Joint Institute for Nuclear Research

JINR Association of Young Scientists and Specialists

Лаборатория ядерных проблем им. В. П. Джелепова

Объединенный институт ядерных исследований

The XXVII International Scientific Conference of Young Scientists and Specialists

Devoted to the 110th anniversary of Bruno Pontecorvo

Theory of neutrino oscillations

Dmitry V.Naumov

Natural units

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Quantum mechanics $$E = \hbar\omega$$

Special Relativity $$E^2 = \mathbf{p}^2\mathbf{c}^2 + \mathbf{m}^2\mathbf{c}^4$$

$$\hbar = c = 1$$

$$[\text{energy}] = [\text{momentum}] = [\text{mass}]$$

$$[\text{time}] = [\text{coordinate}] = [\text{mass}]^{-1}$$

$$[\text{orbital moment}] = [\text{spin}] = [\text{mass}]^{0} = 1$$

Natural units

$$\hbar = c = 1$$

$$[\text{energy}] = [\text{momentum}] = [\text{mass}]$$

$$[\text{time}] = [\text{coordinate}] = [\text{mass}]^{-1}$$

$$[\text{orbital moment}] = [\text{spin}] = [\text{mass}]^{0} = 1$$

Energy measured in electron-volt (eV).

eV to cm^{-1} $\quad \hbar c = 1 = 2 \cdot 10^{-5} \cdot \text{eV} \cdot \text{cm}$

eV to s^{-1} $\quad \hbar = 1 = \frac{2}{3} \cdot 10^{-15} \cdot \text{eV} \cdot \text{s}$

$$\text{keV} = 10^3\ \text{eV}$$

$$\text{MeV} = 10^6\ \text{eV}$$

$$\text{GeV} = 10^9\ \text{eV}$$

Question: dimension of $\mathbf{E}, \mathbf{B}$?

Three lepton numbers

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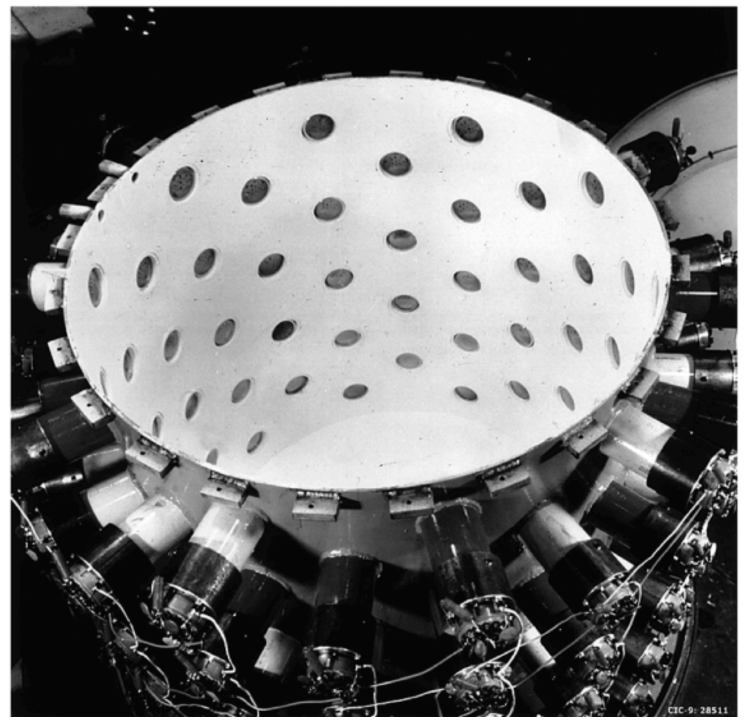
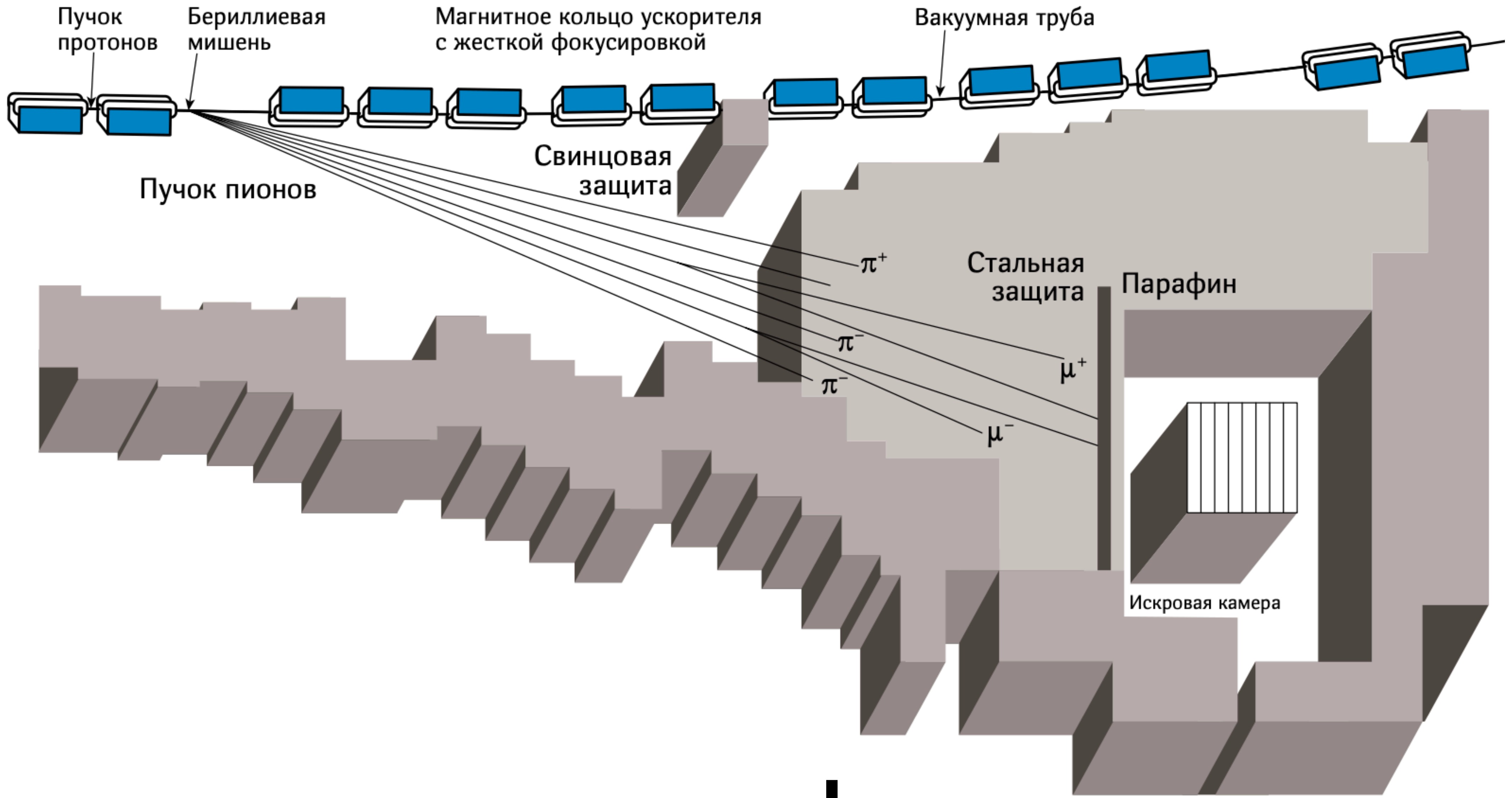
$\overline{\nu}_e$

Reines Cowan



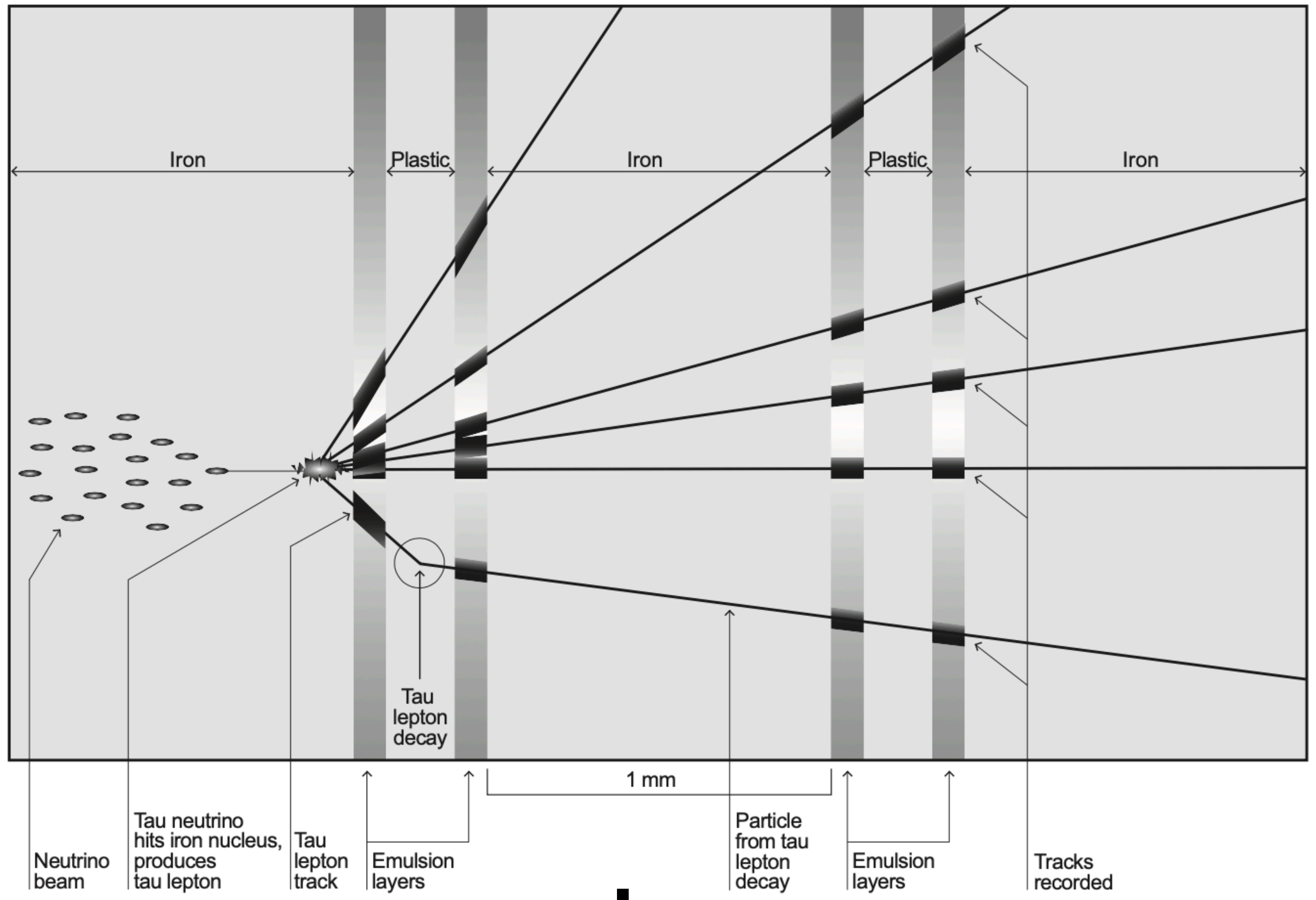
ν_μ

Schwartz, Lederman, Steinberger



ν_τ

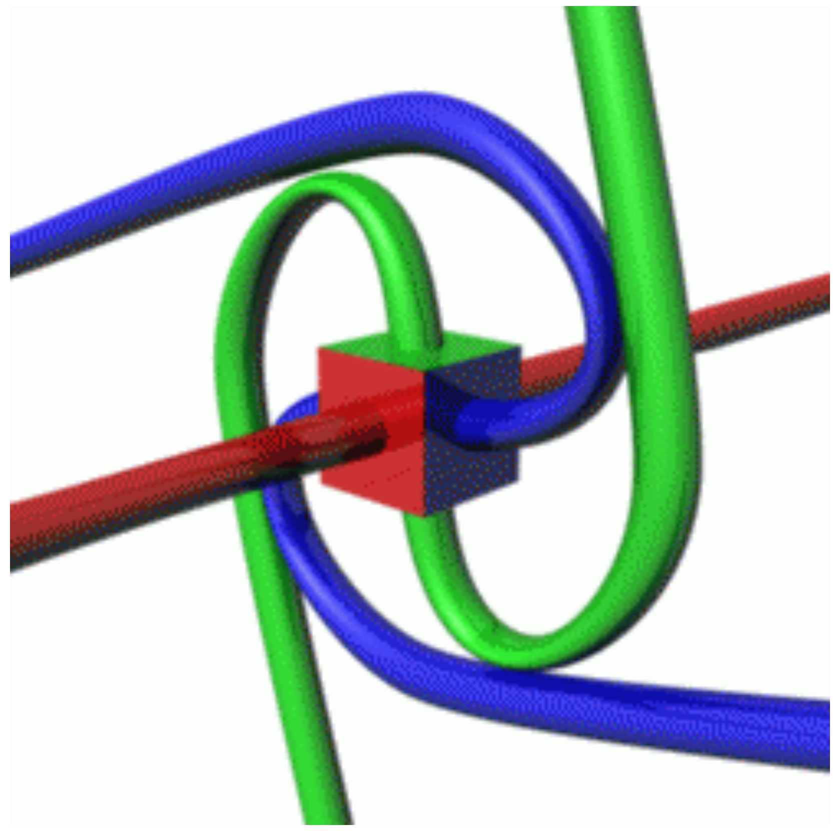
Collaboration DONUT

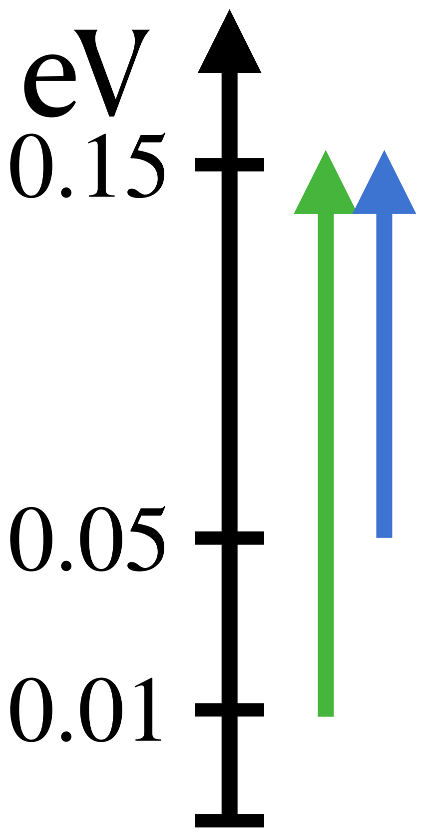


1956 1962 2000

Neutrino properties

- Zero electric charge
- Fermion. Spin = 1/2 ($\cdot\hbar = 1$)
- Known three types ν_1, ν_2, ν_3 and their anti-particles $\bar{\nu}_1, \bar{\nu}_2, \bar{\nu}_3$ with definite masses. Also, their flavor mixtures ν_e (1956) , ν_μ (1962) , ν_τ (2000)
- Neutrino participates in weak and gravitational interactions
- Weak interactions break parity transformation ($\mathbf{r} \to -\mathbf{r}$)





Standard Model (SM)

