Cosmology

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Devoted to the 110th Anniversary of Bruno Pontecorvo

JINR, Dubna, Russia
Standard Model: Major Problems

Gauge fields (interactions): $\gamma, W^\pm, Z, g$
Three generations of matter: $L = (\nu_L, e_L), e_R; Q = (u_L, d_L), d_R, u_R$

- Describes
  - all experiments dealing with electroweak and strong interactions

- Does not describe (PHENO) (THEORY)
  - Neutrino oscillations
  - Dark matter ($\Omega_{DM}$)
  - Baryon asymmetry ($\Omega_B$)
  - Inflationary stage
  - Reheating

Must explain all above ???
Problems in astrophysics... (?)

- Origin of extragalactic magnetic fields
- First stars and reionization of the Universe
- Mechanism of SuperNovae explosion
- Sources of Ultra-high energy cosmic rays (EeV-scale)
- Extremely low IR extragalactic background
- Too old White Dwarfs
- Origin of Fast Radio Bursts
- Origin of ICECUBE neutrinos (PeV-scale)
- Black hole physics
- ...
- Helioseismology vs helioemissivity
- Origin of the heat at the Earth

New Physics and New Cosmology may be either responsible for or testable there.
Each experiment may be unique (unrepeatable):
  – observe only one Universe
  – (so far) registered only one SN explosion
  – might observe only one magnetic monopole (?)
  – can study only one star
  – (so far) can study only one planet
  …
we register photons, neutrinos, gravitational waves, electrons, positrons, protons, nuclei,
  but only photons(?), neutrinos and gravitational waves can point at the source

Can not directly check the model of sources
Can not directly check the media in between
1. General facts and key observables
2. Mystery of Dark Energy
3. Evidences for Dark Matter in astrophysics and cosmology
4. Expanding Universe: mostly useful formulas
5. Neutrino
6. Dark Matter
   - WIMPs
   - Non-thermal mechanisms
"Natural" units in particle physics

\[ \hbar = c = k_B = 1 \]

measured in GeV: energy \( E \), mass \( M \), temperature \( T \)

\[ m_p = 0.938 \text{ GeV}, \quad 1 \text{ K} = 8.6 \times 10^{-14} \text{ GeV} \]

measured in \( \text{GeV}^{-1} \): time \( t \), length \( L \)

\[ 1 \text{ s} = 1.5 \times 10^{24} \text{ GeV}^{-1}, \quad 1 \text{ cm} = 5.1 \times 10^{13} \text{ GeV}^{-1} \]

Gravity (General Relativity): \( V(r) = -G \frac{m_1 m_2}{r} \)

\[ [G] = M^{-2} \]

\[ M_{\text{Pl}} = 1.2 \times 10^{19} \text{ GeV} = 22 \mu g \quad G \equiv \frac{1}{M_{\text{Pl}}^2} \]
“Natural” units in cosmology

<table>
<thead>
<tr>
<th>Unit</th>
<th>Equivalent in CM</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Mpc</td>
<td>$3.1 \times 10^{24}$ cm</td>
<td>distance to object which has a parallax angle of one arcsec</td>
</tr>
<tr>
<td>1 AU</td>
<td>$1.5 \times 10^{13}$ cm</td>
<td>mean Earth-to-Sun distance</td>
</tr>
<tr>
<td>1 ly</td>
<td>$0.95 \times 10^{18}$ cm</td>
<td>distance light travels in one year</td>
</tr>
<tr>
<td>1 pc</td>
<td>$3.3$ ly $= 3.1 \times 10^{18}$ cm</td>
<td>distance to object which has a parallax angle of one arcsec</td>
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</table>

- 100 AU — Solar system size
- 1.3 pc — nearest-to-Sun stars
- 1 kpc — size of dwarf galaxies
- 50 kpc — distance to dwarves
- 0.8 Mpc — distance to Andromeda
- 1-3 Mpc — size of clusters
- 15 Mpc — distance to Virgo
Local Group and nearest galaxies
Outline

1. General facts and key observables
2. Mystery of Dark Energy
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Universe is expanding

Doppler redshift of light

$L \propto a(t)$

$n \propto a^{-3}(t)$

$H(t) = \frac{\dot{a}(t)}{a(t)}$

Hubble parameter

Hubble Law

$H(t_0) r = v_r$
Expansion: redshift $z$

$\lambda_{\text{abs.}}/\lambda_{\text{em.}} \equiv 1 + z$

$z \ll 1$ Hubble law: $z = H_0 r$

$$H_0 = h \cdot 100 \frac{\text{km}}{\text{s} \cdot \text{Mpc}}, \quad h \approx 0.68$$

$z = 2.0841$, $\lambda_{\text{ab}} = 23.38$
Expansion: redshift $z$

$\lambda_{\text{abs.}} / \lambda_{\text{em.}} \equiv 1 + z$

$z \ll 1$ Hubble law: $z = H_0 r$

$H_0 = h \cdot 100 \frac{\text{km}}{\text{s} \cdot \text{Mpc}}$, \hspace{1em} h \approx 0.68$

Hubble Diagram for Cepheids (flow-corrected) standard candles
General facts and key observables

- Type Ia Supernovae → redshift(z)
  - $\sigma = 0.135$ mag

- Cepheids → Type Ia Supernovae
  - $\sigma = 0.130$ mag, $N = 42$

- Geometry → Cepheids
  - 2211.04492

- Geometry: $5 \log D$ [Mpc] + 25
  - Milky Way
  - LMC
  - N4258
  - M31

Dmitry Gorbunov (INR)
Cosmology, 3 November 2023
JINR, AYSS-2023 15/105
General facts and key observables

**Flat - $\Lambda$CDM**

**Early**
- Planck: $67.4^{+0.5}_{-0.5}$
- DES+BAO+BBN: $67.4^{+1.1}_{-1.2}$

**Late**
- SHOES: $74.0^{+1.4}_{-1.4}$
- CCHP: $69.8^{+1.9}_{-1.9}$
- MIRAS: $73.6^{+3.9}_{-3.9}$
- H0LiCOW: $73.3^{+1.7}_{-1.8}$
- MCP: $74.8^{+3.1}_{-3.1}$
- SBF: $76.5^{+4.0}_{-4.0}$

Combining all: $73.3^{+0.8}_{-0.8}$

**Early vs. Late**
- With Cepheids: $73.9^{+1.0}_{-1.0}$ ($5.8\sigma$)
- With TRGB: $72.5^{+1.2}_{-1.2}$ ($4.0\sigma$)
- With MIRAS: $73.8^{+1.4}_{-1.3}$ ($4.4\sigma$)

$H_0$ [km s$^{-1}$ Mpc$^{-1}$]

1907.10625
Universe is homogeneous and isotropic

redshift

\[ z \equiv \frac{\lambda_{\text{detector}}}{\lambda_{\text{source}}} - 1 \]

12434 galaxies
General facts and key observables

Flat $\Lambda$CDM – Growth Tension

- Planck CMB TT, TE, EE + lowE
- Planck CMB TT, TE, EE + lowE + lensing

- WL KiDS–1000
- WL KiDS+VIKING+DES–Y1
- WL KiDS+VIKING+DES–Y1
- WL KiDS+VIKING–450
- WL KiDS+VIKING–450
- WL KiDS–450
- WL KiDS–450
- WL DES Y1 3x2 tpf
- WL DES Y1
- WL HSC–TPCF
- WL HSC–pseudo–C_l

- CC SDSS–DR8
- CC ROSAT
- CC DES Y1
- CC Planck tSZ
- CC Planck tSZ
- CC SPT–tSZ
- CC XMM–XXL

- RSD
- RSD

$S_8 \equiv \sigma_8 \sqrt{\Omega_m / 0.3}$

Planck TT + lowE
Planck TT, TE, EE + lowE
+ DES + lensing
DES $\gamma_t + w$
DES (joint)

$\Omega_m$
The Universe: age & geometry & energy density

\[ [H_0] = L^{-1} = t^{-1} \]

<table>
<thead>
<tr>
<th>time scale: ( t_{H_0} = H_0^{-1} \approx 14 \times 10^9 ) yr</th>
<th>age of our Universe</th>
</tr>
</thead>
<tbody>
<tr>
<td>spatial scale: ( l_{H_0} = H_0^{-1} \approx 4.3 \times 10^3 ) Mpc</td>
<td>size of the visible Universe</td>
</tr>
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</table>

\( t_{H_0} \) is in agreement with various observations

homogeneity and isotropy in 3d:
flat, spherical or hyperbolic

Observations: “very” flat \( R_{\text{curv}} > 10 \times l_{H_0} \)

order-of-magnitude estimate:
\[ \frac{GM_U}{l_U} \sim G\rho_0 l_{H_0}^3/l_{H_0} \sim 1 \]

flat Universe

\[ \rho_c = \frac{3}{8\pi} H_0^2 M_{\text{Pl}}^2 \approx 0.53 \times 10^{-5} \text{GeV/cm}^3 \]

\( \longrightarrow \) 5 protons in each \( 1 \text{ m}^3 \)
Universe is occupied by “thermal” photons

\[ T_0 = 2.726 \text{ K} \]

The spectrum (shape and normalization!) is thermal

\[ n_\gamma = 411 \text{ cm}^{-3} \]
Conclusions from observations

The Universe is homogeneous, isotropic, hot and expanding...

