Relic gravitational wave conversion into photons in the cosmological magnetic field

Конверсия реликтовых гравитационных волн в фотоны под действием космологического магнитного поля

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В работе произведена оценка подавления амплитуды реликтовых гравитационных волн частотой $10^{-16} - 10^{-18}$ Гц за счет их конверсии в электромагнитные волны под действием космологического магнитного поля на протяжении эпохи радиационного доминирования (РД). Вначале получена система уравнений для совместного распространения гравитационной и электромагнитной волны во внешнем магнитном поле на фоне метрики Фридмана-Леметра-Робертсона-Укера и учтено взаимодействие электромагнитных волн с первичной плазмой. Далее полученная система решена численно в приближении однородного космологического магнитного поля с напряженностью, соответствующей современному значению в 1 нГс (в эпоху РД напряженность магнитного поля усиливается обратно пропорционально квадрату масштабного фактора). В результате сделан вывод о незначительном влиянии рассмотренного эффекта конверсии на
1 амплитуду длинноволновых реликтовых гравитационных волн.

The article evaluates the amplitude suppression for the relict gravitational waves with a frequency of $10^{-16} - 10^{-18}$ Hz due to their conversion into electromagnetic waves under the influence of a cosmological magnetic field during the era of radiation dominance (RD). At first, a system of equations was obtained for the joint propagation of gravitational and electromagnetic waves in an external magnetic field in the Friedmann-Lemaitre-Robertson-Walker metric and the interaction of electromagnetic waves with the primary plasma was taken into account. Then the resulting system is solved numerically in the approximation of a uniform cosmological magnetic field with a strength corresponding to the modern value of 1 nG (for the RD epoch the magnetic field strength increases as the inverse scale factor squared). Finally a conclusion was made about the insignificant influence of the considered conversion effect on the amplitude of long-wave relic gravitational
2 waves.

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Introduction

The existence of an intergalactic magnetic field has been proven through observations, and based on the observations theoretical limits are $10^{-16} \lesssim B_0 \lesssim 10^{-9}$ Gs [1]. Even if it is a very weak value, it increases as the inverse square of the scale factor when moving backward in time. Thus, for the considered scale factor interval $a \in [10^{-9}, 10^{-4}]$, present day value of 1 nG is amplified $10^8 - 10^{18}$ times. Hence the conversion of gravitational waves (GWs) into photons induced by the cosmological magnetic field presence may significantly change the relict GW spectrum.

Two more reasons why this issue is worth considering are the interaction of emerging photons with the primary plasma and the long duration of the RD epoch during which GW conversion and these photon interactions occur.

Thus, if the effect of relic GW suppression due to the conversion into photons is sufficiently large, then this may explain why there is no imprint of the primary tensor perturbations on the cosmic microwave background [2–4].

The paper is organized as follows. In the next section notations are introduced and the method of the equation system derivation is described with the list of all effects which are taken into account. Further, the resulting system of differential equations (SoDE) for FLRW background metric is presented. Then the SoDE is solved numerically for GW of frequency 10^{-16}-10^{-18} Hz, taking into account several simplifying assumptions. Finally conclusions and prospects are presented.

Method

Let us expand the full quantities: the metric $\tilde{g}_{\mu\nu}$, the electromagnetic tensor $\tilde{F}_{\mu\nu}$ (EMT) and the electromagnetic field potential $\tilde{A}_\mu$ (EMP)

$$\begin{aligned} &\tilde{g}_{\mu\nu} = g_{\mu\nu} + h_{\mu\nu}, \\ &\tilde{F}_{\mu\nu} = \partial_\mu \tilde{A}_\nu - \partial_\nu \tilde{A}_\mu = F_{\mu\nu} + f_{\mu\nu}, \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \\ &f_{\mu\nu} = \partial_\mu f_\nu - \partial_\nu f_\mu, \qquad \tilde{A}_\mu = A_\mu + f_\mu, \end{aligned} \tag{1}$$

where $g_{\mu\nu}$, $F_{\mu\nu}$ and A_μ are the background quantities, $h_{\mu\nu}$, $f_{\mu\nu}$ and f_μ - the first perturbation corrections to the metric, EMT and EMP respectively.

