Cosmology

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JINR, Dubna, Russia

Standard Model: Major Problems

Gauge fields (interactions): γ, $W^{\pm}$, Z, g

Three generations of matter: $L = \binom{\nu_L}{e_L}$, e_R; $Q = \binom{u_L}{d_L}$, d_R, u_R

- Describes
  - all experiments dealing with electroweak and strong interactions
- Does not describe

(PHENO)

- Neutrino oscillations
- Dark matter (Ω_{DM})
- Baryon asymmetry (Ω_B)
- Inflationary stage
- Reheating

(THEORY)

- Dark energy (Ω_Λ)
- Strong CP-problem
- Gauge hierarchy
- Quantum gravity

Must explain all above

???

Problems in astrophysics. . . (?)

- Origin of extragalactic magnetic fields
- First stars and reionization of the Universe
- Mechanism of SuperNovae explosion
- Sources of Ultra-high energy cosmic rays (EeV-scale)
- Extremely low IR extragalactic background
- Too old White Dwarfs
- Origin of Fast Radio Bursts
- Origin of ICECUBE neutrinos (PeV-scale)
- Black hole physics
- . . .
- Helioseismology vs helioemissivity
- Origin of the heat at the Earth

New Physics and New Cosmology may be

either responsible for

or testable there

Experimental data in Cosmology and Astrophysics

- Each experiment may be unique (unrepeatable):
  – observe only one Universe
  – (so far) registered only one SN explosion
  – might observe only one magnetic monopole (?)
  – can study only one star
  – (so far) can study only one planet
  ...
- we register photons, neutrinos, gravitational waves, electrons, positrons, protons, nuclei,
  but only photons(?), neutrinos and gravitational waves can point at the source
- Can not directly check the model of sources
- Can not directly check the media in between

Outline

1. General facts and key observables
2. Mystery of Dark Energy
3. Evidences for Dark Matter in astrophysics and cosmology
4. Expanding Universe: mostly useful formulas
5. Neutrino
6. Dark Matter
    - WIMPs
    - Non-thermal mechanisms

"Natural" units in particle physics

$$\hbar = c = k_B = 1$$

measured in GeV: energy E, mass M, temperature T

$m_p = 0.938$ GeV, $\quad 1\ \text{K} = 8.6 \times 10^{-14}$ GeV

measured in GeV^{-1}: time t, length L

$1\ \text{s} = 1.5 \times 10^{24}\ \text{GeV}^{-1}, \quad 1\ \text{cm} = 5.1 \times 10^{13}\ \text{GeV}^{-1}$

Gravity (General Relativity): $V(r) = -G\frac{m_1 m_2}{r}$ $\qquad [G] = M^{-2}$

$$M_{\rm Pl} = 1.2 \times 10^{19}\ \text{GeV} = 22\ \mu\text{g} \qquad G \equiv \frac{1}{M_{\rm Pl}^2}$$

"Natural" units in cosmology

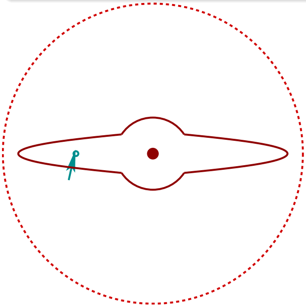
$$1\ \text{Mpc} = 3.1 \times 10^{24}\ \text{cm}$$

$1\ \text{AU} = 1.5 \times 10^{13}\ \text{cm}$ — mean Earth-to-Sun distance

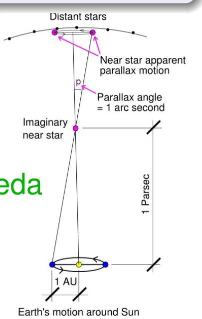
$1\ \text{ly} = 0.95 \times 10^{18}\ \text{cm}$ — distance light travels in one year

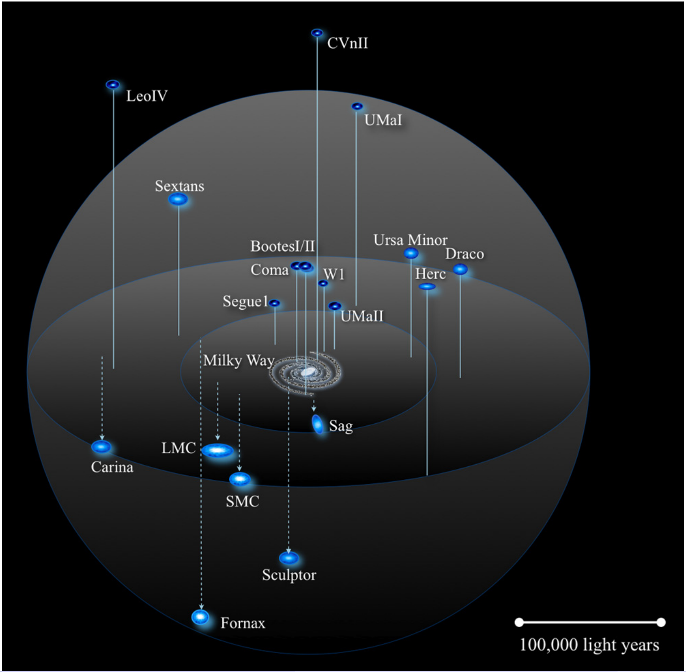
$1\ \text{yr} = 3.16 \times 10^{7}\ \text{s}$

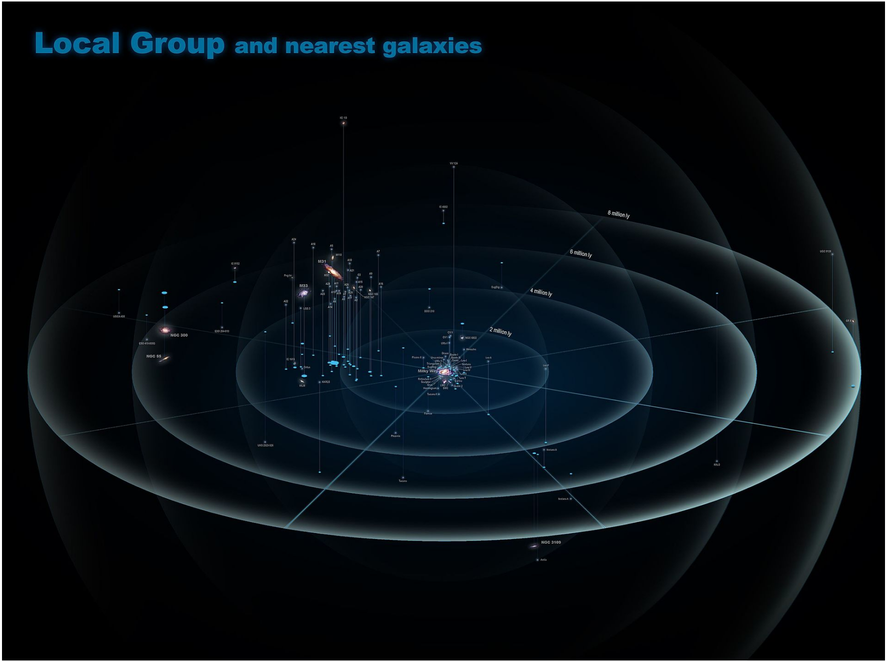
$1\ \text{pc} = 3.3\ \text{ly} = 3.1 \times 10^{18}\ \text{cm}$ — distance to object which has a parallax angle of one arcsec



100 AU — Solar system size
1.3 pc — nearest-to-Sun stars
1 kpc — size of dwarf galaxies
50 kpc — distance to dwarves
0.8 Mpc — distance to Andromeda
1-3 Mpc — size of clusters
15 Mpc — distance to Virgo




CVnII
LeoIV
UMaI
Sextans
BootesI/II
Ursa Minor
Draco
Coma
W1
Herc
Segue1
UMaII
Milky Way
Sag
LMC
Carina
SMC
Sculptor
Fornax
100,000 light years


Local Group and nearest galaxies
8 million ly
6 million ly
4 million ly
2 million ly
M31
M33
NGC 300
NGC 55
Milky Way
NGC 3109

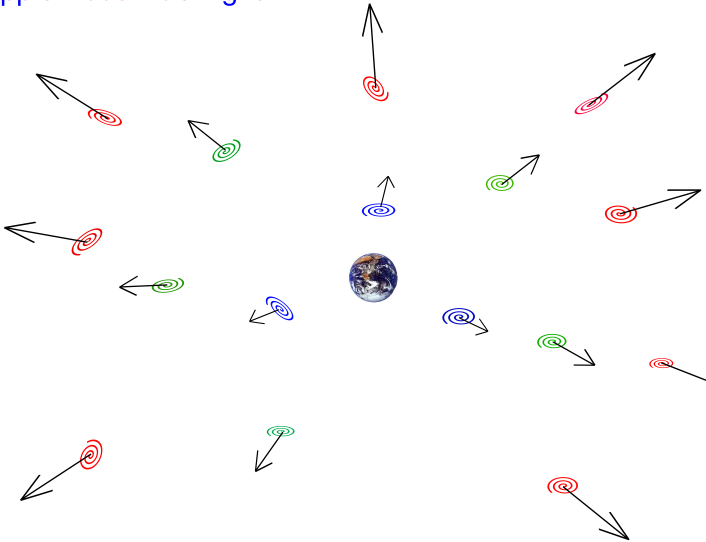
Outline

Universe is expanding

Doppler redshift of light

$L \propto a(t)$

$n \propto a^{-3}(t)$



$H(t) = \frac{\dot{a}(t)}{a(t)}$

Hubble parameter

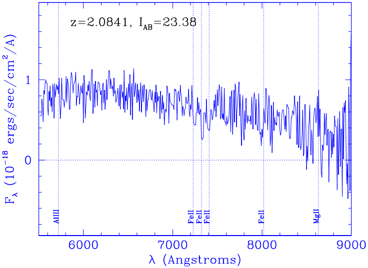
Hubble Law

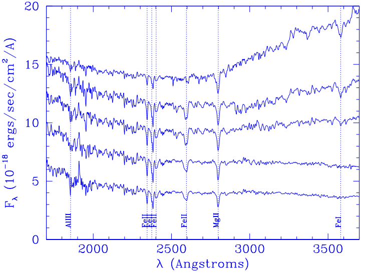
$H(t_0)\, r = v_r$

Expansion: redshift z $\qquad \lambda_{abs.}/\lambda_{em.} \equiv 1 + z$

$z \ll 1$ Hubble law : $z = H_0 r$

$$H_0 = h \cdot 100 \, \frac{\text{km}}{\text{s} \cdot \text{Mpc}} \, , \qquad h \approx 0.68$$





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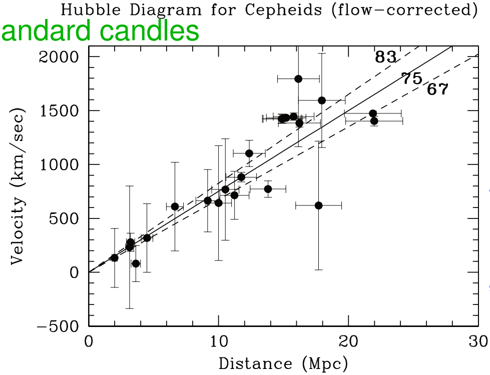
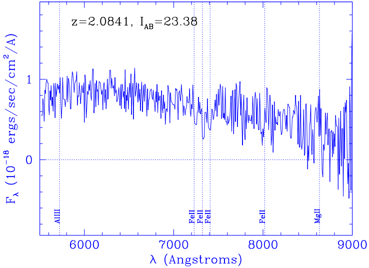
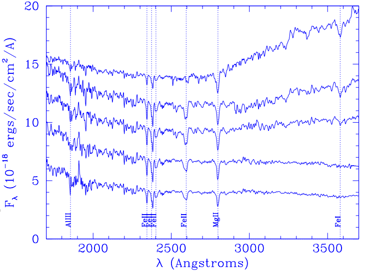
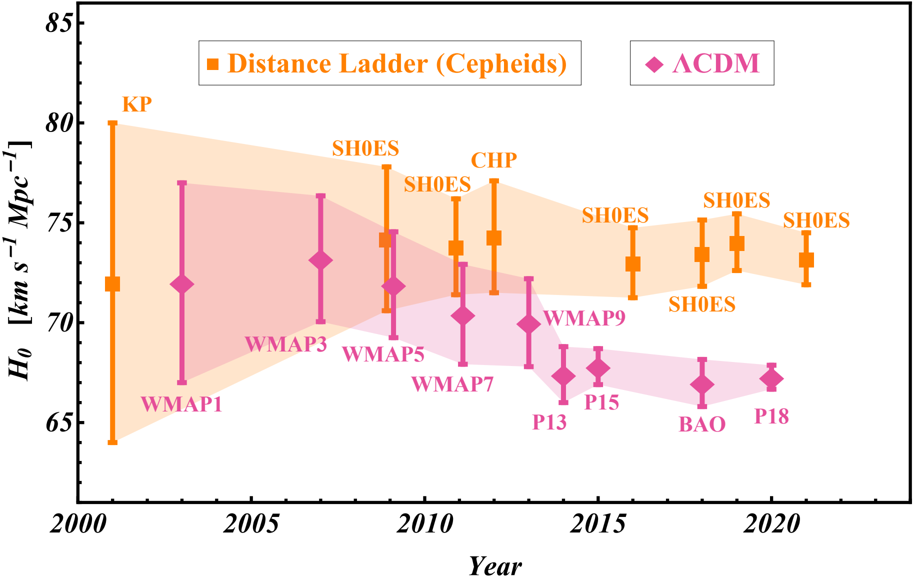
standard candles

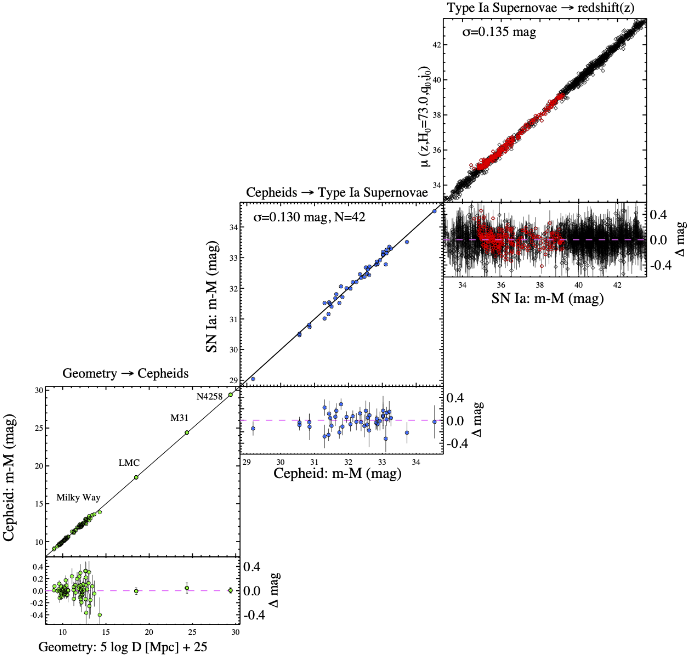






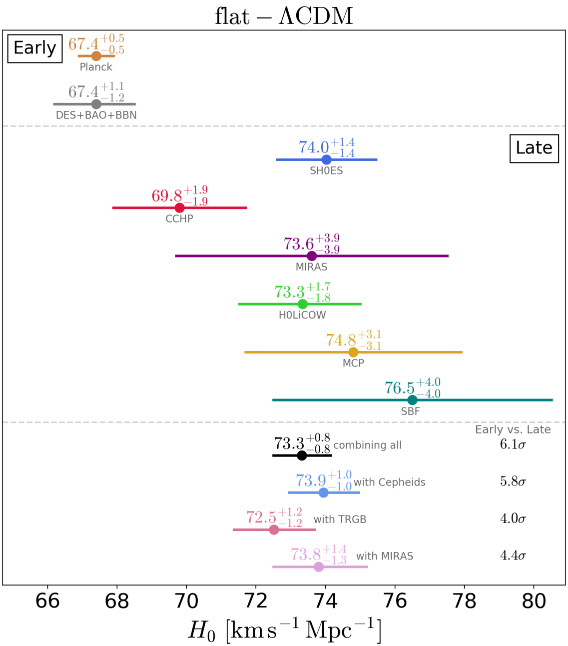

Distance Ladder (Cepheids)
ΛCDM
KP
SH0ES
SH0ES
CHP
SH0ES
SH0ES
SH0ES
SH0ES
WMAP1
WMAP3
WMAP5
WMAP7
WMAP9
P13
P15
BAO
P18
85
80
75
70
65
2000
2005
2010
2015
2020
Year
H_0 [km s^{-1} Mpc^{-1}]

