arXiv > cond-mat > arXiv:2306.00866

Condensed Matter > Superconductivity

[Submitted on 1 Jun 2023]

# Demonstration of nonlocal Josephson effect in Andreev molecules

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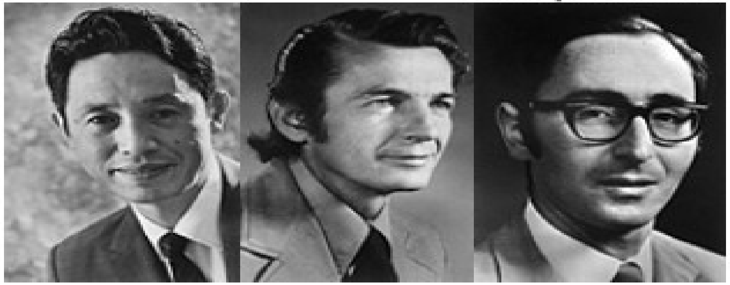
# The Nobel Prize in Physics 1973

"for their experimental discoveries regarding tunneling phenomena in semiconductors and superconductors, respectively"

"for his theoretical predictions of the properties of a supercurrent through a tunnel barrier, in particular those phenomena which are generally known as the Josephson effects"



**Leo Esaki**

1/4 of the prize

Japan

IBM Thomas J. Watson Research Center

**Ivar Giaever**

1/4 of the prize

USA

General Electric Company Schenectady, NY,

**Brian David Josephson**

1/2 of the prize

United Kingdom

