Accretion in a hybrid metric-Palatini f(R)-gravity for spherically symmetric black holes

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Dubna 2023

Hybrid f(R) – gravity

- Action of theory:

$$S = \frac{1}{2\kappa^2}\int \mathrm{d}^4x\sqrt{-g}\,[R + f(\mathcal{R})] + S_m\ ,$$

- Action of theory in scalar-tensor representation:

$$S = \frac{1}{2k^2}\int d^4x\sqrt{-g}\left[(1+\phi)R + \frac{3}{2\phi}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right] + S_m.$$

- Field equations:

$$\mathrm{R}_{\mu\nu} = \frac{1}{1+\phi}\left[k^2\left(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T\right) + \frac{1}{2}g_{\mu\nu}\left(V + \nabla_\alpha\nabla^\alpha\phi\right) + \nabla_\mu\nabla_\nu\phi - \frac{3}{2\phi}\partial_\mu\phi\partial_\nu\phi\right],$$

$$-\nabla_\mu\nabla^\mu\phi + \frac{1}{2\phi}\partial_\mu\phi\partial^\mu\phi + \frac{\phi[2V-(1+\phi)V_\phi]}{3} = \frac{\phi k^2}{3}T.$$

Capozziello S, Harko T, Koivisto TS, Lobo FSN, Olmo GJ. Hybrid Metric-Palatini Gravity. *Universe*. 2015; 1(2):199-238

Spherically-symmetric solution in hybrid f(R)-gravity

- Spherically-symmetric metric:

$$ds^2 = -e^{2\Phi(r)}dt^2 + e^{2\Lambda(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

- Transition to dimentionless variables:

$$\frac{d\phi}{dr} = \frac{c^2}{2GM_\odot n}U(\eta) \qquad \frac{1}{r} = \frac{c^2}{2GM_\odot n}\xi.$$

$$V(\phi) = 2\left(\frac{c^2}{2GM_\odot n}\right)^2 v(\phi)$$

$$e^{-\lambda} = 1 - \frac{2GM_\odot M_{eff}(r)}{c^2 r}.$$

- Bogdan Dănilă, Tiberiu Harko, Francisco S. N. Lobo, and Man Kwong Mak Phys. Rev. D **99**, 064028 – Published 19 March 2019

Field equations in dimentionless variables

$$\frac{d\phi}{d\xi} = -\frac{U}{\xi^2}$$

$$\frac{dM_{eff}}{d\xi} = \frac{(1 - M_{eff}\xi)[\xi^2 dU/d\xi + 3U^2/4\phi - 2\xi U] + M_{eff}\xi^3(1+\phi) - v}{\xi^4(1+\phi+U/2\xi)} - \frac{M_{eff}}{\xi}$$

$$\frac{d\nu}{d\xi} = -\frac{\xi - \{\frac{U(\xi)[8\phi+3U(\xi)/\xi]}{4\phi(1+\phi)} + \xi\}[1 - \xi M_{eff}(\xi)] - \frac{v(\phi)}{\xi(1+\phi)}}{\xi^2[1 - \xi M_{eff}(\xi)][1 + \frac{U(\xi)}{2\xi(1+\phi)}]}$$

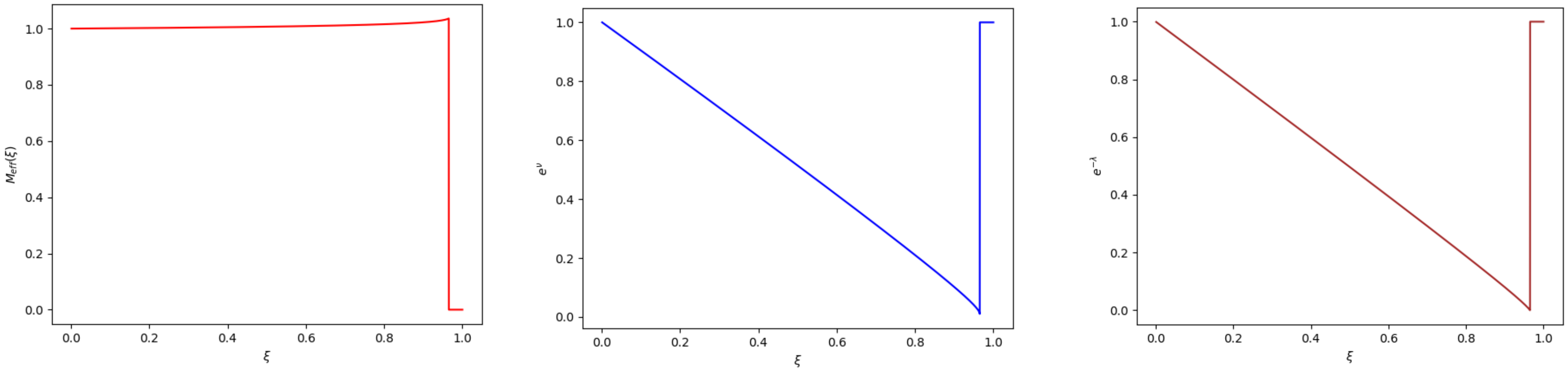
$$\frac{d^2\nu}{d\xi^2} = \frac{(1 - \frac{\xi}{2}\frac{d\nu}{d\xi})(-\xi\frac{dM_{eff}}{d\xi} - M_{eff})}{\xi(1 - \xi M_{eff})} - \frac{5U(\xi)^2}{2\xi^4\phi(1+\phi)} + \frac{2U}{\xi^3(1+\phi)}$$

$$-\frac{2}{\xi^4(1+\phi)(1-\xi M_{eff})}\{\frac{2\phi}{3}[2v - (1+\phi)v_\phi] + v\} - \frac{1}{2}(\frac{d\nu}{d\xi})^2 + \frac{1}{\xi}\frac{d\nu}{d\xi}$$

$$\frac{dU(\xi)}{d\xi} = \frac{\frac{\xi^2 U(\xi)}{2}[\xi\frac{dM_{eff}(\xi)}{d\xi} + M_{eff}(\xi)] - \frac{2\phi}{3}[2v(\phi) - (1+\phi)v_\phi(\phi)]}{\xi^2(1 - \xi M_{eff}(\xi))}$$