Conclusions

- interval between events gets modified

\[
\Delta s^2 = c^2 \Delta t^2 - a^2(t) \Delta x^2
\]

in GR expansion is described by the Friedmann equation

\[
\left( \frac{\dot{a}}{a} \right)^2 = H^2(t) = \frac{8\pi}{3} G \rho_{\text{energy}}
\]

\[
\rho_{\text{density}} = \rho_{\text{radiation}} + \rho_{\text{matter}} + \ldots
\]

- in the past the matter density was higher, our Universe was “hotter” filled with electromagnetic plasma

\[
\rho_{\text{matter}} \propto 1/a^3(t), \quad \rho_{\text{radiation}} \propto 1/a^4(t), \quad \rho_{\text{curvature}} \propto 1/a^2(t)
\]

certainly known up to \( T \sim 1 \text{ MeV} \sim 10^{10} \text{ K} \)
Astrophysical and cosmological data are in agreement

\[ \left( \frac{\dot{a}}{a} \right)^2 = H^2(t) = \frac{8\pi}{3} G \rho_{\text{density}} \]

\[ \rho_{\text{density}} = \rho_{\text{energy}} + \rho_{\text{ordinary}} + \rho_{\text{dark}} + \rho_{\Lambda} \]

\[ \rho_{\text{energy}} \propto \frac{1}{a^4(t)} \propto T^4(t), \quad \rho_{\text{matter}} \propto \frac{1}{a^3(t)} \]

\[ \rho_{\Lambda} = \text{const} \]

\[ \frac{3H_0^2}{8\pi G} = \rho_{\text{energy}}(t_0) = \rho_c \approx 0.53 \times 10^{-5} \text{ GeV cm}^{-3} \]

- **radiation:** \( \Omega_\gamma \equiv \frac{\rho_\gamma}{\rho_c} = 0.5 \times 10^{-4} \)
- **Baryons (H, He):** \( \Omega_B \equiv \frac{\rho_B}{\rho_c} = 0.05 \)
- **Neutrino:** \( \Omega_\nu \equiv \frac{\sum \rho_\nu_i}{\rho_c} < 0.01 \)
- **Dark matter:** \( \Omega_{DM} \equiv \frac{\rho_{DM}}{\rho_c} = 0.27 \)
- **Dark energy:** \( \Omega_\Lambda \equiv \frac{\rho_{\Lambda}}{\rho_c} = 0.68 \)
General facts and key observables

TODAY

- 2.7 K
  - accelerated expansion
  - matter domination

- 4.4 K
  - recombination
  - matter domination
  - radiation domination

- 0.26 eV
  - primordial nucleosynthesis

- 0.8 eV
  - neutrino decoupling

- 50 keV
  - QCD transition

- 1 MeV
  - Electroweak phase transition

- 2.5 MeV
  - confinement ↔ free quarks

- 100 MeV
  - Electroweak phase transition

- 200 MeV
  - baryogenesis

- 1 MeV
  - neutrino decoupling

- 5 min
  - neutrino decoupling

- 0.1 s
  - neutrino decoupling

- 10 µs
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- 0.1 ns
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- 50 keV
  - primordial nucleosynthesis

- 50 ty
  - matter domination

- 50 ty
  - radiation domination

- 7.7 by
  - matter domination

- 4.4 K
  - accelerated expansion

- 14 by
  - radiation domination

Equations:

- \( e + p \rightarrow H + \gamma \)
- \( ^3H + ^4He \rightarrow ^7Li + \gamma \)
- \( ^2H + ^2H \rightarrow n + ^3He \)
- \( p + p \rightarrow ^2H + \gamma \)

- Dark matter production
- Inflation
- Reheating
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Dark Energy: nonclumping matter?

- estimates of Matter contribution confined in galaxies and clusters
  \( \rho_c - \rho_M \neq 0 \) but the Universe is flat, so \( \rho_{\text{curv}} \approx 0 \)
- corrections to the Hubble law: red shift – brightness curves for standard candles (SN Ia)
- The age of the Universe
- CMB anisotropy, large scale structures (galaxy clusters formation), etc

\[ \rho_\Lambda = 0.68 \rho_c \]

\[ \rho_\Lambda \sim 10^{-5} \text{ GeV/cm}^3 \sim (10^{-11.5} \text{ GeV})^4 \]
Dark Energy: all evidences are from cosmology

Working hypothesis is cosmological constant $\Lambda \approx (2.5 \times 10^{-3} \text{eV})^4$:

$\rho = w(t) \rho$, $w = \text{const} = -1$, $\rho = \Lambda$

$$S_\Lambda = -\Lambda \int d^4x \sqrt{-\det g_{\mu\nu}}$$

both parts contribute

$$S_{\text{grav}} = -\frac{1}{16\pi G} \int d^4x \sqrt{-\det g_{\mu\nu}} R,$$

$$S_{\text{matter}} = \int d^4x \sqrt{-\det g_{\mu\nu}} \left( \frac{1}{2} g^{\lambda\rho} \partial_\lambda \phi \partial_\rho \phi - V(\phi) \right)$$

natural values

$$\Lambda_{\text{grav}} \sim 1/G^2 \sim (10^{19} \text{GeV})^4$$,  $$\Lambda_{\text{matter}} \sim V(\phi_{\text{vac}}) \sim (100 \text{GeV})^4, (100 \text{MeV})^4, \ldots$$

Why $\Lambda$ is small?  Why $\Lambda \sim \rho_{\text{matter}}$?  Why $\rho_B \sim \rho_{\text{DM}} \sim \rho_{\Lambda}$ today?
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Astrophysical and cosmological data are in agreement

\[
\left( \frac{\dot{a}}{a} \right)^2 = H^2(t) = \frac{8\pi}{3} G \rho_{\text{density}}
\]

\[
\rho_{\text{density}} = \rho_{\text{radiation}} + \rho_{\text{ordinary matter}} + \rho_{\text{dark matter}} + \rho_{\Lambda}
\]

\[
\rho_{\text{radiation}} \propto 1/a^4(t) \propto T^4(t), \quad \rho_{\text{matter}} \propto 1/a^3(t)
\]

\[
\rho_\Lambda = \text{const}
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**Radiation:**
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**Dark energy:**
\[
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Galactic dark halos: flat rotation curves

\[ v(R) = \sqrt{\frac{G M(R)}{R}} \]

\[ M(R) = 4\pi \int_0^R \rho(r)r^2 dr \]

observations:

\[ v(R) \simeq \text{const} \]

visible matter:

internal regions \( v(R) \propto \sqrt{R} \)

external ("empty") regions \( v(R) \propto \frac{1}{\sqrt{R}} \)
Matter distribution in the Milky Way

\[
\rho (M_{\odot}) / (\text{kpc}^3) \quad \rho (\text{GeV/cm}^3)
\]

- NFW best fit
- stars
- dust and gas
- baryons

$r / \text{kpc}$
Dark Matter in clusters

X-rays from hot gas in clusters

\[
\frac{dP}{dR} = -\mu n_e(R)m_p \frac{GM(R)}{R^2}, \quad M(R) = 4\pi \int_0^R \rho(r)r^2dr, \quad P(R) = n_e(R)T_e(R)
\]

galaxies in clusters

virial theorem

\[
U + 2E_k = 0
\]

\[
3M\langle \nu_r^2 \rangle = \frac{G M^2}{R}
\]

Milky Way: Virgo infall
Gravitational lensing in GR:

\[ \alpha = \frac{4GM}{c^2 b} \]

