Investigating of conformal window in the Litim-Sannino model at 433 order

A. Mukhaeva[a,1]

[a] Joint Institute for Nuclear Research, Joliot-Curie, 6, Dubna 141980, Russia

We consider a four-dimensional quantum field theory with weakly interacting ultraviolet fixed points up to four loop order for gauge, three loop to Yukawa and quartic scalar beta functions. We compute them for a $SU(N_c)$ gauge theory coupled to N_f fundamental fermions and elementary scalars. Moreover, we found fixed point couplings, field and mass anomalous dimensions, and scaling exponents up to the first three non-trivial orders in a small Veneziano parameter. Further, we investigate the size of the conformal window.

PACS: 44.25.+f; 44.90.+c

Introduction

Let us consider possible asymptotic behavior of the dimensionless couplings of Quantum field theory (QFT). As we know, there are two behaviors. The first one called asymptotic freedom [1,2], very characteristic of the Quantum Chromodynamics, which means that with increasing the energy, the value of the some coupling is decreasing. This means that in the deep ultraviolet (UV) this coupling tends to leads to the Gaussian non-interactive fixed point.

Asymptotic safety (AS) [3–6] is some extension of asymptotic freedom, where couplings in the deep UV develop a fixed point. So their value stabilizes at some scale and does not change enough, and this value is not zero. Therefore, the theory remains interactive.

The idea of asymptotic safety has been first proposed by S. Weinberg in the later of 70s [3], as a way of making the four-dimensional theory of gravity non-perturbatively renormalizable. But in recent years AS has been quite extensively used in the framework of Yang-Mills theories as a way of curing pathological behaviors of the $U(1)$ gauge couplings by making them reach the interactive fixed point at some scale.

In this paper we confirm previous studies [7–9] of the Litim-Sannino model at 433 order for gauge, Yukawa and scalar couplings, respectively. Moreover we found the conformal window for finite number of colors, see the full version of paper [10].

Model

field	$SU(N_c)$	$U_L(N_f)$	$U_R(N_f)$
ψ_L	N_c	N_f	1
ψ_R	N_c	1	N_f
H	1	N_f	N_f

Table 1. Model content with corresponding representations under gauge and global symmetry

We consider a four-dimensional, renormalizable QFT with $SU(N_c)$ gauge group and an unbroken $U(N_f)_L \times U(N_f)_R$ global flavour symmetry. We have fermion and scalar fields, as listed in Tab.1 The corresponding Lagrangian consists of a gauge sector with field strength tensor $F_{\mu\nu}$, the coupling to the fermions via the covariant derivative D_μ, the gauge fixing $\mathcal{L}_{gf}$ and ghost terms $\mathcal{L}_{gh}$. The scalar and gauge sector interactions is mediated via the real chiral Yukawa coupling y. In the scalar sector, we have single-trace (u) and double-trace quartic couplings (v). Traces in the Lagrangian run over both flavour and gauge indices.

$$\begin{aligned}\mathcal{L} = &-\frac{1}{4}F^{A\mu\nu}F^{A}_{\mu\nu} + \mathcal{L}_{gf} + \mathcal{L}_{gh} + \mathrm{Tr}(\bar{\psi} i\hat{D}\psi)\\ &+ \mathrm{Tr}(\partial^\mu H^\dagger \partial_\mu H) - y\,\mathrm{Tr}[\bar{\psi}(H\mathcal{P}_R + \ H^\dagger \mathcal{P}_L)\psi]\\ &- m^2\,\mathrm{Tr}(H^\dagger H) - u\,\mathrm{Tr}((H^\dagger H)^2) - v(\mathrm{Tr}(H^\dagger H))^2,\end{aligned} \tag{1}$$

In this work, we are interested in the planar (Veneziano) limit, where field multiplicities N_f and N_c are large and interactions are parametrically weak. The advantage of the Veniziano limit is that it offers perturbative control, allowing expansions in a small parameter.

To prepare for the Veneziano limit, we introduce rescaled couplings, where explicit dependence on (N_c, N_f) drops out and we leave only with a dependence on $\epsilon \equiv \frac{N_f}{N_c} - \frac{11}{2}$:

$$\alpha_g = \frac{g^2 N_c}{(4\pi^2)}, \quad \alpha_y = \frac{y^2 N_c}{(4\pi^2)}, \tag{2}$$

$$\alpha_u = \frac{u N_f}{(4\pi^2)}, \quad \alpha_v = \frac{v N_f^2}{(4\pi^2)}. \tag{3}$$

Moreover, the parameter ϵ becomes continuous in this limit, taking values in the entire range $\epsilon \in [-\frac{11}{2}, \infty)$. We are particularly interested in the regime $|\epsilon| \ll 1$, where we can control perturbativity. The advantage of ϵ is that it is proportional to the one-loop coefficient of the gauge $\beta_g = \alpha_g^2\left(\frac{4}{3}\epsilon + O(\alpha_g)\right)$ beta function, which underlies perturbatively controlled fixed points in any 4D QFT.