2105.05208



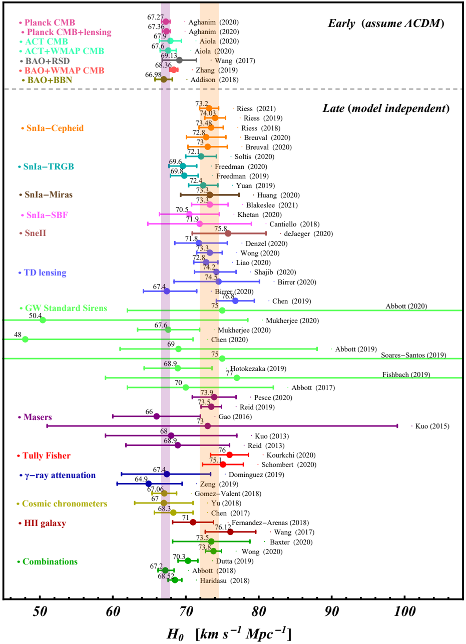
Type Ia Supernovae → redshift(z)
σ=0.135 mag
μ (z,H0=73.0,q0,j0)
Δ mag
SN Ia: m-M (mag)
Cepheids → Type Ia Supernovae
σ=0.130 mag, N=42
SN Ia: m-M (mag)
Cepheid: m-M (mag)
Geometry → Cepheids
N4258
M31
LMC
Milky Way
Cepheid: m-M (mag)
Geometry: 5 log D [Mpc] + 25

2211.04492


flat − ΛCDM
Early
$67.4^{+0.5}_{-0.5}$
Planck
$67.4^{+1.1}_{-1.2}$
DES+BAO+BBN
Late
$74.0^{+1.4}_{-1.4}$
SH0ES
$69.8^{+1.9}_{-1.9}$
CCHP
$73.6^{+3.9}_{-3.9}$
MIRAS
$73.3^{+1.7}_{-1.8}$
H0LiCOW
$74.8^{+3.1}_{-3.1}$
MCP
$76.5^{+4.0}_{-4.0}$
SBF
Early vs. Late
$73.3^{+0.8}_{-0.8}$ combining all
6.1σ
$73.9^{+1.0}_{-1.0}$ with Cepheids
5.8σ
$72.5^{+1.2}_{-1.2}$ with TRGB
4.0σ
$73.8^{+1.4}_{-1.3}$ with MIRAS
4.4σ
66 68 70 72 74 76 78 80
H_0 [km s^{-1} Mpc^{-1}]

1907.10625

Constraints on H_0



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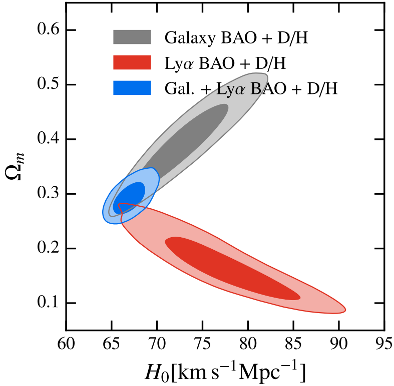
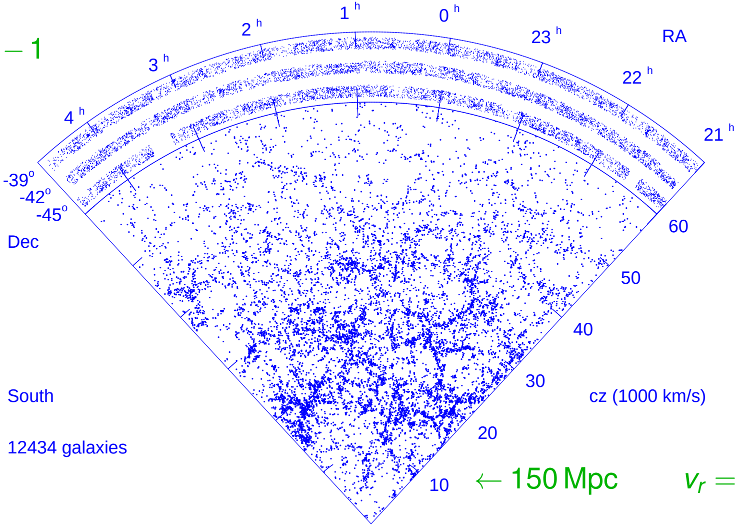


1707.06547

Universe is homogeneous and isotropic

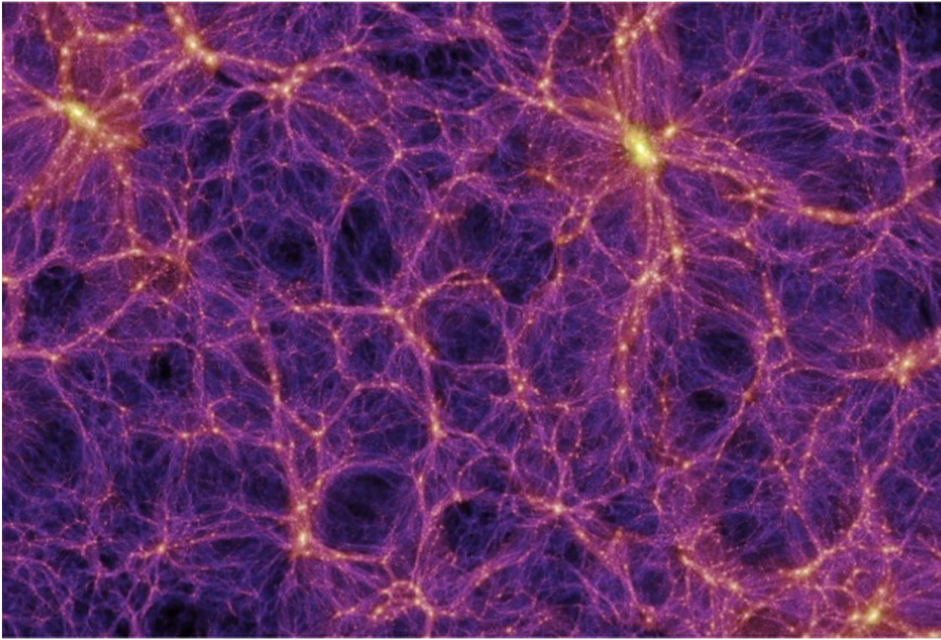
redshift

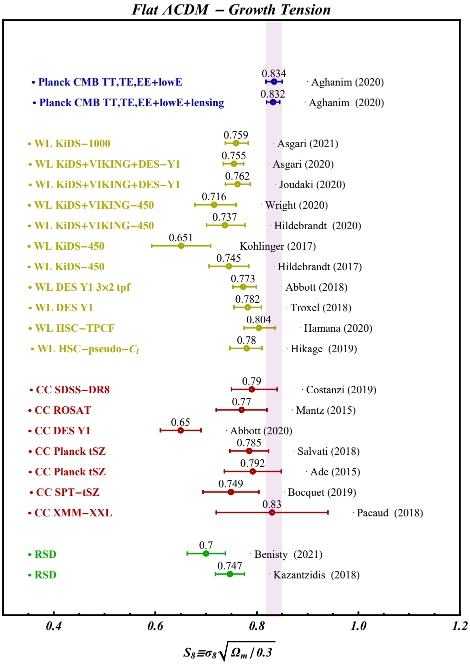
$$z \equiv \frac{\lambda_{\text{detector}}}{\lambda_{\text{source}}} - 1$$



$\leftarrow$ 150 Mpc

$$v_r = cz$$





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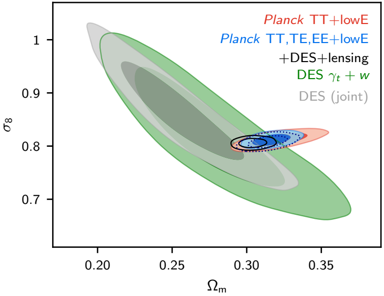


1807.06209

The Universe: age & geometry & energy density

$$[H_0] = L^{-1} = t^{-1}$$

time scale: $t_{H_0} = H_0^{-1} \approx 14 \times 10^9$ yr — age of our Universe

spatial scale: $l_{H_0} = H_0^{-1} \approx 4.3 \times 10^3$ Mpc — size of the visible Universe

t_{H_0} is in agreement with various observations

homogeneity and isotropy in 3d:

flat, spherical or hyperbolic

Observations: "very" flat $R_{curv} > 10 \times l_{H_0}$

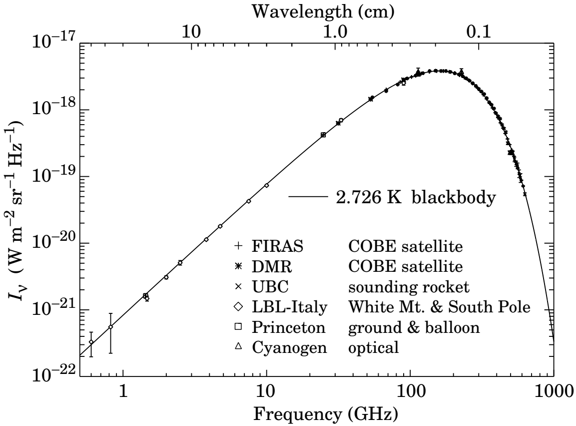
order-of-magnitude estimate: $GM_U/l_U \sim G\rho_0 l_{H_0}^3/l_{H_0} \sim 1$

flat Universe

$$\rho_c = \frac{3}{8\pi} H_0^2 M_{Pl}^2 \approx 0.53 \times 10^{-5} \frac{\text{GeV}}{\text{cm}^3}$$

$\longrightarrow$ 5 protons in each 1 m^3

Universe is occupied by “thermal” photons

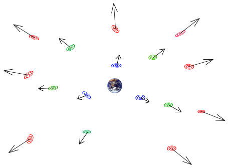


$T_0 = 2.726\,\mathrm{K}$

the spectrum (shape and normalization!) is thermal

$n_\gamma = 411\ \mathrm{cm}^{-3}$

Conclusions from observations



The Universe is homogeneous, isotropic, hot and expanding...

Conclusions

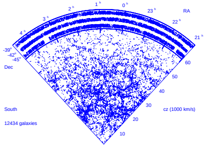
- interval between events gets modified

$$\Delta s^2 = c^2\,\Delta t^2 - a^2(t)\,\Delta \mathbf{x}^2$$

in GR expansion is described by the Friedmann equation

$$\left(\frac{\dot a}{a}\right)^2 = H^2(t) = \frac{8\pi}{3}\,G\rho^{\text{energy}}_{\text{density}}$$

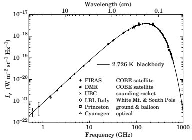
$$\rho^{\text{energy}}_{\text{density}} = \rho_{\text{radiation}} + \rho_{\text{matter}} + \dots$$



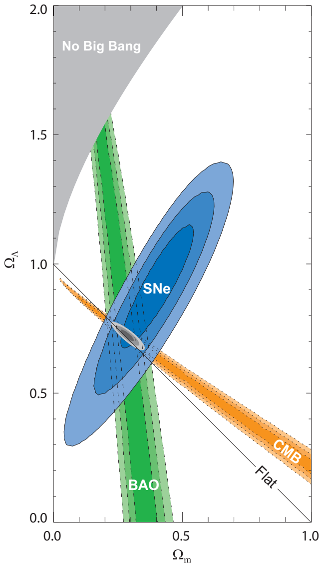
- in the past the matter density was higher, our Universe was "hotter" filled with electromagnetic plasma

$$\rho_{\text{matter}} \propto 1/a^3(t),\;\; \rho_{\text{radiation}} \propto 1/a^4(t),\;\; \rho_{\text{curvature}} \propto 1/a^2(t)$$

certainly known up to $T \sim 1\,\text{MeV} \sim 10^{10}\,\text{K}$



Astrophysical and cosmological data are in agreement



$$\left(\frac{\dot{a}}{a}\right)^2 = H^2(t) = \frac{8\pi}{3} G \rho^{\text{energy}}_{\text{density}}$$

$$\rho^{\text{energy}}_{\text{density}} = \rho_{\text{radiation}} + \rho^{\text{ordinary}}_{\text{matter}} + \rho^{\text{dark}}_{\text{matter}} + \rho_\Lambda$$

$$\rho_{\text{radiation}} \propto 1/a^4(t) \propto T^4(t)\,, \quad \rho_{\text{matter}} \propto 1/a^3(t)$$

$$\rho_\Lambda = \text{const}$$

$$\frac{3H_0^2}{8\pi G} = \rho^{\text{energy}}_{\text{density}}(t_0) \equiv \rho_c \approx 0.53 \times 10^{-5}\,\frac{\text{GeV}}{\text{cm}^3}$$

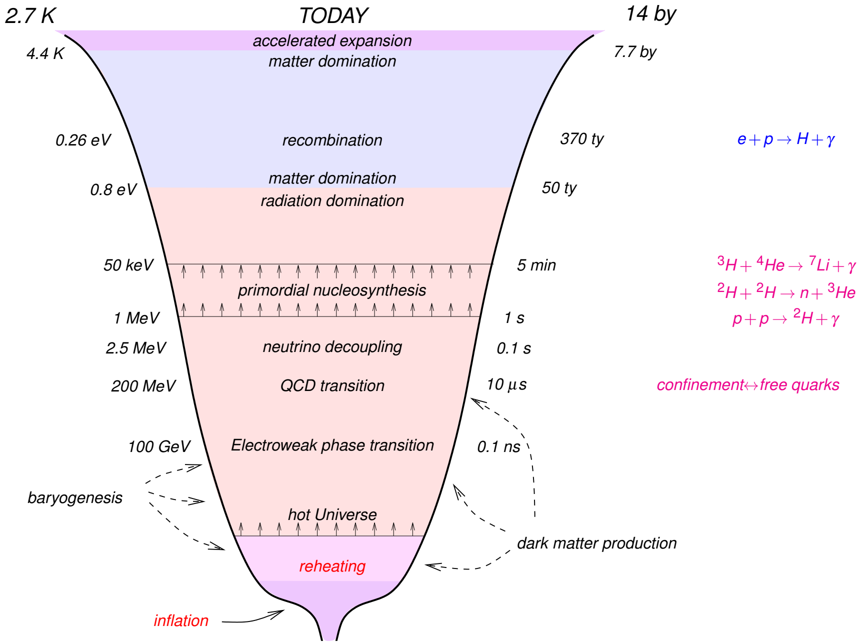
radiation: $\Omega_\gamma \equiv \frac{\rho_\gamma}{\rho_c} = 0.5 \times 10^{-4}$

