Effect of magnetic field on Dual QCD Quark-hadron phase transition

Garima Punetha[1]

LSM Campus Pithoragarh, SSJ University Almora Uttarakhand, India

In order to better understand the dynamics of the quark-hadron phase transition in the presence of a magnetic field, the thermal behaviour of the non-perturbative QCD vacuum has been examined. The dynamical configuration of the resulting dual QCD vacuum and its flux tube configuration have been explored in order to investigate the non-perturbative properties of QCD. Within the context of dual QCD-based hadronic bag, which ensures the critical parameters and the accompanying critical sites for quark-hadron phase transition, related thermodynamic quantities and equation of state (EoS) to define quark matter have also been studied in presence of magnetic field.

PACS: 12.38.-t, 14.80.Hv, 64.60.av, 12.38.Mh

Introduction

High-energy collisions have been effective in producing quark-gluon plasma (QGP), a state of matter thought to have existed shortly after the big bang [1]. During off-center collisions, the charged ions can generate substantial magnetic fields up to $eB(1-15)m_\pi^2$ [2]. This method can produce magnetic fields that last just a brief duration yet are stationary during that time [3]. There has been a flurry of research in this area [4–7], and the theoretical methods used to explore QGP need to be modified to reflect the effects of external magnetic fields. Insights can be gained from measurements at the Large Hadron Collider and relativistic heavy ion collider energies [8,9], which can be used to constrain theoretical modelling. The evolution of QGP is significantly influenced by the equation of state [10]. Therefore, it is crucial to investigate the behaviour of magnetised QGP [11]. Different methods have been used to investigate how magnetic fields affect QGP [12–14]. In the present paper with the help of Grand canonical partition function within the domain of Bag Model, we calculated normalised pressure for magnetized QGP at finite temperature.

Field Decomposition Formulation and Magnetic Symmetry

The mathematical foundation for the dual gauge theory comes from the observation that the non-Abelian gauge symmetry allow an extra internal

[1]E-mail: garimapunetha@gmail.com

symmetry called magnetic symmetry which restricts and reduces the dynamical degrees of the theory [15–18],

$$D_\mu \hat{m} = 0, \;\; i.e. \, (\, \partial_\mu + g \, \mathbf{W}_\mu \times) \, \hat{m} \; = \; 0 \, , \tag{1}$$

where $\hat{m}$ is a multiplet defined by the adjoint representation of the group. The most general gauge potential which satisfies the above constraint is written as,

$$\mathbf{W}_\mu \;=\; A_\mu \, \hat{m} + \, A^{'}_\mu \, \hat{m}^{'} - g^{-1} \, (\, \hat{m} \times \partial_\mu \, \hat{m}) \; - g^{-1} \, (\, \hat{m}^{'} \times \partial_\mu \, \hat{m}^{'}), \tag{2}$$

where, A_μ and $A^{'}_\mu$ are the Abelian component of $\mathbf{W}_\mu$ along $\hat{m}$ and $\hat{m}^{'}$ respectively and are unrestricted by the constraint. The topological structure may be brought into dynamics in a dual symmetric way by imposing magnetic symmetry and for $SU(3)$ the relevant homotopy group is $\pi_2[SU(3)/U(1) \otimes U^{'}(1)]$ so that $\pi_2[SU(3)/U(1) \otimes U^{'}(1)] = \pi_1[U(1) \otimes U^{'}(1)]$, where $U(1) \otimes U^{'}(1)$ is the two Abelian subgroups generated by λ_3 and λ_8. The $\hat{m}$ consist of n winding of the i-spin subgroup followed by $n^{'}$ winding of u-spin subgroup of $SU(3)$ and is given as,

$$\hat{m} = \begin{pmatrix} \sin\alpha \cos\frac{1}{2}\alpha \cos(\beta - \beta^{'}) \\ \sin\alpha \cos\frac{1}{2}\alpha \sin(\beta - \beta^{'}) \\ \frac{1}{4}\cos\alpha(3 + \cos\alpha) \\ \sin\alpha \sin\frac{1}{2}\alpha \cos\beta \\ \sin\alpha \sin\frac{1}{2}\alpha \sin\beta \\ \frac{-1}{2}\sin\alpha \cos\alpha \cos\beta^{'} \\ \frac{-1}{2}\sin\alpha \cos\alpha \sin\beta^{'} \\ \frac{1}{4}\sqrt{3}\cos\alpha(1 - \cos\alpha) \end{pmatrix}, \tag{3}$$