Follow for a dedicated
lecture by A. Bednyakov

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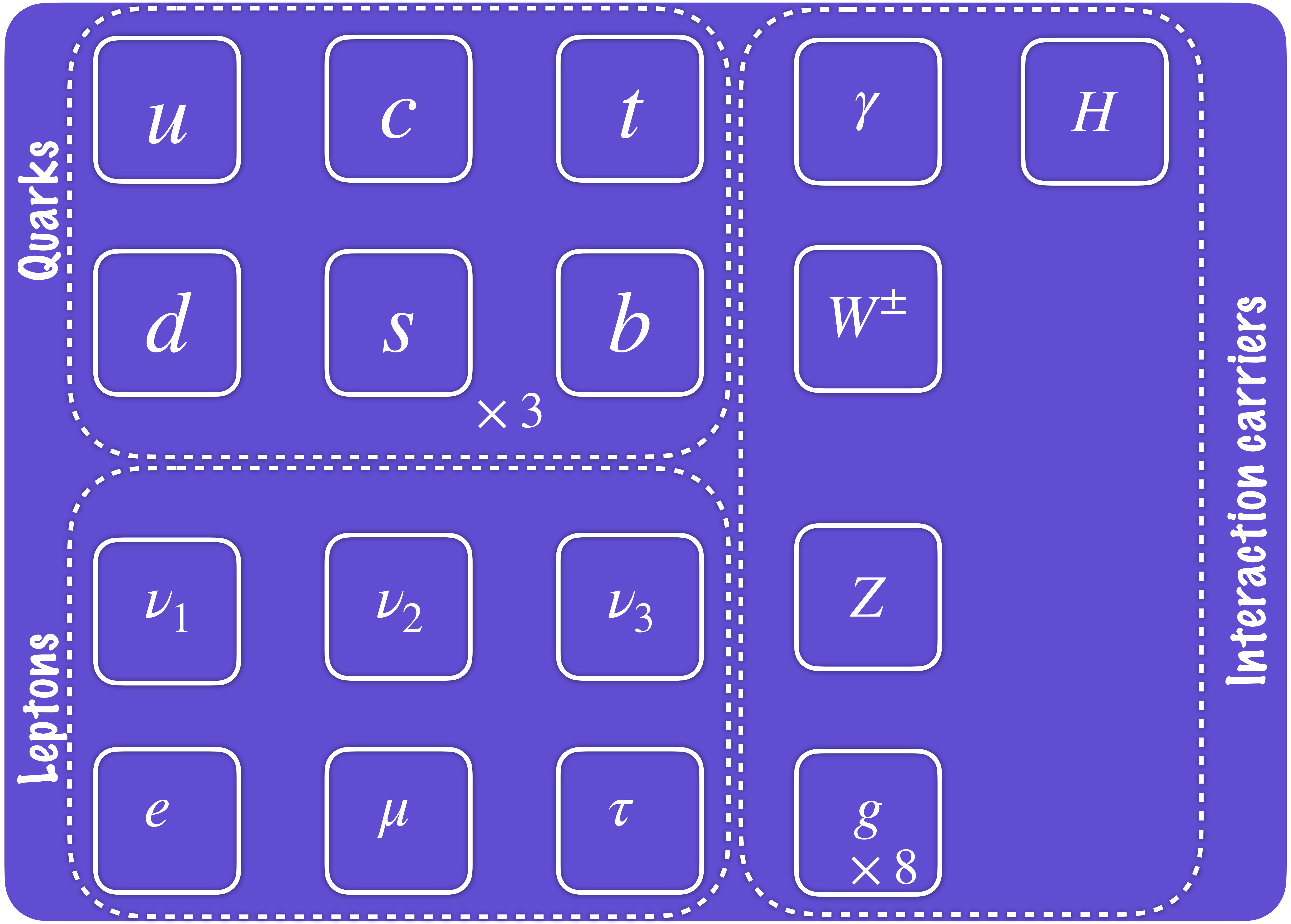
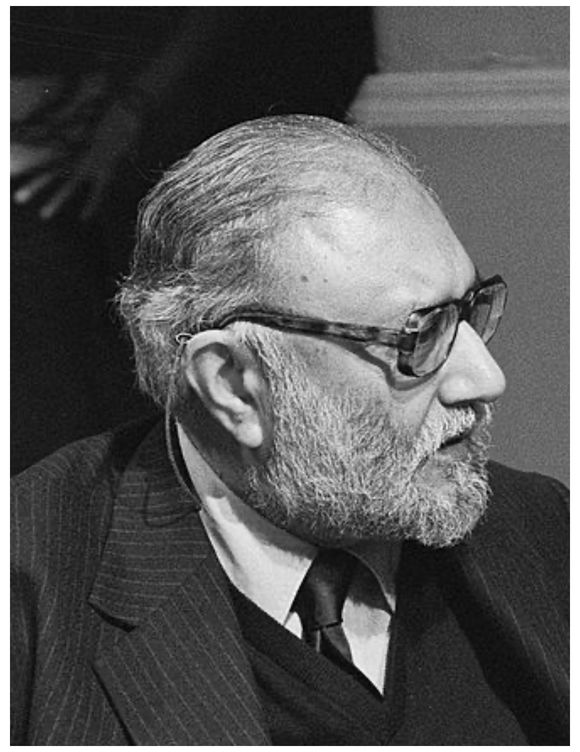


Sheldon Glashow

Steven Weinberg



Abdus Salam

Lepton numbers in the SM

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- Lepton numbers are conserved for massless neutrino

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix} \qquad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \qquad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}$$

$$L_e = 1 \qquad L_\mu = 1 \qquad L_\tau = 1$$

- Possible:

$$\nu_e + n \rightarrow p + e^-$$

- Not possible:

$$\nu_\mu + n \rightarrow p + e^-$$

Massive Neutrino in the SM

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- Three lepton doublets interact with W-field

$$\begin{pmatrix} \nu_1 \\ e \end{pmatrix}_L \qquad \begin{pmatrix} \nu_2 \\ \mu \end{pmatrix}_L \qquad \begin{pmatrix} \nu_3 \\ \tau \end{pmatrix}_L$$

- The interaction amplitude $\mathscr{A} \propto \frac{g}{2\sqrt{2}} V^*_{\alpha i}$ for ν_i and ℓ_α, $\alpha = (e, \mu, \tau)$

- Possible processes:

$$e + W^- \to \nu_i \propto V^*_{ei}$$

$$\mu + W^- \to \nu_i \propto V^*_{\mu i}$$

$$\tau + W^- \to \nu_i \propto V^*_{\tau i}$$

Neutrino in the Standard Model

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- The interaction amplitude $\mathscr{A} \propto \frac{g}{2\sqrt{2}} V^*_{\alpha i}$ for ν_i and ℓ_α

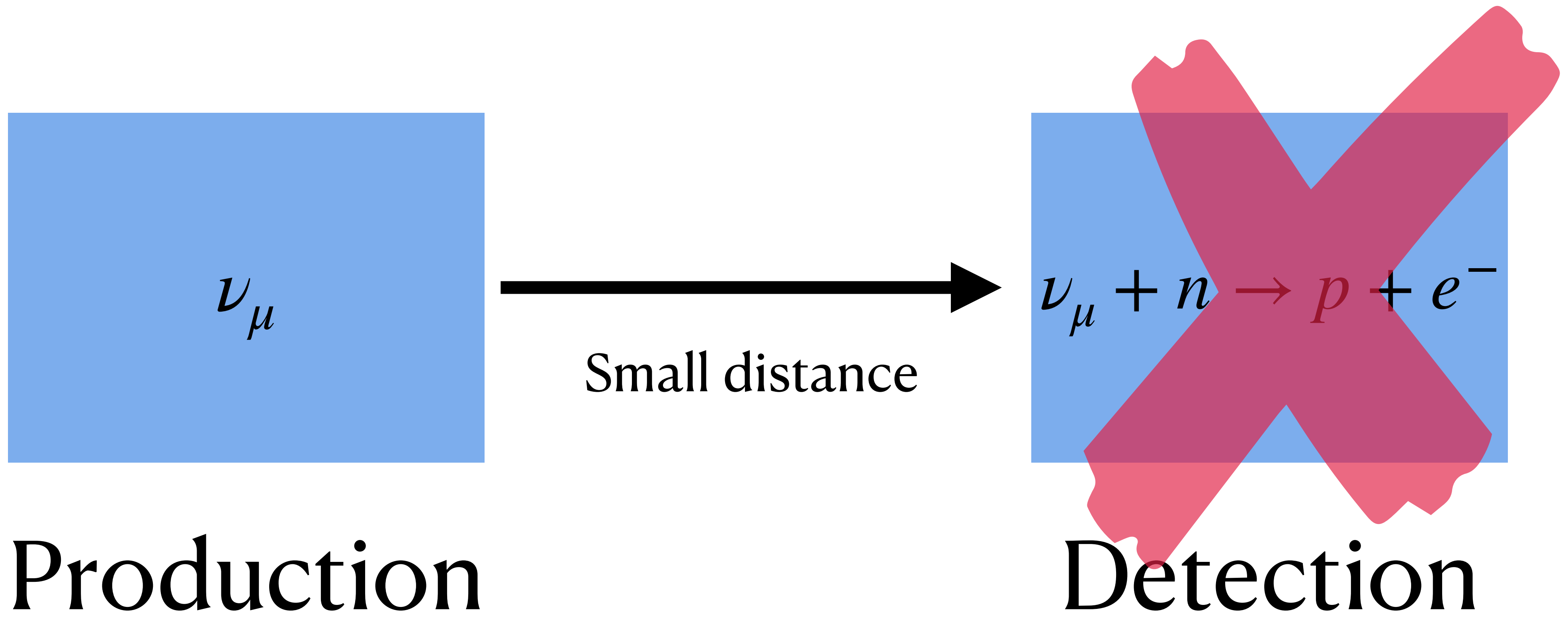
- Nine numbers $V^*_{\alpha i}$ make **unitary** lepton mixing matrix **P**ontecorvo-**M**aki-**N**akagawa-**S**akata

$$\begin{pmatrix} V^*_{e1} & V^*_{e2} & V^*_{e3} \\ V^*_{\mu 1} & V^*_{\mu 2} & V^*_{\mu 3} \\ V^*_{\tau 1} & V^*_{\tau 2} & V^*_{\tau 3} \end{pmatrix}$$

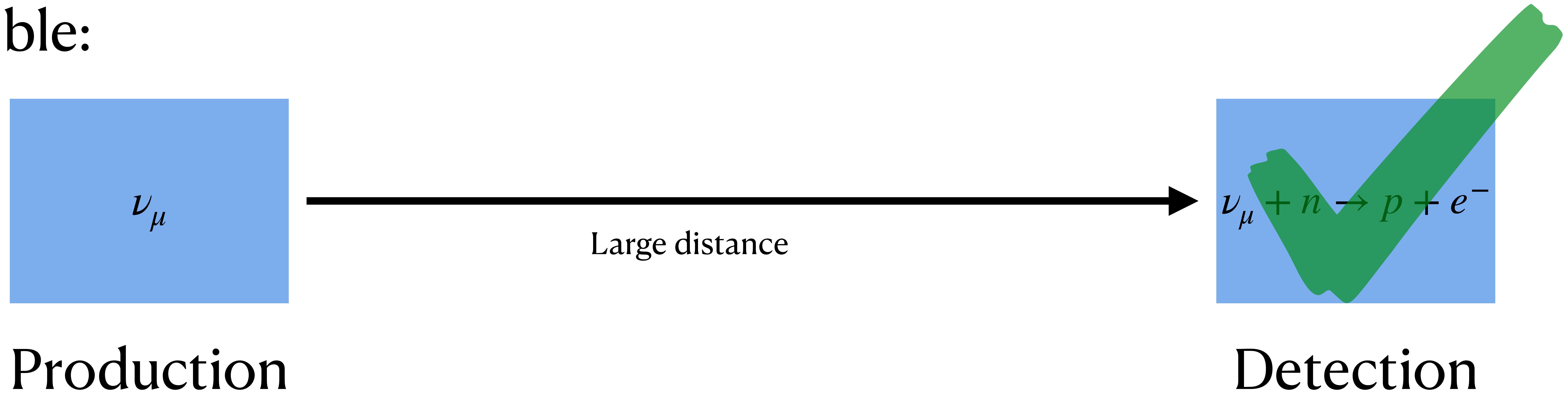
- Non-diagonal V and differing masses m_i lead to *neutrino oscillation* — **macroscopic display of quantum world**

Neutrino oscillations violate lepton numbers

- Not possible:

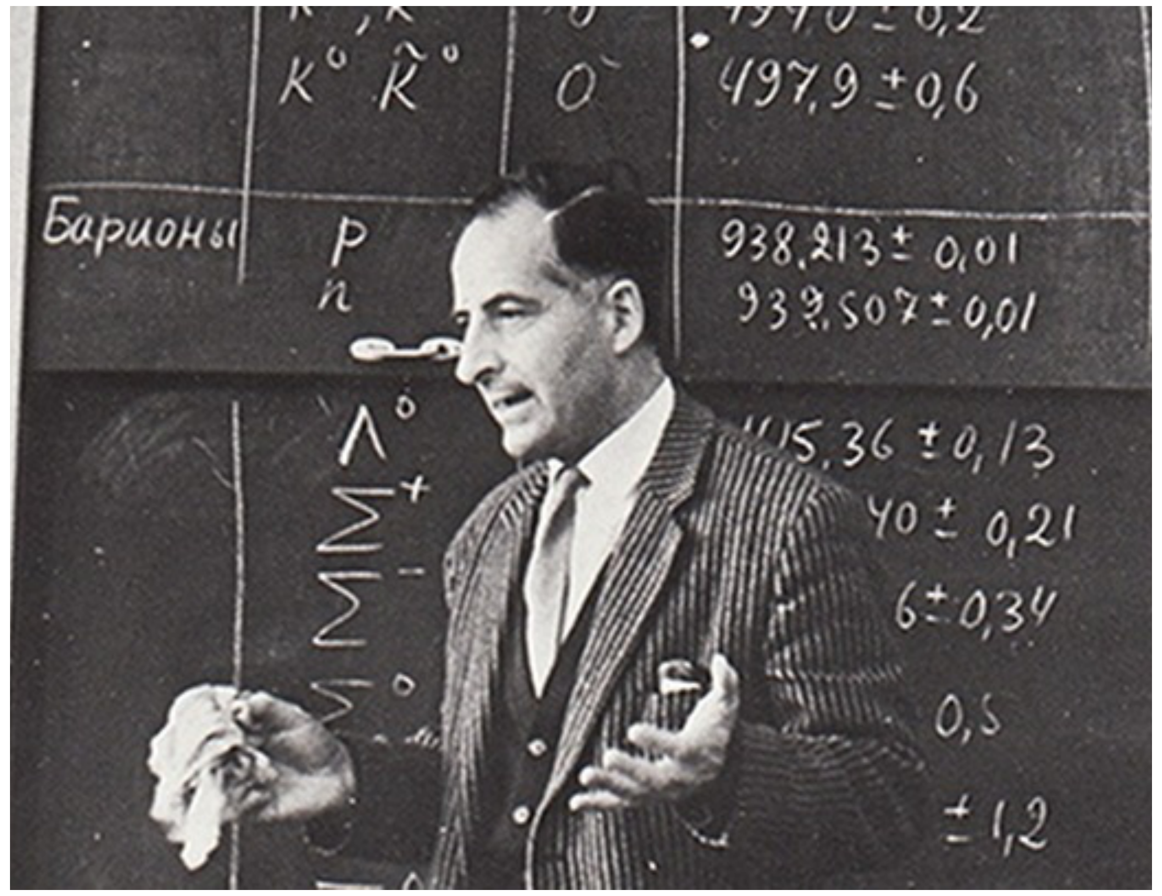


- Periodically possible:

