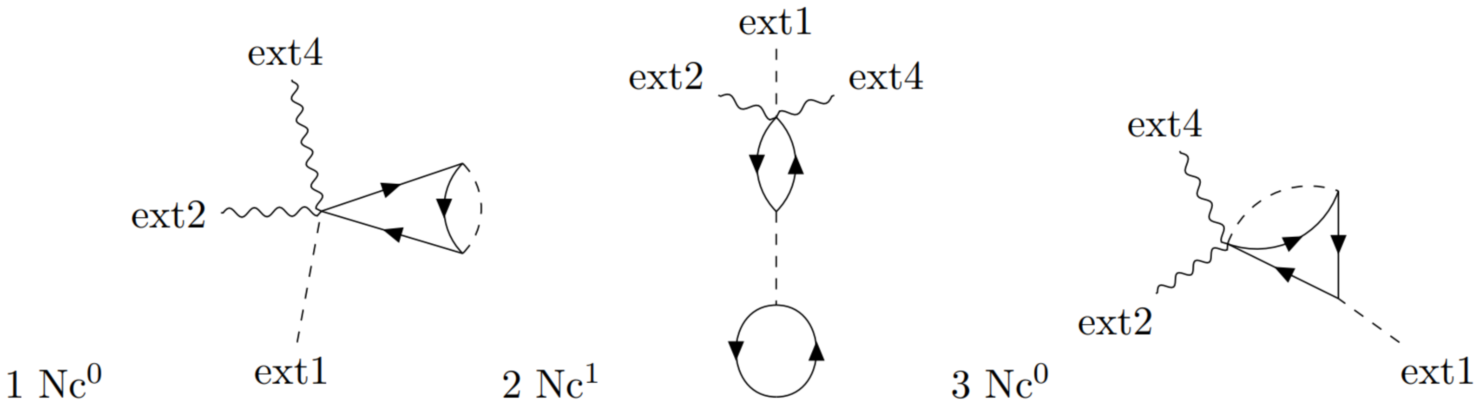
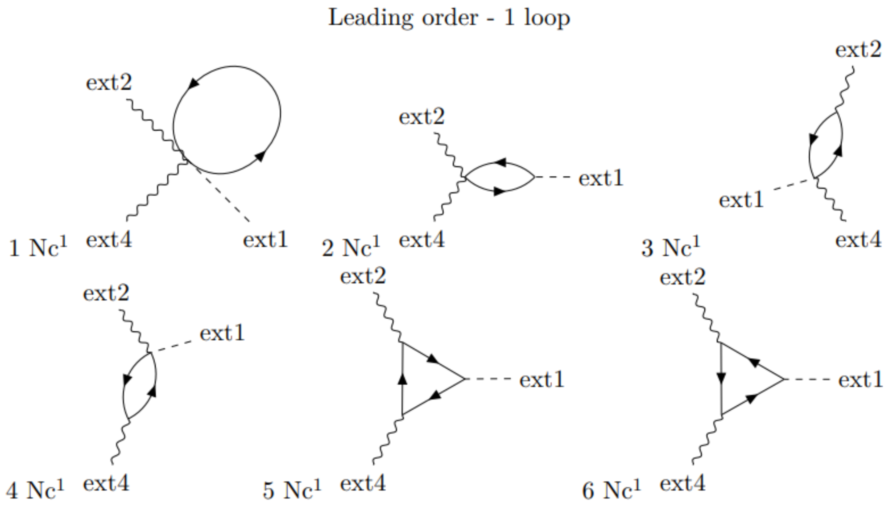