Example of numerical solution

- Case for $\phi_0 = 1, u = 4 \times 10^{-8}$:



- Horizon $\xi = 0.966$ (Horizon for Schwarzschild black hole $\xi_S = 1$)

Thorne-Novikov model

- The time averaged energy flux emitted from the surface of an accretion disk

$$F(r) = -\frac{\dot{M}_0}{4\pi\sqrt{-g}}\frac{\Omega_{,r}}{(\tilde{E}-\Omega\tilde{L})^2}\int_{r_{isco}}^{r}(\tilde{E}-\Omega\tilde{L})\tilde{L}_{,r}r\,dr$$

- Luminosity:

$$L(\nu) = \frac{2h}{c^2}\cos\gamma\int_{r_i}^{r_f}\int_0^{2\pi}\frac{\nu_e^3 r\,d\varphi\,dr}{\exp\left(h\nu_e/kT\right)-1}.$$

where:

$$1+z = \frac{1+\Omega r\sin\varphi\sin\gamma}{\sqrt{-g_{00}-g_{33}\Omega^2}}$$

$$\tilde{E} = -\frac{g_{00}}{\sqrt{-g_{00}-g_{33}\Omega^2}}$$

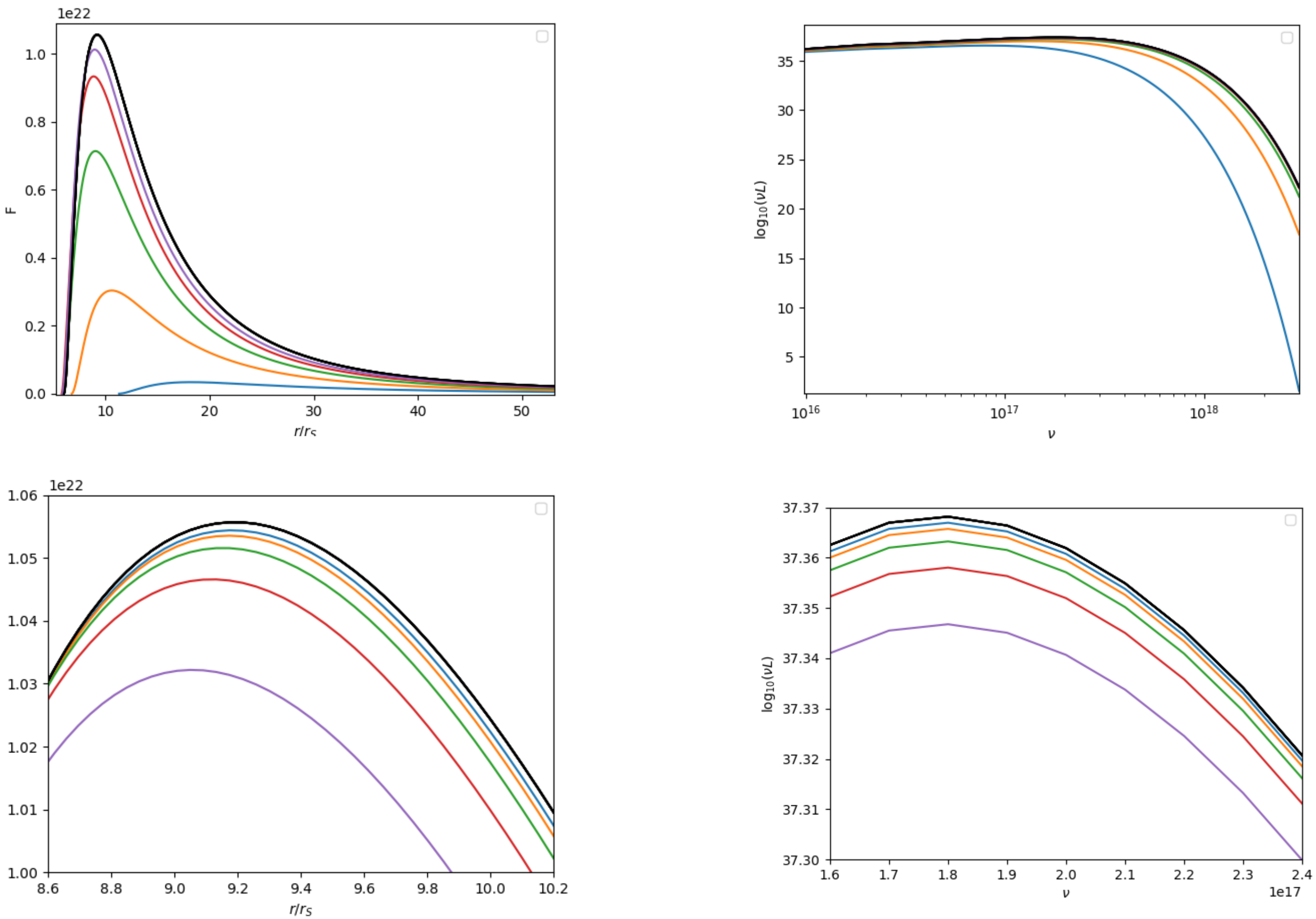
$$\tilde{L} = \frac{g_{33}\Omega}{\sqrt{-g_{00}-g_{33}\Omega^2}}$$

$$\Omega = \frac{d\varphi}{dt} = \sqrt{\frac{-g_{00,r}}{-g_{33,r}}}$$

- D. N. Page and K. S. Thorne, Astrophys. J 191 (1974) 499.