A key feature of non-abelian gauge theories coupled to matter is that fixed point couplings α_i^* can be expanded as a power series in the small parameter

[1]E-mail: mukhaeva@theor.jinr.ru

ϵ. In our case, this means the "conformal expansion" of ϵ. The expansion coefficients $\alpha_i^{(n)}$ are determined using perturbation theory, by performing a perturbative loop expansion up to order $n+1$ in the gauge and up to order n in the Yukawa and quartic β-functions:

$$\alpha_x^* = a_{LO}\epsilon + a_{NLO}\epsilon^2 + a_{NNLO}\epsilon^3 + O(\epsilon^4). \tag{4}$$

It should be noted that generic β-functions for the gauge and Yukawa couplings have been calculated using `RGBeta` [11], while the quartic couplings have been determined using our own tools.

The main reason why we consider $(n+1, n, n)$ loop order is that the one-loop gauge coefficient is of the same order of magnitude as the two-loop gauge coefficient. Therefore we have some correlation between the perturbative and conformal expansions. In the 100 order, the gauge coupling running is slowed down and a fixed point cannot (yet) arise. In the 211 order a fixed point materialises [12], and in order 322, arises bounds on the conformal window [7,8]. In this work, we consider the third order of ϵ corresponding to the 433 approximation [9,10].

Discussion

Let us investigate the size of the UV conformal window for asymptotically safe theories with action (1) using perturbation theory. The main interest of our work is the ϵ^{max} values where the UV fixed point may persist. We can find the UV conformal window directly from β-functions, in this case ϵ value will be called ϵ_{strict}, since we take into account only ϵ to the power of n in the β-functions. The reason for this strict approach is that the higher order coefficients in power expansion (4) are not (yet) accurately determined due to the absence of higher loop terms in β-functions. This scheme is dictated firstly, by the fixed point and eigenvalues:

- as constraints for couplings $0 < |\alpha^*| \lesssim 1$ [13];
- as requirement for vacuum stability $\alpha_u^* > 0$ and $\alpha_u^* + \alpha_v^* > 0$ [14];
- as fixed point merger ($\theta = 0$), which means the collision of the UV fixed point with an infrared (IR) fixed point. We can linearize the RG flow in the region of its UV fixed point $\beta_i = \sum_j M_{ij}(\alpha_j - \alpha_j^*) + subl.$ and find the eigenvalues of M stability matrix ($M_{xx'} = \frac{\partial\beta_x}{\partial\alpha_{x'}}|_{\alpha=\alpha^*}$) which characterize the scaling of couplings in the vicinity of the fixed point. If eigenvalue > 0 then eigendirection called relevant, if < 0 irrelevant. In the 433 order we have 1 relevant and 3 irrelevant directions.

The second strategy employs the loop level approximation, where we retain subleading terms in ϵ and we refer its bounds as ϵ_{subl} (and place the corresponding contributions in the boxes). There we also take into account all the above menthioned constraints from couplings, vacuum stability and critical exponents.

	α_g^*	$\alpha_u^* + \alpha_v^*$	θ_1
ϵ_{strict}^{max}	$(0.117 - 0.457)$	$(0.087 - 0.146)$	$(0.091 - 0.249)$
ϵ_{subl}^{max}	$(0.117 - 0.363)$	$(0.087 - 0.116)$	$(0.091 - 0.234)$

Table 2. Constraints on the UV conformal window for different approximation schemes. Here the first value arises from the Padé approximation, since it gives tighter estimates.