Fig. 9 Diagrams with different (N_c)-factors

Results of Calculating

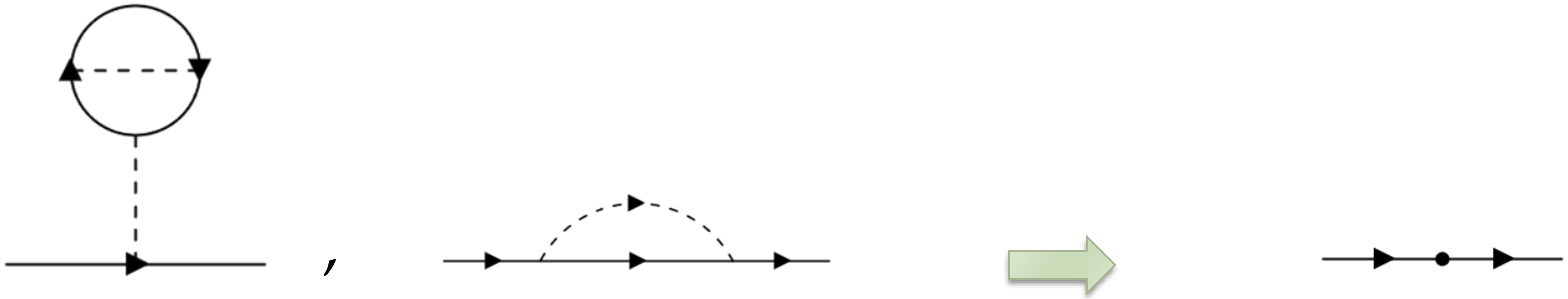
The diagrams data must be input to different files depending on (N_c)-factor:

- $(N_c)^1$ – the leading order
- $(N_c)^0$ – the sub-leading order

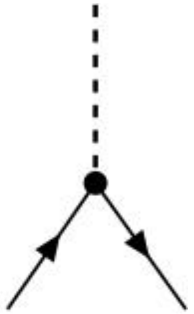


Thus, we selected 162 diagrams that make the main contribution out of 12 thousand and took into account the diagrams that are suppressed by N_c in comparison with the 6 leading order diagrams.