Comparison with accretion on Schwarzschild black hole

- Case with zero potential:



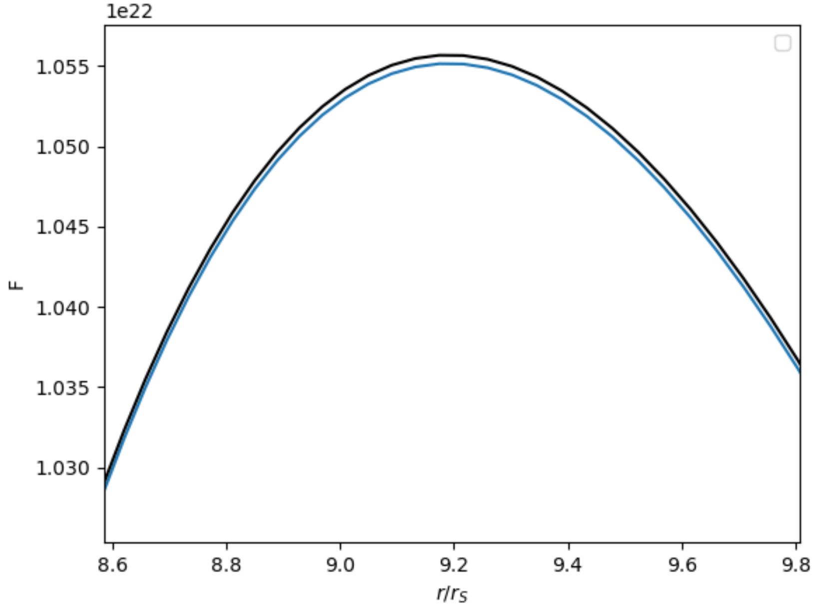
$$u_0 = 5.12 \times 10^{-7}$$

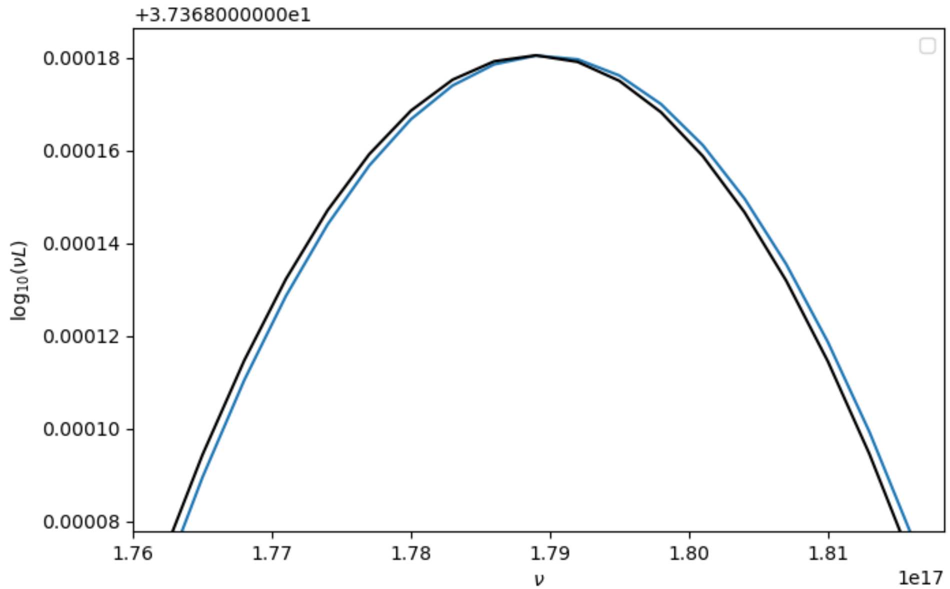
$$\phi_0 = 1$$

- For $\phi_0 = 0.5$: $r_{isco} = 11.3 r_S$

Comparison with accretion on Schwarzschild black hole

- Case with zero potential: $\phi_0 \leqslant 0.00004$

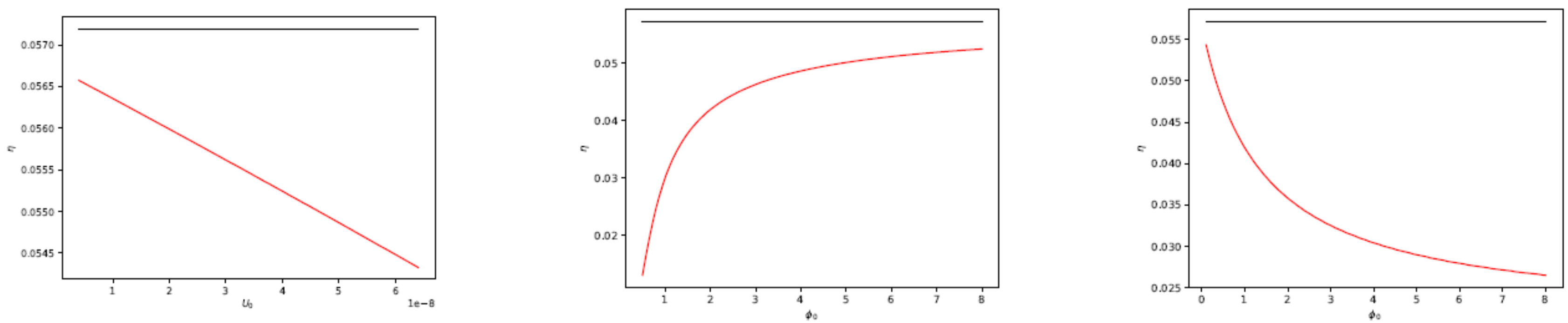




Efficiency for thin accretion disk aroung black hole ($V = 0$)

- Connection between u_0 and ϕ_0:

$$u_0 = -\frac{2GM\phi_0 e^{-m_\phi r} m_\phi}{3c^2 r} - \frac{2GM\phi_0 e^{-m_\phi r}}{3c^2 r^2}. \qquad \epsilon = 1 - \tilde{E}_{\rm isco}.$$



b) c)

Figure 10. Case $V = 0$. The black curve corresponds to the Schwarzschild black hole. a).The efficiency for thin accretion disk around static black hole for different values of u_0 and fixed $\phi_0 = 1$ as functions of u_0. b). The