To derive the equation of motion (EoM) for metric perturbations we expand the Einstein equation, $\tilde{R}_{\mu\nu} - \frac{1}{2}\tilde{g}_{\mu\nu}\tilde{R} = \frac{8\pi}{m_{pl}^2}\tilde{T}_{\mu\nu}$, to the first perturbation order. Also, using the relation $\tilde{T}_{\mu\nu} = \frac{2}{\sqrt{-\tilde{g}}}\frac{\delta \tilde{\mathcal{A}}_{matter}}{\delta \tilde{g}_{\mu\nu}}$, we include into equations the energy-momentum tensor corrections from the Maxwell action and from the Heisenberg-Euler (HE) action in the weak field limit ($B \ll m_e^2$) [5]

$$\tilde{\mathcal{A}}_{Maxwell} = -\frac{1}{4}\int dx^4 \sqrt{-\tilde{g}}\left(\tilde{F}_{\alpha\beta}\tilde{F}^{\alpha\beta} + \tilde{A}_\alpha \tilde{J}^\alpha\right), \tag{2}$$

$$\tilde{\mathcal{A}}_{HE} = \int dx^4 \sqrt{-\tilde{g}} C(T)\left[\left(\tilde{F}_{\alpha\beta}\tilde{F}^{\alpha\beta}\right) + \frac{7}{4}\left(\tilde{F}^{dual}_{\alpha\beta}\tilde{F}^{\alpha\beta}\right)^2\right], \tag{3}$$

where $C(T) = \sum_j \frac{\alpha^2(T) q_j^4}{90 m_j(T)^4}$, where q_j is the charge of the contributing to the loop particles in the electron charge units, and $F^{dual}_{\mu\nu}$ is the dual Maxwell tensor which is defined as $F^{dual}_{\alpha\beta} = \frac{\sqrt{-g}}{2} \epsilon_{\alpha\beta\mu\nu} F^{\mu\nu}$.

The EoM for electromagnetic wave (EMW) is obtained by the action variation over $\delta \tilde{A}_\nu$ followed by the obtained EoM expansion to the first perturbation order. After that we take into account the photon interaction with the plasma and introduce the terms proportional to the damping factor $\Gamma \sim \alpha^2 T$ and to the squared plasma frequency $\omega_{pl}^2 \sim \alpha T^2$ into equation, taking into account the dependence of temperature on the scale factor, $T \sim \frac{1}{a}$.

For more detail see the manuscript [6].

Derivation method and results of solution in FLRW metric

To obtain a qualitative result, the following assumptions are made: magnetic field is homogeneous and directed along the **x** axis, $C(T) = C_0 \sim e^4/m_e^4$ - does not depend on temperature, initial tensor GW propagates along the **z** axis and enters a medium with a magnetic field and a primary plasma. Also we neglect the gravity from the cosmological magnetic field in comparison with the gravity of matter.

We introduce the following initial conditions

$$\mathbf{k} = (0, 0, k_z)\,, \tag{4}$$
$$f^\mu = (0, f^x, f^y, 0)\,, \tag{5}$$
$$f^x(t=0) = 0, \tag{6}$$
$$f^y(t=0) = 0. \tag{7}$$

$$h^\mu_\nu(t=0, z) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & h^0_+ & h^0_\times & 0 \\ 0 & h^0_\times & -h^0_+ & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} e^{i k_z z}, \tag{8}$$

i.e. a clear tensor metric perturbation (gravitational wave) with two polarizations $\{h^0_+, h^0_\times\}$ enters the region with homogeneous magnetic field and with a primary plasma. The GW generates electromagnetic wave with two polarizations f^x and f^y. All the waves propagate along the **z** axis. Let us stress that during propagation in the region with a magnetic field the scalar modes of metric perturbations appear, that means that the tensor $h^\mu_\nu(t, z)$ will have the form

$$h^\mu_\nu(t, z) = \begin{bmatrix} 2\Phi(t) & 0 & 0 & 0 \\ 0 & 2\Psi(t) + h_+(t) & h_\times(t) & 0 \\ 0 & h_\times(t) & 2\Psi(t) - h_+(t) & 0 \\ 0 & 0 & 0 & 2\Psi(t) \end{bmatrix} e^{i k_z z}, \tag{9}$$

where Φ and Ψ are the scalar degrees of freedom.