University of Cambridge Cambridge, United

138 Physics 1973

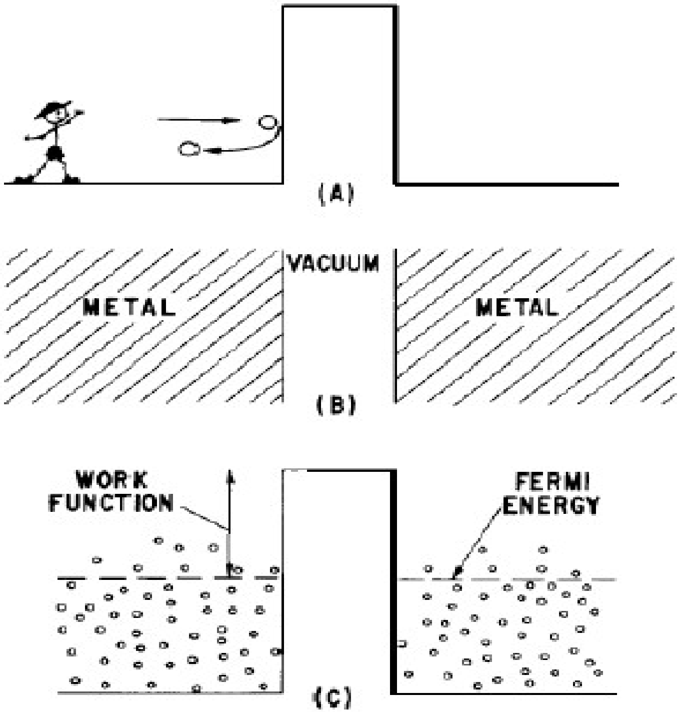



Fig. 1

# Tunneling between a normal metal and another normal metal or a superconductor

140 *Physics 1973*

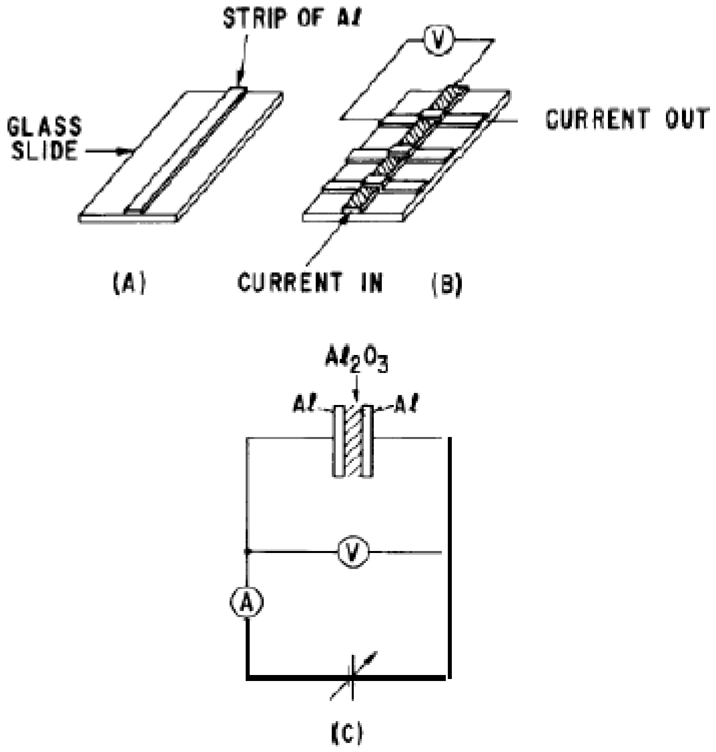



Fig. 3.

142 *Physics 1973*

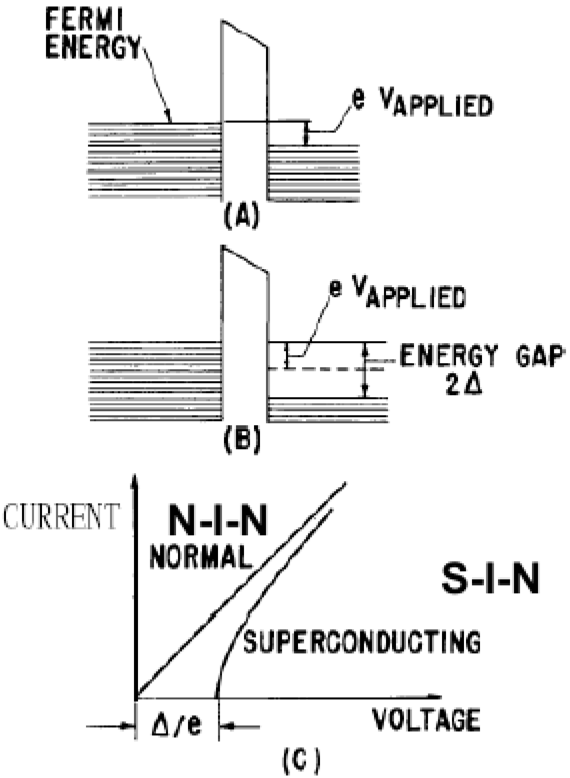
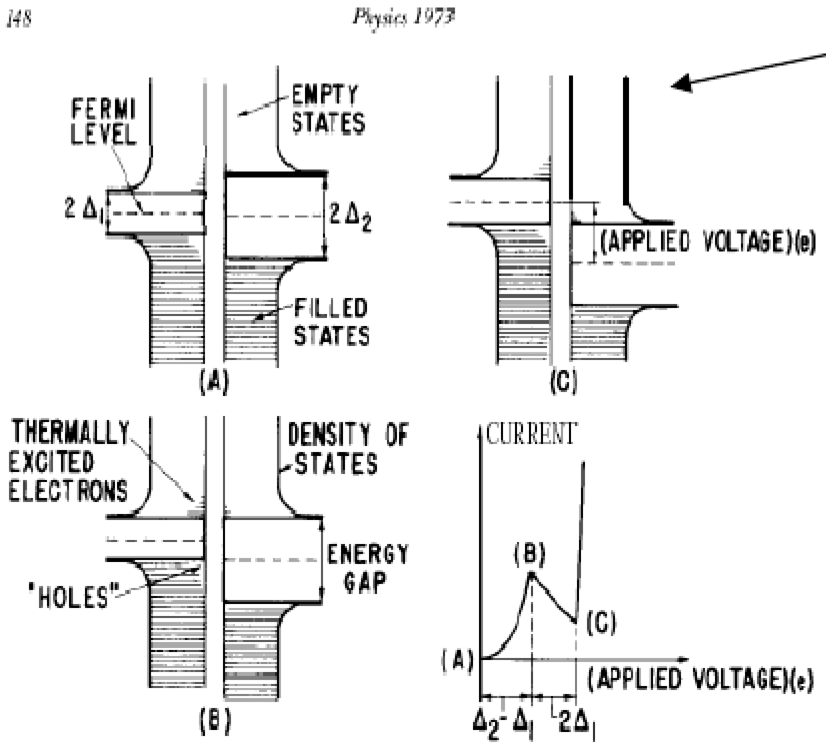
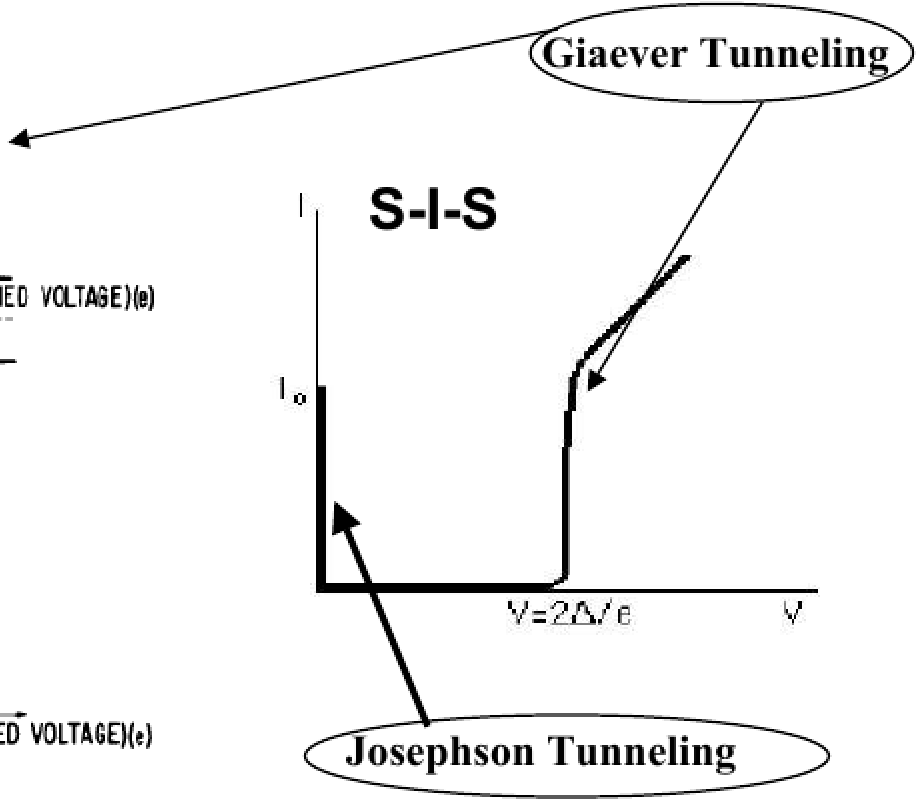