Baryons (H, He): $\Omega_B \equiv \frac{\rho_B}{\rho_c} = 0.05$

Neutrino: $\Omega_\nu \equiv \frac{\sum \rho_{\nu_i}}{\rho_c} < 0.01$

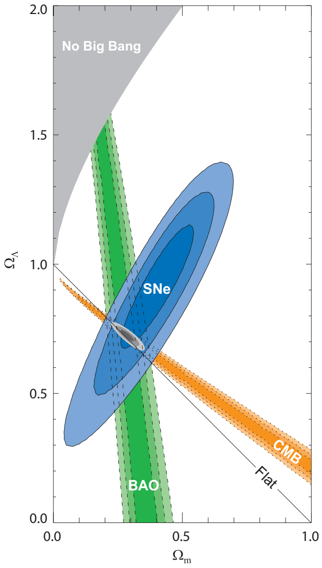
Dark matter: $\Omega_{DM} \equiv \frac{\rho_{DM}}{\rho_c} = 0.27$

Dark energy: $\Omega_\Lambda \equiv \frac{\rho_\Lambda}{\rho_c} = 0.68$


2.7 K
TODAY
14 by
accelerated expansion
4.4 K
matter domination
7.7 by
0.26 eV
recombination
370 ty
$e+p \to H+\gamma$
matter domination
0.8 eV
50 ty
radiation domination
50 keV
5 min
$^3H+{}^4He \to {}^7Li+\gamma$
primordial nucleosynthesis
$^2H+{}^2H \to n+{}^3He$
1 MeV
1 s
$p+p \to {}^2H+\gamma$
2.5 MeV
neutrino decoupling
0.1 s
200 MeV
QCD transition
10 μs
confinement↔free quarks
100 GeV
Electroweak phase transition
0.1 ns
baryogenesis
hot Universe
dark matter production
reheating
inflation

Outline

Dark Energy: nonclumping matter?



- estimates of Matter contribution confined in galaxies and clusters
  $\rho_c - \rho_M \neq 0$ but the Universe is flat, so $\rho_{curv} \simeq 0$
- corrections to the Hubble law : red shift – brightness curves for standard candles (SN Ia)
- The age of the Universe
- CMB anisotropy, large scale structures (galaxy clusters formation), etc

$$\rho_\Lambda = 0.68\rho_c$$

$$\rho_\Lambda \sim 10^{-5}\ \text{GeV/cm}^3 \sim \left(10^{-11.5}\ \text{GeV}\right)^4$$

Dark Energy: all evidences are from cosmology

Working hypothesis is cosmological constant $\Lambda \approx \left(2.5\times 10^{-3}\,\text{eV}\right)^4$:
$p = w(t)\rho$, $w = \text{const} = -1$, $\rho = \Lambda$

$$S_\Lambda = -\Lambda \int d^4x \sqrt{-\det g_{\mu\nu}}$$

both parts contribute

$$S_{\text{grav}} = -\frac{1}{16\pi G}\int d^4x \sqrt{-\det g_{\mu\nu}}\, R\,,$$
$$S_{\text{matter}} = \int d^4x \sqrt{-\det g_{\mu\nu}} \left(\frac{1}{2} g^{\lambda\rho}\partial_\lambda\phi\partial_\rho\phi - V(\phi)\right)$$

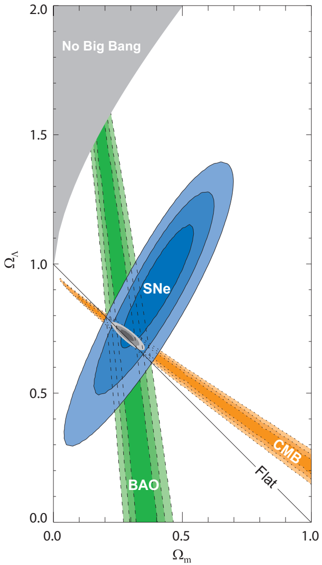
natural values

$$\Lambda_{\text{grav}} \sim 1/G^2 \sim \left(10^{19}\,\text{GeV}\right)^4 , \quad \Lambda_{\text{matter}} \sim V(\phi_{vac}) \sim (100\,\text{GeV})^4, (100\,\text{MeV})^4, \dots$$

Why Λ is small? Why $\Lambda \sim \rho_{\text{matter}}$? Why $\rho_B \sim \rho_{DM} \sim \rho_\Lambda$ today?

Outline

Astrophysical and cosmological data are in agreement



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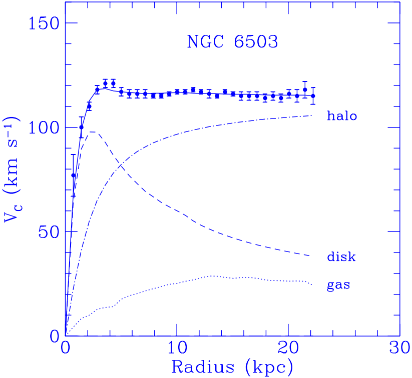
Galactic dark halos: flat rotation curves

$$v(R) = \sqrt{G\frac{M(R)}{R}}$$

$$M(R) = 4\pi \int_0^R \rho(r) r^2 dr$$



observations: $v(R) \simeq \text{const}$

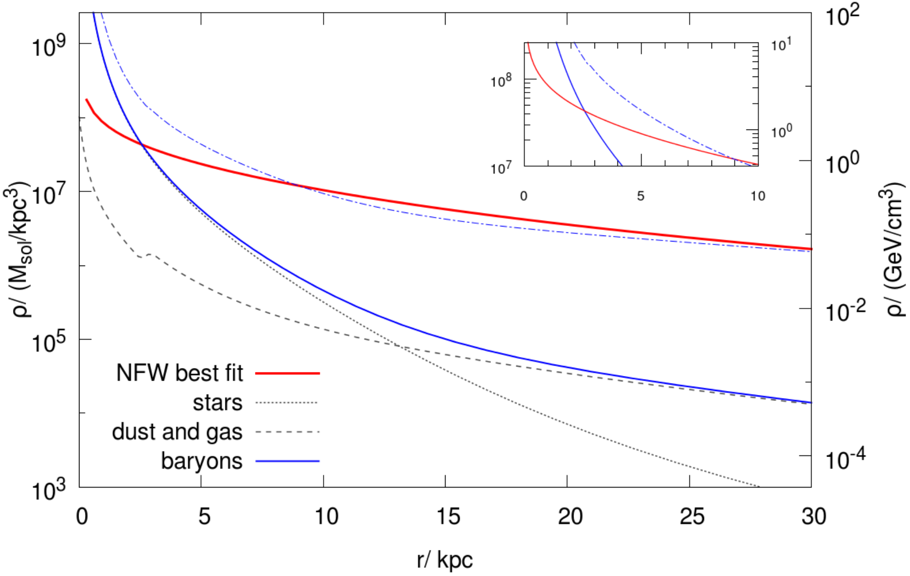
visible matter:

internal regions $v(R) \propto \sqrt{R}$

external ("empty") regions $v(R) \propto 1/\sqrt{R}$

Matter distribution in the Milky Way

1706.09850



Dark Matter in clusters

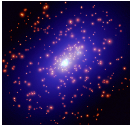
X-rays from hot gas in clusters

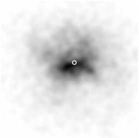
$$\frac{dP}{dR} = -\mu n_e(R) m_p \frac{GM(R)}{R^2}\,, \quad M(R) = 4\pi\int_0^R \rho(r) r^2 dr\,, \quad P(R) = n_e(R) T_e(R)$$

galaxies in clusters — virial theorem

$$U + 2E_k = 0$$

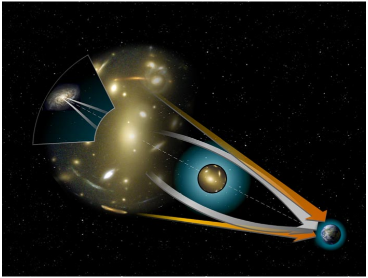
$$3M\langle v_r^2\rangle = G\frac{M^2}{R}$$



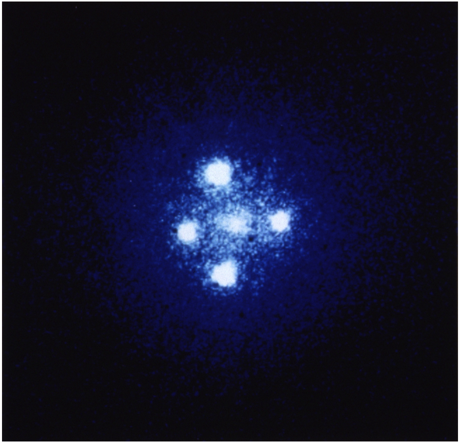


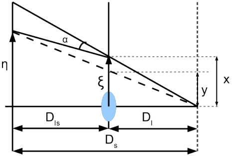
Milky Way: Virgo infall

Gravitational lensing in GR: $\alpha = 4GM/(c^2 b)$



Einstein Cross





$$\vec{\eta} = \frac{D_s}{D_l}\vec{\xi} - D_{ls}\vec{\alpha}\left(\vec{\xi}\right)$$

common lens with specific refraction coefficient

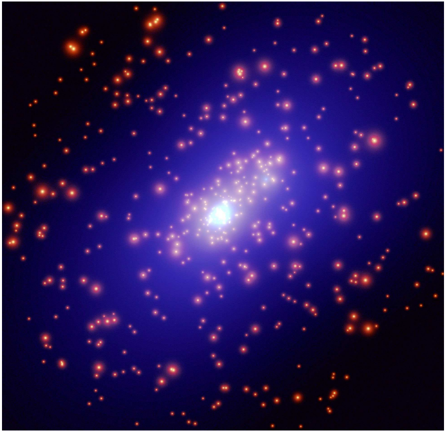
$$\vec{\alpha}\left(\vec{\xi}\right) = \frac{4G}{c}\int \frac{\vec{\xi}-\vec{\xi}'}{\left|\xi-\vec{\xi}'\right|^2} d^2\xi' \int \rho\left(\vec{\xi}',z\right) dz$$

source: quasar $D_s = 2.4$ Gpc

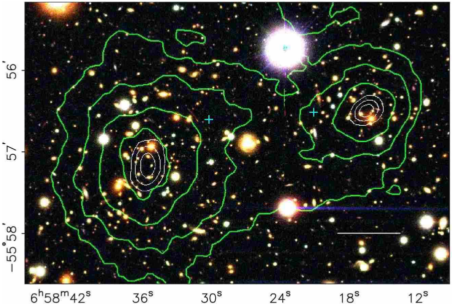
lens: galaxy $D_l = 120$ Mpc

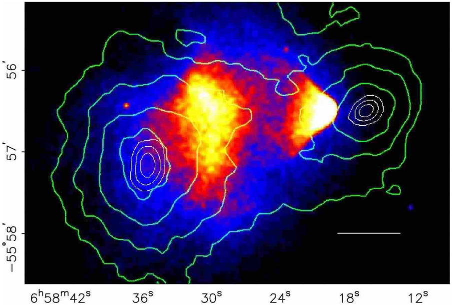
Dark Matter in clusters

gravitational lensing $\rho_B \approx 0.25\rho_{DM}$



Colliding clusters (Bullet clusters 1E0657-558)





gravitational lensing

Observations in X-rays

$M \simeq 10 \times m$

scale is 200 kpc

clusters are at 1.5 Gpc

Dark Matter Properties $p = 0$

(If) particles:

1. stable on cosmological time-scale
2. nonrelativistic long before RD/MD-transition (either Cold or Warm, $v_{RD/MD} \lesssim 10^{-3}$)
3. (almost) collisionless
4. (almost) electrically neutral

If were in thermal equilibrium: $M_X \gtrsim 1$ keV

If not: for bosons

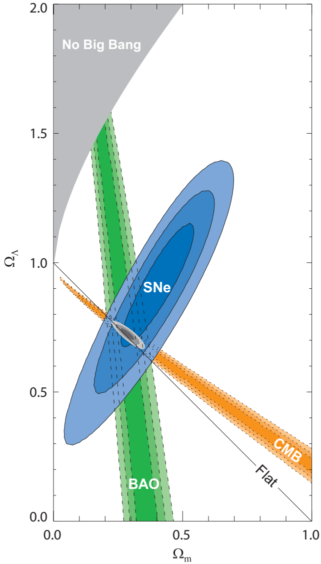
$\lambda = 2\pi/(M_X v_X)$, in a galaxy $v_X \sim 0.5 \cdot 10^{-3} \longrightarrow M_X \gtrsim 3 \cdot 10^{-22}$ eV

for fermions

Pauli blocking: $M_X \gtrsim 750$ eV

$$f(\mathbf{p},\mathbf{x}) = \frac{\rho_X(\mathbf{x})}{M_X} \cdot \frac{1}{\left(\sqrt{2\pi} M_X v_X\right)^3} \cdot e^{-\frac{\mathbf{p}^2}{2M_X^2 v_X^2}} \bigg|_{\mathbf{p}=0} \leq \frac{g_X}{(2\pi)^3}$$

Astrophysical and cosmological data are in agreement



$$\left(\frac{\dot a}{a}\right)^2 = H^2(t) = \frac{8\pi}{3} G \rho_{\text{density}}^{\text{energy}}$$

$$\rho_{\text{density}}^{\text{energy}} = \rho_{\text{radiation}} + \rho_{\text{matter}}^{\text{ordinary}} + \rho_{\text{matter}}^{\text{dark}} + \rho_\Lambda$$

$$\rho_{\text{radiation}} \propto 1/a^4(t) \propto T^4(t)\,, \quad \rho_{\text{matter}} \propto 1/a^3(t)$$

$$\rho_\Lambda = \text{const}$$

$$\frac{3H_0^2}{8\pi G} = \rho_{\text{density}}^{\text{energy}}(t_0) \equiv \rho_c \approx 0.53 \times 10^{-5}\,\frac{\text{GeV}}{\text{cm}^3}$$

radiation: $\Omega_\gamma \equiv \frac{\rho_\gamma}{\rho_c} = 0.5 \times 10^{-4}$

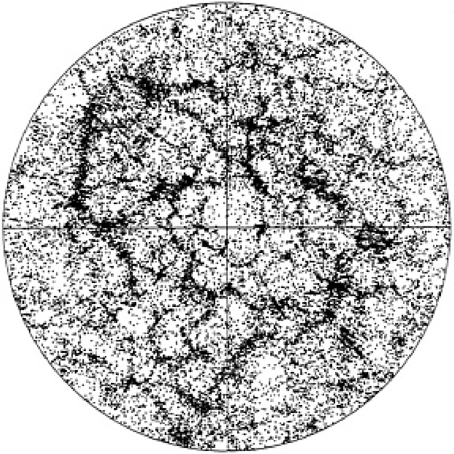
Baryons (H, He): $\Omega_B \equiv \frac{\rho_B}{\rho_c} = 0.05$