The derived system consists of two independent subsystems: $\{f^x, h_\times\}$ and $\{f^y, \Phi, \Psi, h_+\}$. Let us write and solve the first subsystem in terms of the scale factor a:

$$
\begin{aligned}
& f^x : a^2H^2 f^{x\prime\prime} + aH^2\left[1 + a\frac{H'}{H} + 8\frac{2B_0^2C_0 - a^4}{16B_0^2C_0 - a^4} + aH\Gamma\right] f^{x\prime} + \\
& + \left[\frac{k^2}{a^2} + 2aHH' - 8H^2\frac{4B_0^2C_0 + a^4}{16B_0^2C_0 - a^4} + 2\Gamma H + \omega_{pl}^2\right] f^x = -\frac{ikB_0}{a^4 m_{pl}} h_\times, \\
& h_x^y : a^2H^2 h_\times'' + \left(4aH^2 + a^2HH'\right) h_\times' + \left[\frac{k^2}{a^2} - \frac{16\pi G B_0^2}{a^4}\left(\frac{4B_0^2C_0}{a^4} - 1\right)\right] h_\times = \\
& = \frac{16\pi G B_0 ik}{a^2}\left(1 - \frac{16B_0^2C_0}{a^4}\right) m_{pl} f^x,
\end{aligned}
\qquad (10)
$$

where the prime denotes the derivative with respect to the scale factor, H - the Hubble parameter, m_{pl} - the Planck mass, G - the gravitational constant.

It is worth to note that there are poles at $a^4 = 16B_0^2C_0$ in the first equation of system (10). The explanation is that the SoDE is not valid at the singular point. Indeed, we obtained the system using the expantion of electromagnetic field action. If the magnitude of the external magnetic field exceeds the electron mass squared the higher order corrections must be taken into account in the effective Heisenberg-Euler Lagrangian. So the pole will be eliminated.

Let us stress that for the chosen magnetic field strength of $B_0 = 1\,\mathrm{nGs} = 1.95 * 10^{-23}\,\mathrm{MeV}^2$ a pole point is $a_{singular} \sim 10^{-13}$ and lies before the considered scale factor interval $a \in [10^{-9}, 10^{-4}]$.

Finally we use an implicit Runge-Kutta method of order five to solve the SoDE (10). On the solution interval $a \in [10^{-9}, 10^{-4}]$ for the final solution point and for $B_0 = 1$ nGs (present time value) we obtain that the amplitude of GWs with a frequency of $10^{-16} - 10^{-18}$ Hz is suppressed by about 0.01 percent.

Conclusion and prospects

Based on the results we can conclude that the considered phenomenon insignificantly suppresses relic gravitational wave amplitude in low-frequency range. Indeed, even for the upper estimate, in the assumptions of homogeneous magnetic field and orthogonality of the relic GW propagation vector to the magnetic field direction, long-wave relic GW amplitude is suppressed by about 0.01 percent by the effect of conversion into photons during RD epoch.

Anyway the hypothesis had to be tested. Moreover the results of the investigation are applicable for the solution of problems about GW to EMW conversion in the vicinity of the strong magnetic field astrophysical sources. Of course the background metric has to be changed depending on the problem, using the SoDE obtained for the general case of curved space-time in the manuscript [6]. Some qualitative results are new in the context of graviton to photon conversion task. For example - mixing of different metric perturbation modes in the presence of one more selected direction.

In the future works solution of the second part of SoDE with conversion of GW into EMWs and into scalar metric perturbations is planned. And also it is planned to find solutions for relic GW with higher frequencies.

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