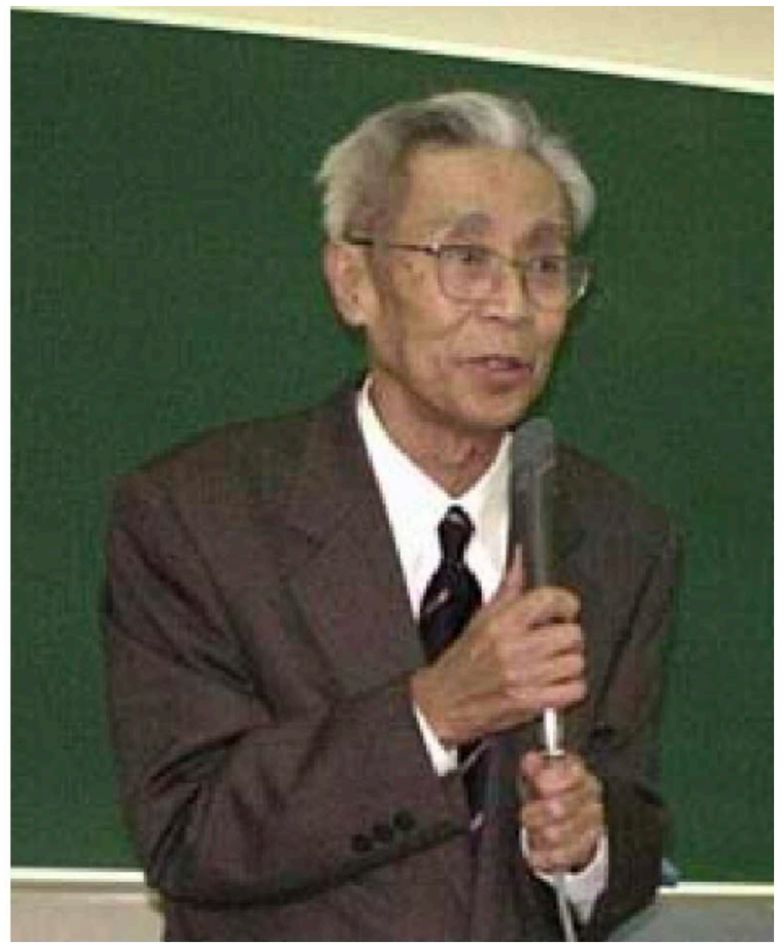


A bit of history of neutrino oscillations

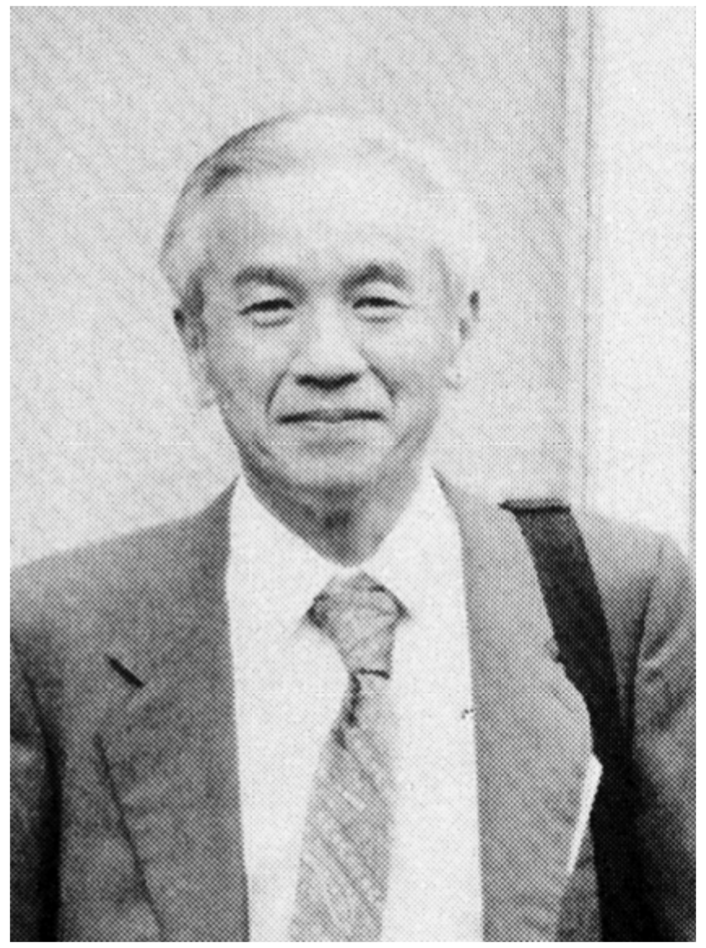
- First idea proposed by Bruno Pontecorvo in 1957:
  - Suggested $\nu \leftrightarrow \bar{\nu}$ oscillations based on analogy with $K^0 \leftrightarrow \overline{K^0}$



- Flavor transitions first considered by Maki-Nakagawa-Sakata in 1962
  - Suggested idea of mixing and $\nu_e \leftrightarrow \nu_\mu$ oscillations



Z. Maki (1929-2005)



M. Nakagawa (1932-2001)

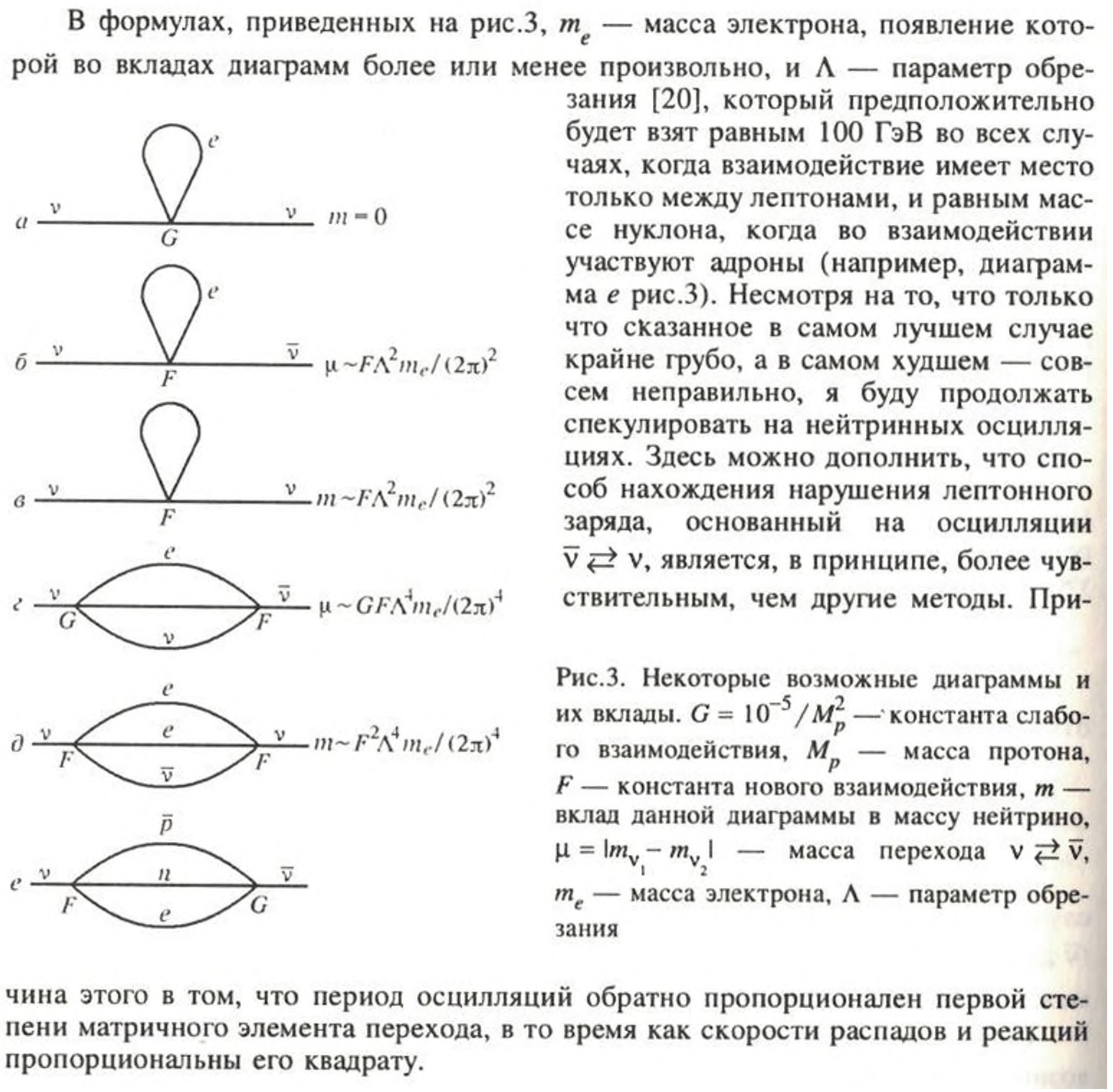
S. Sakata (1911-1970)

A bit of history of neutrino oscillations

- $\nu_e \leftrightarrow \nu_\mu$ oscillations considered by Pontecorvo in 1967
  — Hypotheses about possible mechanisms
  — Hypothesis about solar neutrino deficit (before the experiment!)

- First theory for $\nu_e \leftrightarrow \nu_\mu$ oscillations developed by Gribov and Pontecorvo in 1969.

- Neutrino oscillations firmly discovered experimentally with: solar, reactor, accelerator and atmospheric neutrino. NP in 2015



В формулах, приведенных на рис.3, m_e — масса электрона, появление которой во вкладах диаграмм более или менее произвольно, и Λ — параметр обрезания [20], который предположительно будет взят равным 100 ГэВ во всех случаях, когда взаимодействие имеет место только между лептонами, и равным массе нуклона, когда во взаимодействии участвуют адроны (например, диаграмма *е* рис.3). Несмотря на то, что только что сказанное в самом лучшем случае крайне грубо, а в самом худшем — совсем неправильно, я буду продолжать спекулировать на нейтринных осцилляциях. Здесь можно дополнить, что способ нахождения нарушения лептонного заряда, основанный на осцилляции $\bar{\nu} \rightleftarrows \nu$, является, в принципе, более чувствительным, чем другие методы. Причина этого в том, что период осцилляций обратно пропорционален первой степени матричного элемента перехода, в то время как скорости распадов и реакций пропорциональны его квадрату.

Рис.3. Некоторые возможные диаграммы и их вклады. $G = 10^{-5}/M_p^2$ — константа слабого взаимодействия, M_p — масса протона, F — константа нового взаимодействия, m — вклад данной диаграммы в массу нейтрино, $\mu = |m_{\nu_1} - m_{\nu_2}|$ — масса перехода $\nu \rightleftarrows \bar{\nu}$, m_e — масса электрона, Λ — параметр обрезания

From B. Pontecorvo paper (1957)

What is neutrino oscillation?

What is neutrino oscillation?

- Phenomenon of lepton number transformation:
  - Periodic (oscillation!), quasi-periodic (in vacuum)
  - Complicated function (in matter)

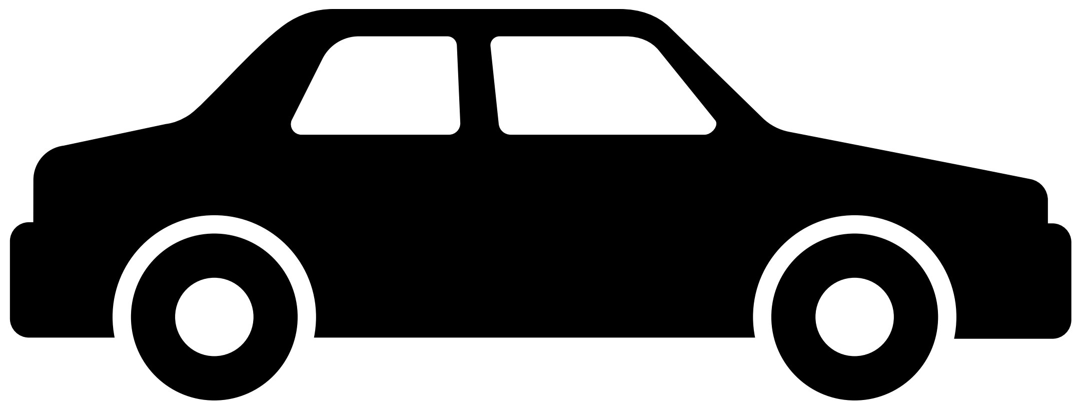
- We explore neutrino oscillations tailored to different expertise levels:
  - Drivers and Pedestrians
  - Life Scientists
  - Experimental Physicists
  - Mechanical Engineers
  - Quantum Mechanics Interested Learners
  - Quantum Field Theorists

Neutrino oscillation:
– Not very good terminology.
– Lepton number oscillation (or transformation) better

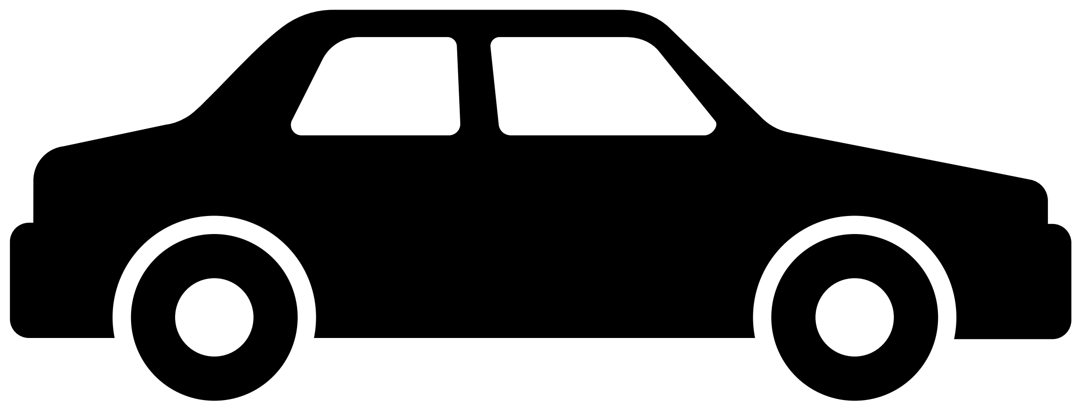
Apologies if you do not find yourself in the list!