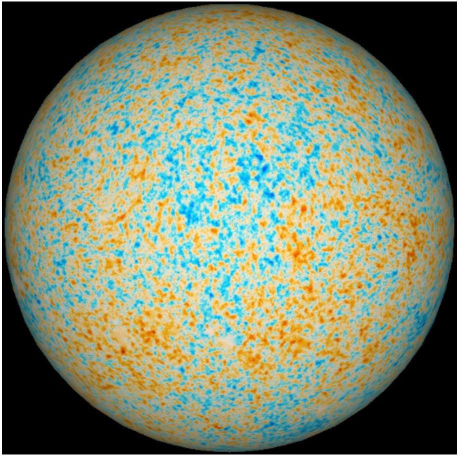
Neutrino: $\Omega_\nu \equiv \frac{\sum \rho_{\nu_i}}{\rho_c} < 0.01$

Dark matter: $\Omega_{DM} \equiv \frac{\rho_{DM}}{\rho_c} = 0.27$

Dark energy: $\Omega_\Lambda \equiv \frac{\rho_\Lambda}{\rho_c} = 0.68$

Inhomogeneous Universe



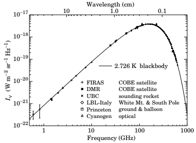


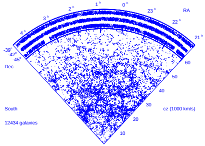
Large Scale Structure

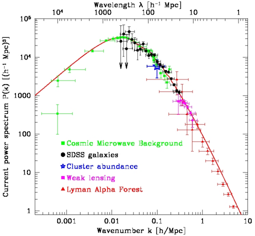
CMB anisotropy

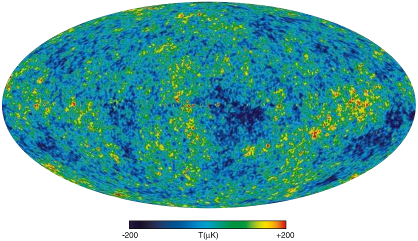
Key observable: matter perturbations

- CMB is isotropic, but "up to corrections, of course..."
  1. Earth movement with respect to CMB
  $$\frac{\Delta T_{\text{dipole}}}{T} \sim 10^{-3}$$
  2. More complex anisotropy!
  $$\frac{\Delta T}{T} \sim 10^{-4} - 10^{-5}$$
- There were matter inhomogenities $\Delta\rho/\rho \sim \Delta T/T$ at the stage of recombination ($e + p \to \gamma + H^*$)
- Jeans instability in the system of gravitating particles at rest $\Longrightarrow$ $\Delta\rho/\rho \nearrow$ $\Longrightarrow$ galaxies (CDM halos)









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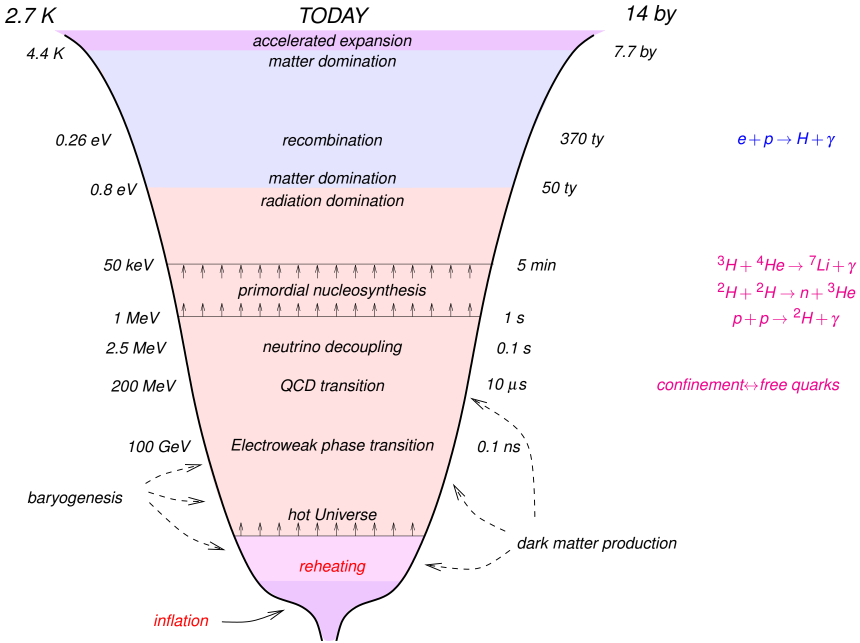
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Outline

Einstein equations

$T_{\mu\nu}$: macroscopic description $\frac{1}{2}\int d^4x\sqrt{-g}T_{\mu\nu}\delta g^{\mu\nu}$

$T_{\mu\nu} = (p+\rho)u_\mu u_\nu - g_{\mu\nu}p$ ideal fluid with $\rho(t)$ and $p(t)$

in the comoving frame $u^0 = 1$, $\mathbf{u} = 0$ (almost) always works

$$T_\mu^\nu = diag(\rho, -p)$$

$$ds^2 = dt^2 - a^2(t)\gamma_{ij}dx^i dx^j \,,$$

$$S_{EH} = -\frac{1}{16\pi G}\int d^4x\sqrt{-g}R \ : \quad R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G T_{\mu\nu}$$

$$(00): \quad \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3}G\rho - \frac{\varkappa}{a^2}$$

Friedmann equation (00) : $\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3}G\rho - \frac{\varkappa}{a^2}$

$$\nabla_\mu T^{\mu 0} = 0 \longrightarrow \dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = 0$$

the equation of state

$$p = p(\rho)$$

many-component liquid, in case of thermal equilibrium

other equations

$$-3d(\ln a) = \frac{d\rho}{p+\rho} = d(\ln s)$$

entropy of cosmic primordial plasma is conserved in a comoving frame

$$sa^3 = \text{const}$$

Examples of cosmological solutions

radiation: $p = \frac{1}{3}\rho$ singular at $t = t_s$

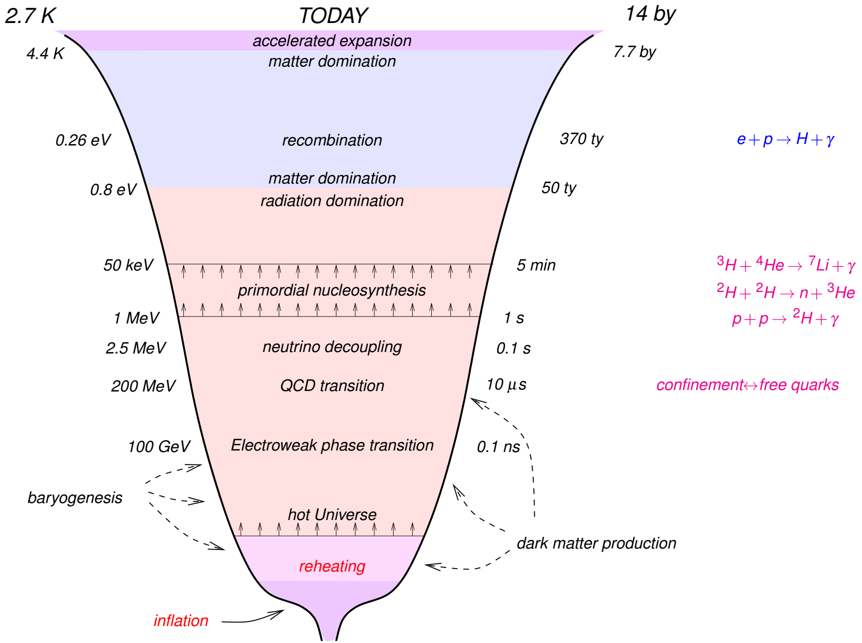
$$\rho = \frac{\text{const}}{a^4}\,, \quad a(t) = const \cdot (t - t_s)^{1/2}\,, \quad \rho(t) = \frac{\text{const}}{(t - t_s)^2}$$

$$t_s = 0\,, \quad H(t) = \frac{\dot{a}}{a}(t) = \frac{1}{2t}\,, \quad \rho = \frac{3}{8\pi G}H^2 = \frac{3}{32\pi G}\frac{1}{t^2}$$

In case of thermal equilibrium $T = \text{const}/a$

$$\rho_b = \frac{\pi^2}{30} g_b T^4\,, \quad \rho_f = \frac{7}{8}\frac{\pi^2}{30} g_f T^4$$

$$\rho = \frac{\pi^2}{30} g_* T^4\,, \qquad g_* = \sum_b g_b + \frac{7}{8}\sum_f g_f = g_*(T)$$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Friedmann equation for the present Universe

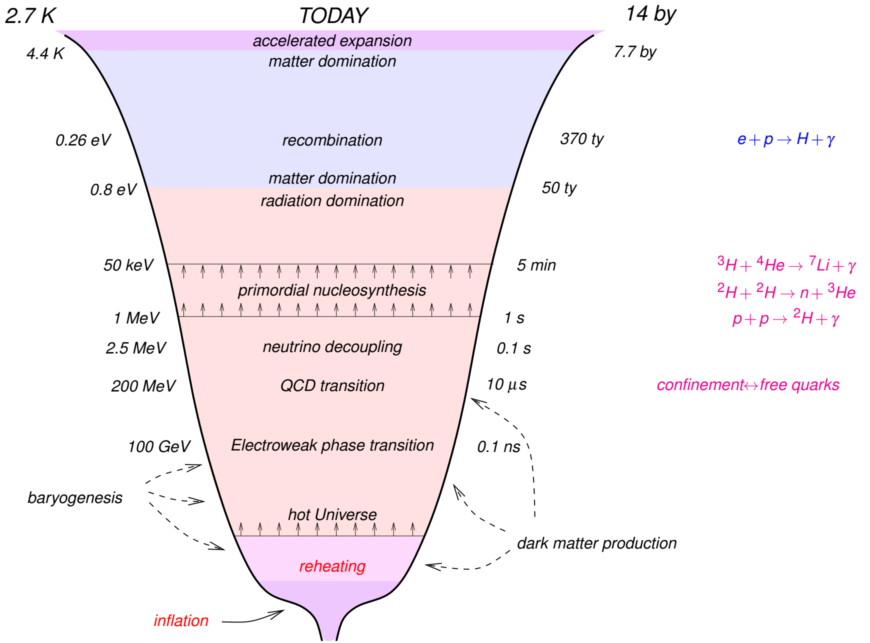
$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3} G\left(\rho_{\mathrm{M}} + \rho_{rad} + \rho_{\Lambda} + \rho_{\mathrm{curv}}\right)$$

$$\frac{8\pi}{3} G\rho_{\mathrm{curv}} = -\frac{\varkappa}{a^2}\,, \quad \rho_c \equiv \frac{3}{8\pi G} H_0^2$$

$$\rho_c = \rho_{\mathrm{M},0} + \rho_{rad,0} + \rho_{\Lambda,0} = \rho_c = 0.52 \cdot 10^{-5} \frac{\mathrm{GeV}}{\mathrm{cm}^3}\,, \qquad \text{for } \; h = 0.7$$

$$\Omega_X \equiv \frac{\rho_{X,0}}{\rho_c}$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3} G\rho_c \left[\Omega_{\mathrm{M}} \left(\frac{a_0}{a}\right)^3 + \Omega_{rad} \left(\frac{a_0}{a}\right)^4 + \Omega_{\Lambda} + \Omega_{curv} \left(\frac{a_0}{a}\right)^2\right]$$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Microscopic processes in the expanding Universe

A competition between scattering, decays, etc and expansion

for general processes one should solve kinetic equations

$$\frac{dn_{X_i}}{dt} + 3Hn_{X_i} = \sum(production - destruction)$$

Boltzmann equation in a comoving volume: $\frac{d}{dt}\left(n\,a^3\right) = a^3 \int \ldots$

production:

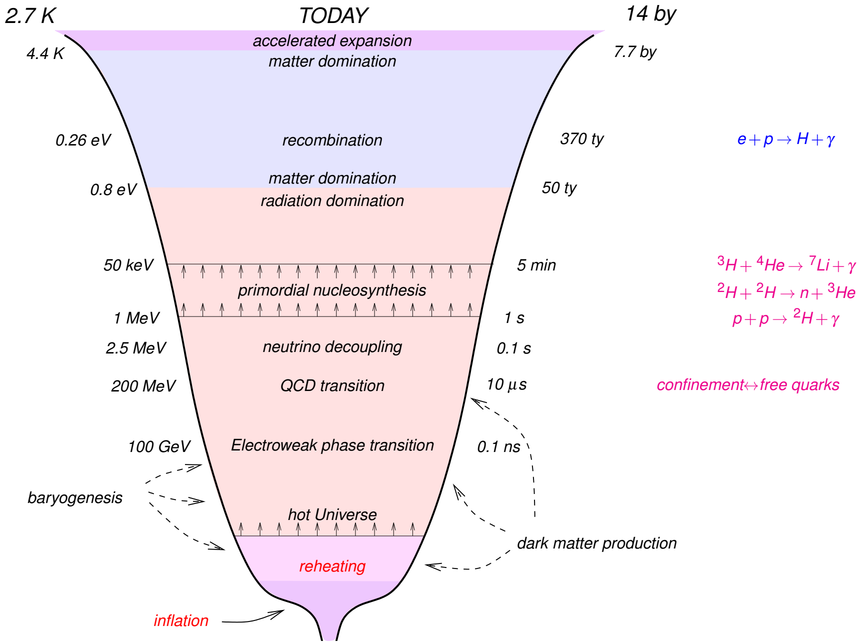
$$\sigma(A+B \to X+C)n_A n_B, \quad \Gamma(D \to E+X)n_D \cdot M_D/E_D, \quad \text{etc}$$

desrtuction:

$$\sigma(A+X \to C+B)n_A n_X, \quad \Gamma(X \to F+G)n_X \cdot M_X/E_X, \quad \text{etc}$$

Fast direct and inverse processes, $\Gamma \gtrsim H$, are in equilibrium,

$\sum(\;) = 0$ and thermalize particles



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Outline

Neutrino freeze-out

$T > m_e$ $\qquad$ $e^+e^- \leftrightarrow \nu\bar{\nu}$, $e\nu \leftrightarrow e\nu$

$$\sigma_\nu \sim G_F^2 E^2$$

neutrino interaction rate

$$\tau_\nu = \frac{1}{\langle \sigma_\nu n v \rangle} \sim \frac{1}{G_F^2 T^5} \qquad\qquad H^2 = \frac{8\pi}{3 M_{Pl}^2} \frac{\pi^2}{30} g_* T^4 \equiv \frac{T^4}{M_{Pl}^{*2}}$$

$$\tau_\nu(T) \sim H^{-1}(T) = \frac{M_{Pl}^*}{T^2}$$

$$T_{\nu,f} \sim \left(\frac{1}{G_F^2 M_{Pl}^*} \right)^{1/3} \sim 2 \div 3\ \mathrm{MeV}$$

Helium abundance (NO chemical equilibrium)