Einstein Cross

Source: quasar \( D_s = 2.4 \) Gpc

Lens: galaxy \( D_l = 120 \) Mpc
Dark Matter in clusters

gravitational lensing

\[ \rho_B \approx 0.25 \rho_{DM} \]
Colliding clusters (Bullet clusters 1E0657-558)

gravitational lensing

scale is 200 kpc
clusters are at 1.5 Gpc

Observations in X-rays

\[ M \sim 10 \times m \]
Dark Matter Properties

(If) particles:

1. stable on cosmological time-scale
2. nonrelativistic long before RD/MD-transition (either Cold or Warm, \( v_{RD/MD} \lesssim 10^{-3} \))
3. (almost) collisionless
4. (almost) electrically neutral

If were in thermal equilibrium:

\[ M_X \gtrsim 1 \text{ keV} \]

If not: for bosons

\[ \lambda = \frac{2\pi}{(M_X v_X)}, \text{ in a galaxy } v_X \sim 0.5 \cdot 10^{-3} \rightarrow M_X \gtrsim 3 \cdot 10^{-22} \text{ eV} \]

for fermions

\[ M_X \gtrsim 750 \text{ eV} \]

Pauli blocking:

\[
f(p, x) = \frac{\rho_X(x)}{M_X} \cdot \frac{1}{\left(\sqrt{2\pi M_X v_X}\right)^3} \cdot e^{-\frac{p^2}{2M_X^2 v_X^2}} \Bigg|_{p=0} \leq \frac{g_X}{(2\pi)^3}
\]
Astrophysical and cosmological data are in agreement

\[
\left( \frac{\dot{a}}{a} \right)^2 = H^2(t) = \frac{8\pi}{3} G \rho_{\text{density}}
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\rho_{\text{density}} = \rho_{\text{energy}} + \rho_{\text{ordinary}} + \rho_{\text{dark}} + \rho_{\Lambda}
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\rho_{\Lambda} = \text{const}
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Dark energy:
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\]
Inhomogeneous Universe

Large Scale Structure

CMB anisotropy
Key observable: matter perturbations

- CMB is isotropic, but “up to corrections, of course...”
  1. Earth movement with respect to CMB
     \[ \Delta T_{\text{dipole}} \sim 10^{-3} \]
  2. More complex anisotropy!
     \[ \Delta T \sim 10^{-4} - 10^{-5} \]

- There were matter inhomogeneities \( \Delta \rho / \rho \sim \Delta T / T \) at the stage of recombination \( (e + p \rightarrow \gamma + H^*) \)

- Jeans instability in the system of gravitating particles at rest \( \Rightarrow \Delta \rho / \rho \uparrow \Rightarrow \) galaxies (CDM halos)
Evidences for Dark Matter in astrophysics and cosmology

TODAY

- 2.7 K
- 4.4 K
- 0.26 eV
- 0.8 eV
- 50 keV
- 1 MeV
- 2.5 MeV
- 200 MeV
- 100 GeV
- Electroweak phase transition
- QCD transition
- Neutrino decoupling
- Primordial nucleosynthesis
- Matter domination
- Radiation domination
- Recombination
- 370 ty
- 50 ty
- 5 min
- 1 s
- 0.1 s
- 10 μs
- 0.1 ns
- Inflation
- Hot Universe
- Reheating
- Dark matter production
- Baryogenesis

14 by

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- $e + p \rightarrow H + \gamma$
- $^3H + ^4He \rightarrow ^7Li + \gamma$
- $^2H + ^2H \rightarrow n + ^3He$
- $p + p \rightarrow ^2H + \gamma$
- Confinement $\leftrightarrow$ free quarks
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Einstein equations

\( T_{\mu \nu} \): macroscopic description

\[
T_{\mu \nu} = (\rho + p) u_\mu u_\nu - g_{\mu \nu} p
\]

in the comoving frame \( u^0 = 1, \ u = 0 \)

(almost) always works

\[
T^\nu_\mu = \text{diag}(\rho, -p)
\]

\[
ds^2 = dt^2 - a^2(t) \gamma_{ij} dx^i dx^j
\]

\[
S_{EH} = -\frac{1}{16\pi G} \int d^4 x \sqrt{-g} R : \ R_{\mu \nu} - \frac{1}{2} g_{\mu \nu} R = 8\pi G T_{\mu \nu}
\]

\[
(00): \ \left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi}{3} G \rho - \frac{\kappa}{a^2}
\]
Friedmann equation \[ (00): \quad \left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi}{3} G\rho - \frac{\kappa}{a^2} \]

\[ \nabla_\mu T^{\mu 0} = 0 \rightarrow \dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = 0 \]

the equation of state

\[ p = p(\rho) \]

many-component liquid,

in case of thermal equilibrium

\[ -3d(\ln a) = \frac{d\rho}{\rho + \rho} = d(\ln s) \]

entropy of cosmic primordial plasma is conserved in a comoving frame

\[ sa^3 = \text{const} \]
Examples of cosmological solutions

**radiation:** \( p = \frac{1}{3} \rho \)  
\( \rho = \frac{\text{const}}{a^4} \), \( a(t) = \text{const} \cdot (t - t_s)^{1/2} \), \( \rho(t) = \frac{\text{const}}{(t - t_s)^2} \)

\( t_s = 0 \), \( H(t) = \frac{\dot{a}}{a}(t) = \frac{1}{2t} \), \( \rho = \frac{3}{8\pi G} H^2 = \frac{3}{32\pi G} \frac{1}{t^2} \)

In case of thermal equilibrium \( T = \text{const}/a \)

\( \rho_b = \frac{\pi^2}{30} g_b T^4 \), \( \rho_f = \frac{7}{8} \frac{\pi^2}{30} g_f T^4 \)

\( \rho = \frac{\pi^2}{30} g_* T^4 \), \( g_* = \sum_b g_b + \frac{7}{8} \sum_f g_f = g_*(T) \)
Expanding Universe: mostly useful formulas

TODAY

4.4 K

accelerated expansion
matter domination

2.7 K

0.26 eV

recombination
matter domination
radiation domination

0.8 eV

370 ty

radiation domination

50 ty

50 keV

3H + 4He → 7Li + γ

1 MeV

2H + 2H → n + 3He

2.5 MeV

p + p → 2H + γ

1 s

neutrino decoupling

5 min

QCD transition

200 MeV

confinement ↔ free quarks

10 µs

Electroweak phase transition

1 MeV

baryogenesis

100 GeV

inflation

hot Universe

reheating

0.1 ns

dark matter production

1 s

primordial nucleosynthesis

5 min

50 ty

30 ty

14 by

7.7 by

300 ty

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Friedmann equation for the present Universe

\[ H^2 \equiv \left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi}{3} G (\rho_M + \rho_{rad} + \rho_{\Lambda} + \rho_{\text{curv}}) \]

\[ \frac{8\pi}{3} G \rho_{\text{curv}} = -\frac{\kappa}{a^2} , \quad \rho_c \equiv \frac{3}{8\pi G} H_0^2 \]

\[ \rho_c = \rho_{M,0} + \rho_{rad,0} + \rho_{\Lambda,0} = \rho_c = 0.52 \cdot 10^{-5} \text{GeV cm}^{-3} , \quad \text{for } h = 0.7 \]

\[ \Omega_X \equiv \frac{\rho_{X,0}}{\rho_c} \]

\[ \left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi}{3} G \rho_c \left[ \Omega_M \left( \frac{a_0}{a} \right)^3 + \Omega_{rad} \left( \frac{a_0}{a} \right)^4 + \Omega_{\Lambda} + \Omega_{\text{curv}} \left( \frac{a_0}{a} \right)^2 \right] \]
Expanding Universe: mostly useful formulas

[TODAY]

2.7 K

14 by

4.4 K

7.7 by

accelerated expansion
matter domination

370 ty

recombination
matter domination
radiation domination

50 ty

0.26 eV

50 keV

0.8 eV

1 MeV

2.5 MeV

100 MeV

5 min

3 H + 4 He → 7 Li + γ

50 ty

e + p → H + γ

radiation domination

2 H + 2 H → n + 3 He

0.1 s

p + p → 2 H + γ

0.1 s

qcd transition

neutrino decoupling

5 min

10 μs

confinement↔free quarks

200 MeV

QCD transition

1 s

3 H + 4 He → 7 Li + γ

Electroweak phase transition

100 GeV

100 GeV

2.5 MeV

1 MeV

0.8 eV

0.26 eV

2.7 K

14 by

4.4 K

7.7 by

accelerated expansion
matter domination

hot universe

baryogenesis

reheating

dark matter production

baryogenesis

inflation

Dmitry Gorbunov (INR) Cosmology, 3 November 2023 JINR, AYSS-2023 48/105
Microscopic processes in the expanding Universe