Neutrino oscillations for drivers and pedestrians

Normal particle



Neutrino



Neutrino oscillations for drivers and pedestrians

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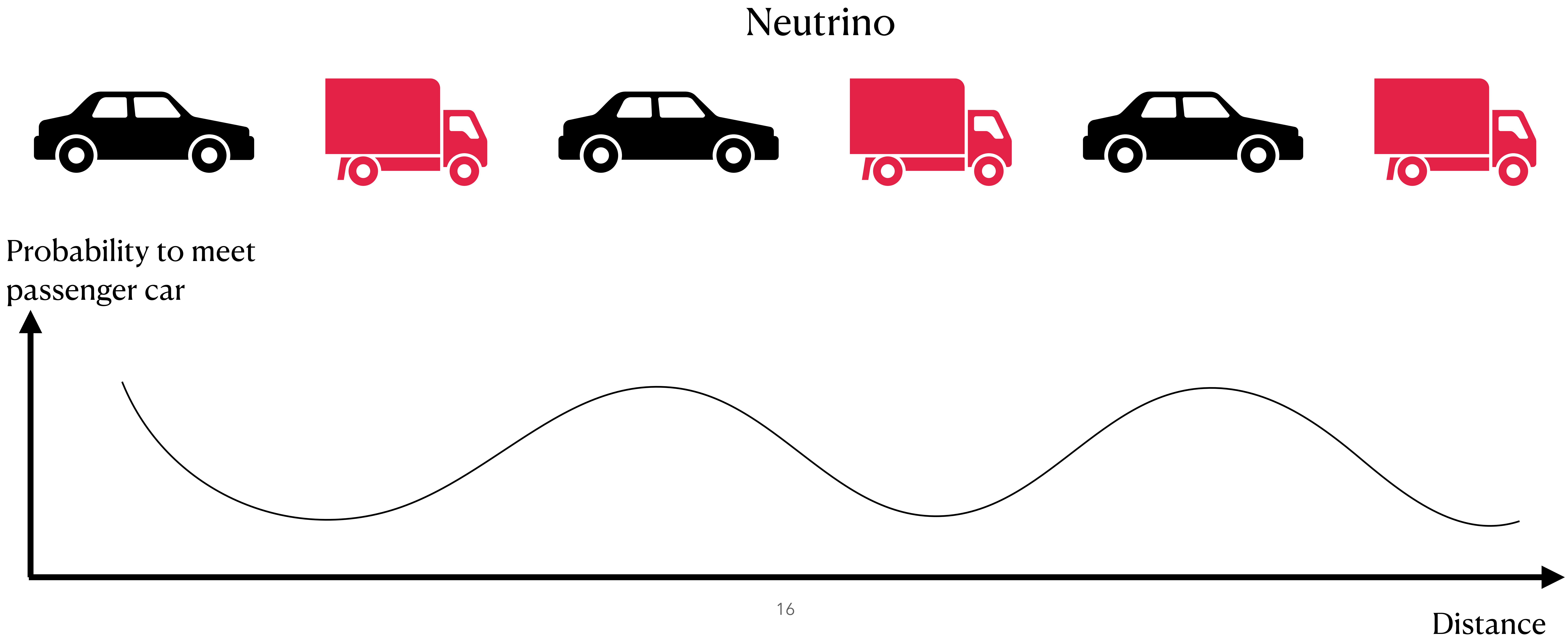
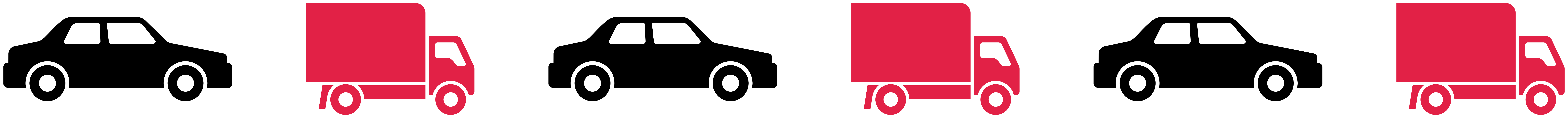
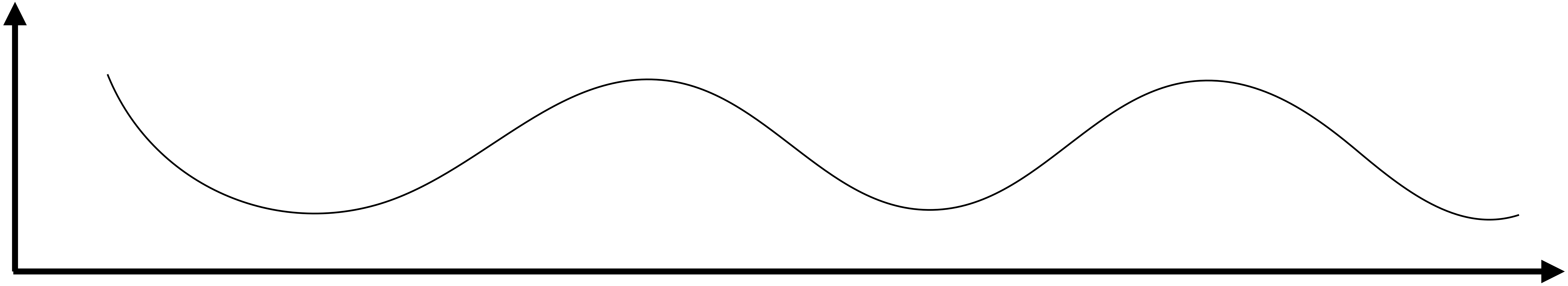


Neutrino oscillations for drivers and pedestrians

Neutrino





Neutrino oscillations for drivers and pedestrians

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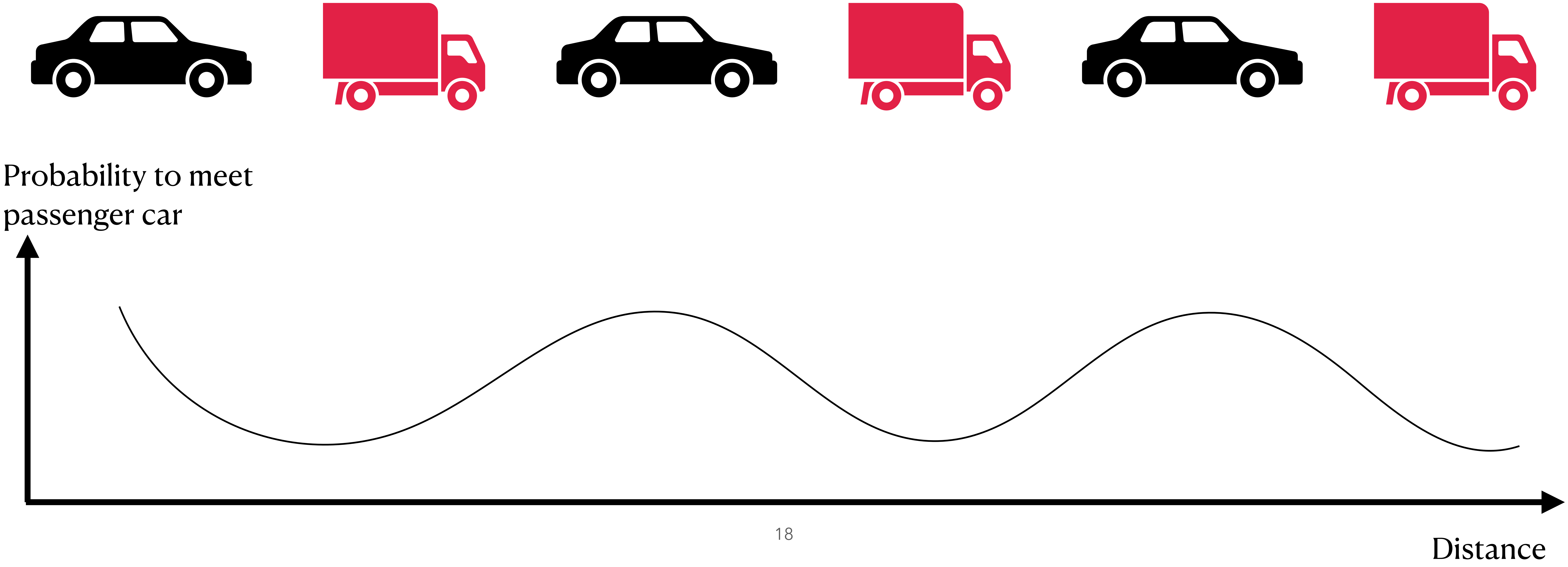
Neutrino



Neutrino oscillations for Life Scientists

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Animation by Marina Mishchenko

Neutrino oscillations for experimental physicists

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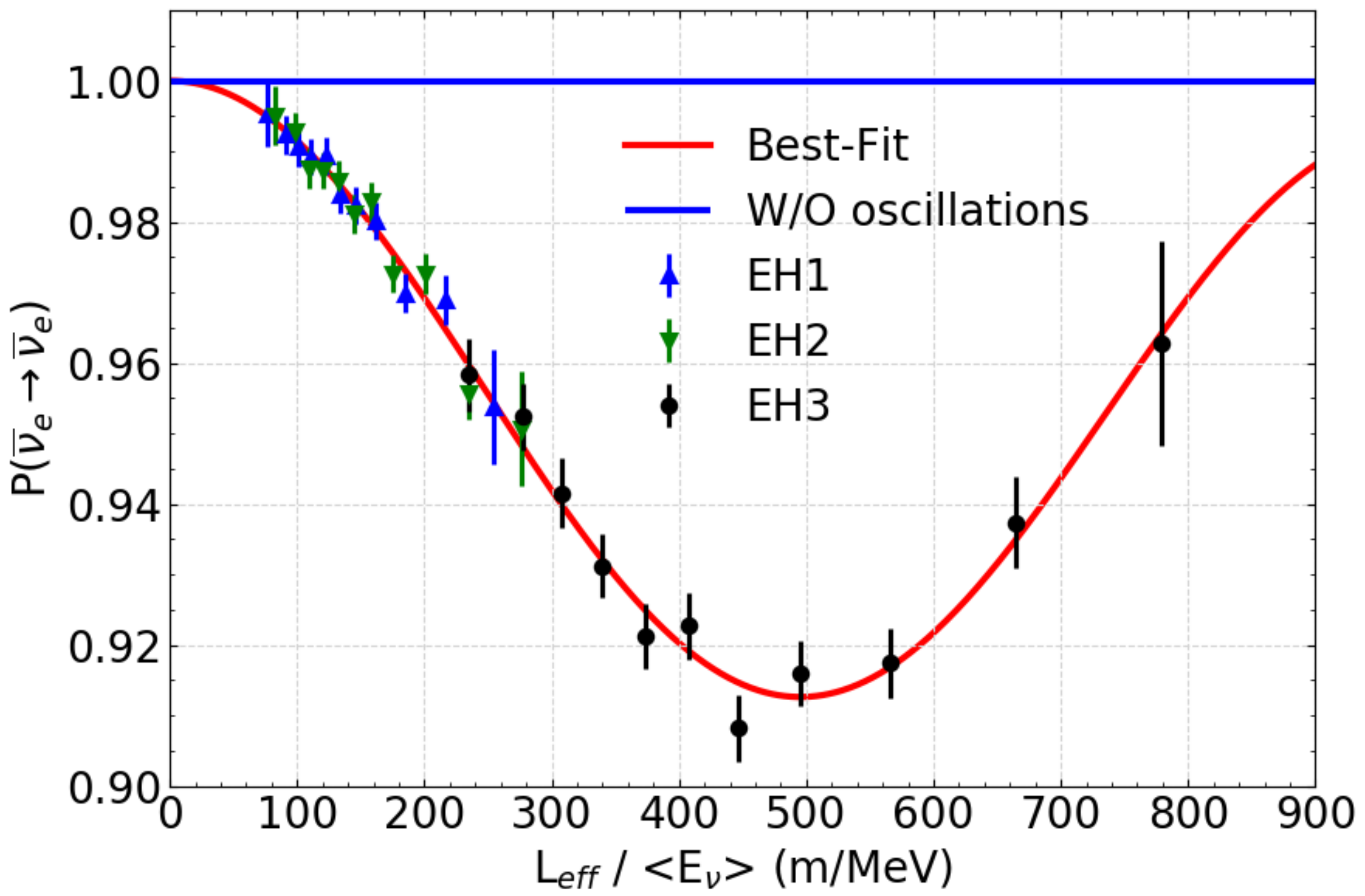
- Prepare ν_e beam (as an example)
- Place your detector at an appropriate distance to measure ν_e deficit and/or ν_μ appearance
- Use formula $P(L/E) = 1 - \sin^2 2\theta \cdot \sin^2 \frac{\Delta m^2}{4} \frac{L}{E}$ for oscillation **survival** probability to measure θ and Δm^2

Neutrino oscillations for experimental physicists

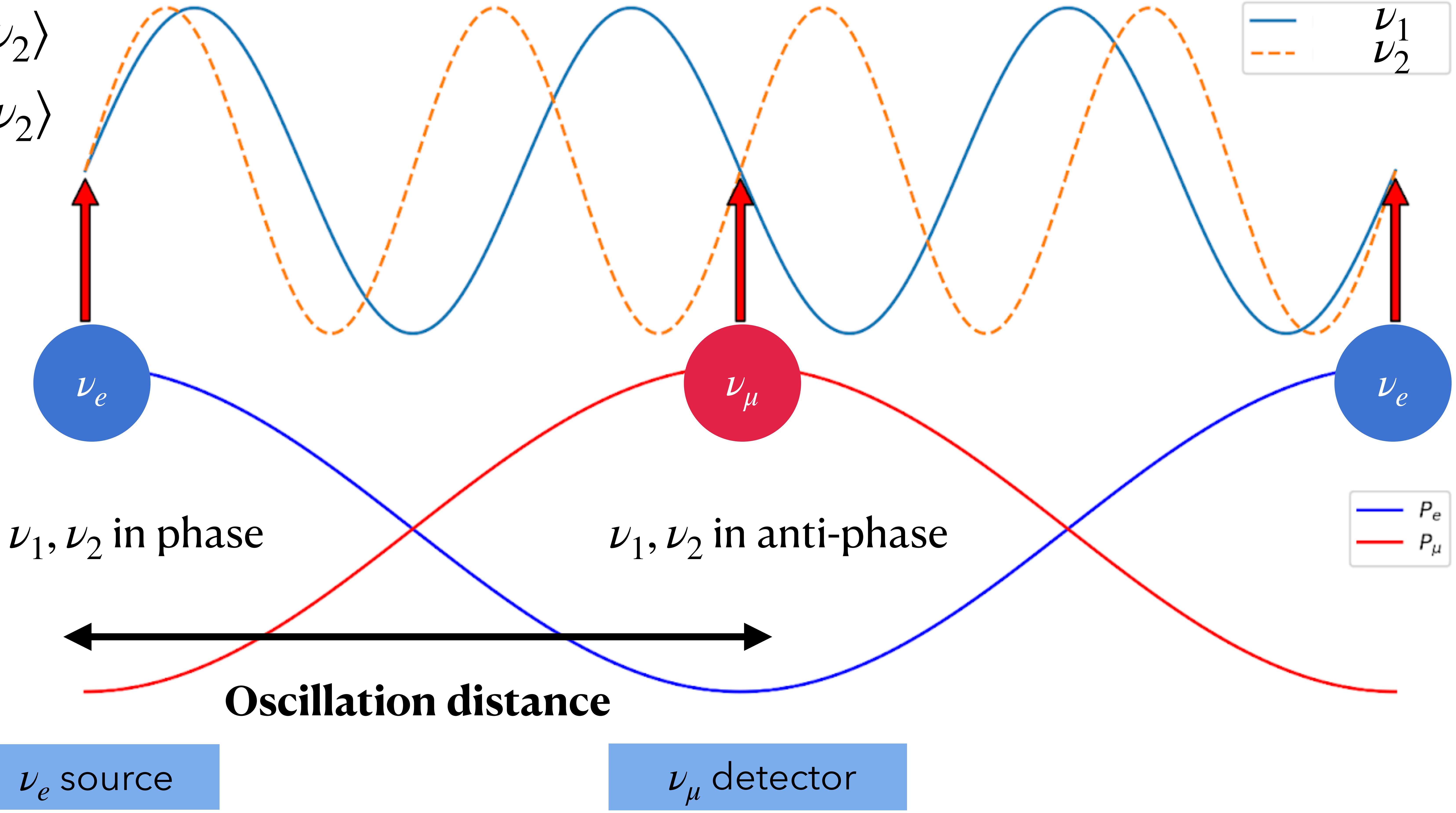
- Assume two massive neutrino states ν_1, ν_2. Then,

$$|\nu_e\rangle = \cos\theta \cdot |\nu_1\rangle - \sin\theta \cdot |\nu_2\rangle$$

$$|\nu_\mu\rangle = \sin\theta \cdot |\nu_1\rangle + \cos\theta \cdot |\nu_2\rangle$$