where α and β are coset variables. Rotating the magnetic vector $\hat{m}$ to a fix time independent direction by a gauge transformation leads to the value of gauge potential as,

$$\mathbf{W}_\mu \xrightarrow{U} g^{-1} \left[\left((\partial_\mu \beta - \frac{1}{2} \partial_\mu \beta^{'}) cos\alpha \right) \hat{\xi}_3 + \frac{1}{2} \sqrt{3} (\partial_\mu \beta^{'} cos\alpha) \hat{\xi}_8 \right], \tag{4}$$

where $\hat{\xi}_3$ and $\hat{\xi}_8$ are the space-time independent unit vectors pointing to λ_3 and λ_8 directions of the internal space respectively and the associated field strength takes the form as,

$$\mathbf{G}_{\mu\nu} \xrightarrow{U} \, (\, F_{\mu\nu} \, + \, B^{(d)}_{\mu\nu} \,) \, \hat{\xi}_3 + \, (\, F^{'}_{\mu\nu} \, + \, B^{'\,(d)}_{\mu\nu} \,) \, \hat{\xi}_8, \tag{5}$$

where $F_{\mu\nu}$ and $B^{(d)}_{\mu\nu}$ are the electric and magnetic field strength respectively. The confinement mechanism of the QCD vacuum can be understood in absence of color electric sources (quarks) and the Lagrangian may be reduced in the following form,

$$\mathcal{L}^{(m)}_d = -\frac{1}{4} B^2_{\mu\nu} - \frac{1}{4} B^{'\,2}_{\mu\nu} + |(\partial_\mu + i \frac{4\pi}{g} B^{(d)}_\mu) \phi|^2 + |(\partial_\mu + i \frac{4\pi\sqrt{(3)}}{g} B^{'\,(d)}_\mu) \phi^{'}|^2 - V, \tag{6}$$

γ	α_s	$\bar{m}_\phi(GeV)$	$\bar{m}_B(GeV)$	$\lambda^{(d)}_{QCD}(GeV^{-1})$	$\xi^{(d)}_{QCD}(GeV^{-1})$	$\kappa^{(d)}_{QCD}$
5.617	0.25	1.21	1.75	0.57	0.83	0.69
6.828	0.24	1.69	1.62	0.61	0.59	0.99
8.093	0.23	2.17	1.52	0.65	0.46	1.42
9.833	0.22	2.90	1.41	0.70	0.34	2.05

Table 1. The masses of vector and scalar glueball as functions of α_s, where $\lambda^{(d)}_{QCD}$, $\xi^{(d)}_{QCD}$ and $\kappa^{(d)}_{QCD}$ are the penetration depth, coherence length and Ginzburg-Landau parameter respectively.

where, V is the effective potential responsible for the dynamical breaking of magnetic symmetry and ϕ is a complex scalar field for monopoles is given below,

$$V = \frac{48\pi^2}{g^4}\lambda(\phi^*\phi - \phi_0^2)^2 + \frac{432\pi^2}{g^4}\lambda^{'}(\phi^*\phi^{'} - \phi_0^{'2})^2. \tag{7}$$

Utilizing the asymptotic solutions, the energy per unit length of the resulting flux tube configuration is obtained as,

$$k_{(n,n^{'})} = 2\pi\int_0^\infty \rho d\rho\left[\frac{n^2g^2}{32\pi^2\rho^2}\left(\frac{dF}{d\rho}\right)^2 + \frac{n^2}{\rho^2}F^2(\rho)\chi^2(\rho) + \left(\frac{d\chi}{d\rho}\right)^2 + 3\lambda\alpha_s^{-2}(\chi^2-\phi_0^2)^2\right] + 2\pi\int_0^\infty \rho d\rho\left[\frac{n^{'2}g^2}{96\pi^2\rho^2}\left(\frac{dG}{d\rho}\right)^2 + \frac{n^{'2}}{\rho^2}G^2(\rho)\chi^{'2}(\rho) + \left(\frac{d\chi^{'}}{d\rho}\right)^2 + 27\lambda^{'}\alpha_s^{-2}(\chi^{'2}-\phi_0^{'2})^2\right], \tag{8}$$