efficiency for thin accretion disk around static black hole for different values of ϕ_0 and fixed $u_0 = 5.12 \times 10^{-7}$ as functions of ϕ_0. c).The efficiency for thin accretion disk around static black hole as functions of ϕ_0. Connection between ϕ_0 and u_0 is taken into accaunt.

Case with potential

- In this work we consider Higgs-type potential:
- $V = -\frac{\mu^2}{2}\phi^2 + \frac{\zeta}{4}\phi^4$
- For more convenience, let's redefine constants in dimensionless form:

$$\alpha = -\frac{1}{4}\left(\frac{2GnM_{BH}}{c^2}\right)^2\mu^2, \qquad \beta = \frac{1}{2}\left(\frac{2GnM_{BH}}{c^2}\right)^2\zeta^2.$$

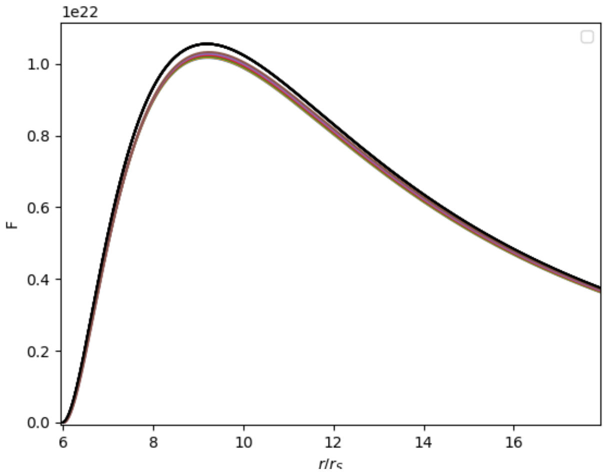
$$v(\phi) = \alpha\phi^2 + \beta\phi^4,$$

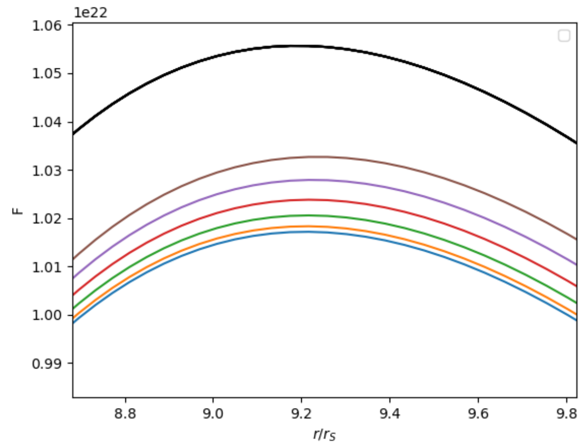
Comparison with accretion on Schwarzschild black hole

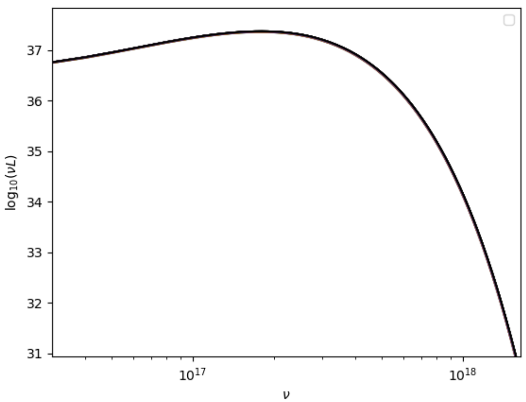
- Case with potential ($\beta = 10^{-10}$):