Neutrons remain mostly in helium

$$n_{^4\text{He}}(T_{NS}) = \frac{1}{2} n_n(T_{NS}) \,,$$

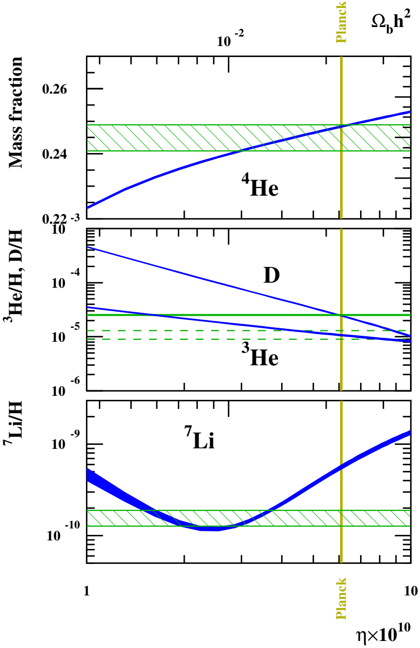
neutron-to-proton ratio $\tau_n \approx 880$ s

$$\frac{n_n(T_{NS})}{n_p(T_{NS})} \approx \frac{1}{5} \cdot \mathrm{e}^{-\frac{t_{NS}}{\tau_n}} \cdot \mathrm{e}^{-\frac{\mu_\nu}{T_n}} \approx \frac{1}{7} \,,$$

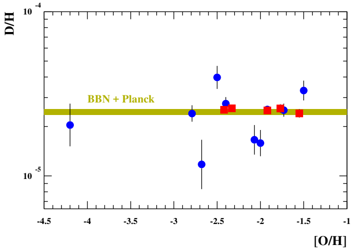
$$Y_p \equiv X_{^4\text{He}} = \frac{m_{^4\text{He}} \cdot n_{^4\text{He}}(T_{NS})}{m_p \left(n_p(T_{NS}) + n_n(T_{NS})\right)} = \frac{2}{\frac{n_p(T_{NS})}{n_n(T_{NS})} + 1} \approx 25\%$$

from observations of relic helium abundance:

$$\Delta N_{\nu,eff} \leq 0.2 \,, \qquad \left|\frac{\mu_\nu}{T_n}\right| \lesssim 0.01$$

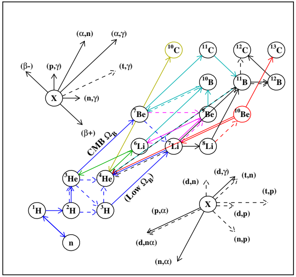


1707.01004 Measurement of $\eta_B = n_B/n_\gamma$ at $T \sim 1$ MeV

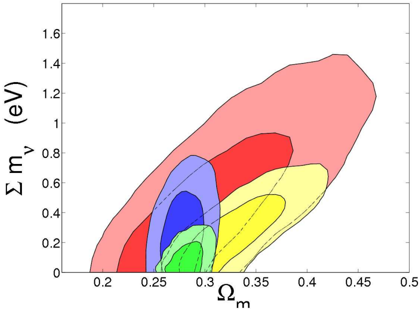


Lack of Lithium. . .

Exotics needed?



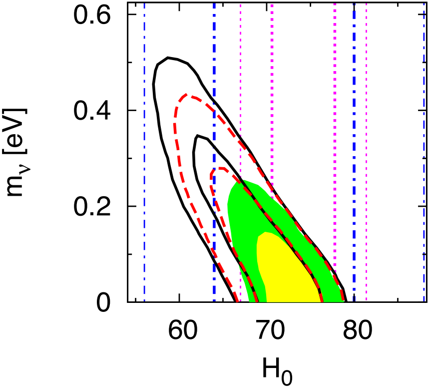
Cosmological limits: sub-eV scale... 14 years ago!!



LRG+BAO+WMAP5+SNe+BAO

$\sum m_\nu < 0.28$ eV (95% CL)

0911.5291



CMB+Hubble measurements

$\sum m_\nu < 0.20$ eV (95% CL)

0911.0976

Baryogenesis

Sakharov conditions of successful baryogenesis

- B-violation $(\Delta B \neq 0)\ XY \cdots \to X'Y' \ldots B$
- C- & CP-violation $(\Delta C \neq 0,\ \Delta CP \neq 0)\ \bar{X}\bar{Y} \cdots \to \bar{X}'\bar{Y}' \ldots \bar{B}$
- processes above are out of equilibrium $X'Y' \ldots B \to XY \ldots$

At 100 GeV$\lesssim T \lesssim 10^{12}$ GeV nonperturbative processes (EW-sphalerons) violate B, L_α, so that only three charges are conserved out of four, e.g.

$$B - L\,, \quad L_e - L_\mu\,, \quad L_e - L_\tau$$

and $B = \alpha \times (B - L)$, $L = (\alpha - 1) \times (B - L)$

Leptogenesis: Baryogenesis from lepton asymmetry of the Universe … due to sterile neutrinos

Why $\Omega_B \sim \Omega_{DM}$? antropic principle?

Outline

Dark Matter Properties $p=0$

(If) particles:

1. stable on cosmological time-scale
2. nonrelativistic long before RD/MD-transition (either Cold or Warm, $v_{RD/MD} \lesssim 10^{-3}$)
3. (almost) collisionless
4. (almost) electrically neutral

If were in thermal equilibrium: $M_X \gtrsim 1$ keV

If not: for bosons

$\lambda = 2\pi/(M_X v_X)$, in a galaxy $v_X \sim 0.5 \cdot 10^{-3} \longrightarrow M_X \gtrsim 3 \cdot 10^{-22}$ eV

for fermions

Pauli blocking: $M_X \gtrsim 750$ eV

$$f(\mathbf{p},\mathbf{x}) = \frac{\rho_X(\mathbf{x})}{M_X} \cdot \frac{1}{\left(\sqrt{2\pi} M_X v_X\right)^3} \cdot e^{-\frac{\mathbf{p}^2}{2M_X^2 v_X^2}} \bigg|_{\mathbf{p}=0} \leq \frac{g_X}{(2\pi)^3}$$

Dark Matter Candidates

- WIMPs (neutralino, . . .)
- sterile neutrinos
- gravitino
- axion
- Heavy relics
- (Topological) defects
- Massive Astrophysical Compact Halo Objects
- Primordial black hole remnants

Freeze-out of nonrelativistic Dark Matter

Assumptions:

1. no $X - \bar{X}$ asymmetry — either $X = \bar{X}$ or $n_X = n_{\bar{X}}$
2. @ $T \lesssim M_X$ in thermal equilibrium with plasma — (e.g. neutrons)

$$n_X = n_{\bar{X}} = g_X \left(\frac{M_X T}{2\pi}\right)^{3/2} \mathrm{e}^{-M_X/T}$$

$$X\bar{X} \longrightarrow \text{light particles}$$

freeze-out temperature T_f — $H \equiv T^2/M_{Pl}^*$, $M_{Pl}^* = M_{Pl}/1.66\sqrt{g_*}$

$$n_X \langle \sigma_{ann} v \rangle = H(T_f) \longrightarrow T_f = \frac{M_X}{\ln\left(\frac{g_X M_X M_{Pl}^* \sigma_0}{(2\pi)^{3/2}}\right)} .$$

Bethe formula: — s-wave: $\sigma_{ann} = \frac{\sigma_0}{v}$

Weakly Interacting Massive Particles

density after freeze-out: $n_X(T_f) = \frac{T_f^2}{M_{Pl}^* \sigma_0}$

present density: $n_X(T_0) = \left(\frac{a(T_f)}{a(T_0)}\right)^3 n_X(T_f) = \left(\frac{s_0}{s(T_f)}\right) n_X(T_f) \propto \frac{1}{T_f}$

$X + \bar{X}$ contribution to critical density:

$$\Omega_X = 2\frac{M_X n_X(T_0)}{\rho_c} = 7.6\frac{s_0 \ln\left(\frac{g_X M_{Pl}^* M_X \sigma_0}{(2\pi)^{3/2}}\right)}{\rho_c \sigma_0 M_{Pl}\sqrt{g_*(T_f)}}$$

$$= 0.1 \cdot \left(\frac{(10\ \text{TeV})^{-2}}{\sigma_0}\right) \frac{10}{\sqrt{g_*(T_f)}} \ln\left(\frac{g_X M_{Pl}^* M_X \sigma_0}{(2\pi)^{3/2}}\right) \cdot \frac{1}{2h^2}$$

WIMPs: discussion

$$\Omega_X = 0.1 \cdot \left(\frac{(10\ \text{TeV})^{-2}}{\sigma_0} \right) \frac{10}{\sqrt{g_*(T_f)}} \ln \left(\frac{g_X M_{Pl}^* M_X \sigma_0}{(2\pi)^{3/2}} \right) \cdot \frac{1}{2h^2}$$

- natural DM: subweak-scale cross section $\sigma_0 \sim 0.01 \times \sigma_W$
  say, $M_X \sim 1$ TeV or X is not a weak gauge eigenstate
- naturaly "light" unitarity $\sigma_0 \lesssim \frac{4\pi}{M_X^2} \longrightarrow M_X \lesssim 100$ TeV
- all stable particles with smaller σ_0 are forbidden !!
- WIMPs remain in kinetic equilibrium with plasma till $T \sim 10$ MeV

  this is Cold Dark Matter, $v_{RD/MD} \ll 10^{-3}$

  WIMPs may form dark halos (clumps) much lighter than dwarf galaxies

Weakly IMPs are mostly welcome (e.g. LSP in SUSY)

We can fully explore the model !!

- Direct searches for Galactic Dark Matter ($v \sim 10^{-3}$) a hit

$$X + \text{nuclei} \to X + \text{nuclei} + \Delta E$$

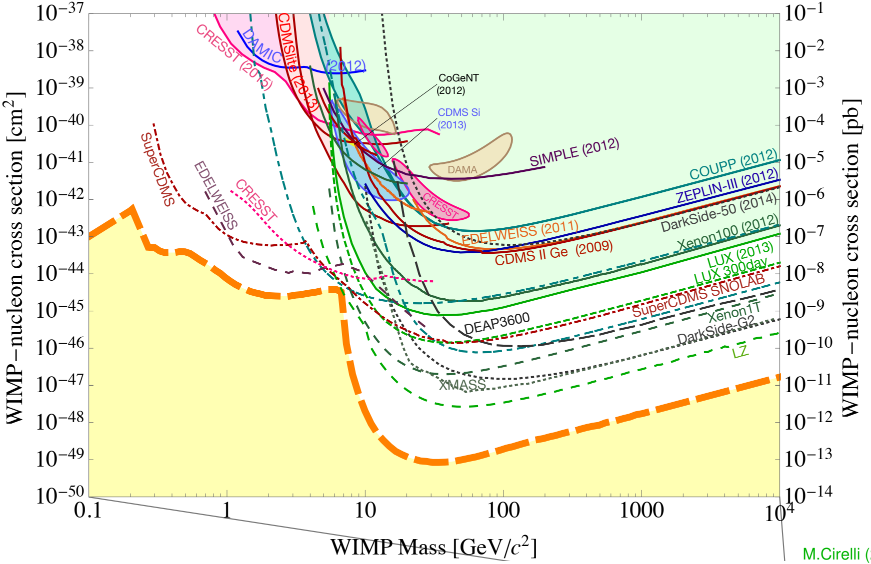
- Can search for WIMPs in cosmic rays: products of WIMPs annihilation (in Galactic center, dwarf galaxies, Sun) $\propto n^2$

$$X + \bar{X} \to p\bar{p}\,,\ \ e^+e^-\,,\ \ \nu, \gamma, \ldots$$

- Can search for WIMPs in collision experiments (LHC): missing

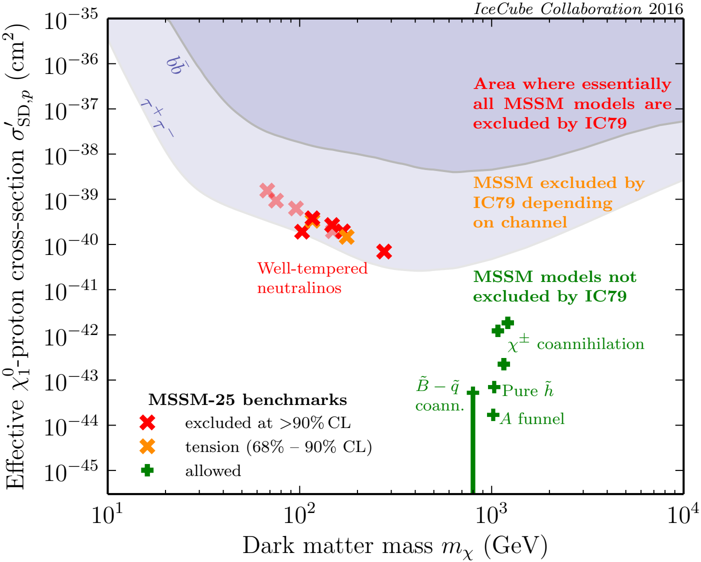
$$X + \bar{X} \leftrightarrow \text{SM} + \text{SM'} + \ldots$$

Prospects in WIMP searches



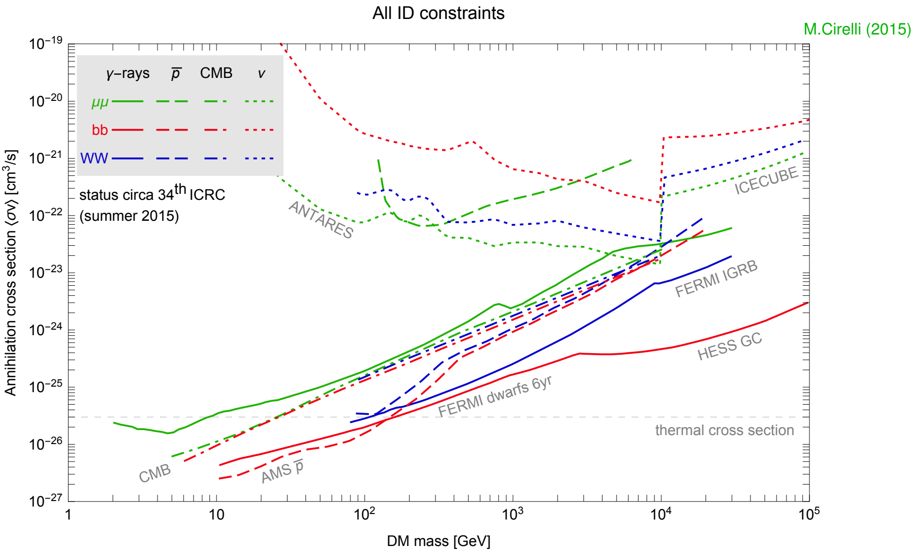
M.Cirelli (2015)

Constraining the DM model parameter space



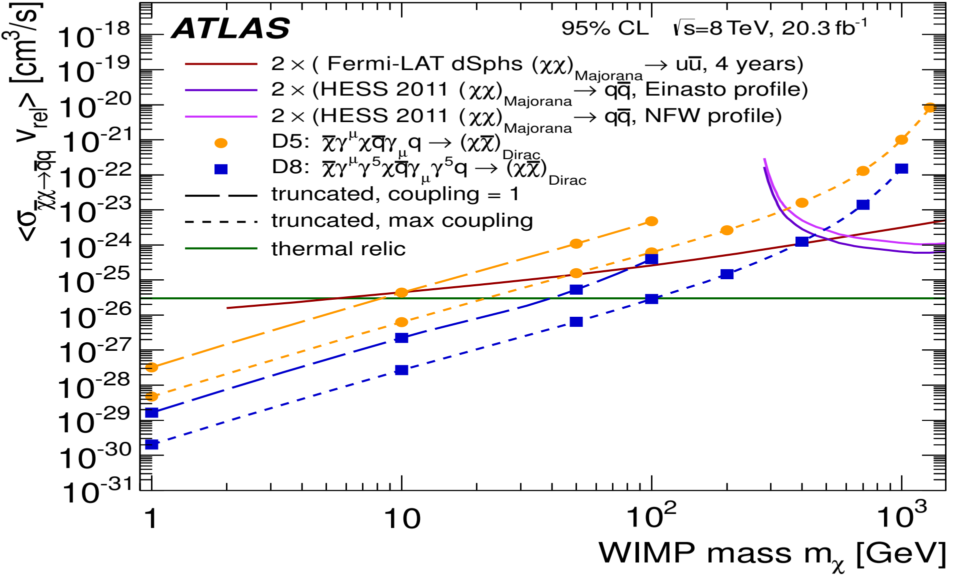
M.G. Aartsen at al (2016)

Present indirect limits on DM annihilation (clumps..)