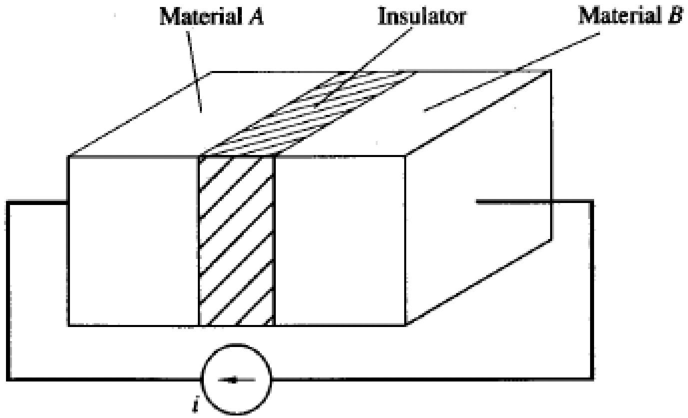

Fig. 5.

# Tunneling between two superconductors

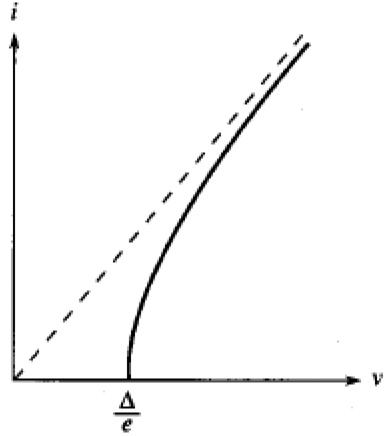


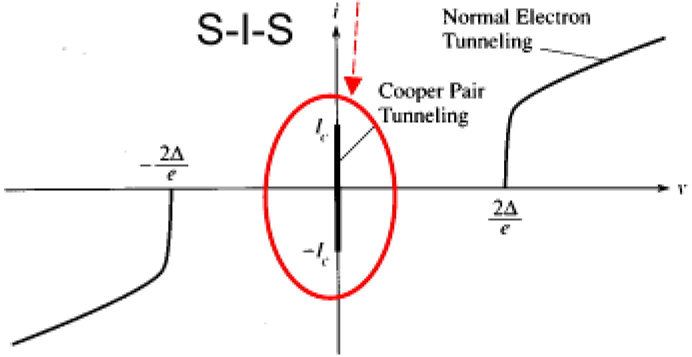
# Tunneling Summary: We are only concerned with the Josephson Tunneling in a *Basic Junction*




S-I-N




S-I-S



# Macroscopic Quantum Model

**1. The wavefunction describes the whole ensemble of superelectrons such that**

$$\Psi^*(\mathbf{r},t)\Psi(\mathbf{r},t) = n^\star(\mathbf{r},t) \quad \longrightarrow \quad \text{density}$$

and $$\int d\mathbf{r}\,\Psi^*(\mathbf{r},t)\Psi(\mathbf{r},t) = \mathsf{N}^\star \quad \longrightarrow \quad \text{Total number}$$

**2. The flow of probability becomes the flow of particles, with the physical current density given by**

$$\mathbf{J}_\mathsf{S} = q^\star\,\mathsf{Re}\left\{\Psi^*\left(\frac{\hbar}{im^\star}\nabla - \frac{q^\star}{m^\star}\mathbf{A}\right)\Psi\right\}$$

**3. This macroscopic quantum wavefunction follows**

$$i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{r},t) = \frac{1}{2m^\star}\left(\frac{\hbar}{i}\nabla - q^\star\mathbf{A}(\mathbf{r},t)\right)^2\Psi(\mathbf{r},t) + q^\star\phi(\mathbf{r},t)\,\Psi(\mathbf{r},t)$$

# Wave function

Writing $\Psi(\mathbf{r},t) = \sqrt{n^\star(\mathbf{r},t)}\, e^{i\theta(\mathbf{r},t)}$ , we find

The real part of the S-Eqn gives

$$-\hbar\frac{\partial}{\partial t}\theta(\mathbf{r},t) \;=\; \frac{\hbar^2 n_s^\star}{2m^\star}\left(\nabla\theta(\mathbf{r},t) - \frac{q^\star}{\hbar}\mathbf{A}(\mathbf{r},t)\right)^2 + \frac{\hbar^2}{8m^\star n_s^\star(\mathbf{r},t)}\left(\nabla^2 n_s^\star(\mathbf{r},t)\right)^2 + q^\star\phi(\mathbf{r},t)$$

The imaginary part of the S-Eqn gives the supercurrent equation:

$$\mathbf{J}_\mathsf{S} = q^\star n^\star(\mathbf{r},t)\left(\frac{\hbar}{m^\star}\nabla\theta(\mathbf{r},t) - \frac{q^\star}{m^\star}\mathbf{A}(\mathbf{r},t)\right)$$

# Supercurrent Equation with n* constant

Let $n^\star(\mathbf{r},t) = n^\star$ be a constant, so that $\Psi(\mathbf{r},t) = \sqrt{n^\star}\, e^{i\theta(\mathbf{r},t)}$

we find

$$-\hbar \frac{\partial\theta}{\partial t} = \underbrace{\frac{1}{2n^\star}\Lambda\, \mathbf{J}_\mathsf{S}{}^2 + q^\star\phi}_{\text{Energy of a superelectron}} \quad \text{with} \quad \Lambda \equiv \frac{m^\star}{n^\star(q^\star)^2}$$

and

$$\Lambda\mathbf{J}_\mathsf{S} = -\left(\mathbf{A}(\mathbf{r},t) - \frac{\hbar}{q^\star}\nabla\theta(\mathbf{r},t)\right)$$