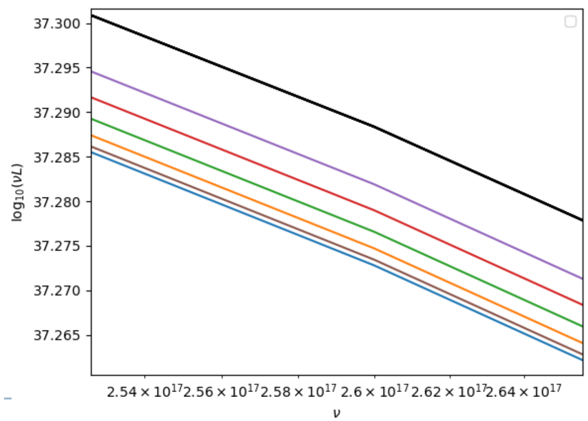
$$v(\phi) = \alpha\phi^2 + \beta\phi^4$$

$$\phi_0 = 1,\ u_0 = 10^{-8}$$





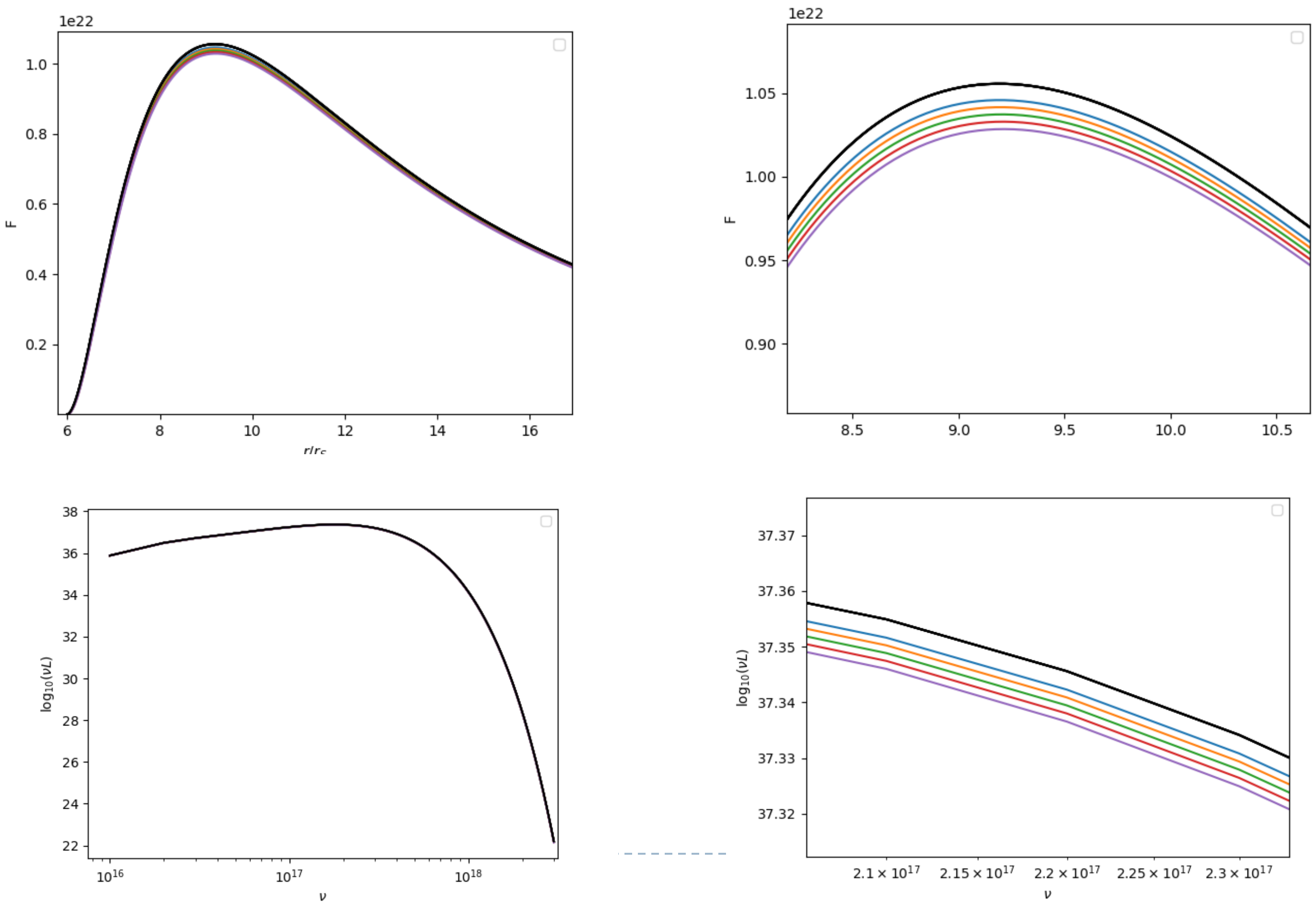




Comparison with accretion on Schwarzschild black hole

- Case with potential ($\alpha = -10^{-10}$): $v(\phi) = \alpha\phi^2 + \beta\phi^4$