LHC limits for annihilation

1502.01518



If thermal CDM but not Weakly IMPs?

We still can study the model if DM annihilates (partly) into SM particles

- But DM particle X can be light and feebly coupled (t-channel)

$$\sigma_0 \sim \frac{\xi^4}{M_X^2}$$

ξ is not a gauge coupling within GUT !

- With small σ_0 one needs entropy production
- σ_0 may be increased by s-channel resonance, $M_Y \approx 2M_X$
- annihilation can be amplified by co-annihilation channels, $X + A \to SM$
- With light messangers between Dark and Visible sectors many estimates change, say $\sigma_0 = \sigma_0(v)$
- DM interaction at freeze-out and now are not the same
  say, Sommerfield enhancement of the annihilation of slow particles $v \sim 10^{-3}$

Dark Matter: non-thermal production

1. in the primordial plasma of SM particles
   (via scatterings (freeze-in), gravitino
   via oscillations): sterile neutrino of 1-50 keV
2. at phase transitions:
   axion of $10^{-4} - 10^{-7}$ eV
   Q-balls
   strangelets (?)
3. during reheating (after inflation?): black holes
   any guy coupled (only) to inflaton
   - perturbatively: inflaton decays
     production by external (inflaton) field
   - non-perturbatively: Bose-enhancement of
     coherent production by external field
4. while the Universe expands:
   gravity produces any particles at $H \sim M_X$

Illustration with a simple example of scalar DM

most general renormalizable coupled to SM:

Z_2-invariant Higgs (Φ) portal

$$\Delta\mathscr{L} = \frac{1}{2}g^{\mu\nu}\partial_\mu S\partial_\nu S - \frac{1}{2}m^2S^2 + g^2S^2\Phi^\dagger\Phi - \frac{\lambda}{4}S^4$$

Options:

- freeze-out: sufficiently large g^2

$$\sigma_{hh\to SS}\times n_h \gtrsim H \;\;\to\;\; \sigma_{SS\to\ldots} = \sigma_0, \;\; \text{e.g. } \frac{g^4}{(4\pi\ldots)^2m_S^2} = \sigma_0$$

- freeze-in: intermediate g^2

$$\dot{n}_S + 3Hn_S = \sigma_{hh\to SS}n_h^2 \;\;\to\;\; \frac{n_S}{s} = \#\int dT\,\frac{n_h^2}{sHT}\times\frac{g^4}{T^2} \sim g^4\frac{M_{Pl}}{m_S} \;\;\to$$

$\Omega_S \propto g^4 \;\;\to\;\; g^2 \approx 10^{-11}$ still natural...

Free massive scalar field $g^2 = 0$

$$\mathscr{L} = \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} m_\phi^2 \phi^2$$

Homogeneous scalar field in the expanding Universe

$$\ddot{\phi} + 3H\dot{\phi} + m_\phi^2 \phi = 0$$

Two-stage evolution:

$$m_\phi < H(t) \quad \Longrightarrow \quad \phi = \phi_i = \text{const}$$

$$m_\phi > H(t) \quad \Longrightarrow \quad p = \langle E_k \rangle - \langle E_p \rangle = 0\,, \quad \rho \sim m_\phi^2 \phi^2 \propto 1/a^3$$

- dust-like substance in the late Universe, $\Omega \propto m_\phi^{1/2} \phi_i^2$
  depends on initial conditions
- presureless at spatial scales $l > M_{Pl}^{1/2}/\rho^{1/4} m_\phi^{1/2}$ fuzzy DM
- isocurvature mode: $\delta\rho_\phi \propto \delta H, \; \delta f_i$

scalar DM without dependence on initial field

$0 \neq g^2 < 10^{-11}$ Z_2-invariant Higgs (Φ) portal

$$\Delta\mathscr{L} = \frac{1}{2}g^{\mu\nu}\partial_\mu S\partial_\nu S - \frac{1}{2}m^2S^2 + g^2S^2\Phi^\dagger\Phi - \frac{\lambda}{4}S^4$$

Higgs particles in plasma change the potential:

$$g^2S^2\Phi^\dagger\Phi \;\to\; g^2S^2T^2/3$$

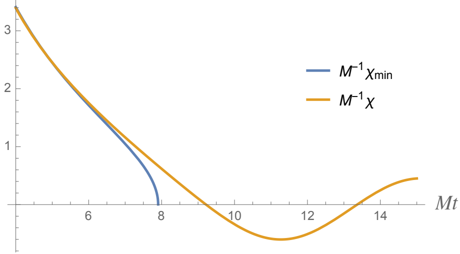
Z_2 symmerty is broken after reheating by the plasma contribution

Temperature decrease restores Z_2

2004.03410

$$\Delta\mathcal{L} = \frac{1}{2}g^{\mu\nu}\partial_\mu S\partial_\nu S - \frac{1}{2}m^2S^2 + g^2S^2T^2/3 - \frac{\lambda}{4}S^4$$

S starts from the false vacuum



at $g^2T_*^2 \simeq m^2$ sign changes and *S* starts to oscillate

gravitational misalignment

$$\rho_{DM}(t_*) = \frac{m^2 \cdot S_*^2}{2} \simeq \frac{(m^5H_*)^{2/3}}{4\lambda}$$

And the correct amount of DM by classical oscillating field

$$g^2 \simeq 10^{-12} \times \left(\frac{\lambda}{10^{-6}}\right)^{6/5} \times \left(\frac{10^6\,\text{GeV}}{m}\right)^2$$

Dark Matter: many well-motivated candidates

- WIMPs — related to EW scale, SUSY
- sterile neutrinos — active neutrino oscillations
- light scalar field — string theory
- axion — strong CP-problem
- gravitino — local SUSY
- Heavy relics — GUTs
- (Topological) defects — GUTs
- Massive Astrophysical Compact Heavy Objects
- Primordial black hole (remnants) — Phase transitions exotic inflation, reheating

Multicomponent Dark Matter ?

γ, ν, H, He

Standard cosmological model $ds^2 = dt^2 - a^2(t)dx^2$

$$\left(\frac{\dot{a}}{a}\right)^2 \equiv H^2 = H_0^2 \left[\Omega_\Lambda + (\Omega_{DM} + \Omega_B + \Omega_{\nu,m\neq 0})\left(\frac{a_0}{a}\right)^3 + (\Omega_\gamma + \Omega_{\nu,m=0})\left(\frac{a_0}{a}\right)^4\right]$$

- $T_\gamma = 2.735\,\mathrm{K}, \quad \Longrightarrow \quad \Omega_\gamma \sim 10^{-5}$
- $N_\nu \approx 3, \quad \sum m_\nu < 0.2\,\mathrm{eV} \quad \Longrightarrow \quad \Omega_{\nu,\neq 0}, \Omega_{\nu,0} \sim 10^{-5}$?
- $\Omega_B = 4.5\% \quad \Longrightarrow \quad \eta_B \equiv n_B/n_\gamma = 6\times 10^{-10}$
- $\Omega_{DM} = 27.5\%$
- $H_0 = 67\,\mathrm{km/s/Mpc} \quad \Longrightarrow \quad \rho_0 = 5\,\mathrm{GeV/m^3}$
- $\Omega_\Lambda = 68\% \quad \Longrightarrow$ flat space
- adiabatic, gaussian matter perturbations

$$\left\langle \left(\frac{\delta\rho}{\rho}\right)^2 \right\rangle \sim A_S \int \frac{dk}{k}\left(\frac{k}{k_*}\right)^{n_S-1}$$

  with $A_S = 3\times 10^{-9}$ and $n_S = 0.97$
- no tensor perturbations, $r \equiv A_T/A_S < 0.05$
- reionization at $z \equiv a_0/a = 10$

Weakly Interacting Massive Particles

Assumptions:

1. no $X - \bar{X}$ asymmetry $\qquad n_X = n_{\bar{X}}$
2. @ $T < M_X$ in thermal equilibrium with plasma

$$n_X = n_{\bar{X}} = g_X \left(\frac{M_X T}{2\pi}\right)^{3/2} e^{-M_X/T}$$

$$X\bar{X} \longrightarrow \text{light particles}$$

freeze-out temperature T_f $\qquad M_{Pl}^* = M_{Pl}/1.66\sqrt{g_*}$

$$\frac{1}{n_X}\frac{1}{\langle\sigma_{ann} v\rangle} = H^{-1}(T_f) \longrightarrow T_f = \frac{M_X}{\ln\left(\frac{g_X M_X M_{Pl}^* \sigma_0}{(2\pi)^{3/2}}\right)}\,.$$

Bethe formulae: $\qquad$ s-wave: $\sigma_{ann} = \frac{\sigma_0}{v}$

Weakly Interacting Massive Particles

density after freeze-out: $n_X(T_f) = \frac{T_f^2}{M_{Pl}^* \sigma_0}$

present density: $n_X(T_0) = \left(\frac{a(T_f)}{a(T_0)}\right)^3 n_X(T_f) = \left(\frac{s_0}{s(T_f)}\right) n_X(T_f) \propto \frac{1}{T_f} \propto \frac{1}{M_X}$

$X + \bar{X}$ contribution to critical density:

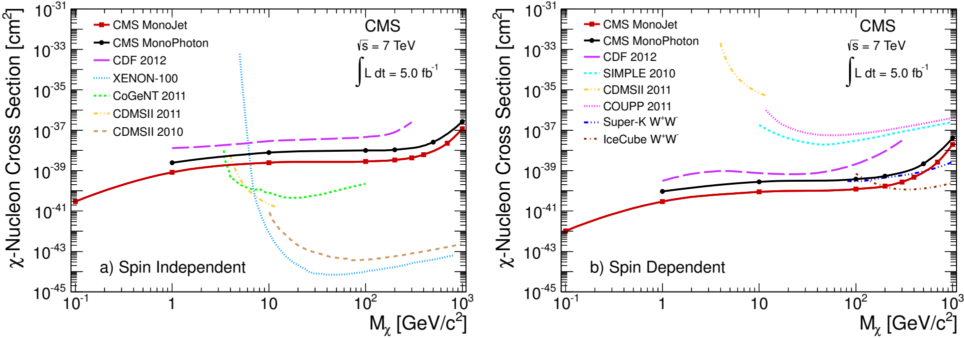
$$\Omega_X = 2\frac{M_X n_X(T_0)}{\rho_c} = 7.6 \frac{s_0 \ln\left(\frac{g_X M_{Pl}^* M_X \sigma_0}{(2\pi)^{3/2}}\right)}{\rho_c \sigma_0 M_{Pl}\sqrt{g_*(T_f)}}$$

$$= 0.1 \cdot \left(\frac{(10\ \text{TeV})^{-2}}{\sigma_0}\right) \frac{0.3}{\sqrt{g_*(T_f)}} \ln\left(\frac{g_X M_{Pl}^* M_X \sigma_0}{(2\pi)^{3/2}}\right) \cdot \frac{1}{2h^2}$$

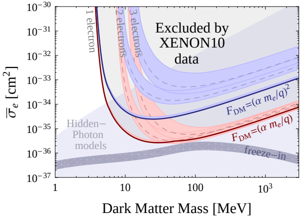
natural dark matter: $\sigma_0 \sim 0.01 \times \sigma_W$

naturaly “light” $\sigma_0 \lesssim \frac{4\pi}{M_X^2} \longrightarrow M_X \lesssim 100$ TeV

Recent results of (in)direct searches



for WIMPs 1206.5663



there are analyses for lower mass ranges and other type of interactions:

e.g. 1206.2644

Decoupling of relativistic specia (DM?)

Thermal equilibrium is forbidden:

$T_d \gg M_X$, and then $n_X/s = \text{const}$

$$\Omega_{3/2} = \frac{m_X \cdot n_{X,0}}{\rho_c} = \frac{m_X \cdot s_0}{\rho_c} \frac{n_{X,0}}{s_0} = 0.2 \frac{M_X}{100\text{ eV}} \left(\frac{g_X}{2}\right) \cdot \left(\frac{100}{g_*(T_d)}\right) \cdot \frac{1}{2h^2}$$

- If fermions: limit from Pauli-blocking
- Generally: too hot at Equality:
  from structure formation we need at $T_{Eq} \sim 1$ eV, $v_{DM} \lesssim 10^{-3}$

NB: for $M_X = 100$ eV at Equality ($T_{Eq} \sim 1$ eV) X-particle velocities are

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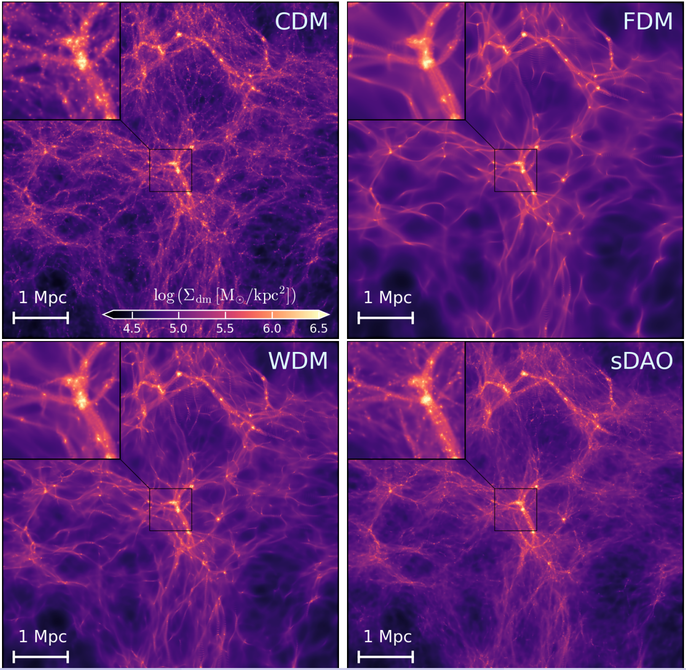
NB: for $M_X = 100$ eV at Equality ($T_{Eq} \sim 1$ eV) X-particle velocities are $v \simeq 10^{-2}$

Other Dark Matter candidates are not in equilibrium!