A competition between scattering, decays, etc and expansion for general processes one should solve kinetic equations

\[
\frac{dn_{X_i}}{dt} + 3Hn_{X_i} = \sum (\text{production} - \text{destruction})
\]

Boltzmann equation in a comoving volume: \(\frac{d}{dt} (na^3) = a^3 \int \ldots\)

**production:**
\[\sigma(A + B \rightarrow X + C)n_An_B, \quad \Gamma(D \rightarrow E + X)n_D \cdot M_D/E_D, \quad \text{etc}\]

**destruction:**
\[\sigma(A + X \rightarrow C + B)n_An_X, \quad \Gamma(X \rightarrow F + G)n_X \cdot M_X/E_X, \quad \text{etc}\]

Fast direct and inverse processes, \(\Gamma \gtrsim H\), are in equilibrium, \(\Sigma(\ldots) = 0\) and thermalize particles.
Expanding Universe: mostly useful formulas

Today
- accelerated expansion
- matter domination

Recombination
- recombination
- matter domination
- radiation domination

Primordial nucleosynthesis
- $^3\text{H} + ^4\text{He} \rightarrow ^7\text{Li} + \gamma$
- $^2\text{H} + ^2\text{H} \rightarrow n + ^3\text{He}$
- $p + p \rightarrow ^2\text{H} + \gamma$

Neutrino decoupling
- QCD transition
- $\text{confinement} \leftrightarrow \text{free quarks}$

Hot Universe
- Electroweak phase transition
- $\text{1 MeV}$
- $\text{200 MeV}$
- $\text{50 keV}$

Radiation domination
- $\text{0.8 eV}$

Matter domination
- $\text{0.26 eV}$

Reheating
- $\text{2.7 K}$

Baryogenesis
- $\text{4.4 K}$

Inflation
- $\text{14 by}$

Dark matter production
- $\text{7.7 by}$

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Outline

1. General facts and key observables
2. Mystery of Dark Energy
3. Evidences for Dark Matter in astrophysics and cosmology
4. Expanding Universe: mostly useful formulas
5. Neutrino
   - WIMPs
   - Non-thermal mechanisms
6. Dark Matter
Neutrino freeze-out

\( T > m_e \)

\[ e^+ e^- \leftrightarrow \nu \bar{\nu}, \; ev \leftrightarrow ev \]

\[ \sigma_v \sim G_F^2 E^2 \]

neutrino interaction rate

\[ \tau_v = \frac{1}{\langle \sigma_v n v \rangle} \sim \frac{1}{G_F^2 T^5} \]

\[ H^2 = \frac{8\pi}{3 M_{Pl}^2} \frac{\pi^2}{30} g_* T^4 \equiv \frac{T^4}{M_{Pl}^*} \]

\[ \tau_v(T) \sim H^{-1}(T) = \frac{M_{Pl}^*}{T^2} \]

\[ T_{v,f} \sim \left( \frac{1}{G_F^2 M_{Pl}^*} \right)^{1/3} \sim 2 \div 3 \text{ MeV} \]
Helium abundance (NO chemical equilibrium)

Neutrons remain mostly in helium

\[ n_{4\text{He}}(T_{NS}) = \frac{1}{2} n_n(T_{NS}), \]

neutron-to-proton ratio \( \tau_n \approx 880 \text{ s} \)

\[
\frac{n_n(T_{NS})}{n_p(T_{NS})} \approx \frac{1}{5} \cdot e^{-\frac{t_{NS}}{\tau_n}} \cdot e^{-\frac{\mu_v}{T_n}} \approx \frac{1}{7},
\]

\[ Y_p \equiv X_{4\text{He}} = \frac{m_{4\text{He}} \cdot n_{4\text{He}}(T_{NS})}{m_p (n_p(T_{NS}) + n_n(T_{NS}))} = \frac{2}{\frac{n_p(T_{NS})}{n_n(T_{NS})} + 1} \approx 25\%.
\]

from observations of relic helium abundance:

\[ \Delta N_{\nu,\text{eff}} \leq 0.2, \quad \left| \frac{\mu_v}{T_n} \right| \lesssim 0.01. \]
Neutrino

$\Omega_B h^2$

Planck

Mass fraction

$^4\text{He}$

$^3\text{He}$

$^7\text{Li}$

$\eta\times10^{10}$

$Lack of Lithium... Exotics needed?

Measurement of $\eta_B = n_B/n_\gamma$ at $T \sim 1 \text{ MeV}$
Cosmological limits: sub-eV scale... 14 years ago!!

$\sum m_\nu < 0.28$ eV (95% CL)

LRG+BAO+WMAP5+SNe+BAO

CMB+Hubble measurements
$\sum m_\nu < 0.20$ eV (95% CL)
Baryogenesis

Sakharov conditions of successful baryogenesis

- **B-violation**
  
  \[(\Delta B \neq 0) \quad X Y \ldots \rightarrow X' Y' \ldots B\]

- **C- & CP-violation**
  
  \[(\Delta C \neq 0, \Delta CP \neq 0) \quad \bar{X} \bar{Y} \ldots \rightarrow \bar{X}' \bar{Y}' \ldots \bar{B}\]

- processes above are out of equilibrium
  
  \[X' Y' \ldots B \rightarrow XY\ldots\]

At 100 GeV ≲ T ≲ 10^{12} GeV nonperturbative processes (EW-sphalerons) violate $B, \ L_\alpha$, so that only three charges are conserved out of four, e.g.

\[B - L, \quad L_e - L_\mu, \quad L_e - L_\tau\]

and

\[B = \alpha \times (B - L), \quad L = (\alpha - 1) \times (B - L)\]

Leptogenesis: Baryogenesis from lepton asymmetry of the Universe \ldots due to sterile neutrinos

Why $\Omega_B \sim \Omega_{DM}$?

antropic principle?
Outline

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6. Dark Matter
   - WIMPs
   - Non-thermal mechanisms
Dark Matter Properties

(If) particles:
1. stable on cosmological time-scale
2. nonrelativistic long before RD/MD-transition (either Cold or Warm, $\nu_{RD/MD} \lesssim 10^{-3}$)
3. (almost) collisionless
4. (almost) electrically neutral

If were in thermal equilibrium: $M_X \gtrsim 1 \text{ keV}$

If not: for bosons

$$\lambda = \frac{2\pi}{(M_X \nu_X)} \text{, in a galaxy } \nu_X \sim 0.5 \cdot 10^{-3} \rightarrow M_X \gtrsim 3 \cdot 10^{-22} \text{ eV}$$

for fermions

$$M_X \gtrsim 750 \text{ eV}$$

Pauli blocking:

$$f(p, x) = \frac{\rho_X(x)}{M_X} \cdot \frac{1}{\left(\sqrt{2\pi} M_X \nu_X\right)^3} \cdot e^{-\frac{p^2}{2M_X^2 \nu_X^2}} \bigg|_{p=0} \leq \frac{g_X}{(2\pi)^3}$$
Dark Matter Candidates

- WIMPs (neutralino, ...)
- sterile neutrinos
- gravitino
- axion
- Heavy relics
- (Topological) defects
- Massive Astrophysical Compact Halo Objects
- Primordial black hole remnants
Freeze-out of nonrelativistic Dark Matter

Assumptions:

1. no $X - \bar{X}$ asymmetry
   - either $X = \bar{X}$ or $n_X = n_{\bar{X}}$

2. @ $T \lesssim M_X$ in thermal equilibrium with plasma (e.g. neutrons)

$$n_X = n_{\bar{X}} = g_X \left( \frac{M_X T}{2\pi} \right)^{3/2} e^{-M_X/T}$$

$X\bar{X} \rightarrow$ light particles

freeze-out temperature $T_f$

$$n_X \langle \sigma_{\text{ann}} v \rangle = H(T_f) \quad \Rightarrow \quad T_f = \frac{M_X}{\ln \left( \frac{g_X M_X M^*_\text{Pl} \sigma_0}{(2\pi)^{3/2}} \right)}$$

Bethe formula:

$s$-wave: $\sigma_{\text{ann}} = \frac{\sigma_0}{v}$
Weakly Interacting Massive Particles

density after freeze-out:

\[ n_X(T_f) = \frac{T_f^2}{M_{Pl}^2 \sigma_0} \]

present density:

\[ n_X(T_0) = \left( \frac{a(T_f)}{a(T_0)} \right)^3 n_X(T_f) = \left( \frac{s_0}{s(T_f)} \right) n_X(T_f) \propto \frac{1}{T_f} \]

