The Multiple Compton process in a strongly magnetized plasma

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The process of multiple Compton scattering, $e\gamma \to e(N\gamma)$, is considered in a strongly magnetized, charge-asymmetric, cold electron-positron plasma. The special case of process – the Double Compton scattering is considered. The amplitudes for both processes are derived in the general case, where electrons can occupy arbitrary Landau levels. An expressions for the amplitude of the Double Compton process in the magnetar case have been obtained. The double Compton scattering under considered conditions is shown to be an efficient process for the production of polarized photons.

Рассмотрен процесс множественного комптоновского рассеяния $e\gamma \to e(N\ \gamma)$ в сильно замагниченной, зарядово-асимметричной, холодной электрон-позитронной плазме. Рассмотрен частный случай процесса – двойное комтпоновское рассеяние. Получены амплитуды обоих процессов в общем случае, когда электроны могут занимать произвольный уровень Ландау. Получено выражение амплитуды двойного комптоновского процесса в случае магнитара. Показано, что двойное рассеяние Комптона в рассматриваемых условиях является эффективным процессом для получения поляризованных фотонов.

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Introduction

The isolated neutron stars has attracted considerable scientific interest since its discovery. They possess strong magnetic fields, i.e., fields whose values are orders of magnitude greater than the critical value $B \gg B_e = (m^2c^3)/(e\hbar) \simeq 4.41 \cdot 10^{13}$ G, with radio pulsars and magnetars being of particular interest.

The estimates have shown that magnetar's magnetic fields can reach values of $B \sim 10^{14} - 10^{15}$ G (see review [1]). The analysis of the emission spectra has demonstrated the presence of the electron-positron plasma in their shells.

The problem of the radiation from such objects [2] remains one of the unresolved challenges in their study. Observations indicate that their radiation spectra contain a greater number of soft gamma photons compared to the

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predictions of existing models [3]. One potential approach for addressing this issue entails an investigation of the mechanisms governing the generation of polarized photons in these objects. Previously, multiple photon re-emission in the magnetospheres of such stars, photon splitting in magnetic fields and plasma [4, 5] were considered in the literature. Nevertheless, as the analysis has revealed, these mechanisms are currently insufficient to fully describe the observed spectra. In this work, the process of multiple Compton scattering in a strong magnetic field, taking into account the plasma dispersion effects [6], will be investigated. The Melrose-Parle technique for calculating multivertex processes in magnetic fields, developed in [7], will be applied for calculations in magnetic fields, which simplifies the calculations by reducing the number of operations. Throughout the work, a similar approach will be applied to the process of double Compton scattering in a strong magnetic field, that considered as one of the possible sources of soft photons in NS (neutron stars). The Heaviside system of units will also be used: $\hbar = c = k_{Bol} = 1$. Here, $\hbar$ – Planck constant, c – speed of light, and k_{Bol} – Boltzmann constant, m – electron mass, $e > 0$, e – elementary charge.

Amplitude of Multiple Compton process

Begin the analysis of the process $e\gamma \to e(N\gamma)$, where N – number of the final photons, by constructing S-matrix element of process.
The process is depicted by the following Feynman diagrams:

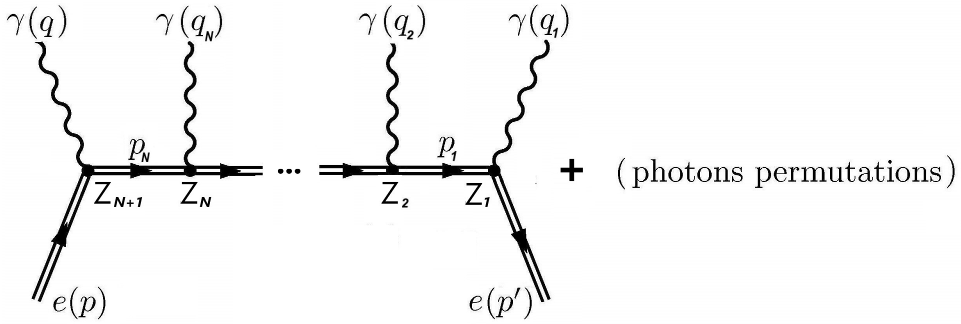

Fig. 1. The Feynman diagrams of $e\gamma \to e(N\gamma)$ process

In accordance with standard Feynman diagram techniques, the S-matrix element can be expressed as:

$$S^{s,s'}_{p,p'} = (ie)^{N+1} \int d^4Z_1...d^4Z_{N+1}\bar{\Psi}^{s'}_{\ell',p'}(Z_1)\hat{A}^*_{q_1}(Z_1)\hat{S}_{p_1}(Z_1,Z_2)\hat{A}^*_{q_2}(Z_2)...\times$$
$$\hat{S}_{p_N}(Z_{N-1},Z_N)\hat{A}_q(Z_{N+1})\Psi^s_{\ell,p}(Z_{N+1}) + \text{(photon permutations)} \tag{1}$$