Daya Bay discovered non-zero θ_{13}



Follow lecture by M. Gonchar

Neutrino oscillations for mechanical engineers

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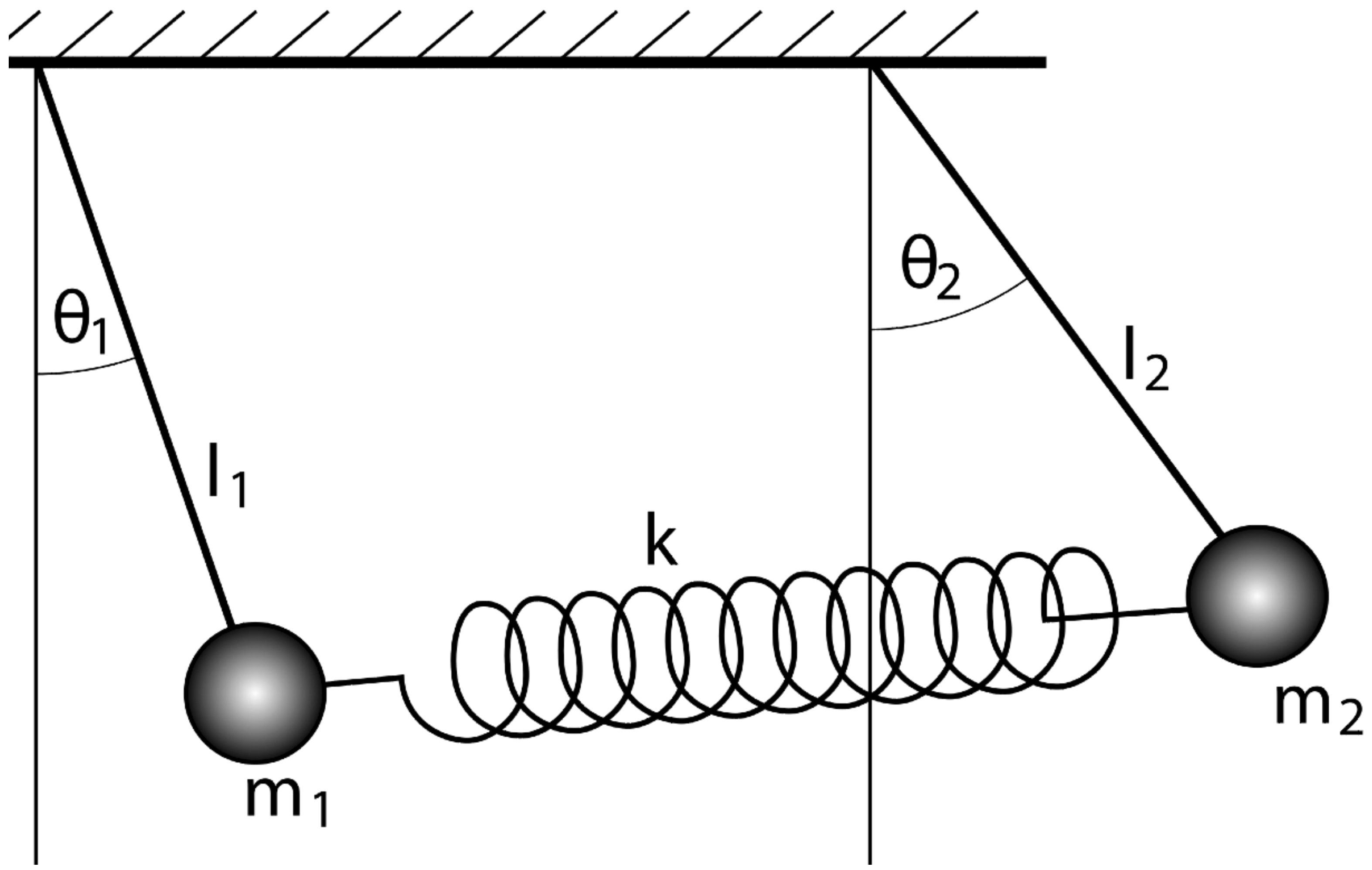


- Consider two coupled pendulums
- Potential energy

$$V = \frac{m}{2}\left(\frac{g}{l_1}x_1^2 + \frac{g}{l_2}x_2^2 + \frac{k}{m}(x_1 - x_2)^2\right)$$

$$= \frac{m}{2}(x_1, x_2)\begin{pmatrix} \frac{g}{l_1} + \frac{k}{m} & -\frac{k}{m} \\ -\frac{k}{m} & \frac{g}{l_2} + \frac{k}{m} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

- Change variables to diagonalize V

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix}\begin{pmatrix} x_1' \\ x_2' \end{pmatrix}$$

$$V = \frac{m}{2}(x_1', x_2')\begin{pmatrix} \omega_1^2 & 0 \\ 0 & \omega_2^2 \end{pmatrix}\begin{pmatrix} x_1' \\ x_2' \end{pmatrix}$$

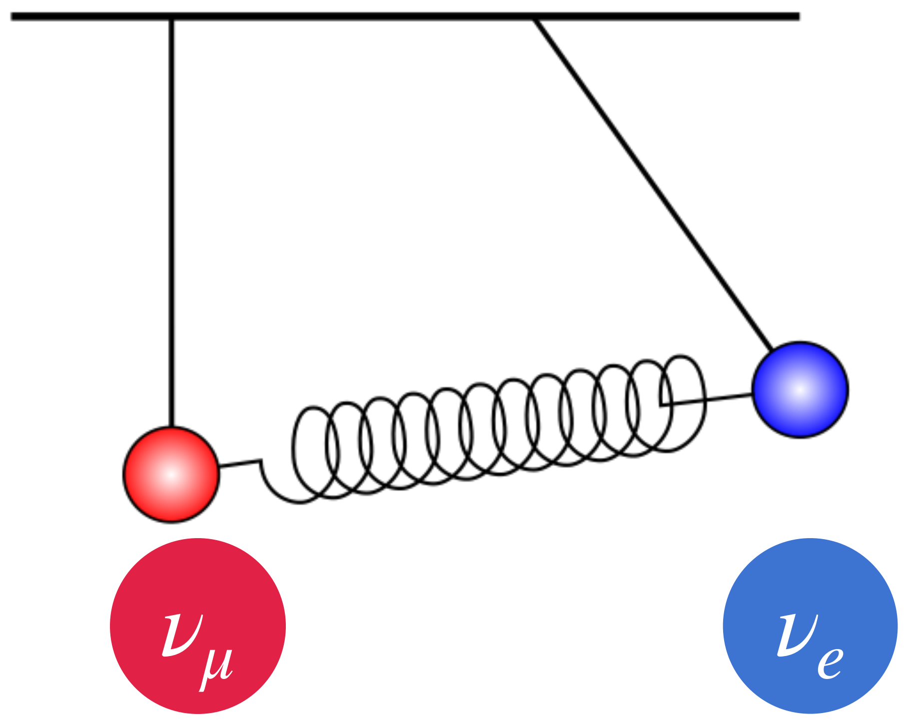
Neutrino oscillations for mechanical engineers

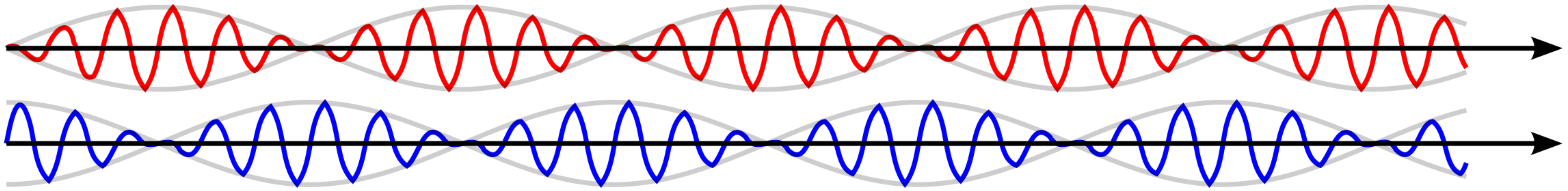
- Normal mode. Small frequency

- Normal mode. Large frequency

Neutrino oscillations for mechanical engineers

- Begin with blue pendulum given total energy E_0





- Energy oscillates between the pendulums

$$\frac{E(t)}{E_0} = 1 - \underbrace{\frac{4r}{(1+r)^2}}_{\sin^2 2\theta} \cdot \sin^2 \frac{\Delta\omega \cdot t}{2}$$

$$\Delta\omega = \omega_2 - \omega_1 \qquad r = m_2/m_1$$

- Great analogy with neutrino oscillations

$$P(L/E) = 1 - \sin^2 2\theta \cdot \sin^2 \frac{\Delta E \cdot L}{2}$$

$$\sin^2 2\theta = \frac{4r}{(1+r)^2} \qquad \Delta\omega = \Delta E$$

Neutrino oscillations for Quantum Mechanics Learners

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- Consider quantum system to be in a **pure** state $|\Psi_i\rangle$ with definite energy E_i. Its time evolution:

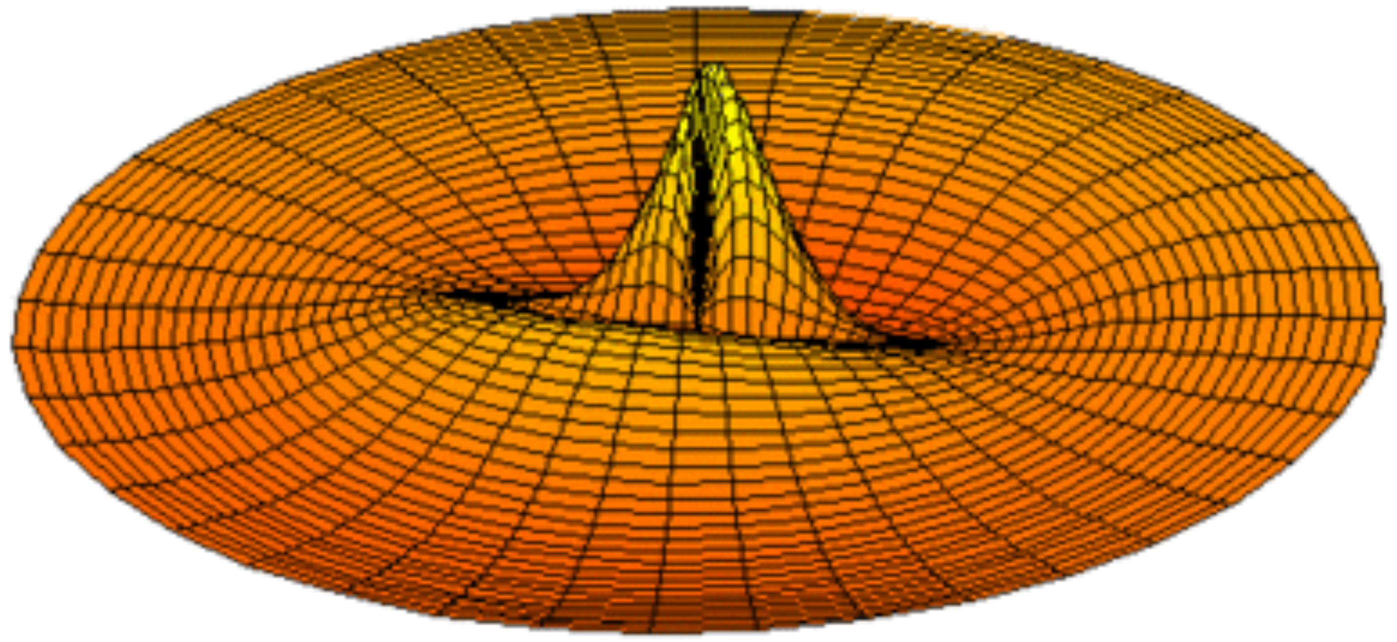
$$|\Psi_i(t)\rangle = e^{-iE_it}\,|\Psi_i(0)\rangle$$

- The wave function oscillates but the system remains in the same state: **survival probability**=**1**
- Consider a superposition $|\Psi\rangle = a \cdot |\Psi_1\rangle + b \cdot |\Psi_2\rangle$.

Its time evolution:

$$|\Psi(t)\rangle = a \cdot e^{-iE_1t}|\Psi_1(0)\rangle + be^{-iE_2t} \cdot |\Psi_2(0)\rangle$$

$(|a|^2 + |b|^2 = 1)$



Hydrogen ψ_{410}. Source: wikipedia

Neutrino oscillations for Quantum Mechanics Learners

- Consider a superposition $|\Psi\rangle = a \cdot |\Psi_1\rangle + b \cdot |\Psi_2\rangle$. Its time evolution:

$$|\Psi(t)\rangle = a \cdot e^{-iE_1 t}|\Psi_1(0)\rangle + b e^{-iE_2 t} \cdot |\Psi_2(0)\rangle$$

- Survival probability after time t reads:

$$P = |\langle\Psi(0)|\Psi(t)\rangle|^2 = \left| |a|^2 \cdot e^{-iE_1 t} + |b|^2 e^{-iE_2 t} \right|^2$$

$$= 1 - 4|a|^2|b|^2 \sin^2 \frac{(E_1 - E_2) \cdot t}{2}$$

Neutrino oscillations for Quantum Field Theorists

- Let us draft a Feynman diagram for lepton number violating process

Neutrino oscillations for Quantum Field Theorists

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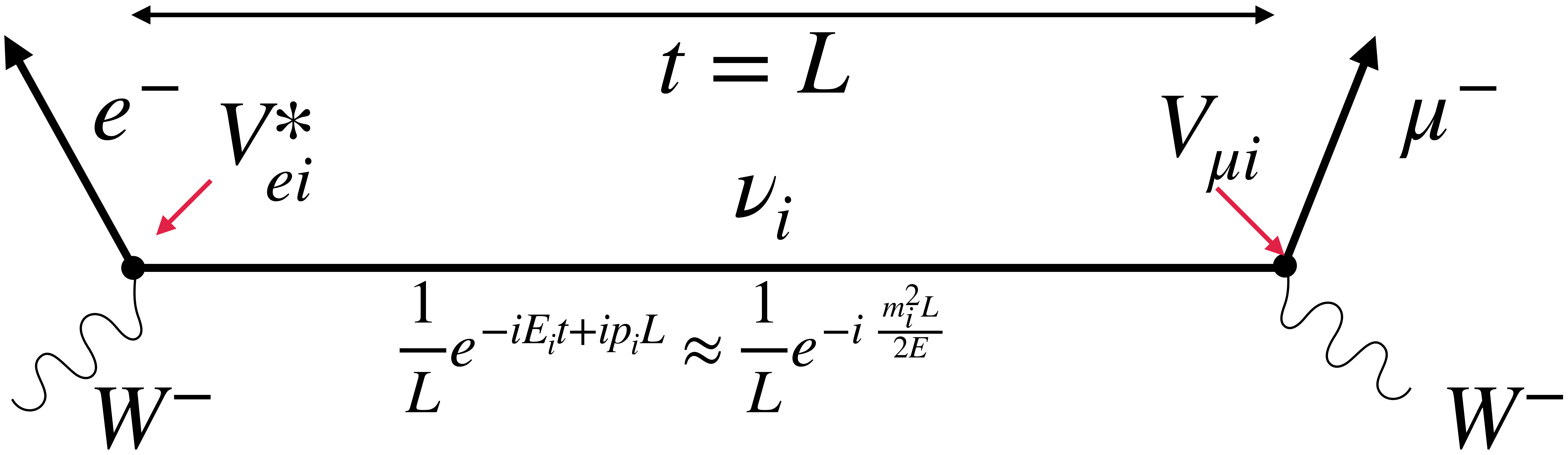
$$\mathscr{A} \propto \sum_i$$



$$\mathscr{A} \propto \sum_i V^*_{ei} V_{\mu i}\ \frac{1}{L} e^{-i\,\frac{m_i^2 L}{2E}}$$

$$\left|\mathscr{A}\right|^2 \propto \frac{1}{L^2} \left| \sum_i V^*_{ei} V_{\mu i} e^{-i\frac{L m_i^2}{2E}} \right|^2$$

Neutrino oscillations for Quantum Field Theorists

$$|\mathscr{A}|^2 \propto \frac{1}{L^2}\left|\sum_i V^*_{ei} V_{\mu i} e^{-i\frac{Lm_i^2}{2E}}\right|^2$$

$$P_{e\mu}(L/E) = \sum_{i,j} V^*_{ei} V^*_{\mu j} V_{\mu i} V_{ej}\, e^{-iL\,\frac{\Delta m^2_{ij}}{2E}}$$

- (Quasi)periodic dependence («oscillations») of the probability
- Non-zero non-diagonal $V_{\alpha i}$ required and $\Delta m^2_{ij} \equiv m_i^2 - m_j^2 \neq 0$
- What is oscillating? The lepton flavor $L_e \leftrightarrow L_\mu$

Three neutrino oscillations in vacuum

$$\begin{pmatrix} V_{e1}^* & V_{e2}^* & V_{e3}^* \\ V_{\mu 1}^* & V_{\mu 2}^* & V_{\mu 3}^* \\ V_{\tau 1}^* & V_{\tau 2}^* & V_{\tau 3}^* \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{23} & \sin\theta_{23} \\ 0 & -\sin\theta_{23} & \cos\theta_{23} \end{pmatrix} \begin{pmatrix} \cos\theta_{13} & 0 & \sin\theta_{13} e^{-i\delta} \\ 0 & 1 & \\ -\sin\theta_{13} e^{i\delta} & 0 & \cos\theta_{13} \end{pmatrix} \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ -\sin\theta_{12} & \cos\theta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- Three mixing angles $\theta_{12}, \theta_{13}, \theta_{23}$
- One phase: δ.
  - If $\delta \neq 0, \pi$: **CP violation,** which can be observed as:

$$P(\nu_\alpha \to \nu_\beta) \neq P(\bar{\nu}_\alpha \to \bar{\nu}_\beta),\ \text{for } \alpha \neq \beta$$