\( X + \bar{X} \) contribution to critical density:

\[ \Omega_X = 2 \frac{M_X n_X(T_0)}{\rho_c} = 7.6 \frac{s_0 \ln \left( \frac{g_X M_{Pl}^* M_X \sigma_0}{(2\pi)^{3/2}} \right)}{\rho_c \sigma_0 M_{Pl} \sqrt{g^*_f(T_f)}} \]

\[ = 0.1 \cdot \left( \frac{(10 \text{ TeV})^{-2}}{\sigma_0} \right) \frac{10}{\sqrt{g^*_f(T_f)}} \ln \left( \frac{g_X M_{Pl}^* M_X \sigma_0}{(2\pi)^{3/2}} \right) \cdot \frac{1}{2h^2} \]
WIMPs: discussion

\[ \Omega_X = 0.1 \cdot \left( \frac{(10 \text{ TeV})^{-2}}{\sigma_0} \right) \cdot \frac{10}{\sqrt{g^* (T_f)}} \ln \left( \frac{g_X M_{Pl}^* M_X \sigma_0}{(2\pi)^{3/2}} \right) \cdot \frac{1}{2h^2} \]

- natural DM: subweak-scale cross section \( \sigma_0 \sim 0.01 \times \sigma_W \)
  - say, \( M_X \sim 1 \text{ TeV} \) or \( X \) is not a weak gauge eigenstate
- naturally “light” unitarity \( \sigma_0 \lesssim \frac{4\pi}{M_X^2} \rightarrow M_X \lesssim 100 \text{ TeV} \)
- all stable particles with smaller \( \sigma_0 \) are forbidden !!
- WIMPs remain in kinetic equilibrium with plasma till \( T \sim 10 \text{ MeV} \)
  - this is Cold Dark Matter, \( \nu_{RD/MD} \ll 10^{-3} \)
- WIMPs may form dark halos (clumps) much lighter than dwarf galaxies
Weakly IMPs are mostly welcome (e.g. LSP in SUSY)

We can fully explore the model !!

- Direct searches for Galactic Dark Matter ($\nu \sim 10^{-3}$)
  
  \[ X + \text{nuclei} \rightarrow X + \text{nuclei} + \Delta E \]

- Can search for WIMPs in cosmic rays: products of WIMPs annihilation (in Galactic center, dwarf galaxies, Sun)
  
  \[ X + \bar{X} \rightarrow p\bar{p}, \ e^+ e^-, \ \nu, \gamma, \ldots \]

- Can search for WIMPs in collision experiments (LHC): missing
  
  \[ X + \bar{X} \leftrightarrow \text{SM} + \text{SM'} + \ldots \]
Prospects in WIMP searches

- Sensitivity Projections: Spin-Independent Interactions

- DarkSide-50 (2014)
- SuperCDMS SNOLAB
- DEAP3600
- XMASS
- DarkSide-G2
- SuperCDMS
- EDELWEISS
- CRESST

- M.Cirelli (2015)
Constraining the DM model parameter space

![Graph showing the effective $\chi^0_1$-proton cross-section versus dark matter mass $m_{\chi}$ (GeV)].

- **Effective $\chi^0_1$-proton cross-section $\sigma'_{SD,p}$ (cm$^2$)**
- **Dark matter mass $m_{\chi}$ (GeV)**

Key Features:
- **Well-tempered neutralinos**
- **MSSM models not excluded by IC79**
- **MSSM excluded by IC79 depending on channel**
- **M.G. Aartsen et al. (2016)**

Legend:
- **MSSM-25 benchmarks**
  - Excluded at >90% CL
  - Tension (68% – 90% CL)
  - Allowed
- **$\tilde{B} - \tilde{q}$ coann.**
- **Pure $\tilde{h}$**
- **A funnel**

IceCube Collaboration 2016

Area where essentially all MSSM models are excluded by IC79

MSSM excluded by IC79 depending on channel

IceCube Collaboration 2016

M.G. Aartsen et al. (2016)
Present indirect limits on DM annihilation (clumps..)

All ID constraints

status circa 34\textsuperscript{th} ICRC (summer 2015)

DM mass \[\text{GeV}\]

Annihilation cross section \(\langle \sigma v \rangle\) \[\text{cm}^3/\text{s}\]

\(\gamma\)-rays \(\bar{p}\) CMB \(\nu\)

\(\mu\mu\) \(\bar{p}\) CMB \(\nu\)

\(bb\) \(\bar{p}\) CMB \(\nu\)

WW

FERMI

dwarfs

6 yr

CMB

HESS

ICECUBE

ANTARES

AMS

FERMI IGRB

H ESS GC

thermal cross section

M. Cirelli (2015)
LHC limits for annihilation

\[ \sigma \frac{d \sigma}{d m_{\chi}} \text{ [cm}^3\text{s}^{-1}] \]

\( \langle \sigma_{\chi \chi \rightarrow qq} V_{\text{rel}} \rangle \)

95% CL, \( \sqrt{s}=8\text{ TeV}, 20.3\text{ fb}^{-1} \)

- 2 \times (Fermi-LAT dSphs (\chi\chi)\rightarrow u\bar{u}, 4 years)
- 2 \times (HESS 2011 (\chi\chi)\rightarrow q\bar{q}, Einasto profile)
- 2 \times (HESS 2011 (\chi\chi)\rightarrow q\bar{q}, NFW profile)
- D5: \( \chi^\mu \chi^\nu \gamma q \rightarrow (\chi\chi) \)
- D8: \( \chi^\mu \gamma \chi^\nu \gamma q \gamma \rightarrow (\chi\chi) \)

- truncated, coupling = 1
- truncated, max coupling
- thermal relic

Dmitry Gorbunov (INR)
Cosmology, 3 November 2023
JINR, AYSS-2023
If thermal CDM but not **Weakly** IMPs?

We still can study the model if DM annihilates (partly) into SM particles

- But DM particle $X$ can be light and feebly coupled ($t$-channel)

$$\sigma_0 \sim \frac{\xi^4}{M_X^2}$$

$\xi$ is not a gauge coupling within GUT!

- With small $\sigma_0$ one needs entropy production
- $\sigma_0$ may be increased by $s$-channel resonance, $M_Y \approx 2M_X$
- Annihilation can be amplified by co-annihilation channels, $X + A \rightarrow SM$
- With light messengers between Dark and Visible sectors many estimates change, say $\sigma_0 = \sigma_0(\nu)$
- DM interaction at freeze-out and now are not the same
  say, **Sommerfield enhancement** of the annihilation of slow particles $\nu \sim 10^{-3}$
Dark Matter: non-thermal production

1. in the primordial plasma of SM particles (via scatterings (freeze-in), via oscillations):
   - gravitino
   - sterile neutrino of 1-50 keV

2. at phase transitions:
   - axion of $10^{-4} - 10^{-7}$ eV
   - Q-balls
   - strangelets (?)

3. during reheating (after inflation?):
   - black holes
   - any guy coupled (only) to inflaton
   - inflaton decays
   - production by external (inflaton) field
   - Bose-enhancement of coherent production by external field

4. while the Universe expands:
   - gravity produces any particles at $H \sim M_X$
Illustration with a simple example of scalar DM

most general renormalizable coupled to SM:

$$\Delta \mathcal{L} = \frac{1}{2} g_{\mu\nu} \partial_{\mu} S \partial_{\nu} S - \frac{1}{2} m^2 S^2 + g^2 S^2 \Phi^\dagger \Phi - \frac{\lambda}{4} S^4$$

Options:

- **freeze-out:** sufficiently large $g^2$

  $$\sigma_{hh\rightarrow SS} \times n_h \gtrsim H \rightarrow \sigma_{SS\rightarrow \ldots} = \sigma_0, \quad \text{e.g.} \quad \frac{g^4}{(4\pi \ldots)^2 m_S^2} = \sigma_0$$

- **freeze-in:** intermediate $g^2$

  $$\dot{n}_S + 3Hn_S = \sigma_{hh\rightarrow SS} n_h^2 \rightarrow \frac{n_S}{s} = \# \int dT \frac{n_h^2}{sHT} \times \frac{g^4}{T^2} \sim g^4 \frac{M_{Pl}}{m_S} \rightarrow$$

  $$\Omega_S \propto g^4 \rightarrow g^2 \approx 10^{-11}$$

still natural...
Free massive scalar field \( g^2 = 0 \)

\[
\mathcal{L} = \frac{1}{2} g^{\mu \nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} m_\phi^2 \phi^2
\]