- WIMPs (neutralino, . . .) $\Longleftarrow$ thermal !
- sterile neutrinos $\Longleftarrow$ Price: sensitive to mass and couplings!
- axion $\Longleftarrow$ Price: sensitive to mass and (=couplings)!
- gravitino $\Longleftarrow$ Price: sensitive to mass, couplings and reheating temperature !!!
- Heavy relics
- (Topological) defects
- Massive Astrophysical Compact Halo Objects
- Primordial black hole remnants


CDM
FDM
WDM
sDAO
1 Mpc
1 Mpc
1 Mpc
1 Mpc
$\log(\Sigma_{\rm dm}\,[M_\odot/{\rm kpc}^2])$
4.5
5.0
5.5
6.0
6.5

2304.06742

DM keV sterile neutrino

Sterile neutrino of keV scale mass provides the Warm Dark Matter

Relevant parameters: mass $M_N \sim$1-10 keV and active-sterile neutrino mixing angle $\theta \ll 1$

Bounds on mass

- Phase space density (refined Pauli-blocking): $M_N \gtrsim 0.3$ keV
- Lyman-α forest: $M_N \gtrsim 10$ keV

Bound on mass M_N and mixing angle θ

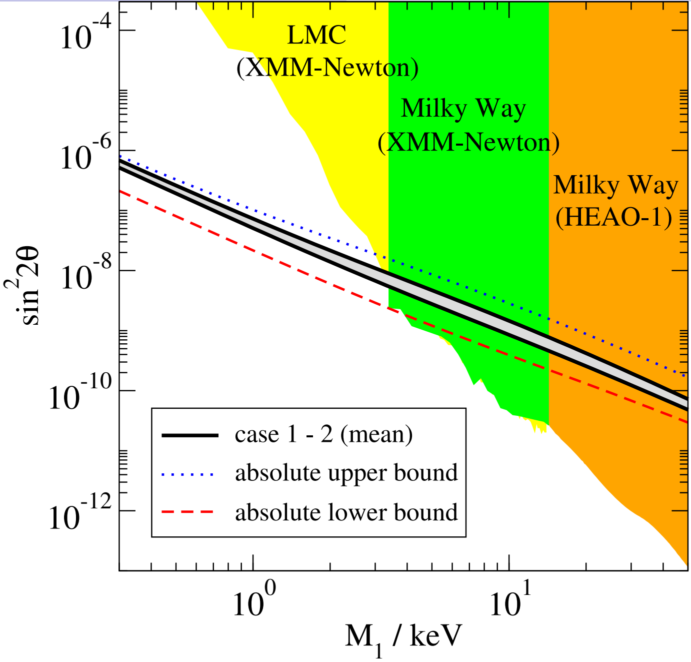
- X-ray observation: $N \to \nu + \gamma$, a peak at $\omega_\gamma = M_N/2$ of intensity $\propto \theta^2$

Production mechanism

- Dodelson-Widrow (thermal) scenario: $\nu_a \to N$ due to mixing,

$$\rho_N \propto \theta^2$$

- Primordial abundance: physics at higher energies
  - Lepton asymmetries
  - Production from inflaton decay
  - etc.


LMC
(XMM-Newton)
Milky Way
(XMM-Newton)
Milky Way
(HEAO-1)
case 1 - 2 (mean)
absolute upper bound
absolute lower bound
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^{-12}
$\sin^2 2\theta$
10^0
10^1
M_1 / keV

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- Dodelson-Widrow (thermal) scenario: $\nu_a \to N$ due to mixing,

$$\rho_N \propto \theta^2$$

  is ruled out
- Primordial abundance: physics at higher energies
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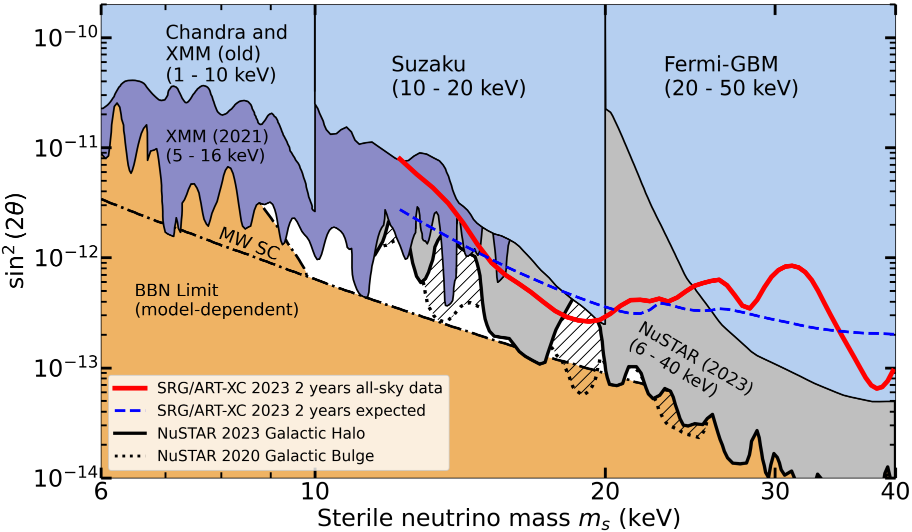


Chandra and
XMM (old)
(1 - 10 keV)
Suzaku
(10 - 20 keV)
Fermi-GBM
(20 - 50 keV)
XMM (2021)
(5 - 16 keV)
MW SC
BBN Limit
(model-dependent)
NuSTAR (2023)
(6 - 40 keV)
SRG/ART-XC 2023 2 years all-sky data
SRG/ART-XC 2023 2 years expected
NuSTAR 2023 Galactic Halo
NuSTAR 2020 Galactic Bulge
$\sin^2(2\theta)$
10^{-10}
10^{-11}
10^{-12}
10^{-13}
10^{-14}
6
10
20
30
40
Sterile neutrino mass m_s (keV)

2303.12673

Free scalar field as Cold Dark Matter (axion)

Homogeneous scalar field

$$\ddot{\phi} + 3H\dot{\phi} + m^2\phi = 0$$

at $m \ll H$ no evolution: $\phi = \text{const}$, at $m \gg H$ it oscillates, so

$$\rho = \frac{1}{2}\left(\frac{d\phi}{dt}\right)^2 + \frac{m^2}{2}\phi^2 = \langle E_k\rangle + \langle E_p\rangle = 2\langle E_p\rangle\,, \quad p = \frac{1}{2}\left(\frac{d\phi}{dt}\right)^2 - \frac{m^2}{2}\phi^2 = \langle E_k\rangle - \langle E_p\rangle = 0\,,$$

behaves as nonrelativistic (dark) matter (dust-like component) !!

nonperturbative CP-violation in QCD

$$L_\theta = \frac{\alpha_s}{8\pi}\left(\theta_0 + \text{Arg}\left(\text{Det}\hat{M}_q\right)\right) G^a_{\mu\nu}\tilde{G}^{\mu\nu\,a} \equiv \frac{\alpha_s}{8\pi}\cdot\theta\cdot G^a_{\mu\nu}\tilde{G}^{\mu\nu\,a}\,.$$

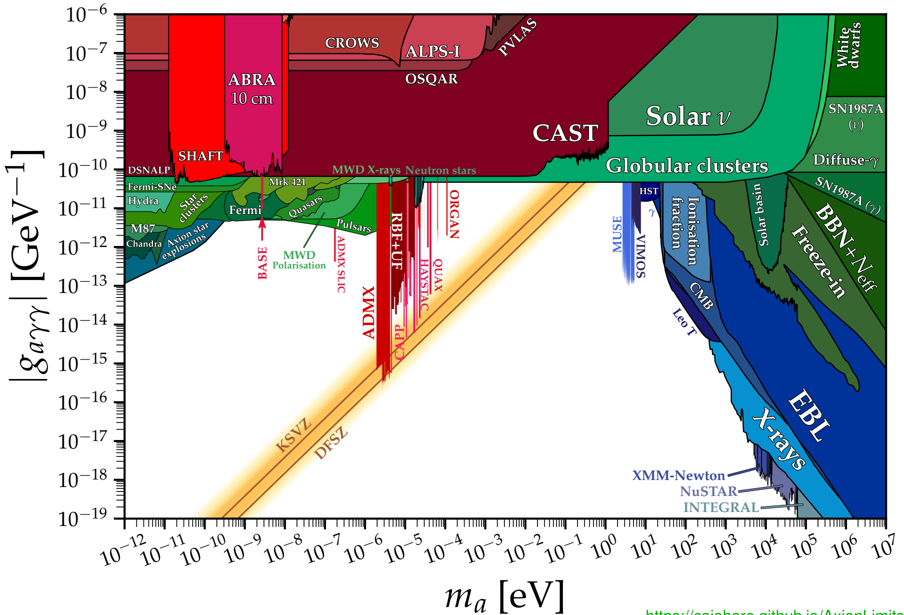
$$\theta \to \bar{\theta}(x) = \theta + C_g\frac{a(x)}{f_{PQ}}\,.$$

$$\mathscr{L} = \frac{f_{PQ}^2}{2}\cdot\left(\frac{d\bar{\theta}}{dt}\right)^2 - \frac{m_a^2(T)}{2}f_{PQ}^2\bar{\theta}^2\,,$$

$m_a(T) \simeq 0\,, T > \Lambda_{QCD}$ and $m_a(T) \simeq m_a \simeq m_\pi f_\pi / f_{PQ}$

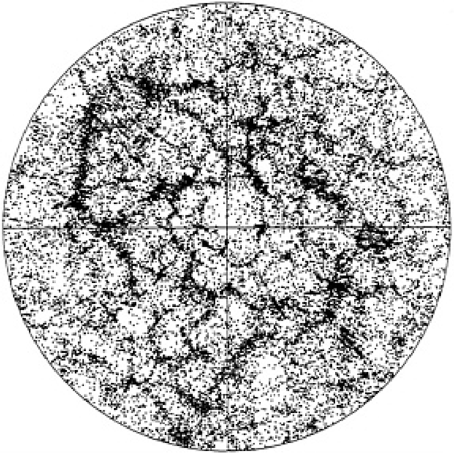
Check this $\Longrightarrow$

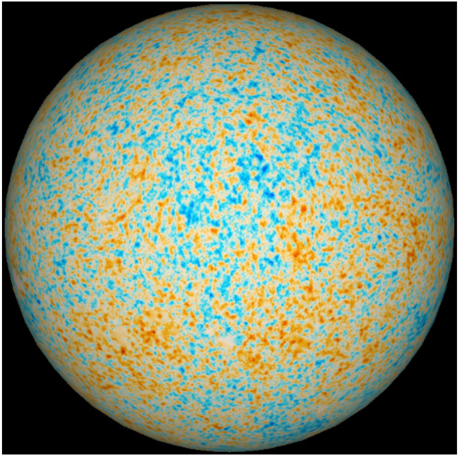
$$\Omega_a \simeq 0.2\cdot\bar{\theta}_i^2\cdot\left(\frac{4\cdot 10^{-6}\text{ eV}}{m_a}\right)\cdot\frac{1}{2h^2}$$



https://cajohare.github.io/AxionLimits/

Inhomogeneous Universe





Large Scale Structure CMB anisotropy

Small inhomogeneities in the expanding Universe

matter perturbations (perfect fluid approximation)

$$T_0^0 \to \rho(t) + \delta\rho(\eta, \mathbf{x}), \quad T_i^0 \to \partial_i v(\eta, \mathbf{x}), \quad T_j^i \to \delta p(\eta, \mathbf{x})$$

gravitational perturbations (scalar and tensor modes)

$$ds^2 = a^2(\eta)\left[(1 + 2\Phi(\eta, \mathbf{x}))\, d\eta^2 - (1 + 2\Psi(\eta, \mathbf{x}))\, \mathbf{dx}^2 - h_{ij}^{TT}(\eta, \mathbf{x}) dx^i dx^j\right]$$

Equations for linear perturbations, $\delta\rho/\rho \equiv \delta \ll 1$, $\Phi \ll 1$, etc

$$R_{\mu\nu} + \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu} \;\; \to \;\; \dots$$

$$\nabla_\mu T^{\mu\nu} = 0 \;\; \to \;\; \dots$$

These inhomogeneities (matter perturbations)

originate from the initial matter density (scalar) perturbations

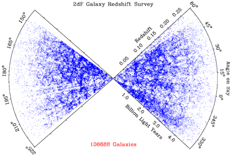
$\delta\rho/\rho \sim \delta T/T \sim 10^{-4}$, which are

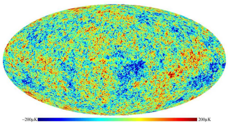
adiabatic $\delta\left(\frac{n_B}{s}\right) = \delta\left(\frac{n_{DM}}{s}\right) = \delta\left(\frac{n_L}{s}\right)$

Gaussian $\langle \frac{\delta\rho}{\rho}(\mathbf{k}) \frac{\delta\rho}{\rho}(\mathbf{k}') \rangle \propto \left(\frac{\delta\rho}{\rho}(\mathbf{k})\right)^2 \times \delta(\mathbf{k}+\mathbf{k}')$

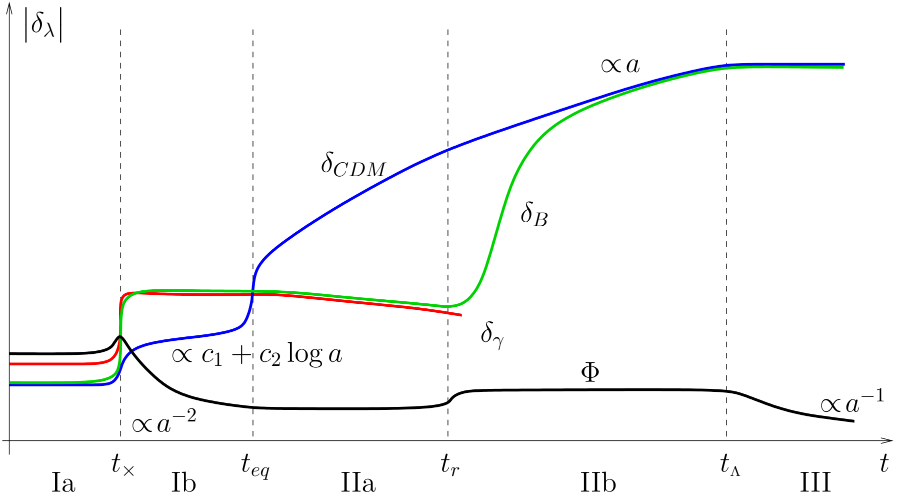
flat spectrum $\langle \left(\frac{\delta\rho}{\rho}(\mathbf{x})\right)^2 \rangle = \int_0^\infty \frac{\mathbf{dk}}{\mathbf{k}} \mathscr{P}_S(\mathbf{k})$ $\mathscr{P}_S(\mathbf{k}) \approx \text{const}$

LSS and CMB $\mathscr{P}_S \equiv A_S \times \left(\frac{k}{k_*}\right)^{n_s-1}$ $A_S \approx 2.5 \times 10^{-9}$, $n_S \approx 0.97$





Subhorizon modes ($k/a > H$) at various stages



On formulas...