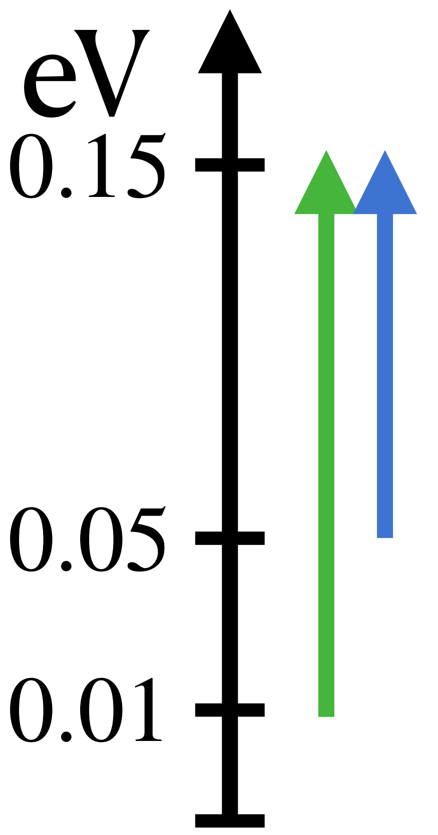
Three neutrino oscillations in vacuum. Experimental summary

$$\begin{pmatrix} V^*_{e1} & V^*_{e2} & V^*_{e3} \\ V^*_{\mu 1} & V^*_{\mu 2} & V^*_{\mu 3} \\ V^*_{\tau 1} & V^*_{\tau 2} & V^*_{\tau 3} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{23} & \sin\theta_{23} \\ 0 & -\sin\theta_{23} & \cos\theta_{23} \end{pmatrix} \begin{pmatrix} \cos\theta_{13} & 0 & \sin\theta_{13}e^{-i\delta} \\ 0 & 1 & \\ -\sin\theta_{13}e^{i\delta} & 0 & \cos\theta_{13} \end{pmatrix} \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ -\sin\theta_{12} & \cos\theta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- Solar and reactor neutrino: $\theta_{12}, \Delta m^2_{21}$
- Atmospheric and accelerator neutrino: $\theta_{23}, \Delta m^2_{32}$
- Reactor neutrino at 2 km (Daya Bay, RENO, DC): $\theta_{13}, \Delta m^2_{32}$

$$\Delta m^2_{21} \approx 7.5 \cdot 10^{-5} \mathrm{eV}^2$$

$$|\Delta m^2_{32}| \approx 2.4 \cdot 10^{-3} \mathrm{eV}^2$$



- Neutrino mass measurements:

$$\Delta m^2_{21} = m^2_2 - m^2_1, m^2_2 = m^2_1 + \Delta m^2_{21}$$

$$m_2 = \sqrt{m^2_1 + \Delta m^2_{21}} \geq \sqrt{\Delta m^2_{21}} \approx 0.01 \text{ eV}$$

Normal ordering $\quad m_3 = \sqrt{m^2_2 + |\Delta m^2_{32}|} \geq 0.05 \text{ eV}, m_3 > m_2$

Inverse ordering $\quad m_2 = \sqrt{m^2_3 + |\Delta m^2_{32}|} \geq 0.05 \text{ eV}, m_2 > m_3$

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Neutrino oscillations in matter

If matter matters?

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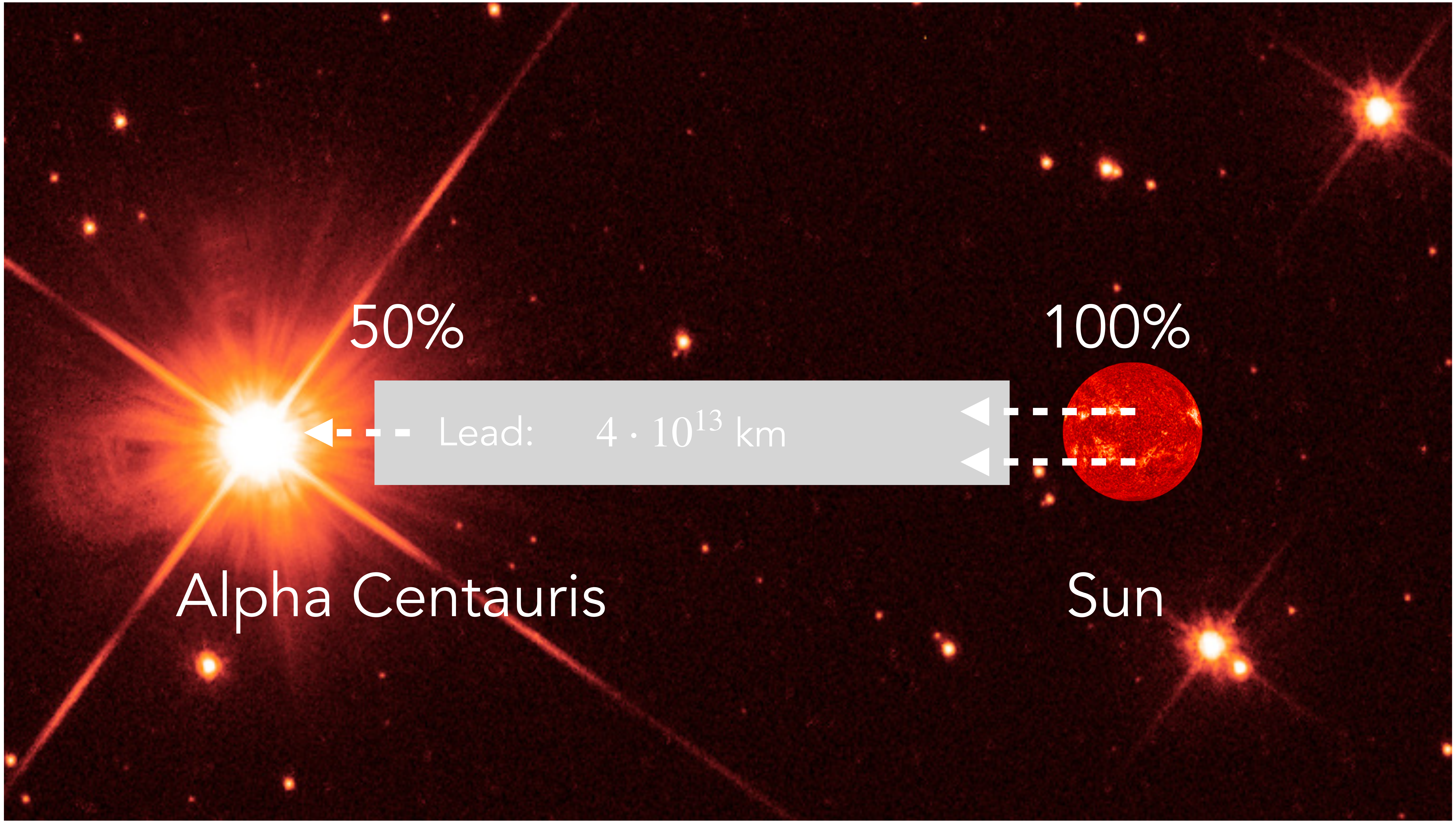
Weak interactions are very weak



If matter matters?

Weak interactions are very weak

- If neutrino interacts so weakly, then Sun is just a **transparent** medium.
  - — Why it can matter?

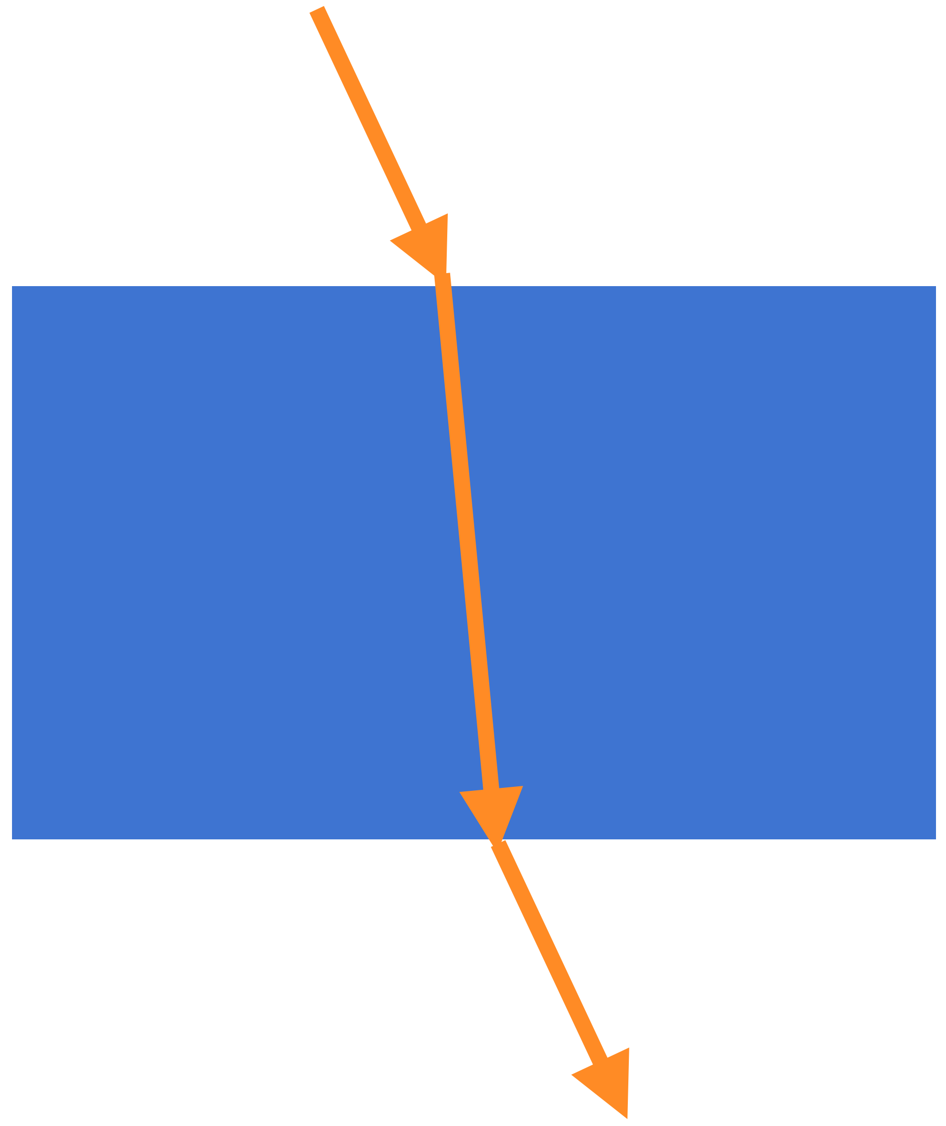
- Glass or drop of water are also **transparent** media ... but they do matter on light propagation because of **refraction**

Do you understand light refraction?

Check yourself

Find the right answer

- Light slows down in matter because:
  - It is absorbed by atoms and re-emitted with some delay
  - It is scattered by atoms and takes a longer path
  - It experiences a friction

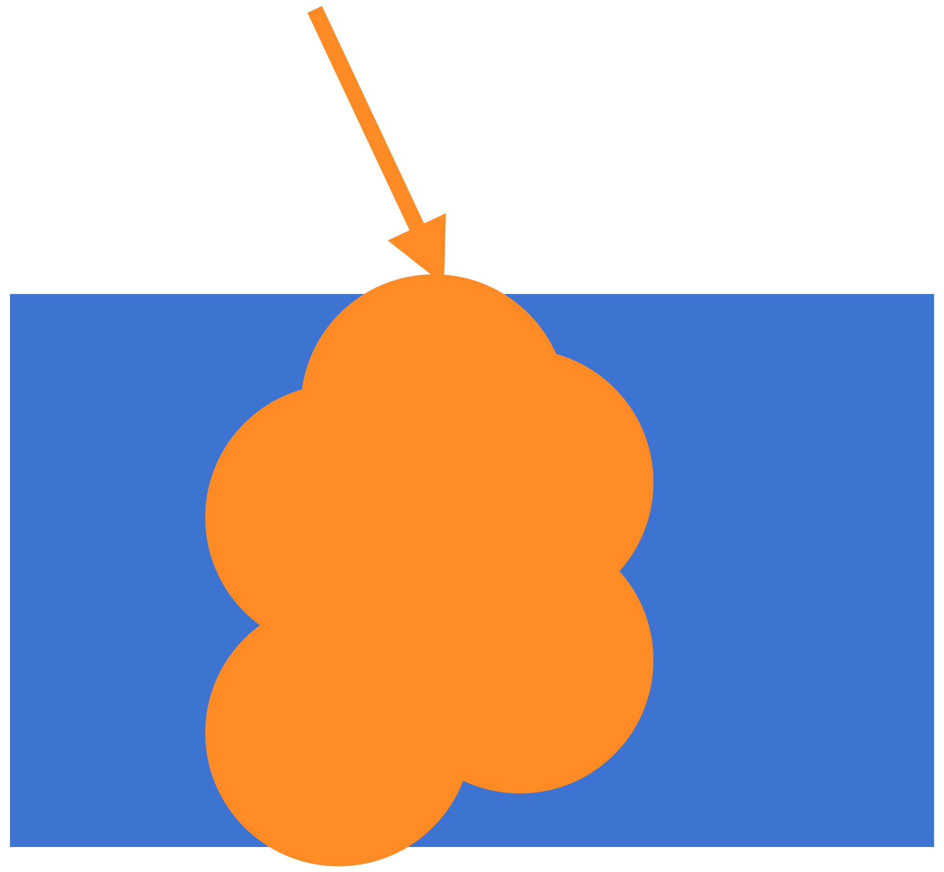


Do you understand light refraction?

Check yourself

Find the right answer

- Light slows down in matter because:
  - ✗ **It is absorbed by atoms and re-emitted with some delay**
  - — It is scattered by atoms and takes a longer path
  - — It experiences a friction



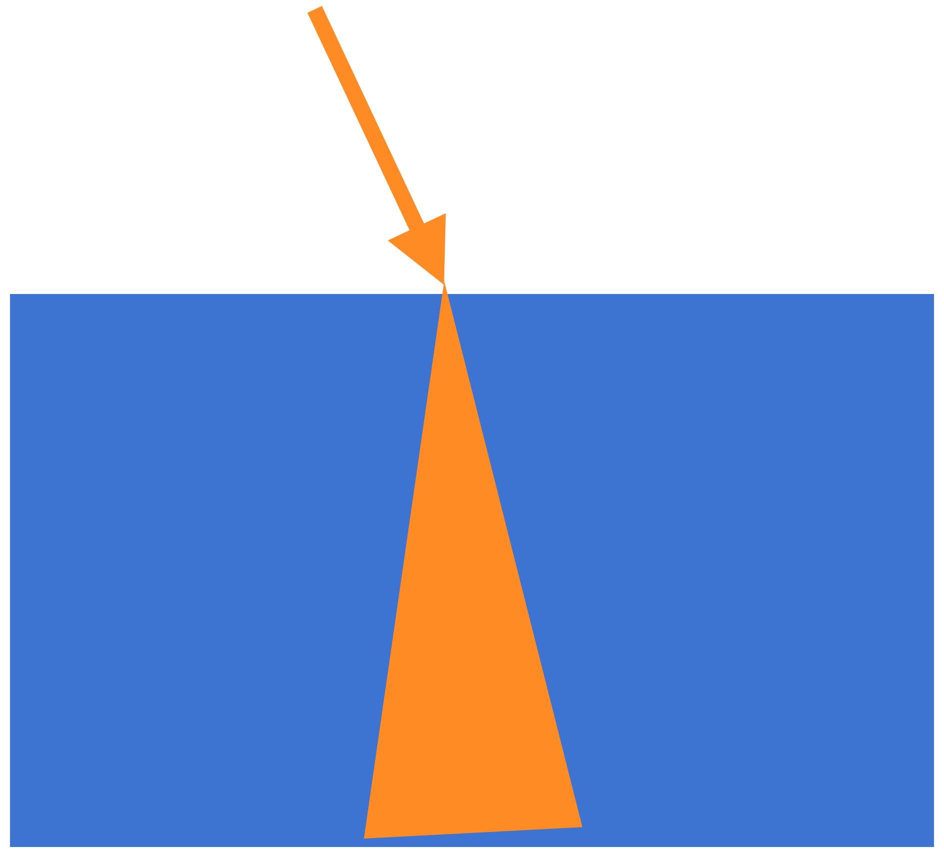
Re-emission does not keep the original direction

Do you understand light refraction?