Homogeneous scalar field in the expanding Universe

\[
\ddot{\phi} + 3H \dot{\phi} + m_\phi^2 \phi = 0
\]

Two-stage evolution:

\[
m_\phi < H(t) \implies \phi = \phi_i = \text{const}
\]

\[
m_\phi > H(t) \implies p = \langle E_k \rangle - \langle E_p \rangle = 0, \quad \rho \sim m_\phi^2 \phi^2 \propto 1/a^3
\]

- dust-like substance in the late Universe, \( \Omega \propto m_\phi^{1/2} \phi_i^2 \)
  - depends on initial conditions
- pressureless at spatial scales \( l > M_{Pl}^{1/2}/\rho^{1/4} m_\phi^{1/2} \)
  - fuzzy DM
- isocurvature mode: \( \delta \rho_\phi \propto \delta H, \; \delta f_i \)
scalar DM without dependence on initial field

$0 \neq g^2 < 10^{-11}$

$\Delta \mathcal{L} = \frac{1}{2} g^{\mu \nu} \partial_\mu S \partial_\nu S - \frac{1}{2} m^2 S^2 + g^2 S^2 \Phi \Phi^\dagger - \frac{\lambda}{4} S^4$

Higgs particles in plasma change the potential:

$g^2 S^2 \Phi \Phi \rightarrow g^2 S^2 \frac{T^2}{3}$

$Z_2$ symmetry is broken after reheating by the plasma contribution
Temperature decrease restores $Z_2$

\[
\Delta L = \frac{1}{2} g^{\mu \nu} \partial_\mu S \partial_\nu S - \frac{1}{2} m^2 S^2 + g^2 S^2 T^2 / 3 - \frac{\lambda}{4} S^4
\]

$S$ starts from the false vacuum

at $g^2 T^*_2 \simeq m^2$ sign changes and $S$ starts to oscillate gravitational misalignment

And the correct amount of DM by classical oscillating field

\[
g^2 \simeq 10^{-12} \times \left( \frac{\lambda}{10^{-6}} \right)^{6/5} \times \left( \frac{10^6 \text{GeV}}{m} \right)^2
\]
Dark Matter: many well-motivated candidates

- WIMPs related to EW scale, SUSY
- sterile neutrinos active neutrino oscillations
- light scalar field string theory
- axion strong CP-problem
- gravitino local SUSY
- Heavy relics GUTs
- (Topological) defects GUTs
- Massive Astrophysical Compact Heavy Objects Phase transitions
- Primordial black hole (remnants) exotic inflation, reheating

Multicomponent Dark Matter?

$\gamma, \nu, H, He$
Standard cosmological model \( ds^2 = dt^2 - a^2(t)dx^2 \)

\[
\left( \frac{\dot{a}}{a} \right)^2 \equiv H^2 = H_0^2 \left[ \Omega_\Lambda + (\Omega_{DM} + \Omega_B + \Omega_{\nu,m\neq0}) \left( \frac{a_0}{a} \right)^3 + (\Omega_\gamma + \Omega_{\nu,m=0}) \left( \frac{a_0}{a} \right)^4 \right]
\]

- \( T_\gamma = 2.735 \text{ K} \), \( \implies \Omega_\gamma \sim 10^{-5} \)
- \( N_\nu \approx 3, \sum m_\nu < 0.2 \text{ eV} \implies \Omega_\nu,\neq0, \Omega_{\nu,0} \sim 10^{-5} \)?
- \( \Omega_B = 4.5\% \implies \eta_B = n_B/n_\gamma = 6 \times 10^{-10} \)
- \( \Omega_{DM} = 27.5\% \)
- \( H_0 = 67 \text{ km/s/Mpc} \implies \rho_0 = 5 \text{ GeV/m}^3 \)
- \( \Omega_\Lambda = 68\% \implies \text{flat space} \)
- adiabatic, gaussian matter perturbations
  \[
  \left\langle \left( \frac{\delta \rho}{\rho} \right)^2 \right\rangle \sim A_S \int \frac{dk}{k} \left( \frac{k}{k_*} \right)^{n_S - 1}
  \]
  with \( A_S = 3 \times 10^{-9} \) and \( n_S = 0.97 \)
- no tensor perturbations, \( r = A_T/A_S < 0.05 \)
- reionization at \( z = a_0/a = 10 \)
Weakly Interacting Massive Particles

Assumptions:

1. no $X - \bar{X}$ asymmetry
2. @ $T < M_X$ in thermal equilibrium with plasma

\[ n_X = n_{\bar{X}} = n_X = g_X \left( \frac{M_X T}{2\pi} \right)^{3/2} e^{-M_X/T} \]

$X\bar{X} \rightarrow$ light particles

freeze-out temperature $T_f$

\[ \frac{1}{n_X} \frac{1}{\langle \sigma_{\text{ann}} v \rangle} = H^{-1} (T_f) \rightarrow T_f = \frac{M_X}{\ln \left( \frac{g_X M_X M_{Pl}^* \sigma_0}{(2\pi)^{3/2}} \right)} . \]

Bethe formulae: $s$-wave: $\sigma_{\text{ann}} = \frac{\sigma_0}{v}$
Weakly Interacting Massive Particles

density after freeze-out: 

\[ n_X(T_f) = \frac{T_f^2}{M_p^2 \sigma_0} \]

present density: 

\[ n_X(T_0) = \left( \frac{a(T_f)}{a(T_0)} \right)^3 n_X(T_f) = \left( \frac{s_0}{s(T_f)} \right) n_X(T_f) \propto \frac{1}{T_f} \propto \frac{1}{M_X} \]

\[ X + \bar{X} \] contribution to critical density:

\[ \Omega_X = 2 \frac{M_X n_X(T_0)}{\rho_c} = 7.6 \frac{s_0 \ln \left( \frac{g_X M_p^* M_X \sigma_0}{(2\pi)^{3/2}} \right)}{\rho_c \sigma_0 M_p \sqrt{g^*(T_f)}} = 0.1 \cdot \left( \frac{(10 \text{ TeV})^{-2}}{\sigma_0} \right) \frac{0.3}{\sqrt{g^*(T_f)}} \ln \left( \frac{g_X M_p^* M_X \sigma_0}{(2\pi)^{3/2}} \right) \cdot \frac{1}{2h^2} \]

natural dark matter: \[ \sigma_0 \sim 0.01 \times \sigma_W \]

naturally “light” \[ \sigma_0 \lesssim \frac{4\pi}{M_X^2} \rightarrow M_X \lesssim 100 \text{ TeV} \]
Recent results of (in)direct searches

**a) Spin Independent**

- CMS MonoJet
- CMS MonoPhoton
- CDF 2012
- XENON-100
- CoGeNT 2011
- CDMSII 2011
- CDMSII 2010

**b) Spin Dependent**

- CMS MonoJet
- CMS MonoPhoton
- CDF 2012
- SIMPLE 2010
- CDMSII 2010
- COUPP 2011
- Super-K W+W
- IceCube W+W

for WIMPs

1206.5663

there are analyses for lower mass ranges and other type of interactions:

e.g. 1206.2644
Decoupling of relativistic specia (DM?)

Thermal equilibrium is forbidden:

\[ T_d \gg M_X, \text{ and then } \frac{n_X}{s} = \text{const} \]

\[
\frac{\Omega_{3/2}}{\rho_c} = \frac{m_X \cdot n_{X,0}}{\rho_c} = \frac{m_X \cdot s_0}{s_0} \frac{n_{X,0}}{s_0} = 0.2 \frac{M_X}{100 \text{ eV}} \left( \frac{g_X}{2} \right) \cdot \left( \frac{100}{g^*_X(T_d)} \right) \cdot \frac{1}{2h^2}
\]

- If fermions: limit from Pauli-blocking
- Generally: too hot at Equality:
  from structure formation we need at \( T_{Eq} \sim 1 \text{ eV}, \) \( v_{DM} \lesssim 10^{-3} \)

NB: for \( M_X = 100 \text{ eV} \) at Equality \( (T_{Eq} \sim 1 \text{ eV}) \) \( X \)-particle velocities are ▢
Decoupling of relativistic species (DM?)