# London's Equations

1. Take the curl of the supercurrent equation

$$\Lambda \mathbf{J}_S = -\left(\mathbf{A}(\mathbf{r},t) - \frac{\hbar}{q^\star}\nabla\theta(\mathbf{r},t)\right)$$

gives the Second London Equation: $\nabla \times (\Lambda \mathbf{J}_S) = -\nabla \times \mathbf{A} = -\mathbf{B}$

2. Take the time derivative of the supercurrent equation:

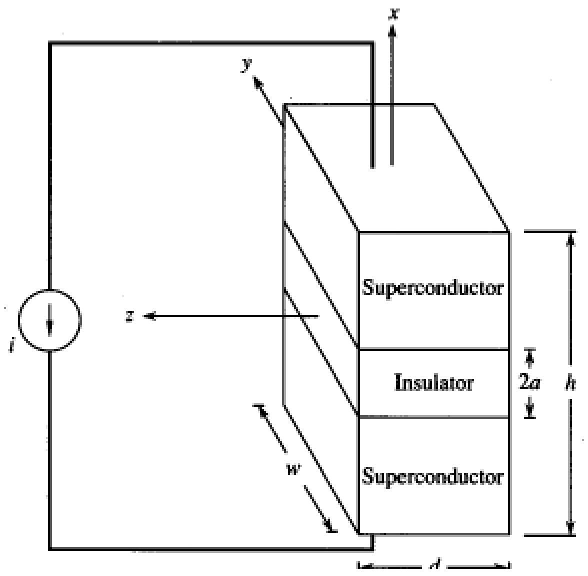
$$\frac{\partial}{\partial t}(\Lambda \mathbf{J}_S) = -\left[\frac{\partial \mathbf{A}}{\partial t} - \frac{\hbar}{q^\star}\nabla\left(\frac{\partial\theta}{\partial t}\right)\right]$$

with $-\hbar\frac{\partial\theta}{\partial t} = \frac{1}{2n^\star}\Lambda \mathbf{J}_S{}^2 + q^\star\phi$ gives

$$\frac{\partial}{\partial t}(\Lambda \mathbf{J}_S) = \mathbf{E} - \frac{1}{n^\star q^\star}\nabla\left(\frac{1}{2}\Lambda \mathbf{J}_S{}^2\right)$$

Something more than First London Equation?

# In the superconducting electrodes:




The Supercurrent Equations govern the electrodes,

$$\mathbf{J}_s(\mathbf{r},t) = -\frac{1}{\Lambda}\left(\mathbf{A}(\mathbf{r},t) + \frac{\Phi_o}{2\pi}\nabla\theta(\mathbf{r},t)\right)$$

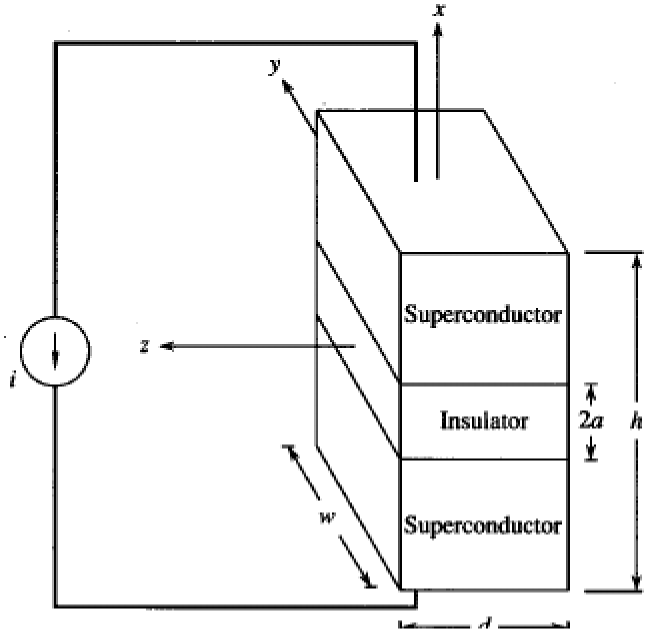
$$\frac{\partial}{\partial t}\theta(\mathbf{r},t) = -\frac{1}{\hbar}\left(\frac{\Lambda \mathbf{J}_S^2}{2n^\star} + q^\star\phi(\mathbf{r},t)\right)$$