Results

Using the expansion (4), and solving $\beta_i(\alpha_j^*) = 0$, we find the numerical expansion for fixed points

$$\begin{aligned} \alpha_g^* &= 0.456\epsilon + 0.781\epsilon^2 + 6.610\epsilon^3 + \boxed{24.137\epsilon^4}, \\ \alpha_y^* &= 0.211\epsilon + 0.508\epsilon^2 + 3.322\epsilon^3 + \boxed{15.212\ \epsilon^4}, \\ \alpha_u^* &= 0.200\epsilon + 0.440\epsilon^2 + 2.693\epsilon^3 + \boxed{12.119\ \epsilon^4}, \\ \alpha_v^* &= -0.137\epsilon - 0.632\epsilon^2 - 4.313\epsilon^3 - \boxed{24.147\ \epsilon^4}. \end{aligned} \tag{5}$$

The corresponding scaling exponents have the following form

$$\begin{aligned} \theta_1 &= -0.608\epsilon^2 + 0.707\epsilon^3 + 6.947\epsilon^4 + \boxed{4.825\ \epsilon^5}, \\ \theta_2 &= 2.737\epsilon + 6.676\epsilon^2 + 22.120\epsilon^3 + \boxed{102.55\ \epsilon^4}, \\ \theta_3 &= 2.941\epsilon + 1.041\epsilon^2 + 5.137\epsilon^3 - \boxed{62.340\ \epsilon^4}, \\ \theta_4 &= 4.039\epsilon + 9.107\epsilon^2 + 38.646\epsilon^3 + \boxed{87.016\ \epsilon^4}. \end{aligned} \tag{6}$$

Making the dimensional analysis with couplings [13], we note that the tightest bound on ϵ arises from the gauge coupling (5). In the same manner we can find the constraints on ϵ using the vacuum stability condition [14]. At the end, taking into account the expressions for scaling exponents (6), we notice that the series expansion for $\theta_{2,3,4}$ are monotonous, with same-sign corrections at every order. However the LO sign for relevant scaling exponent θ_1 has opposite value compared other loop terms, which is the indication of FP merger. Thus, we can extract the constraints from the relevant scaling exponent.

We summarize our results on the UV conformal window in Tab.2.

Therefore, we 1. confirmed the results [9]; 2. estimated conformal window in the Veneziano limit and set the upper bound for $\epsilon^{max} \approx (0.09 \pm 0.01)$.

Acknowledgements

We are thankful to Alexander Bednyakov for help at every step of the work and Tom Steudtner for corresponding discussion.

REFERENCES

1. *Gross D.J., Wilczek F.* Ultraviolet Behavior of Nonabelian Gauge Theories // Phys. Rev. Lett. — 1973. — V. 30. — P. 1343–1346.

2. *Politzer H.D.* Reliable Perturbative Results for Strong Interactions? // Phys. Rev. Lett. — 1973. — V. 30. — P. 1346–1349.

3. *Weinberg S.* ULTRAVIOLET DIVERGENCES IN QUANTUM THEORIES OF GRAVITATION // General Relativity: An Einstein Centenary Survey. — 1980. — P. 790–831.

4. *Eichhorn A.* An asymptotically safe guide to quantum gravity and matter // Front. Astron. Space Sci. — 2019. — V. 5. — P. 47. — arXiv:1810.07615.

5. *Eichhorn A., Schiffer M.* Asymptotic safety of gravity with matter. — 2022. — 12. — arXiv:2212.07456.

6. *Bednyakov A., Mukhaeva A.* Perturbative Asymptotic Safety and Its Phenomenological Applications // Symmetry. — 2023. — V. 15, no. 8. — P. 1497. — arXiv:2309.08258.

7. *Bond A.D., Litim D.F., Medina Vazquez G., Steudtner T.* UV conformal window for asymptotic safety // Phys. Rev. D. — 2018. — V. 97, no. 3. — P. 036019. — arXiv:1710.07615.

8. *Bond A.D., Litim D.F., Vazquez G.M.* Conformal windows beyond asymptotic freedom // Phys. Rev. D. — 2021. — V. 104, no. 10. — P. 105002. — arXiv:2107.13020.

9. *Litim D.F., Riyaz N., Stamou E., Steudtner T.* Asymptotic Safety Guaranteed at Four Loop. — 2023. — 7. — arXiv:2307.08747.

10. *Bednyakov A.V., Mukhaeva A.I.* in preparation. — 2023.

11. *Thomsen A.E.* Introducing RGBeta: a Mathematica package for the evaluation of renormalization group β-functions // Eur. Phys. J. C. — 2021. — V. 81, no. 5. — P. 408. — arXiv:2101.08265.

12. *Litim D.F., Sannino F.* Asymptotic safety guaranteed // JHEP. — 2014. — V. 12. — P. 178. — arXiv:1406.2337 [hep-th].

13. *Weinberg S.* Phenomenological Lagrangians // Physica A. — 1979. — V. 96, no. 1-2. — P. 327–340.

14. *Paterson A.J.* Coleman-Weinberg Symmetry Breaking in the Chiral SU(N) X SU(N) Linear Sigma Model // Nucl. Phys. B. — 1981. — V. 190. — P. 188–204.