Thermal equilibrium is forbidden:

\[ T_d \gg M_X, \text{ and then } n_X/s = \text{const} \]

\[ \Omega_{3/2} = \frac{m_X \cdot n_{X,0}}{\rho_c} = \frac{m_X \cdot s_0}{\rho_c} \cdot \frac{n_{X,0}}{s_0} = 0.2 \frac{M_X}{100 \text{ eV}} \left( \frac{g_X}{2} \right) \cdot \left( \frac{100}{g^* (T_d)} \right) \cdot \frac{1}{2h^2} \]

- If fermions: limit from Pauli-blocking
- Generally: too hot at Equality:
  from structure formation we need at \( T_{\text{Eq}} \sim 1 \text{ eV} \), \( v_{\text{DM}} \lesssim 10^{-3} \)

\textbf{NB}: for \( M_X = 100 \text{ eV} \) at Equality \( (T_{\text{Eq}} \sim 1 \text{ eV}) \) \( X \)-particle velocities are \( v \sim 10^{-2} \)
Other Dark Matter candidates are not in equilibrium!

- **WIMPs (neutralino, ...)** ⇐ thermal !
- **sterile neutrinos** ⇐ Price: sensitive to mass and couplings!
- **axion** ⇐ Price: sensitive to mass and (=couplings)!
- **gravitino** ⇐ Price: sensitive to mass, couplings and reheating temperature !!!
- **Heavy relics**
- **(Topological) defects**
- **Massive Astrophysical Compact Halo Objects**
- **Primordial black hole remnants**
Dark Matter

Non-thermal mechanisms

CDM

FDM

WDM

sDAO

$\log (\Sigma_{dm} [M_\odot / kpc^2])$

1 Mpc

1 Mpc

1 Mpc

1 Mpc

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Cosmology, 3 November 2023

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DM keV sterile neutrino

Sterile neutrino of keV scale mass provides the Warm Dark Matter

Relevant parameters: mass $M_N \sim 1\text{-}10\text{ keV}$ and active-sterile neutrino mixing angle $\theta \ll 1$

Bounds on mass

- Phase space density (refined Pauli-blocking): $M_N \gtrsim 0.3\text{ keV}$
- Lyman-$\alpha$ forest: $M_N \gtrsim 10\text{ keV}$

Bound on mass $M_N$ and mixing angle $\theta$

- $X$-ray observation: $N \rightarrow \nu + \gamma$, a peak at $\omega_\gamma = M_N/2$ of intensity $\propto \theta^2$

Production mechanism

- Dodelson-Widrow (thermal) scenario: $\nu_a \rightarrow N$ due to mixing,

$$\rho_N \propto \theta^2$$

- Primordial abundance: physics at higher energies
  - Lepton asymmetries
  - Production from inflaton decay
  - etc.
Dark Matter
Non-thermal mechanisms

\[ \sin^2 2\theta \]

- LMC (XMM-Newton)
- Milky Way (XMM-Newton)
- Milky Way (HEAO-1)

Case 1 - 2 (mean)
Absolute upper bound
Absolute lower bound

\[ M_1 / \text{keV} \]
DM keV sterile neutrino

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- Phase space density (refined Pauli-blocking): $M_N \gtrsim 0.3$ keV
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Production mechanism

- Dodelson-Widrow (thermal) scenario: $\nu_a \rightarrow N$ due to mixing,

  $\rho_N \propto \theta^2$

  is ruled out

- Primordial abundance: physics at higher energies
  - Lepton asymmetries
  - Production from inflaton decay
  - etc.
Free scalar field as Cold Dark Matter (axion)

Homogeneous scalar field

\[ \ddot{\phi} + 3H \dot{\phi} + m^2 \phi = 0 \]

at \( m \ll H \) no evolution: \( \phi = \text{const} \), at \( m \gg H \) it oscillates, so

\[ \rho = \frac{1}{2} \left( \frac{d\phi}{dt} \right)^2 + \frac{m^2}{2} \phi^2 = \langle E_k \rangle + \langle E_p \rangle = 2\langle E_p \rangle, \quad p = \frac{1}{2} \left( \frac{d\phi}{dt} \right)^2 - \frac{m^2}{2} \phi^2 = \langle E_k \rangle - \langle E_p \rangle = 0, \]

behaves as nonrelativistic (dark) matter (dust-like component) !!

nonperturbative CP-violation in QCD

\[ L_\theta = \frac{\alpha_s}{8\pi} \left( \theta_0 + \text{Arg} \left( \text{Det} \hat{M}_q \right) \right) G^a_{\mu\nu} \tilde{G}^{\mu\nu a} \equiv \frac{\alpha_s}{8\pi} \cdot \theta \cdot G^a_{\mu\nu} \tilde{G}^{\mu\nu a}. \]

\[ \theta \to \bar{\theta}(x) = \theta + C_g \frac{a(x)}{f_{PQ}}. \]

\[ \mathcal{L} = \frac{f_{PQ}^2}{2} \left( \frac{d\bar{\theta}}{dt} \right)^2 - \frac{m_a(T)}{2} f_{PQ}^2 \bar{\theta}^2, \]

\[ m_a(T) \simeq 0, \quad T > \Lambda_{QCD} \quad \text{and} \quad m_a(T) \simeq m_a \simeq m_\pi f_\pi / f_{PQ} \]

Check this \[ \Omega_a \simeq 0.2 \cdot \bar{\theta}_i^2 \cdot \left( \frac{4 \cdot 10^{-6} \text{eV}}{m_a} \right) \cdot \frac{1}{2h^2} \]
Inhomogeneous Universe

Large Scale Structure

CMB anisotropy
Small inhomogeneities in the expanding Universe

matter perturbations (perfect fluid approximation)

\[ T_0^0 \rightarrow \rho(t) + \delta \rho(\eta, \mathbf{x}), \quad T_i^0 \rightarrow \partial_i \nu(\eta, \mathbf{x}), \quad T_j^i \rightarrow \delta \rho(\eta, \mathbf{x}) \]

gravitational perturbations (scalar and tensor modes)

\[ ds^2 = a^2(\eta) \left[ (1 + 2 \Phi(\eta, \mathbf{x})) d\eta^2 - (1 + 2 \psi(\eta, \mathbf{x})) d\mathbf{x}^2 - h^{TT}_{ij}(\eta, \mathbf{x}) d\mathbf{x}^i d\mathbf{x}^j \right] \]

Equations for linear perturbations, \( \delta \rho / \rho \equiv \delta \ll 1, \Phi \ll 1 \), etc

\[ R_{\mu \nu} + \frac{1}{2} R g_{\mu \nu} = 8 \pi G T_{\mu \nu} \rightarrow \ldots \]

\[ \nabla_\mu T^{\mu \nu} = 0 \rightarrow \ldots \]
These inhomogeneities (matter perturbations) originate from the initial matter density (scalar) perturbations

\[ \frac{\delta \rho}{\rho} \sim \frac{\delta T}{T} \sim 10^{-4}, \text{ which are} \]

adiabatic

\[ \delta \left( \frac{n_B}{s} \right) = \delta \left( \frac{n_{DM}}{s} \right) = \delta \left( \frac{n_L}{s} \right) \]

Gaussian

\[ \langle \frac{\delta \rho}{\rho} (\mathbf{k}) \frac{\delta \rho}{\rho} (\mathbf{k}') \rangle \propto \left( \frac{\delta \rho}{\rho} (\mathbf{k}) \right)^2 \times \delta (\mathbf{k} + \mathbf{k}') \]

flat spectrum

\[ \langle \left( \frac{\delta \rho}{\rho} (\mathbf{x}) \right)^2 \rangle = \int_0^{\infty} \frac{dk}{\mathbf{k}} \, \mathcal{P}_S(\mathbf{k}) \quad \mathcal{P}_S(\mathbf{k}) \approx \text{const} \]

LSS and CMB

\[ \mathcal{P}_S \equiv A_S \times \left( \frac{k}{k_*} \right)^{n_s - 1} \quad A_S \approx 2.5 \times 10^{-9}, \quad n_s \approx 0.97 \]
Subhorizon modes \( (k/a > H) \) at various stages

\[ |\delta_\lambda| \]

- \( \propto c_1 + c_2 \log a \)
- \( \propto a^{-2} \)
- \( \propto a \)
- \( \propto a^{-1} \)

\[ \delta_{CDM}, \delta_B, \delta_\gamma, \Phi \]
On formulas...