Even in the absence of E&M fields, a gradient of the phase can cause a current and the time change of that phase can cause a voltage. For example, for a constant current $\mathbf{J}_o$, at the boundaries we find

$$\mathbf{J}_S(\pm a,t) = -\frac{\Phi_o}{2\pi\Lambda}\nabla\theta(\pm a,t) = \mathbf{J}_o \quad \& \quad \frac{\partial}{\partial t}\theta(\pm a,t) = -\frac{1}{\hbar}\left(\frac{\Lambda \mathbf{J}_o^2}{2n^\star}\right) = -\frac{\mathcal{E}_o}{\hbar}$$

So that the wavefunction in the electrode is $\Psi(\mathbf{r},t) = \Psi(\mathbf{r})e^{-i(\mathcal{E}_o t/\hbar)}$

# In the insulator

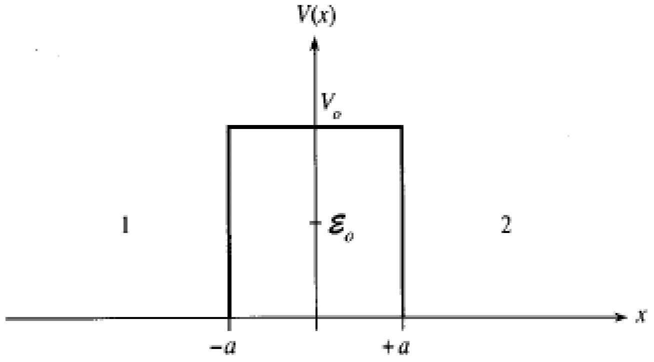



The current must be continuous, so it must flux through the insulating barrier; a process which is not allowed classically. But quantum mechanically the superelectrons can tunnel through the insulating barrier as a supecurrent with zero voltage. This is the Josephson current.

Because the supercurrent equation does not hold in the insulating region, the full macroscopic wave equation must be used to find Y in the insulating region, with the boundary conditions given by the wavefunction at the electrodes.

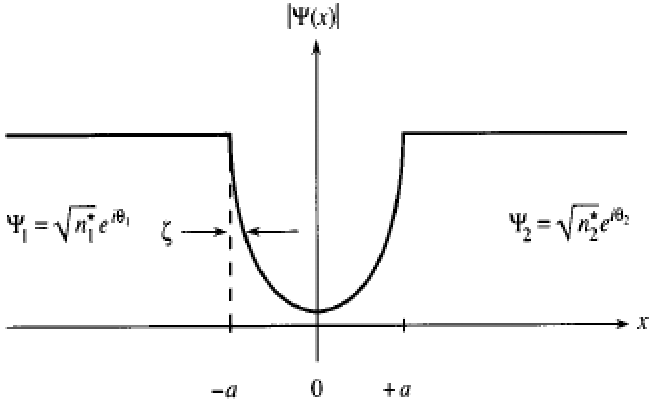
$$i\hbar \frac{\partial}{\partial t}\Psi(\mathbf{r},t) = \frac{1}{2m^\star}\left(\frac{\hbar}{i}\nabla - q^\star \mathbf{A}(\mathbf{r},t)\right)^2 \Psi(\mathbf{r},t) + q^\star \phi(\mathbf{r},t)\,\Psi(\mathbf{r},t) + \underbrace{V(x)}_{\text{Tunneling Potential Barrier}}\Psi(\mathbf{r},t)$$

# Tunneling through the Barrier




The energy of the superelectron is less than the barrier height, so that no classical particles flow.

$$-\frac{\hbar^2}{2m^\star}\nabla^2\Psi(\mathbf{r}) = \underbrace{(\mathcal{E}_o - V_o)}_{\text{constant}}\Psi(\mathbf{r}) \qquad \text{for } |x| \le a$$