Where Z_k – 4-coordinate of k-th vortex, $\hat{S}_{p_k}(Z_k, Z_{k-1})$ – propagator with momenta p_k, $\Psi^s_{\ell,p}$ – electron wave function with polarization state – s, Landau level – ℓ, momenta – p. A_μ – the photon field potential (for more detail about notations see [8]), q – photon momenta connected with vortex Z_k. Star denotes the complex conjugation.

The method employed will necessitate the use of a propagator with electrons located on a mass surface, therefore the propagator should be expressed in a more convenient form:

$$\hat{S}(X,X') = \sum_{\ell} \int \frac{dq_0 dq_y dq_z}{(2\pi)^3} \tag{2}$$

$$\frac{\exp[-i(q(X-X'))_{\|} + iq_y(X-X')_y]}{q_{\|}^2 - M_\ell^2 - Re(q)} \sum_{s=\pm 1} U_\ell^s(\xi(X_x))\bar{U}_\ell^s(\xi(X_x^{'}))$$

Here, $U_\ell^s(\xi(X_x))$ represents fermionic bispinors associated with the electron wave function, which can be found in [8], and the bar signifies Dirac conjugation. The amplitude of the process at an arbitrary Landau level can be deduced from the S-matrix, and it is:

$$M_{\ell,\ell'}^{s,s'} = \sum_{n_1,\ldots,n_N=0}^{\infty} \frac{\exp\left[-\frac{i}{2eB}\left\{q_x(p_N+p)_y\right\}\right]}{\sqrt{4M_\ell M_{\ell'}(m+M_\ell)(m+M_{\ell'})}} \left[\frac{q_y - iq_x}{\sqrt{q_\perp^2}}\right]^{n_N-\ell} \sum_{s_1,\ldots,s_N=\pm 1} T_{n_N,\ell}^{s,s_N}(q,p_N,p) \times$$

$$\prod_{k=1}^{N} \frac{T_{n_{k-1},n_k}^{s_{k-1},s_k}(q_k,p_{k-1},p_k)}{[(p_k)_{\|}^2 - M_{n_k}^2]} \left[\frac{(q_k)_y + i(q_k)_x}{\sqrt{(q_k)_\perp^2}}\right]^{n_{k-1}-n_k} \frac{\exp\left[\frac{i}{2eB}\left\{(q_k)_x(p_{k-1}+p_k)_y\right\}\right]}{2M_{n_k}(m+M_{n_k})} \tag{3}$$

Where $q_{i\alpha} = (\omega_i, \mathbf{k}_i)$ – 4-momenta of photon, $p_0 = p'$, $n_0 = \ell'$, $s_0 = s'$, n_N – Landau level of electron from N-th propagator, $M_\ell = \sqrt{m^2 + 2eB\ell}$ – effective electron mass in the magnetic field. The four-vectors with indices $\perp$ and $\|$ belong to the Euclidean $\{1, 2\}$-subspace and the Minkowski $\{0, 3\}$-subspace correspondingly in the frame were the magnetic field is directed along third axis; $(ab)_\perp = (a\Lambda b) = a_1b_1 + a_2b_2$, $(ab)_{\|} = (a\tilde{\Lambda}b) = a_0b_0 - a_3b_3$ and $\Lambda_{\alpha\beta} = \mathrm{diag}\{0,1,1,0\}$, $\tilde{\Lambda}_{\alpha\beta} = \mathrm{diag}\{1,0,0,-1\}$.

$T_{n,m}^{s,s'}(q,p,p')$ is an invariant expression connected with each vortexes, it's has a complex form and could found in [8].