where n, $n^{'}$ are color quantum numbers and $F(\rho) \overset{\rho\to\infty}{\longrightarrow} C\sqrt{\rho}exp\left(-m_B\rho\right)$, $G(\rho) \overset{\rho\to\infty}{\longrightarrow} C^{'}\sqrt{\rho}exp\left(-m^{'}_B\rho\right)$ and $\phi(x) = \exp(in\varphi)\chi(\rho)$, $\phi^{'}(x) = \exp(in^{'}\varphi)\chi^{'}(\rho)$. Incorporating color reflection invariance the masses of the magnetic glueballs is estimated by evaluating the string tension $k_{(n,n^{'})}$ of the resulting flux tube written as, $k_{(n,n^{'})} = \frac{1}{2\pi\alpha^{'}} = \gamma_{(n,n^{'})}\phi_0^2$. As a result, using the numerical computation of equation (8), for $\gamma_{(n,n^{'})}$, one may obtain the vector ($\bar{m}_B$) and scalar ($\bar{m}_\phi$) magnetic glueball masses as a function of α_s and these results are obtained in table (1).

Equation of State in presence of Magnetic Field

The transition from a confined hadronic phase to a QGP phase, is foreseen at high temperatures and/or baryonic chemical potentials and for some decades, the bag model equation of state (EoS) has been accustomed to characterize QGP. Employing the dual QCD formalism, the bag energy (B_{su3})

may be identified and manifest to perform an significant role in determining the phase structure of the QCD vacuum. The dominant part of the energy related with the confinement regime is explicated from equation (8) and may be recognized as bag energy expressed as,

$$B_{su3}^{1/4} = \left(\frac{\lambda}{\pi^2}\right)^{1/4} \frac{\bar{m_B}}{4}, \tag{9}$$

where $\bar{m_B}$ is the non-thermal contribution to the vector glueball mass. To contemplate the QGP phase, we utilize a Bag model EoS with bag energy. We have chosen the limit of large B in order to make the numerical evaluation of thermal integrals less complicated. The pressure for the QGP phase in presence of magnetic field may be expressed in the following form [19],

$$P_{QGP} = \frac{37}{90}\pi^2 T^4 + \ T\Sigma_f \frac{q_f B}{2\pi^2} \int dk_z ln\left[1 + e^{-\frac{\sqrt{k_z^2+m_f^2}}{T}}\right] - B_{su3}, \tag{10}$$

where B is the external magnetic field in the z-direction, f is the flavor index and q_f is the electric charge. The pressure for hadronic phase may be expressed as [18],

$$P_h = \frac{7}{180}\pi^2 T^4. \tag{11}$$

Utilizing the grand canonical ensemble formalism, the EoS for quark-hadron system at non-vanishing external magnetic field has been constructed within the framework of $SU(3)$ dual QCD hadronic bag in which the bag pressure has been shown to depend on the vector glueball mass mode of the magnetically condensed QCD vacuum. In fig. 1 3-D plot of normalised pressure for hadronic and QGP phase given by equations (10) and (11) respectively and has been plotted in the $T - eB$ plane for different couplings.

Conclusion

The effect of magnetic field (B) on the equation of state of hot quark matter is investigated. A magnetic field induces phase-space Landau-level quantization and for lowest Landau-level ($l = 0$) normalised pressure for QGP and hadronic phase have been obtained within the framework of dual QCD hadronic bag at non-zero external magnetic field. With increasing magnetic field strength the critical temperature reduces which leads to inverse magnetic catalysis. The normalised pressure increases with increasing magnetic field especially at low temperatures. At high temperature the normalised pressure is not affected by magnetic field and is apparently limited to the Stefan Boltzmann limits.

ACKNOWLEDGEMENT

The author is grateful to the Uttarakhand State Council for Science and Technology (UCOST), Dehradun (India) for the financial assistance in terms of the research project for the study.