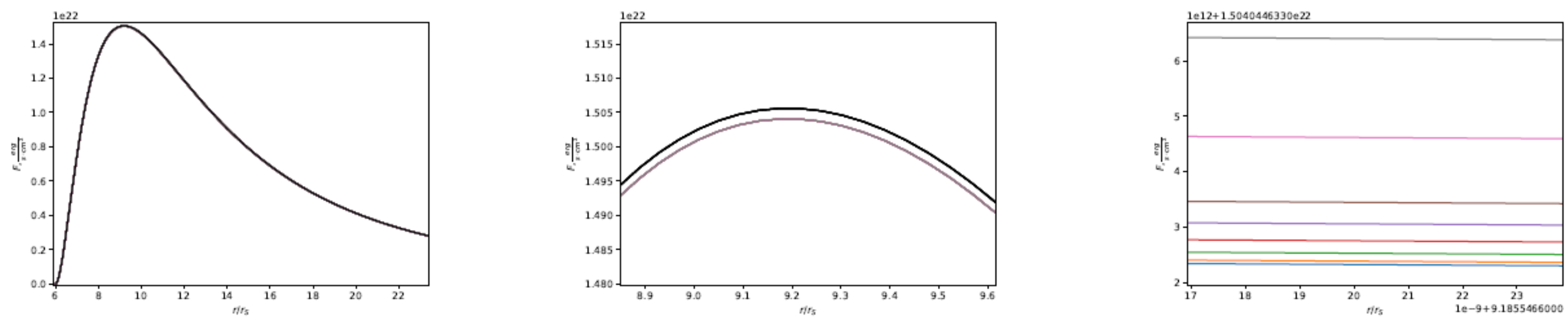
$\phi_0 = 1,\ u_0 = 10^{-8}$



Comparison with accretion on Schwarzschild black hole

- Case with potential: $v(\phi) = \alpha\phi^2 + \beta\phi^4$
- Connection between m_φ and α and β:

$$m_\varphi^2 = [2V_0 - V_\phi - (1+\phi)\phi V_{\phi\phi}]/3|_{\phi=\phi_0},$$



a) b) c)

Hyggs-type potential case. The energy flux $F(r)$ of a disk around a static black hole with mass accretion rate $\dot{M} = 2.21 \times 10^{18} g/s$ and mass $M = 8.48 M_\odot$ for different values of α and fixed $\phi_0 = 4 \times 10^{-5}$, $\beta = 10^{-20}$ as functions of the normalized radial coordinate r/r_s. The connections (5.6) and (5.7) are taken into accaunt. b) The black curve corresponds to the Schwarzschild black hole, the brown curve corresponds different parameters α. c) Zoom version. The parameter α is taken to be: $\alpha = -4 \times 10^{-5}$ (grey curve), $\alpha = -3 \times 10^{-5}$ (pink curve), $\alpha = -2.1 \times 10^{-5}$ (brown curve), $\alpha = -1.7 \times 10^{-5}$ (purple curve), $\alpha = -1.3 \times 10^{-5}$ (red curve), $\alpha = -9 \times 10^{-6}$ (green curve), $\alpha = -5 \times 10^{-6}$ (orange curve), $\alpha = -10^{-6}$ (blue curve).

Comparison with accretion on Schwarzschild black hole

- Case with potential: $v(\phi) = \alpha\phi^2 + \beta\phi^4$
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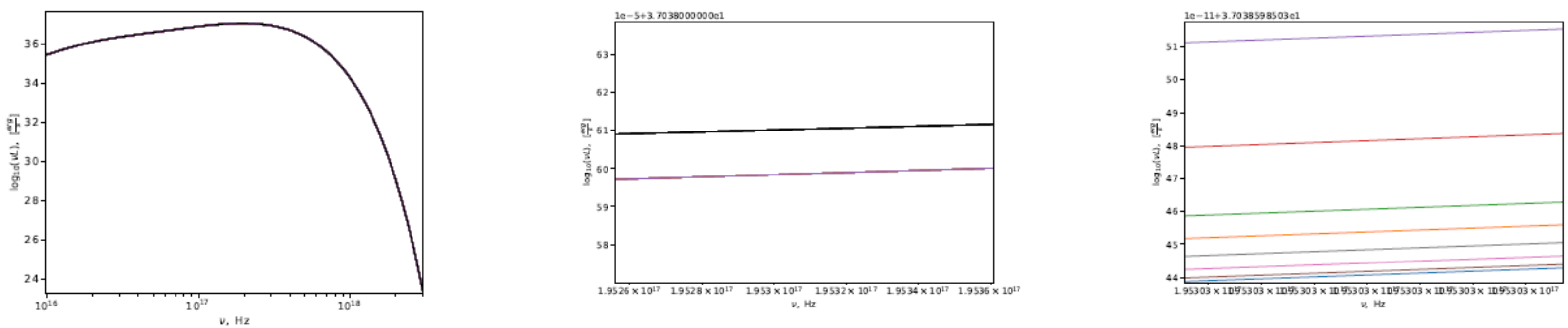
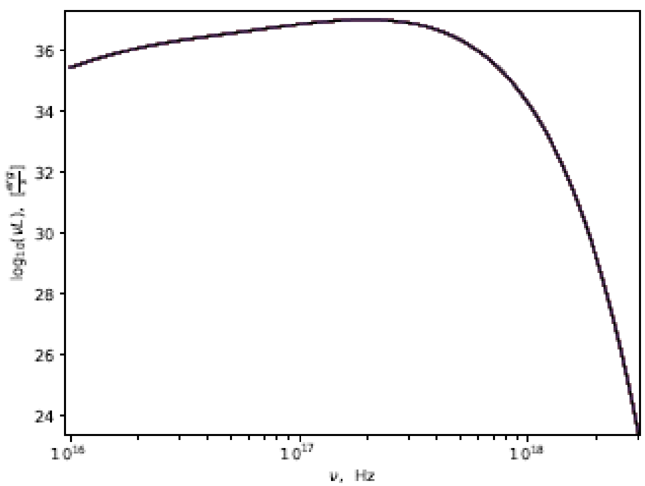


a) b) c)

