Analysis of $1/N_c$ corrections in the quark model for the pion transition form factor
Анализ $1/N_c$ поправок в кварковой модели для переходного форм-фактора пиона

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Проанализированы $1/N_c$ поправки в рамках нелокальной кварковой модели. Начальный этап генерации диаграмм выполняется в программе QGRAF. Данные, сгенерированные QGRAF, обрабатываются, и диаграммы отбираются на основе правил $1/N_c$ счета. Диаграммы классифицируются по определенным типам. В дальнейшем выражения передаются в программу аналитических расчетов FORM. Проведен анализ диаграмм Фейнмана для переходного форм-фактора пиона.

$1/N_c$ corrections in the framework of the nonlocal quark model are analyzed. The initial stage of generating Feynman diagrams is carried out in the QGRAF program. The data generated by QGRAF is processed and the diagrams are selected with the $1/N_c$ counting rules. The diagrams are classified to certain types. Subsequently, the expressions are transferred to the FORM analytical calculation program. The analysis of the Feynman diagrams for the pion transition form factor is performed.

PACS:

Introduction

The fundamental theory of the strong interactions is quantum chromodynamics. Due to the property of asymptotic freedom, the coupling constant α_s is small at large momentum transfers, and therefore well-developed perturbation theory methods can be used. In the low-energy region α_s is no longer a small parameter and the usage of methods beyond perturbation theory is necessary. Therefore one can use the non-pertrubative effective models which are based on symmetries of QCD Lagrangian in such domain.

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Nonlocal quark model

One of the widely used effective models is the quark version of the Nambu-Jona-Lasinio model [1]. The simplest $SU(2)\times SU(2)$ The Lagrangian of this model looks as follows [2]:

$$\mathcal{L}=\mathcal{L}_{free}+\mathcal{L}_{P,S}+\mathcal{L}_{V,A},\quad \mathcal{L}_{free}=\bar{q}(x)(i\hat{\partial}-M_c)q(x), \tag{1}$$

$$\mathcal{L}_{P,S}=\frac{G_1}{2}\left(\Big(J_S^a(x)\Big)^2+\Big(J_P^a(x)\Big)^2\right). \tag{2}$$

$\mathcal{L}_{free}$ – corresponds to "free" quarks, $\mathcal{L}_{P,S}$ – scalar and pseudoscalar mesons, G_1 is the four-quark coupling constant, $J_M^{(a)}$ are quark currents. In the nonlocal version of the NJL model motivated by the instanton interactions [3,4] these currents are

$$J_M^a(x)=\int d^4x_1 d^4x_2\, f(x_1)f(x_2)\,\bar{q}(x-x_1)\,\Gamma_M^a q(x+x_2). \tag{3}$$

Dynamic mass

Due to the spontaneous breaking of chiral symmetry the current quarks are transformed to the constituent ones. One can obtain the effective Lagrangian by performing bosonization

$$\mathcal{L}_{eff}=\mathcal{L}_{free}-\frac{1}{2G_1}\left(\Big(P^a(x)\Big)^2+\Big(S^a(x)\Big)^2\right)+P^a(x)J_P^a(x)+S^a(x)J_S^a(x).$$

Here $P^a(x)$ and $S^a(x)$ are the meson fields (pseudoscalar and scalar, respectively).The field $S^a(x)$ has a non-zero vacuum value $\langle S^0\rangle_0=\sigma_0\neq 0$. In order to obtain a physical scalar field with zero vacuum value, it is necessary to shift the scalar field $S^0=\tilde{S}^0+\sigma^0$ which leads to the appearance of the dynamical quark mass. In the nonlocal model it depends on the quark momentum

$$m(p)=m_c+m_d f^2(p),\quad m_d=G_1\frac{8N_c}{(2\pi)^4}\int d_E^4k\frac{f^2(k^2)m(k^2)}{k^2+m^2(k^2)}. \tag{4}$$

$1/N_c$ expansion

In order to have a correspondence for the weak pion decay constant with QCD calculations ($f_\pi\sim\sqrt{N_c}$) the four-quark coupling constant G_1 should behave as $G_1\sim 1/N_c$ [5,6]. Since the meson polarization operator $\Pi(p^2)$

$$\Pi(p^2)=i\frac{N_c}{(2\pi)^4}\int d^4k f^2(k_+^2)f^2(k_-^2)\,\mathrm{Tr}_{d,f}\left[\mathrm{S}(k_-)\Gamma_{M_1}^a\mathrm{S}(k_+)\Gamma_{M_2}^b\right],$$

also depends linearly on N_c, the meson propagator

$$D(p^2) = (-G^{-1} + \Pi(p^2))^{-1} \tag{5}$$

is suppressed by the $1/N_c$ factor $D \sim 1/N_c$. Thus, each meson propagator inside the loop leads to the addition of the $1/N_c$ factor. In the left part in Fig.1 the diagram for the leading contribution to the pion transition form factor is shown, while the right diagram is the possible $1/N_c$ correction to the leading quark triangle diagram. This is the basis for the $1/N_c$ expansion and the calculation of the corresponding corrections.