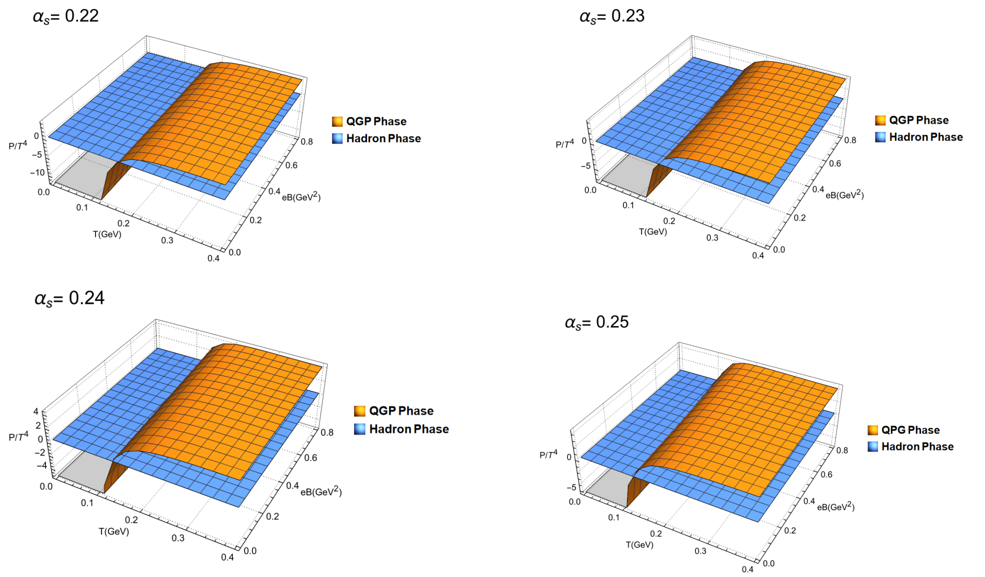

Fig. 1. The 3-D plot of normalized pressure (hadron phase and QGP phase) in $T - eB$ plane for different couplings.

REFERENCES

1. B. B. Back et al., “The PHOBOS Perspective on Discoveries at RHIC,” Nucl.Phys. A **757**, 28 (2005) doi.org/10.1016/j.nuclphysa.2005.03.084 [arXiv:nucl-ex/0410022]; J. Adams et al., “Experimental and Theoretical Challenges in the Search for the Quark Gluon Plasma: The STAR Collaboration’s Critical Assessment of the Evidence from RHIC Collisions,” ibid. **757**, 102 (2005) doi.org/10.1016/j.nuclphysa.2005.03.085 [arXiv:nucl-ex/0501009]; K. Adcoxet al., “Formation of dense partonic matter in relativistic nucleus-nucleus collisions at RHIC: Experimental evaluation by the PHENIX collaboration,” ibid. **757**, 184 (2005) doi.org/10.1016/j.nuclphysa.2005.03.086 [arXiv:nucl-ex/0410003]; I. Arseneet al., “Quark Gluon Plasma an Color Glass Condensate at RHIC? The perspective from the BRAHMS experiment,” ibid. **757**, 1 (2005) doi.org/10.1016/j.nuclphysa.2005.02.130 [arXiv:nucl-ex/0410020].

2. V.V.Skokov, A. Y. Illarionov and V. D. Toneev, “Estimate of the magnetic field strength in heavy-ion collisions,” IJMPA **24**, 5925 (2009) doi.org/10.1142/S0217751X09047570

3. K. Tuchin, “Electromagnetic radiation by quark-gluon plasma in magnetic field,” Phys. Rev. C **83**, 017901 (2011) doi: 10.1103/PhysRevC.87.024912 [arXiv:1206.0485 [hep-ph]]

4. B. Karmakar, R. Ghosh, A. Bandyopadhyay, N. Haque, and M. G. Mustafa, “Anisotropic pressure of deconfined QCD matter in presence of strong magnetic field within one-loop approximation,” Phys. Rev. D **99**,

094002 (2019) doi.org/10.1103/PhysRevD.99.094002 [arXiv:1902.02607 [hep-ph]]