- short waves, $k\eta_{eq} \gg 1$ $\qquad R_B \equiv 3\rho_B/4\rho_\gamma$

$$\delta_\gamma = \Phi_{(i)} \cdot \left[-324 \cdot (1+R_B)\, l^2(\Omega_M)\, \frac{\Omega_{CDM}}{\Omega_M}\, (1+z_{eq})\, \frac{\log(0.2k\eta_{eq})}{(k\eta_0)^2} + \frac{6}{(1+R_B)^{1/4}} \cos\left(k \int_0^\eta d\tilde{\eta}\, u_s \right) \right],$$

- long waves, $k\eta_{rec} \ll 1$

$$\delta_\gamma = -\frac{12}{5}\Phi_{(i)} = \text{const}$$

- intermediate waves ...

$$\delta_\gamma(\mathbf{k},\eta) = -4\,[1+R_B(\eta)]\,\Phi(\mathbf{k},\eta) + 4\Phi_{(i)}(\mathbf{k}) \cdot A(k,\eta) \cos\left(k \int_0^\eta u_s d\tilde{\eta} \right),$$

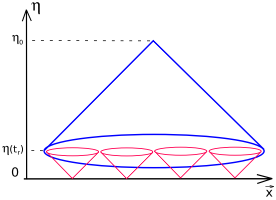
Cosmological (particle) horizon $l_H(t)$

distance covered by photons emitted at $t = 0$

the size of causally-connected region — the size of the visible part of the Universe

in conformal coordinates:
$ds^2 = 0 \longrightarrow |d\mathbf{x}| = d\eta$
coordinate size of the horizon equals
$\eta(t) = \int d\eta$

$$l_H(t) = a(t)\eta(t) = a(t)\int_0^t \frac{dt'}{a(t')}$$



dust

$$l_H(t) = 3t = \frac{2}{H(t)}\,, \qquad l_{H,0} = 2.6\times 10^{28}\text{cm} \quad (h = 0.7)$$

Last scattering: $\gamma e \to \gamma e$

$$\sigma_{\rm T} = \frac{8\pi}{3}\frac{\alpha^2}{m_e^2} \approx 0.67\cdot 10^{-24}\,\text{cm}^2\,, \qquad \tau_\gamma = \frac{1}{\sigma_{\rm T}\cdot n_e(T)}$$

last scattering: $\tau_\gamma(T_f) \simeq H^{-1}(T_f) \simeq t_f$

$$T_f = 0.26\ \text{eV}\,, \quad z = 1100\,, \quad t_f = 370\,000\ \text{yr}$$

for general processes one should solve kinetic equations

$$\frac{dn_{X_i}}{dt} + 3Hn_{X_i} = \int (production - destruction)$$

Boltzmann equation in a comoving volume: $\frac{d}{dt}\left(n\,a^3\right) = a^3 \int \dots$

Recombination: horizon

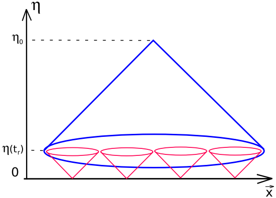
matter domination: $l_{H,r} = 2H_r^{-1}$

$$H_r^2 = \frac{8\pi}{3} G\rho_M(t_r) = \frac{8\pi}{3} G\rho_{M,0}\left(\frac{a_0}{a_r}\right)^3 = \frac{8\pi}{3} G\rho_c \Omega_{M,0}(1+z_r)^3 \, .$$

at recombination: $l_{H,r} = \frac{2}{H_0\sqrt{\Omega_M}} \frac{1}{(1+z_r)^{3/2}}$

today: $l_{H,r}(t_0) = l_{H,r} \times \frac{a_0}{a_r} = \frac{2}{H_0\sqrt{\Omega_M}} \frac{1}{\sqrt{1+z_r}}$

$$\frac{l_{H_0}}{l_{H,r}(t_0)} \sim \sqrt{1+z_r} \simeq 30$$



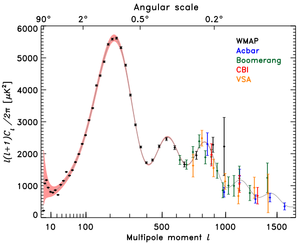
Recombination: angle

angular distance: $$d_{ph} = r_a(z)\,\Delta\theta$$

$$\chi_r = \int_{t_r}^{t_0} \frac{dt}{a(t)}\,, \quad \Delta\theta_r = \frac{l_{H,r}}{r_a(z_r)}$$

$$d_{conf} = \sinh\chi_r\,\Delta\theta$$

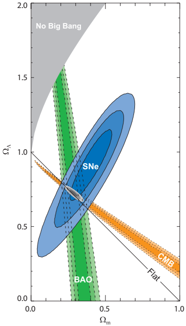
$$r_a(z_r) = (1+z_r)^{-1}\cdot a_0 \cdot \sinh\chi_r$$



$$\Delta\theta_r = \frac{1}{\sqrt{z_r+1}}\,, \quad \Omega_{curv} = \Omega_\Lambda = 0\,.$$

$$\Delta\theta_r = \frac{1}{\sqrt{z_r+1}} \frac{2\sqrt{\Omega_{curv}/\Omega_M}}{\sinh\left(2\sqrt{\Omega_{curv}/\Omega_M}\, I\right)}\,.$$

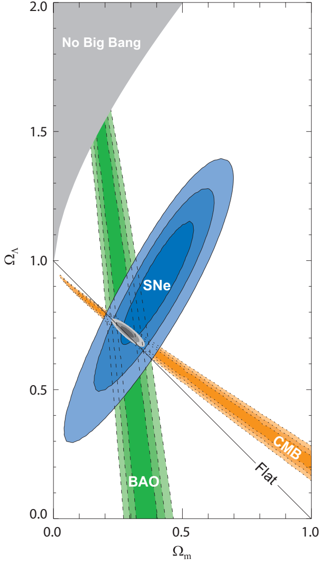
$$I = \int_0^1 \frac{dy}{\sqrt{1+\frac{\Omega_\Lambda}{\Omega_M}y^6}}$$

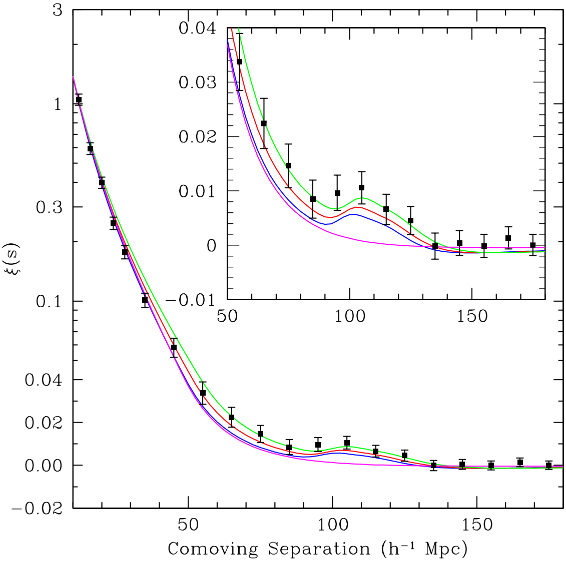


Acoustic oscillations in relativistic plasma:
What matters is the sound horizon:

$$l_{s,r} = l_{H,r}\cdot v_s \approx l_{H,r}/\sqrt{3}$$

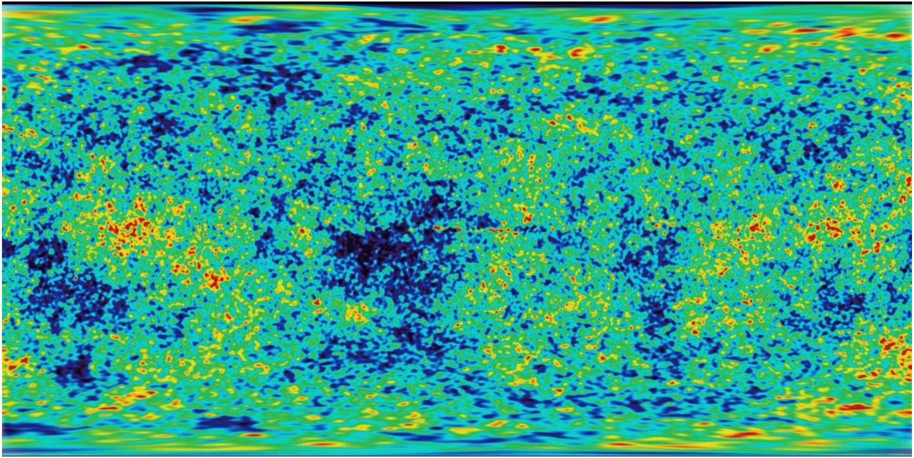
Then $$\Delta\theta_{r,s} = \frac{1}{\sqrt{3}}\frac{1}{\sqrt{1+z}} \times \frac{180^\circ}{\pi} \simeq 1^\circ$$





$$110/0.7\,\text{Mpc} \simeq l_{H,r}(t_0) \times \sqrt{v_s^2} \simeq l_{H_0}/\sqrt{3}/\sqrt{1+z_r}$$

CMB map



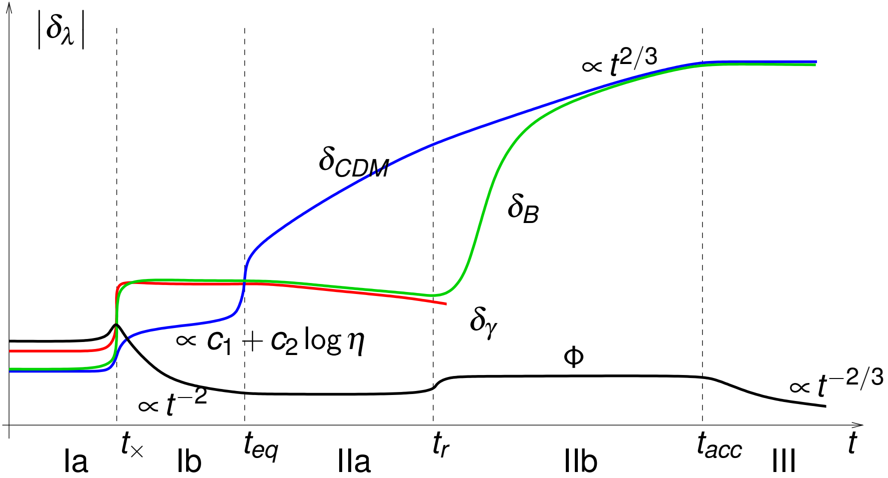
Mode evolution

- Amplitude remains constant, while superhorizon, e.g. $k/a < H$
- Subhorizon Inhomogeneities of DM start to grow at MD-stage, $\delta\rho_{CDM}/\rho_{CDM} \propto a$ from $T \approx 0.8\,\text{eV}$
  Smaller objects (first stars, dwarf galaxies) are first to form
- Subhorizon Inhomogeneities of baryons join those of DM only after recombination, $\delta\rho_{CDM}/\rho_{CDM} \propto a$ from $T_{rec} \approx 0.25\,\text{eV}$
- at recombination $\delta\rho_B/\rho_B \sim \delta T/T \sim 10^{-4}$ and would grow only by a factor $T_{rec}/T_0 \sim 10^3$ without DM
- Subhorizon Inhomogeneities of photons $\delta\rho_\gamma/\rho_\gamma$ oscillate with constant amplitude at RD and with decreasing amplitude at MD, thus we can measure $T_{RD/MD}/T_{rec}$
- Phase of oscillations decoupled after recombination depends on the wave-length, recombination time and sound speed

$$\delta\rho_\gamma/\rho_\gamma \propto \cos\left(k\int_0^{t_r} \frac{v_s\,dt}{a(t)}\right) = cos(kl_{sound})$$

- $$\delta T(\theta,\varphi) = \sum a_{lm} Y_{lm}(\theta,\varphi)\,, \qquad \langle a^*_{lm} a_{lm}\rangle = C_l \equiv 2\pi\mathscr{D}_l/(l(l+1))$$

Mode evolution at various stages



On formulas...

- short waves, $k\eta_{eq} \gg 1$ $\qquad R_B \equiv 3\rho_B/4\rho_\gamma$

$$\delta_\gamma = \Phi_{(i)} \cdot \left[-324 \cdot (1+R_B)\, l^2 \frac{\Omega_{CDM}}{\Omega_M} (1+z_{eq}) \frac{\log(0.2k\eta_{eq})}{(k\eta_0)^2} + \frac{6}{(1+R_B)^{1/4}} \cos\left(k \int_0^\eta d\tilde{\eta}\, u_s \right) \right] ,$$

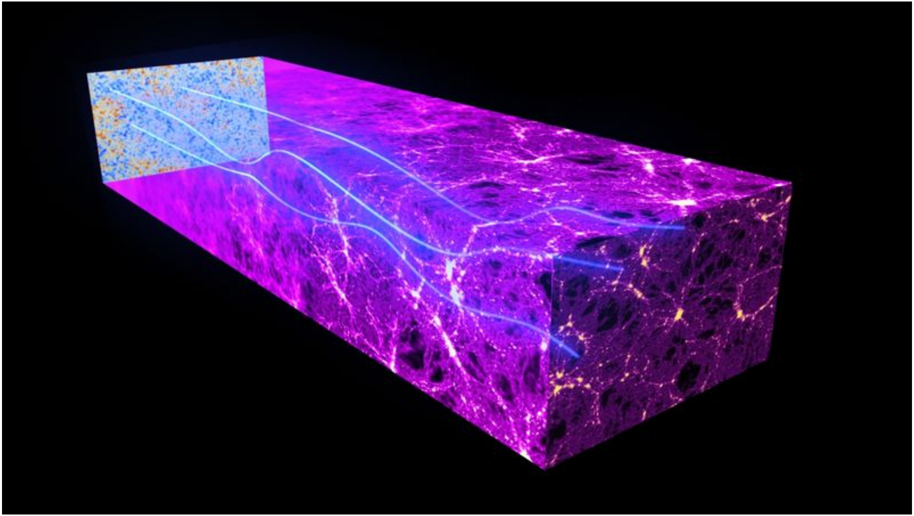
- long waves, $k\eta_{rec} \ll 1$

$$\delta_\gamma = -\frac{12}{5} \Phi_{(i)} = \text{const}$$

- intermediate waves ...

$$\delta_\gamma(\mathbf{k},\eta) = -4\,[1+R_B(\eta)]\, \Phi(\mathbf{k},\eta) + 4\Phi_{(i)}(\mathbf{k}) \cdot A(k,\eta) \cos\left(k \int_0^\eta u_s d\tilde{\eta} \right) ,$$

On top of that: propagation in expanding Universe



On formulas. . .

From linear approximation to the geodesic equation. . .
for scalar perturbations

$$\frac{\delta T}{T}(\mathbf{n},\eta_0) = \frac{1}{4}\delta_\gamma(\eta_r) + (\Phi(\eta_r) - \Phi(\eta_0)) + \int_{\eta_r}^{\eta_0}\left(\Phi' - \Psi'\right) d\eta + \mathbf{nv}(\eta_r) - \mathbf{nv}(\eta_0)\,.$$

for tensor perturbations

$$\frac{\delta T}{T}(\mathbf{n},\eta_0) = \frac{1}{2}\int_{\eta_r}^{\eta_0} d\eta\, n_i h_{ij}^{TT\,\prime} n_j\,,$$

CMB measurements (Planck) $\theta, \Omega_{DM}, \Omega_B, \tau, \Delta_{\mathscr{R}}, n_s$