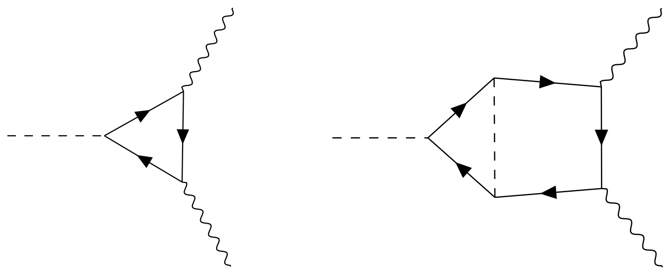

Fig. 1. Diagram with meson propagator (right) is suppressed in comparison with diagram without meson (left).

Feynman diagrams

Generally, Feynman diagrams consist of: external lines corresponding to free particles in initial and final states; internal lines – propagators; vertices with three or more lines, indicating the (non)local interactions of particles. In the paper the only processes with bosons in the initial or final states are considered and therefore the quark lines always form loops. Since it is necessary to take a trace along the internal degrees of freedom during the calculation of the fermionic loop, the additional factor N_c appears.

Therefore, in order to determine the N_c order of the diagram the following rules should be used:

- the closed fermionic cycle contributes as $\sim N_c$,
- the meson propagator as $\sim 1/N_c$.

There are two corrections to the quark propagator as it is shown in Fig.2.

One can see that these contributions have different numbers of loops. Possible $1/N_c$ corrections for a vertex with a photon can be obtained from corrections to a quark propagator by attaching a photon line to an internal quark or a meson propagator, or to a quark-meson vertex. At the end some of these diagrams should be ignored.

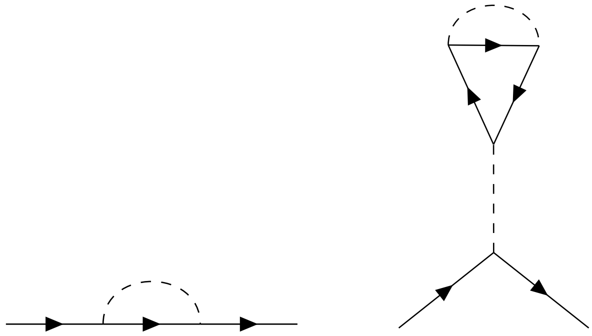

Fig. 2. $1/N_c$ corrections to the quark propagator.

Results

For the pion transition form factor the contributions of the leading and sub-leading orders are considered. For the pion transition form factor at the leading order there is the $(N_c)^1$ factor (apart from the pion coupling constant $g_\pi \sim 1/\sqrt{N_c}$). The sub-leading contributions is proportional to $(N_c)^0$. In the leading order there are only single-loop diagrams. Due to the nonlocal vertices with photons there are 6 possible diagrams (in the local model only two quark triangles are present) [7, 8]. In the sub-leading order the number of loops should be at least one larger, i.e. 2, than the leading contribution. However, the three and four loops contributions of the order $(N_c)^0$ are also possible. Totally, there are following number of diagrams in different loops:

- 2-loops – 42
- 3-loops – 88
- 4-loops – 32.

So, one can see that from the 6 diagrams in the leading order, the 162 diagrams appears at the sub-leading order. The typical leading order diagram is given in Fig.1(left), while the subleading 2-,3-,4-loops diagrams in Fig.1(right), Fig.3(left) and Fig.3(right), correspondingly.

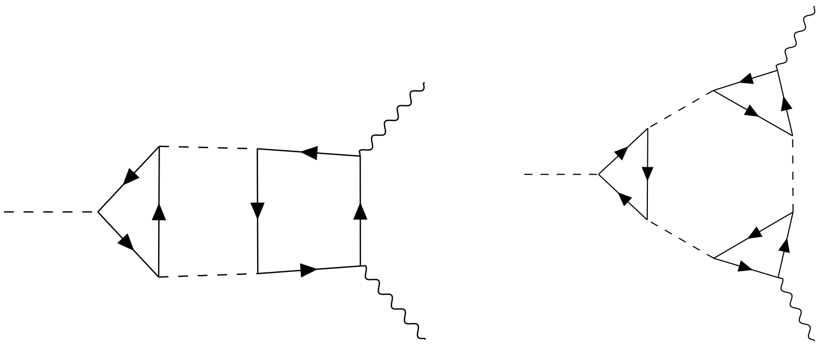

Fig. 3. Sub leading contribution to pion transition form factor: three loop (left), four loop (right).

Conclusion

Summing up the work the following results are obtained:

- the diagrams generated by QGRAF is processed by Python code and then transferred to FORM,
- the $1/N_c$ selection of the diagrams is performed,
- the diagrams for pion transition form factor are obtained.

In future we plan to calculate the obtained diagrams.

The work of A.E.R. is funded by the Ministry of Education and Science of the Russian Federation within the framework of the project "Analytical and numerical methods of mathematical physics in problems of tomography, quantum field theory,and fluid and gas mechanics" (no. of state registration: 121041300058-1).

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