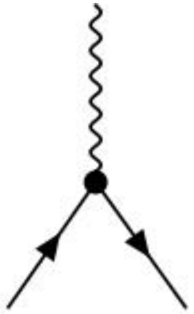
Corrections to the quark propagator



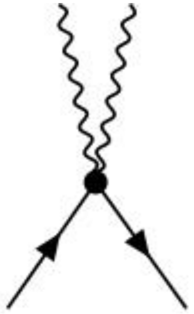
Vertices



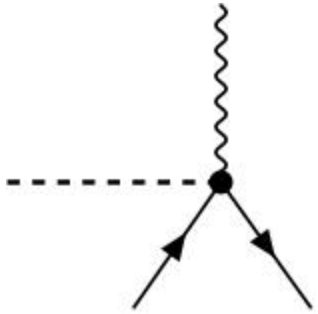
1. meson - quark- antiquark



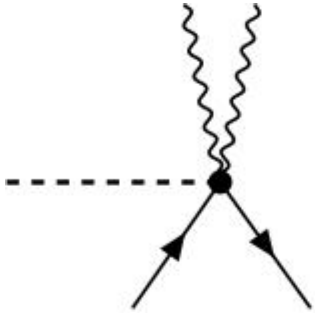
2. photon - quark- antiquark



3. photon- photon - quark- antiquark

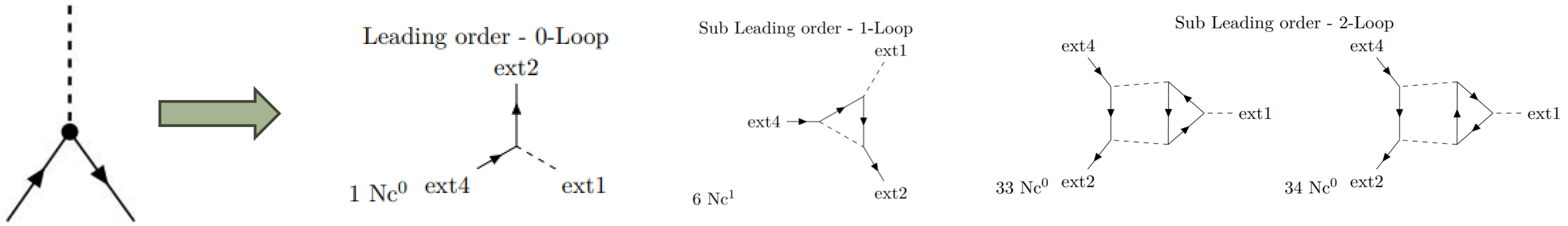


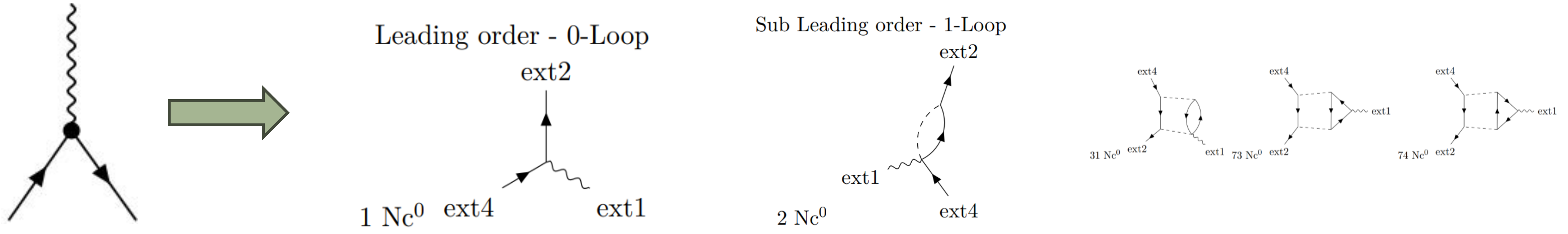
4. meson - photon - quark- antiquark

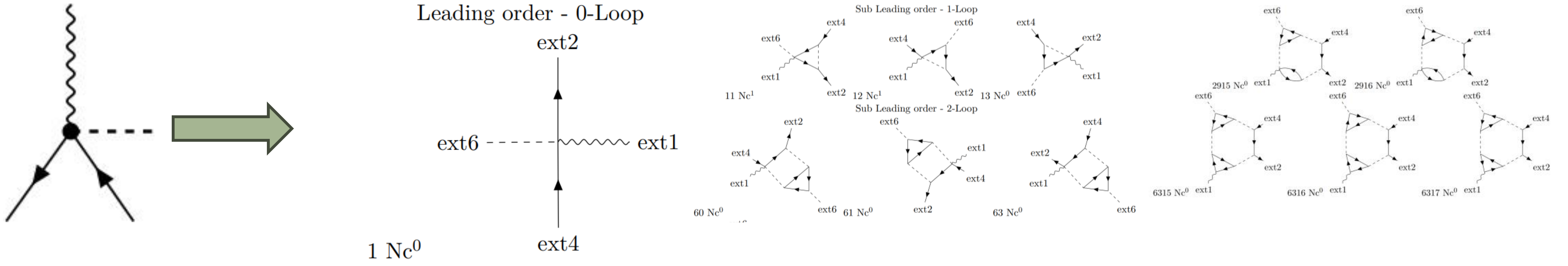


5. meson - photon- photon - quark- antiquark

Corrections to the vertices







Conclusion

Summing up the work, I would like to emphasize the following results:

- We have compiled an algorithm to process data generated by QGRAF and transfer diagrams to FORM
- Implemented a selection algorithm based on the $1/N_c$ index
- Calculated the pion transition form factor
- Analyzed structural elements' corrections

Sources

- Noguera P. QGRAF 3.5.0 / P. Noguera // CeFEMA, Instituto Superior T´ecnico, Universidade de Lisboa (ULisboa), Lisbon, Portugal. , 2021.
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Thank you for attention!!