5. A. Bandyopadhyay, C. A. Islam, and M. G. Mustafa, "Electromagnetic spectral properties and Debye screening of a strongly magnetized hot medium," Phys. Rev. D **94**, 114034 (2016) doi.org/10.1103/PhysRevD.94.114034 [arXiv:1602.06769 [hep-ph]]

6. L. Levkova and C. DeTar, "Quark-gluon plasma in an external magnetic field," Phys. Rev. Lett. **112**, 012002 (2014) doi.org/10.1103/PhysRevLett.112.012002 [arXiv:1309.1142 [hep-lat]]

7. A. Bandyopadhyay, R. L. S. Farias, B. S. Lopes, and R. O. Ramos, "Quantum chromodynamics axion in a hot and magnetized medium," Phys. Rev. D **100**, 076021 (2019) doi.org/10.1103/PhysRevD.100.076021 [arXiv:1906.09250 [hep-ph]]

8. S. Acharya et al., "Probing the Effects of Strong Electromagnetic Fields with Charge-Dependent Directed Flow in Pb-Pb Collisions at the LHC," Phys. Rev. Lett. **125**, 022301 (2020) doi.org/10.1103/PhysRevLett.125.022301

9. J. Adam et al., "First Observation of the Directed Flow of D^0 and $\bar{D}^0$ in Au + Au collision at $\sqrt{S_{NN}} = 200$ GeV," Phys. Rev. Lett. **123**, 162301 (2019) https://doi.org/10.1103/PhysRevLett.123.162301 [arXiv:1905.02052 [nucl-ex]]

10. S. Bhadury, M. Kurian, V. Chandra, and A. Jaiswal, "First order dissipative hydrodynamics and viscous corrections to the entropy four-current from an effective covariant kinetic theory," J. Phys. G: Nucl. Part. Phys. **47**, 085108 (2020) doi 10.1088/1361-6471/ab907b [arXiv:1902.05285 [hep-ph]]

11. S. S. Avancini, V. Dexheimer, R. L. S. Farias, and V. S. Timteo, "Anisotropy in the equation of state of strongly magnetized quark matter within the Nambu–Jona-Lasinio model," Phys. Rev. C **97**, 035207 (2018) doi:10.1103/PhysRevC.97.035207

12. M. Kurian and V. Chandra, "Effective description of hot QCD medium in strong magnetic field and longitudinal conductivity," Phys. Rev. D **96**, 114026 (2017) doi.org/10.1103/PhysRevD.96.114026 [arXiv:1709.08320 [nucl-th]]

13. S. Koothottil and V. M. Bannur, "Thermodynamic Behaviour of Magnetized QGP within the Self-Consistent Quasiparticle Model," Phys. Rev. C **99**, 035210 (2019) doi: 10.1103/PhysRevC.99.035210 [arXiv:1811.05377 [nucl-th]]

14. R. L. S. Farias, V. S. Timteo, S. S. Avancini, M. B. Pinto and G. Krein, "Thermo-magnetic effects in quark matter: Nambu-Jona-Lasinio model constrained by lattice QCD" Eur. Phys. J. A **53**, 101 (2017) doi: 10.1140/epja/i2017-12320-8 [arXiv:1603.03847 [hep-ph]]

15. Y. M. Cho, "Restricted gauge theory," Phys. Rev. D **21** (1980) 1080 doi.org/10.1103/PhysRevD.21.1080

16. Y. M. Cho, "Extended gauge theory and its mass spectrum" Phys. Rev. D **23** (1981) 2415 doi.org/10.1103/PhysRevD.23.2415

17. Y. M. Cho, "Glueball Spectrum in Extended Quantum Chromodynamics," Phys. Rev. Lett. **46** (1981) 302 doi.org/10.1103/PhysRevLett.46.302

18. Garima Punetha, Aritra Bandhyopadhay and Shuchi Bisht, "Dual QCD thermodynamics at finite temperature and chemical potential, " IJMPA **37** (2022) doi.org/10.1142/S0217751X22501755 2250175.

19. Eduardo S. Fraga, Leticia F. Palhares, "Deconfinement in the presence of a strong magnetic background: an exercise within the MIT bag model," Phys. Rev. D **86**, 016008 (2012) doi.org/10.1103/PhysRevD.86.016008 [arXiv:1201.5881 [hep-ph]].