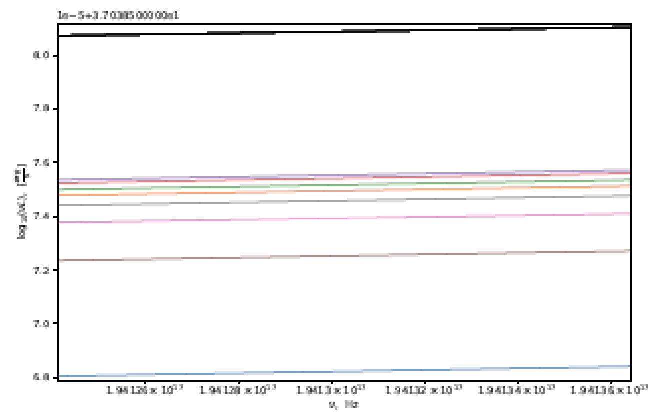
Hyggs-type potential case. The emission spectrum $\nu L(\nu)$ of the accretion disk around a static black hole with mass accretion rate $\dot{M} = 2.21 \times 10^{18} g/s$ and mass $M = 8.48 M_\odot$ for different values of α and fixed $\phi_0 = 4 \times 10^{-5}$, $\beta = 10^{-20}$ as functions of frequency ν. The connections (5.6) and (5.7) are taken into accaunt. b) The black curve corresponds to the Schwarzschild black hole, the brown curve corresponds different parameters α. c) Zoom version. The parameter α is taken to be $\alpha = -4 \times 10^{-5}$ (purple curve), $\alpha = -3 \times 10^{-5}$ (red curve), $\alpha = -2.1 \times 10^{-5}$ (green curve), $\alpha = -1.7 \times 10^{-5}$ (orange curve), $\alpha = -1.3 \times 10^{-5}$ (grey curve), $\alpha = -9 \times 10^{-6}$ (pink curve), $\alpha = -5 \times 10^{-6}$ (brown curve), $\alpha = -10^{-6}$ (blue curve).

Comparison with accretion on Schwarzschild black hole

- Case with potential: $v(\phi) = \alpha\phi^2 + \beta\phi^4$
- Connection between m_φ and α and β: $m_\varphi^2 = -\mu^2$



a)



b)

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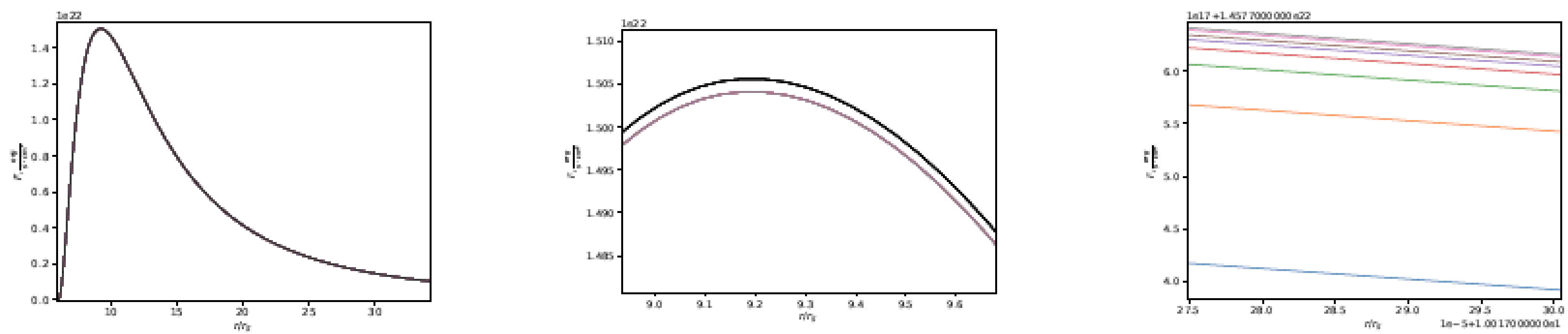
Comparison with accretion on Schwarzschild black hole

- Case with potential: $v(\phi) = \alpha\phi^2 + \beta\phi^4$
- Connection between m_φ and α and β: $m_\varphi^2 = -\mu^2$