Check yourself

Find the right answer

- Light slows down in matter because:
  - ✗ It is absorbed by atoms and re-emitted with some delay
  - ✗ **It is scattered by atoms and takes a longer path**
  - — It experiences a friction



More material —> wider the beam

Do you understand light refraction?

Check yourself

Find the right answer

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- Light slows down in matter because:
  - It is absorbed by atoms and re-emitted with some delay
  - It is scattered by atoms and takes a longer path
  - **It experiences a friction**

Just NO!

Do you understand light refraction?

Check yourself

Find the right answer

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- Light slows down in matter because:

— Incident electromagnetic wave forces electrons to vibrate and emit the secondary wave

— Both the incident and the secondary waves move at the speed of light.

— The secondary wave is delayed in phase by about $\pi/2$ and the front of resulting wave moves slower

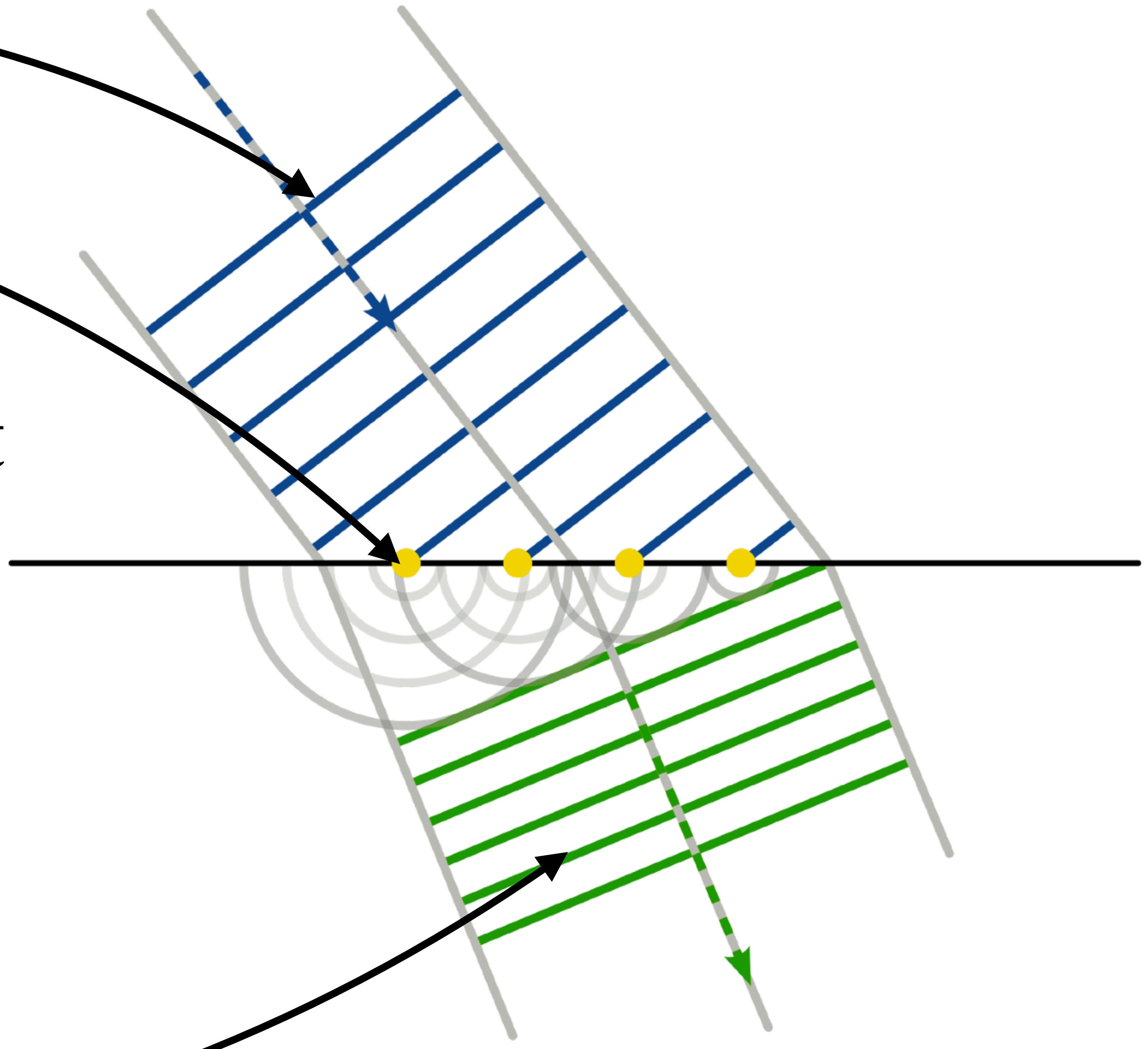

Image source: Wikipedia

Refraction index

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- These complex phenomena can be conveniently described by a refraction index n
- Consider a wave $\cos(\omega \cdot t - k \cdot x)$ in vacuum.
  - The phase velocity can be found from $\omega \cdot t - k \cdot x = 0$ as

$$c = \frac{x}{t} = \frac{\omega}{k}$$

- Consider a wave $\cos(\omega \cdot t - n \cdot k \cdot x)$ in matter.
  - The phase velocity can be found from $\omega \cdot t - n \cdot k \cdot x = 0$ as

$$v = \frac{x}{t} = \frac{\omega}{n \cdot k} = \frac{c}{n}$$

Refraction index

- Microscopic consideration yields

$$n = 1 + \frac{V}{k},$$

Where V is a photon-matter potential due to $\gamma + e \to \gamma + e$

- Neutrino also experiences the refraction in matter due to $\nu_{e,\mu} + e \to \nu_{e,\mu} + e$

If matter matters? YES!

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- Due to $\nu_e + e \to \nu_e + e$ reaction ν_e experience the potential (calculated in the SM)

$$V = \sqrt{2} G_F n_e \approx 10^{-10} - 10^{-11} \text{ эВ}$$

- The potential is negligibly small compared to the neutrino energy

$$V \ll E_\nu \simeq (0.1 - 10) \cdot 10^6 \text{ eV}$$

- However it is comparable with

$$\Delta E = \frac{\Delta m^2}{2E} \simeq \frac{10^{-5} \text{ eV}^2}{10^6 \text{ eV}} = 10^{-11} \text{ eV}$$

- Sun refracts neutrino like a glass ball refracts the light

Neutrino oscillations in matter

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- ν_μ do not feel the electrons (via W exchange)
- ν_e can pass without interactions with electrons
- ν_e can interact with electrons in $\nu_e + e \rightarrow \nu_e + e$

Refraction

Neutrino oscillations in matter

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- Sun refracts neutrino like a glass ball refracts the light

Neutrino oscillations in matter

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- Sun refracts neutrino like a glass ball refracts the light
- Refraction index for ν_e

$$n_{\nu_e} = 1 - \sqrt{2} G_F n_e / E$$

- Refraction index for ν_μ

$$n_{\nu_\mu} = 1$$

- The difference in refraction indices for ν_e and ν_μ drastically changes the oscillation pattern

$$n_{\nu_e} - n_{\nu_\mu} = -\sqrt{2} G_F n_e / E_\nu$$

Neutrino oscillations in matter

Follow lecture
by O. Smirnov

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Vacuum MSW resonance Adiabatic transition

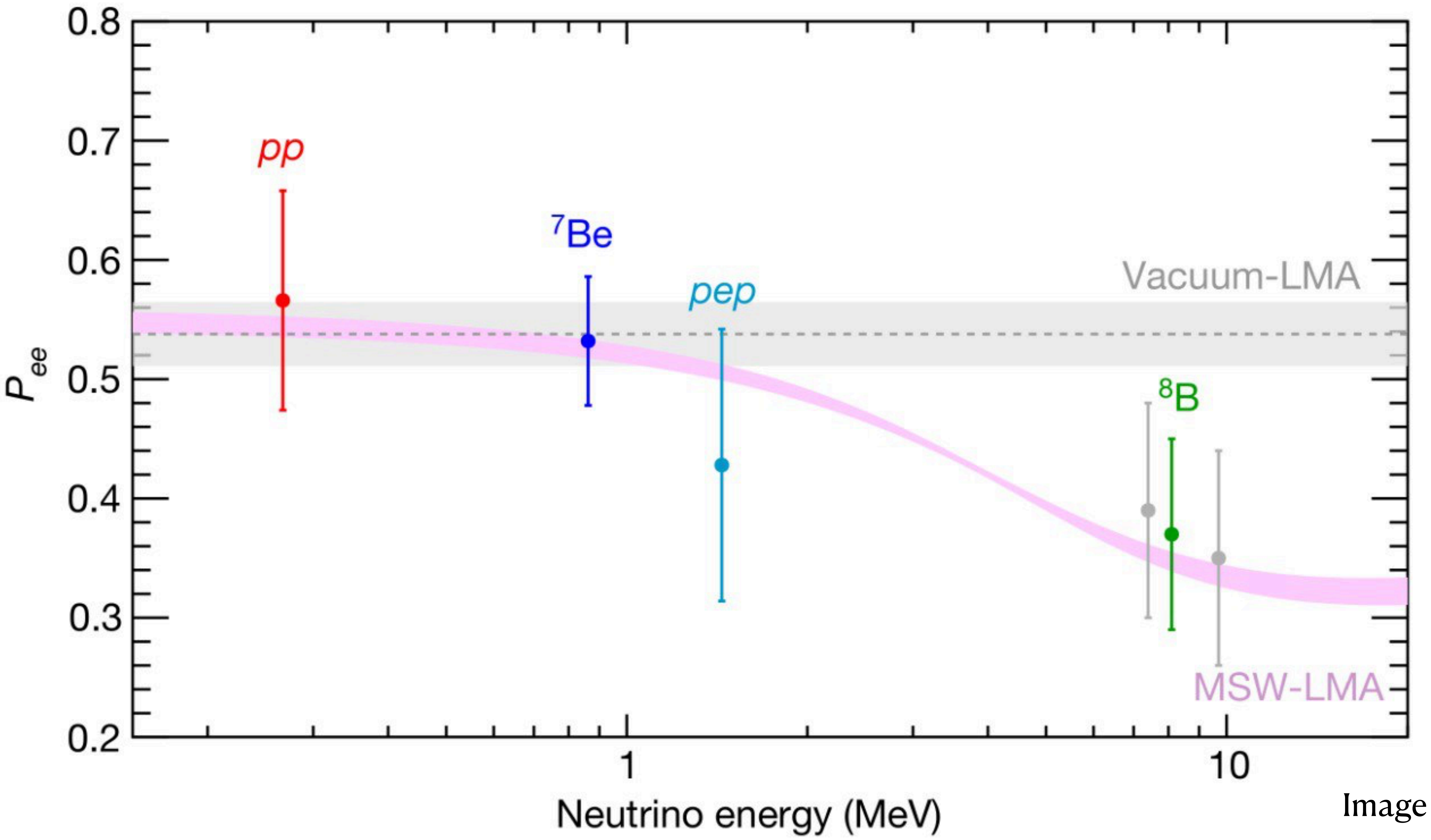

Image courtesy: BOREXINO Coll.


Critical
Review

(Implicit) hypotheses

Of neutrino oscillation within plane wave model

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- A coherent superposition $\nu_\alpha = \sum_i V^*_{\alpha i} \nu_i$ is produced and interacted
- Quantum states $|\nu_i\rangle$ have definite momenta with $\delta p_i = 0$
- Momenta of all $|\nu_i\rangle$ are the same $p_1 = p_2 = p_3 = p$
- Neutrino are ultra-relativistic particles $|p_i| \gg m_i$
- Time t equals to the distance L:

$$t = L$$

(Implicit) hypotheses

Of neutrino oscillation within plane wave model

A coherent superposition $\nu_\alpha = \sum_i V^*_{\alpha i} \nu_i$ is produced and interacted

- Then, why massive neutrino ν_i are produced coherently, while charged leptons seem not? If charged leptons oscillate?
- In the SM charged leptons and neutrino fields are symmetric

$$\mathscr{L} = -\frac{g}{2\sqrt{2}} \sum_{\alpha=e,\mu,\tau} \sum_{i=1}^{3} \boxed{V_{\alpha i}}\, \overline{\ell}_\alpha O^\mu \nu_i W_\mu + \text{эс}$$

Lepton mixing matrix (not neutrino mixing matrix!)

(Implicit) hypotheses

Of neutrino oscillation within plane wave model

Quantum states $|\nu_i\rangle$ have definite momenta with $\delta p_i = 0$

- Then, position uncertainty reads: $\delta x_\nu = \dfrac{1}{\delta p_\nu} = \infty$
- What is the distance L in the oscillation formula then?

(Implicit) hypotheses

Of neutrino oscillation within plane wave model

Momenta of all $|\nu_i\rangle$ are the same $p_1 = p_2 = p_3 = p$

- Breaks Lorentz invariance

- Contradicts to kinematics of decays

Try to prove these statements yourself

(Implicit) hypotheses

Of neutrino oscillation within plane wave model

Neutrino are ultra-relativistic particles $|p_i| \gg m_i$

- True for all experiments so far
- Not true for relic neutrinos

(Implicit) hypotheses

Of neutrino oscillation within plane wave model

Time t equals to the distance L:

$$t = L$$

- Let us make it better $L = vt = \frac{p_i}{E_\nu} t$

$$\varphi = E_i t - p_i L = E_i t - \frac{p_i^2}{E_i} t = \frac{E_i^2 - p_i^2}{E_i} t = \frac{m_i^2}{E_i} t$$

- The phase difference then:

$$\varphi_{ij} = \varphi_i - \varphi_j = \frac{m_i^2 - m_j^2}{E_i} t = 2\frac{m_i^2 - m_j^2}{2E_i} t$$

- The phase difference is TWO times larger than the standard!