- **short waves,** \( k \eta_{eq} \gg 1 \)

\[
\delta_{\gamma} = \Phi(i) \cdot \left[ -324 \cdot (1 + R_B) I^2(\Omega_M) \frac{\Omega_{CDM}}{\Omega_M} (1 + z_{eq}) \frac{\log(0.2k\eta_{eq})}{(k\eta_0)^2} \right. \\
\left. + \frac{6}{(1 + R_B)^{1/4}} \cos \left( k \int_0^\eta d\tilde{\eta} u_s \right) \right],
\]

- **long waves,** \( k \eta_{rec} \ll 1 \)

\[
\delta_{\gamma} = -\frac{12}{5} \Phi(i) = \text{const}
\]

- **intermediate waves ...**

\[
\delta_{\gamma}(k, \eta) = -4 \left[ 1 + R_B(\eta) \right] \Phi(k, \eta) + 4\Phi(i)(k) \cdot A(k, \eta) \cos \left( k \int_0^\eta u_s d\tilde{\eta} \right),
\]

\[ R_B \equiv 3\rho_B/4\rho_\gamma \]
Cosmological (particle) horizon $l_H(t)$

distance covered by photons emitted at $t = 0$

the size of causally-connected region — the size of the visible part of the Universe

in conformal coordinates:

$ds^2 = 0 \rightarrow |d\mathbf{x}| = d\eta$

coordinate size of the horizon equals

$\eta(t) = \int d\eta$

$$l_H(t) = a(t)\eta(t) = a(t) \int_0^t \frac{dt'}{a(t')}$$

**dust**

$$l_H(t) = 3t = \frac{2}{H(t)} , \quad l_{H,0} = 2.6 \times 10^{28} \text{cm} \quad (h = 0.7)$$
Last scattering: $\gamma e \rightarrow \gamma e$

$$\sigma_T = \frac{8\pi \alpha^2}{3} \frac{1}{m_e^2} \approx 0.67 \times 10^{-24} \text{ cm}^2,$$

$$\tau_\gamma = \frac{1}{\sigma_T \cdot n_e(T)}$$

last scattering: $$\tau_\gamma(T_f) \approx H^{-1}(T_f) \approx t_f$$

$$T_f = 0.26 \text{ eV}, \quad z = 1100, \quad t_f = 370 \,000 \text{ yr}$$

for general processes one should solve kinetic equations

$$\frac{dn_{X_i}}{dt} + 3Hn_{X_i} = \int (\text{production} - \text{destruction})$$

Boltzmann equation in a comoving volume: $$\frac{d}{dt} (n a^3) = a^3 \int \ldots$$
Recombination: horizon

**matter domination:**

\[ H_r^2 = \frac{8\pi}{3} G \rho_{M}(t_r) = \frac{8\pi}{3} G \rho_{M,0} \left( \frac{a_0}{a_r} \right)^3 = \frac{8\pi}{3} G \rho_c \Omega_{M,0} (1 + z_r)^3. \]

**at recombination:**

\[ l_{H,r} = 2 H_r^{-1} \]

**today:**

\[ l_{H,r}(t_0) = l_{H,r} \times \frac{a_0}{a_r} = \frac{2}{H_0 \sqrt{\Omega_M}} \frac{1}{\sqrt{1+z_r}} \]

\[ \frac{l_{H_0}}{l_{H,r}(t_0)} \sim \sqrt{1 + z_r} \simeq 30 \]
Recombination: angle

angular distance:

\[ d_{ph} = r_a(z) \Delta \theta \]

\[ \chi_r = \int_{t_r}^{t_0} \frac{dt}{a(t)}, \quad \Delta \theta_r = \frac{l_{H,r}}{r_a(z_r)} \]

\[ d_{conf} = \sinh \chi_r \Delta \theta \]

\[ r_a(z_r) = (1 + z_r)^{-1} \cdot a_0 \cdot \sinh \chi_r \]

\[ \Delta \theta_r = \frac{1}{\sqrt{z_r + 1}}, \quad \Omega_{curv} = \Omega_\Lambda = 0 . \]

\[ \Delta \theta_r = \frac{1}{\sqrt{z_r + 1}} \frac{2 \sqrt{\Omega_{curv}/\Omega_M}}{\sinh \left( 2 \sqrt{\Omega_{curv}/\Omega_M} \right)} . \]

\[ l = \int_0^1 \frac{dy}{\sqrt{1 + \frac{\Omega_\Lambda}{\Omega_M} y^6}} \]

Acoustic oscillations in relativistic plasma:
What matters is the sound horizon:

\[ l_{s,r} = l_{H,r} \cdot v_s \approx l_{H,r}/\sqrt{3} \]

Then

\[ \Delta \theta_{r,s} = \frac{1}{\sqrt{3}} \frac{1}{\sqrt{1 + \frac{\Omega_\Lambda}{\Omega_M}}} \times \frac{180^\circ}{\pi} \sim 1^\circ \]
$$110/0.7\text{Mpc} \simeq l_{H,r}(t_0) \times \sqrt{v_s^2} \simeq l_{H_0}/\sqrt{3}/\sqrt{1+z_r}$$
CMB map
Mode evolution

- Amplitude remains constant, while superhorizon, e.g. $k/a < H$
- Subhorizon inhomogeneities of DM start to grow at MD-stage, $\delta \rho_{\text{CDM}}/\rho_{\text{CDM}} \propto a$ from $T \approx 0.8$ eV
- Smaller objects (first stars, dwarf galaxies) are first to form
- Subhorizon inhomogeneities of baryons join those of DM only after recombination, $\delta \rho_{\text{CDM}}/\rho_{\text{CDM}} \propto a$ from $T_{\text{rec}} \approx 0.25$ eV
- At recombination $\delta \rho_B/\rho_B \sim \delta T/T \sim 10^{-4}$ and would grow only by a factor $T_{\text{rec}}/T_0 \sim 10^3$ without DM
- Subhorizon inhomogeneities of photons $\delta \rho_\gamma/\rho_\gamma$ oscillate with constant amplitude at RD and with decreasing amplitude at MD, thus we can measure $T_{\text{RD}/\text{MD}}/T_{\text{rec}}$
- Phase of oscillations decoupled after recombination depends on the wave-length, recombination time and sound speed

$$\delta \rho_\gamma/\rho_\gamma \propto \cos \left( k \int_0^{t_r} \frac{v_s dt}{a(t)} \right) = \cos(kl_{\text{sound}})$$

$$\delta T(\theta, \varphi) = \sum a_{lm} Y_{lm}(\theta, \varphi), \quad \langle a_{lm}^* a_{lm} \rangle = C_l \equiv 2\pi D_l/(l(l+1))$$
Mode evolution at various stages

\[ |\delta_\lambda| \]

\[ \delta_{CDM} \]

\[ \delta_B \]

\[ \delta_\gamma \]

\[ \Phi \]

\[ \propto t^{-2} \]

\[ \propto c_1 + c_2 \log \eta \]

\[ \propto t^{2/3} \]

\[ \propto t^{-2/3} \]

\[ I_a \quad I_b \quad II_a \quad II_b \quad III \]

\[ t_X \quad t_{eq} \quad t_r \quad t_{acc} \]
On formulas...

- **short waves,** $k \eta_{eq} \gg 1$

  $$
  \delta_\gamma = \Phi(i) \cdot \left[ -324 \cdot (1 + R_B) I^2 \frac{\Omega_{CDM}}{\Omega_M} (1 + z_{eq}) \log(0.2k \eta_{eq}) \right] \frac{(k \eta_0)^2}{(1 + R_B)^{1/4}} \cos \left( k \int_0^\eta d\tilde{\eta} u_s \right),
  $$

- **long waves,** $k \eta_{rec} \ll 1$

  $$
  \delta_\gamma = -\frac{12}{5} \Phi(i) = \text{const}
  $$

- **intermediate waves** ...

  $$
  \delta_\gamma(k, \eta) = -4 \left[ 1 + R_B(\eta) \right] \Phi(k, \eta) + 4 \Phi(i)(k) \cdot A(k, \eta) \cos \left( k \int_0^\eta u_s d\tilde{\eta} \right),
  $$

  $$
  R_B \equiv 3\rho_B/4\rho_\gamma
  $$
On top of that: propagation in expanding Universe
On formulas... 

From linear approximation to the geodesic equation... for scalar perturbations

\[
\frac{\delta T}{T} (n, \eta_0) = \frac{1}{4} \delta \gamma (\eta_r) + (\Phi (\eta_r) - \Phi (\eta_0)) \\
\quad + \int_{\eta_r}^{\eta_0} (\Phi' - \Psi') \, d\eta \\
\quad + nv (\eta_r) - nv (\eta_0).
\]

for tensor perturbations

\[
\frac{\delta T}{T} (n, \eta_0) = \frac{1}{2} \int_{\eta_r}^{\eta_0} d\eta \, n_i h_{ij}^{TT'} n_j,
\]
CMB measurements (Planck) \( \theta, \Omega_{DM}, \Omega_B, \tau, \Delta R, n_s \)

Angular scale

Multipole moment, \( \ell \)