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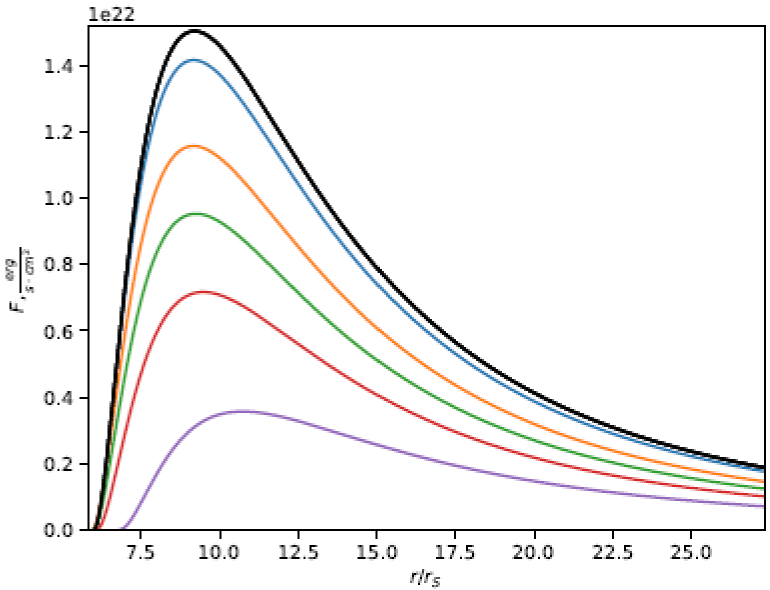
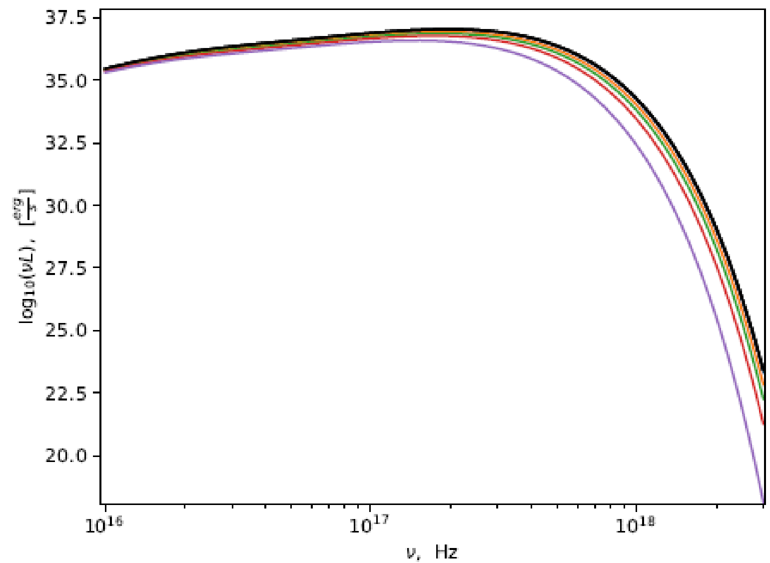
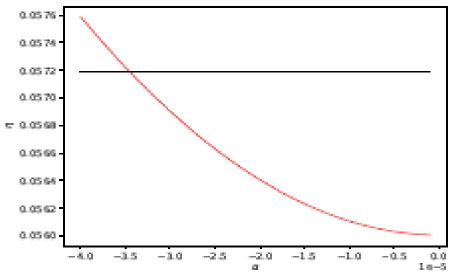
Comparison with accretion on Schwarzschild black hole

- Case with potential: $v(\phi) = \alpha\phi^2 + \beta\phi^4$
- Connection between m_φ and α and β: $m_\varphi^2 = -\mu^2$

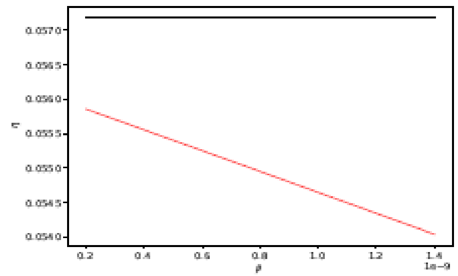




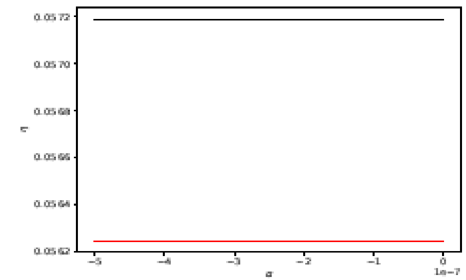
Efficiency for thin accretion disk aroung black hole ($v = \alpha\phi^2 + \beta\phi^4$)



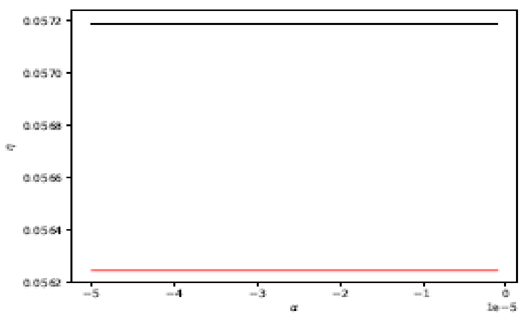
a)



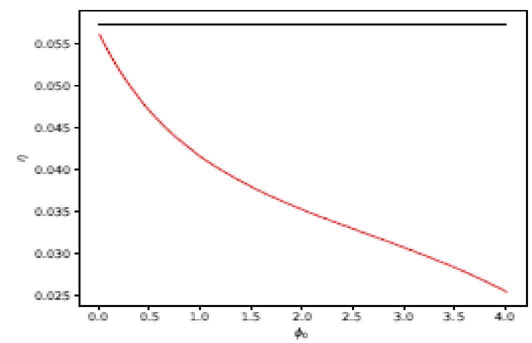
b)



c)



d)



e)

Hyggs-type potential case. The black curve corresponds to the Schwarzschild black hole in all figures. The efficiency for thin accretion disk around static black hole a) for different values of α and fixed $\phi_0 = 1$, $u_0 = 10^{-8}$, $\beta = 10^{-10}$; b) for different values of β and fixed $\phi_0 = 1$, $u_0 = 10^{-8}$, $\alpha = -10^{-10}$; c) for different values of α and fixed $\phi_0 = 4 \times 10^{-5}$, $\beta = 10^{-20}$. The connections (5.6) and (5.7) are taken into accaunt; d) for different values of α and fixed $\phi_0 = 4 \times 10^{-5}$, $\beta = 10^{-11}$. The connection $m_\varphi^2 = -\mu^2$ is taken into accaunt; e) for different values of ϕ_0 and fixed $\alpha = -10^{-10}$, $\beta = 10^{-11}$. The connection $m_\varphi^2 = -\mu^2$ is taken into accaunt.

Conclusion

- Our results are very close to the same results for accretion onto Schwarzschild black hole with taking into account limitations from Solar System test
- The existence of adequate inflation regimes indicates the viability of the hybrid f(R)-gravity
- Accretion disks in hybrid f(R)-gravity are dimmer and cooler than in general relativity and metric f(R)-gravity

Thank you for your attention!