Resume

Plane wave model

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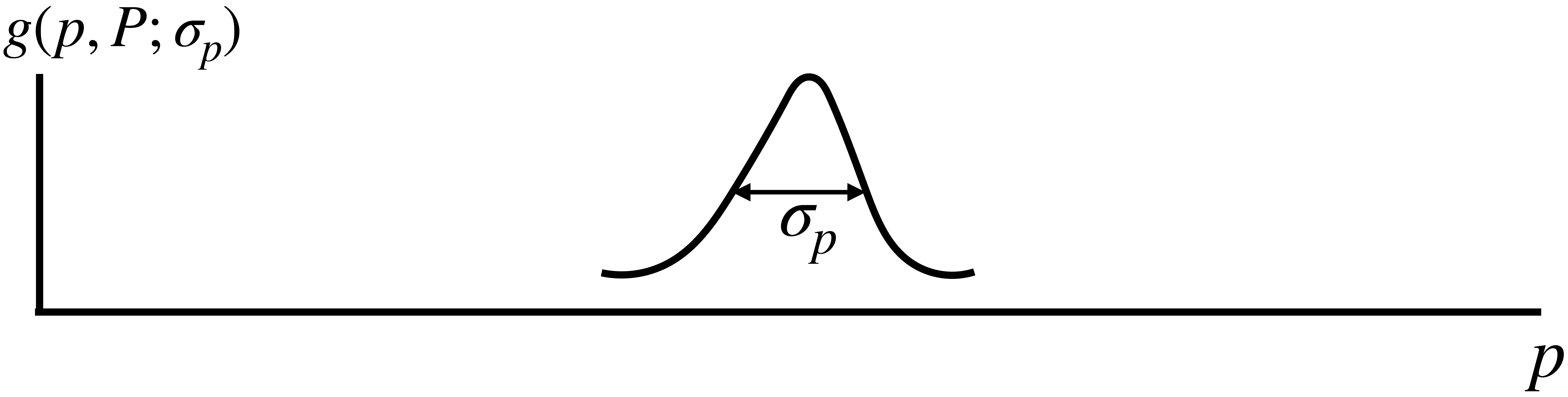
- A coherent superposition $\nu_\alpha = \sum_i V^*_{\alpha i} \nu_i$ is produced and interacted ✗
- Quantum states $|\nu_i\rangle$ have definite momenta with $\delta p_i = 0$ ✗
- Momenta of all $|\nu_i\rangle$ are the same $p_1 = p_2 = p_3 = p$ ✗
- Neutrino are ultra-relativistic particles $|p_i| \gg m_i$ ✓
- Time t equals to the distance L: ✗

$$t = L$$

Wave packet model

Wave packet model

$$|p\rangle \rightarrow \int \frac{dp}{2\pi} g(p, P; \sigma_p) \, |p\rangle$$



Momentum space

Wave packet model

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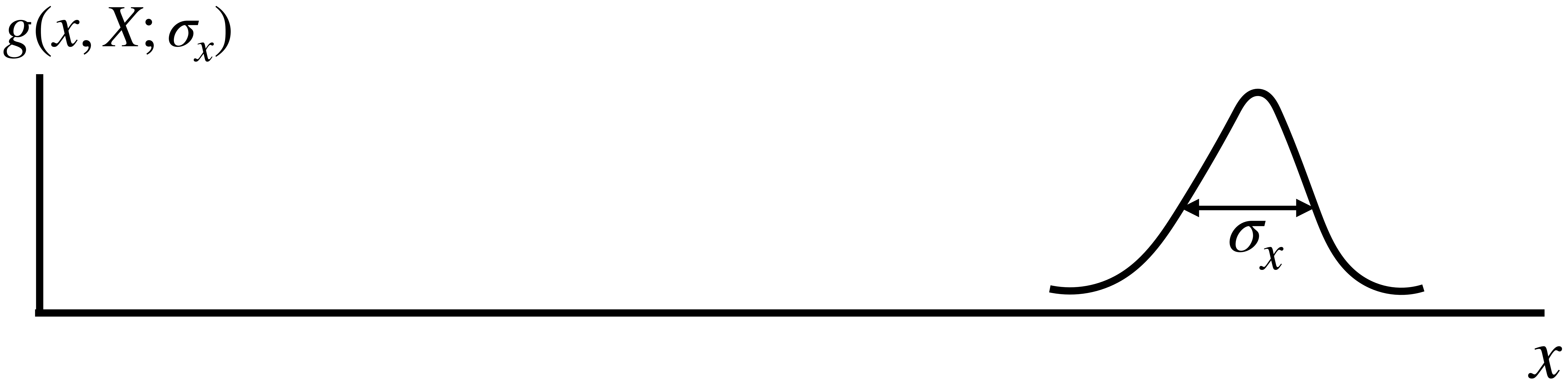
$$|p\rangle \to \int \frac{dp}{2\pi} g(p, P; \sigma_p) |p\rangle$$

Coordinate space

Wave packet model

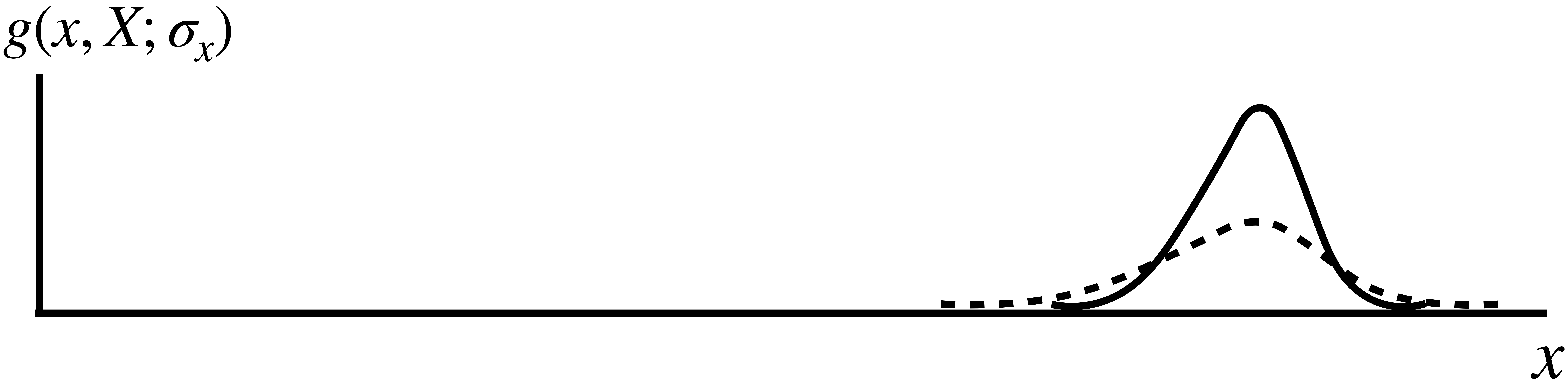
$$|p\rangle \to \int \frac{dp}{2\pi} g(p, P; \sigma_p)\,|p\rangle$$



Coordinate space

Wave packet model

$$|p\rangle \to \int \frac{dp}{2\pi} g(p, P; \sigma_p)\,|p\rangle$$



Coordinate space

Wave packet disperses (ignore it here)

Vacuum neutrino oscillations In wave packet model

Oscillation probability

Plane wave model

$$P_{e\mu}(L/E) = 1 - \sin^2 2\theta \sin^2 \frac{\Delta m^2 L}{4E}$$

Wave packet model

$$P_{e\mu}(L/E) = 1 - \frac{1}{2}\sin^2 2\theta \left(1 - \exp\left[-(L/L_{coh})^2 - 1/4(\Delta m^2/\sigma_{m^2})\right] \cos \frac{\Delta m^2 L}{2E} \right)$$

Resume

Plane wave model

- A coherent superposition $\nu_\alpha = \sum_i V^*_{\alpha i} \nu_i$ is produced and interacted only if $\Delta m^2 \ll \sigma_{m^2}$ ✓
- Quantum states $|\nu_i\rangle$ do not have definite momenta ✓
- Momenta of all $|\nu_i\rangle$ are not the same ✓
- Neutrino can be ultra-relativistic or non-relativistic particles ✓
- Time t is not equal to the distance L: ✓

$$t = \frac{2L}{\frac{1}{v_1} + \frac{1}{v_2}}$$

✓

Oscillation probability

In wave packet model

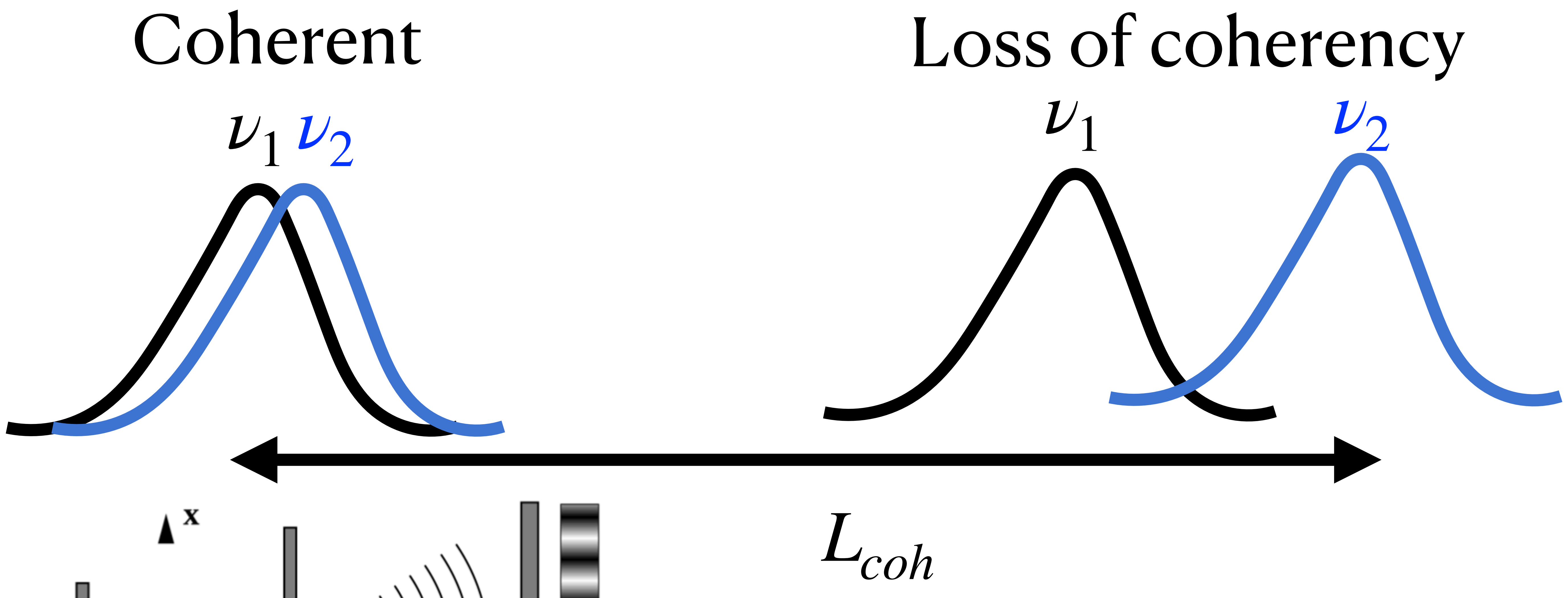
$$P_{e\mu}(L/E) = 1 - \frac{1}{2}\sin^2 2\theta \left(1 - \exp\left[-(L/L_{coh})^2 - 1/4(\Delta m^2/\sigma_{m^2})\right]\cos\frac{\Delta m^2 L}{2E}\right)$$

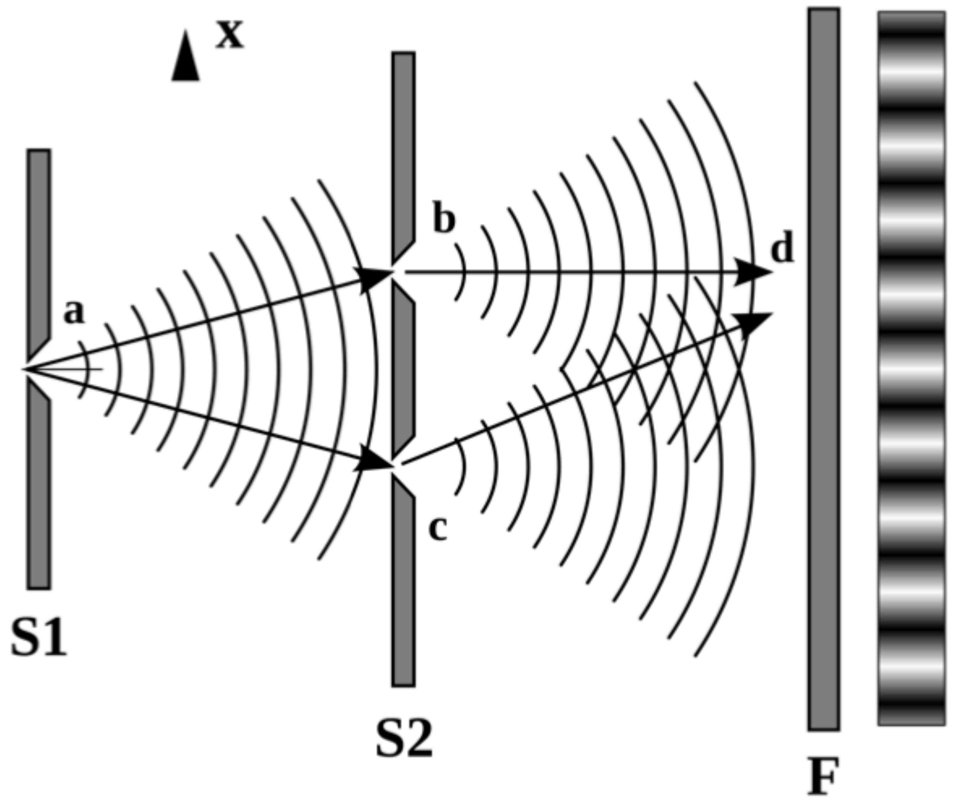
$$L_{coh} = L_{osc}\frac{p}{\sqrt{2}\pi\sigma_p}$$

Coherence length

$$\sigma_{m^2} = 2\sqrt{2}p\sigma_p$$

Uncertainty in mass²





D.N. & V.Naumov, J.Phys.G 37 (2010) 105014
D.N. & V.Naumov, JPhys.Part.Nucl. 51 (2020) 1, 1-106

Do charged leptons oscillate?

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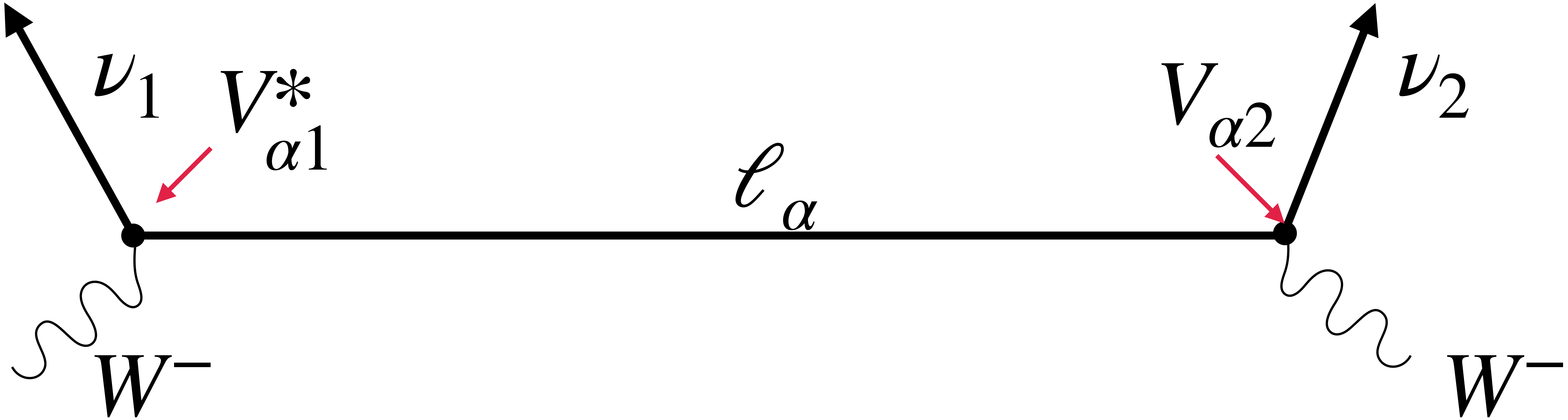
YES, in principle.
NO practically

See also:
E. Akhmedov JHEP09(2007)116

$$\mathscr{A} \propto \sum_{\alpha=e,\mu,\tau}$$



$$P_{\nu_1\nu_2}(L/E) = 1 - \frac{1}{2}\sin^2 2\theta \left(1 - e^{-(L/L_{coh})^2} e^{-1/4(\Delta m^2_{e\mu}/\sigma_{m^2})} \cos\frac{\Delta m^2 L}{2E} \right)$$

$$\underbrace{e^{-(L/L_{coh})^2} e^{-1/4(\Delta m^2_{e\mu}/\sigma_{m^2})}}_{\ll 1}$$

Summary

Summary

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$$\mathscr{A} \propto \sum_i$$