Amplitude of Double Compton process

The amplitude of the process $e\gamma \to e\gamma\gamma$ at an arbitrary Landau level can be got from (3) by choosing $N = 2$:

$$M_{\ell,\ell'}^{s,s'} = \sum_{n_1,n_2=0}^{\infty} \frac{1}{[(p_1)_{\|}^2 - M_{n_1}^2][(p_2)_{\|}^2 - M_{n_2}^2]} \frac{1}{\sqrt{4M_\ell M_{\ell'}(m+M_\ell)(m+M_{\ell'})}} \times$$

$$\frac{\exp\left[\frac{i}{2eB}\left\{q_{1x}(p'_y + (p_1)_y) + (q_2)_x((p_1)_y + (p_2)_y) - q_x((p_2)_y + p_y)\right\}\right]}{4M_{n_1}M_{n_2}(m+M_{n_1})(m+M_{n_2})} \times$$

$$\left[\frac{(q_1)_y + i(q_1)_x}{\sqrt{(q_1)_\perp^2}}\right]^{\ell'-n_1} \left[\frac{(q_2)_y + i(q_2)_x}{\sqrt{(q_2)_\perp^2}}\right]^{n_1-n_2} \left[\frac{q_y - iq_x}{\sqrt{q_\perp^2)}}\right]^{n_2-\ell} \times$$

$$\sum_{s_1,s_2=\pm 1} T_{\ell,n_2}^{s,s_2}(q,p,p_2) T_{n_2,n_1}^{s_2,s_1}(q_2,p_2,p_1) T_{n_1,\ell'}^{s_1,s'}(q_1,p_1,p') \tag{4}$$

In the conditional of magnetar the electrons has to occupy the ground Landau level. The amplitude in the zero approximation will be described by the case when all electrons are at the ground level, and it could be expressed in the form

$$\mathcal{M}^{tot}_{0,0} \simeq \exp\left[-\frac{(q_1)^2_\perp + (q_2)^2_\perp + (q^2_\perp)}{4eB}\right] \times$$
$$\frac{(K_1(p', p_1)\varepsilon_{q_1})(K_1(p_1, p_2)\varepsilon_{q_2})(K_1(p_2, p)\varepsilon_q)}{[(p_1)^2_\parallel + m][(p_2)^2_\parallel + m]} + (\text{photon permutations}) \quad (5)$$

Where

$$K_{1\alpha}(p, p') = \sqrt{\frac{2}{(p\tilde{\Lambda}p') + M_n M_m}} \left\{ M_n(\tilde{\Lambda}p')_\alpha + M_m(\tilde{\Lambda}p)_\alpha \right\} \quad (6)$$

Photon production efficiency

To analyze the efficiency of the process, $e\gamma \to e\gamma\gamma$ under consideration and to compare it with other competitive reactions we calculate the photon absorption rates which can be defined according [9]

$$W_{e\gamma\to e\gamma\gamma} = \frac{eB}{64(2\pi)^7\omega_\lambda} \int |\ \mathcal{M}^{tot}_{0,0}\ |^2\ Z_q Z_{q_1} Z_{q_2}(1 - f_{E'}) \times$$
$$f_E(1 + f_{\omega_1})(1 + f_{\omega_2})\delta(\omega + E - \omega_1 - \omega_2 - E') \frac{dp_z\, d^3q_1 d^3q_2}{EE'\omega'\omega''} \quad (7)$$

$f_E = [e^{(E-\mu)/T} - 1]^{-1}$ – equilibrium electron distribution function,
$f_\omega = [e^{\omega/T} - 1]^{-1}$ – equilibrium photon distribution function.
$Z_q^{-1} = 1 - \frac{\partial \varkappa^{(2)}(q)}{\partial \omega^2}$ – renormalization coefficient of the wave function of photon. It is considered facts: $\hat{\varepsilon}^{(2)}_\alpha(q) \to \hat{\varepsilon}^{(2)}_\alpha(q)\sqrt{Z_q}$ and fact that the eigenvalue of the polarization operator $\varkappa^{(2)}$ (see [6]) becomes large near the electron-positron pair production threshold.

The analysis provides an estimate of the number of mode 2 photons produced in the process $e\gamma \to e\gamma\gamma$ in the magnetosphere of strongly magnetized neutron star:

$$\frac{dN}{dV dt} \simeq 1.6 \cdot 10^{17} \left(\frac{1}{\text{cm}^3\text{s}}\right) \left(\frac{n_e}{3 \cdot 10^{13}\text{cm}^{-3}}\right) \left(\frac{T}{5\text{KeV}}\right)^5 \quad (8)$$

Conclusion

In work the multiple Compton process, $e\gamma \to e(N\gamma)$, in the presence of a strongly magnetized, charge asymmetric, cold plasma is considered.

As an application of the obtained results the double compton process is got. The amplitudes of the processes $e\gamma \to e(N\gamma)$, $e\gamma \to e\gamma\gamma$ are obtained. The amplitude of Double Compton process is obtained both in general form and special cease that impotent in magnetars. The number of mode 2 photons production in the process is estimated. It's shown that the magnetic field suppress the efficiency of photon production in the double Compton process, it's shown that process can be an effective source of polarized photons in NS conditions.

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