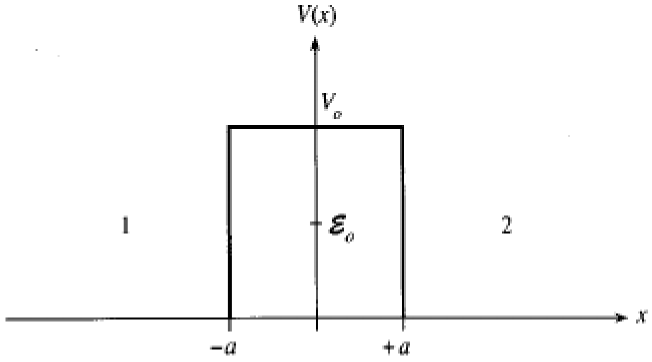

Therefore, in the insulating region

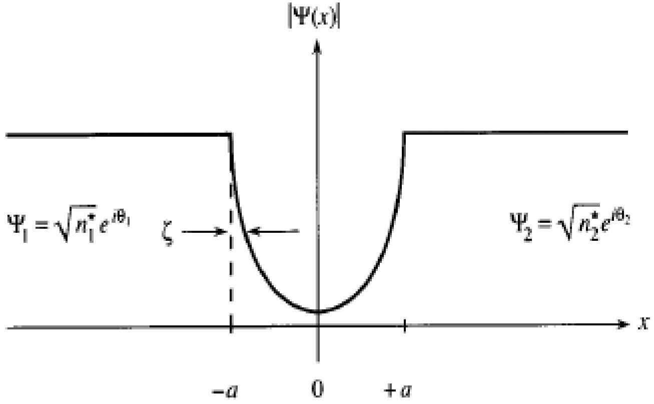
$$\Psi(x) = C_1 \cosh x/\zeta + C_2 \sinh x/\zeta$$

Where $\zeta \equiv \sqrt{\dfrac{\hbar^2}{2m^\star(V_o - \mathcal{E}_o)}}$ so that

$$\mathbf{J}_\mathsf{S} = \frac{2q^\star}{m^\star}\mathrm{Re}\left\{\Psi^*\frac{\hbar}{i}\nabla\Psi\right\} = \frac{q^\star\hbar}{m^\star\zeta}\mathrm{Im}\left\{C_1{}^*C_2\right\}$$

# Tunneling through the Barrier




$$\mathbf{J}_\mathsf{S} = \frac{2q^\star}{m^\star} \mathrm{Re}\left\{\Psi^* \frac{\hbar}{i} \nabla \Psi\right\} = \frac{q^\star \hbar}{m^\star \zeta} \mathrm{Im}\left\{C_1{}^* C_2\right\}$$

At the boundaries.

$$\Psi(-a) = \sqrt{n_1^\star}\, e^{i\theta_1} \quad \& \quad \Psi(+a) = \sqrt{n_2^\star}\, e^{i\theta_2}$$

So that

$$C_1 = \frac{\sqrt{n_1^\star}\, e^{i\theta_1} + \sqrt{n_2^\star}\, e^{i\theta_2}}{2\cosh(a/\zeta)} \quad \& C_2 = -\frac{\sqrt{n_1^\star}\, e^{i\theta_1} - \sqrt{n_2^\star}\, e^{i\theta_2}}{2\sinh(a/\zeta)}$$

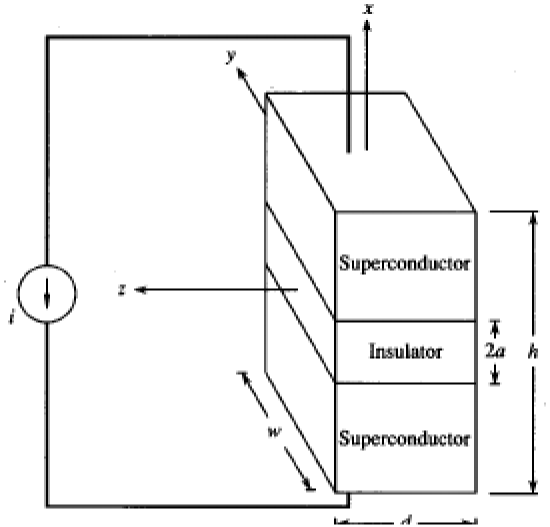
Therefore,

$$\mathbf{J}_\mathsf{S} = \mathbf{J}_\mathsf{C} \sin(\theta_1 - \theta_2)$$

with $$\mathbf{J}_\mathsf{C} = \frac{e\hbar\sqrt{n_1 n_2}}{m\zeta \sinh(2a/\zeta)}$$

# Josephson Current-Phase relation

$$\mathbf{J}_S = \mathbf{J}_C \sin(\theta_1 - \theta_2)$$




In the presence of and electromagnetic field, the Josephson current-phase relation generalizes to

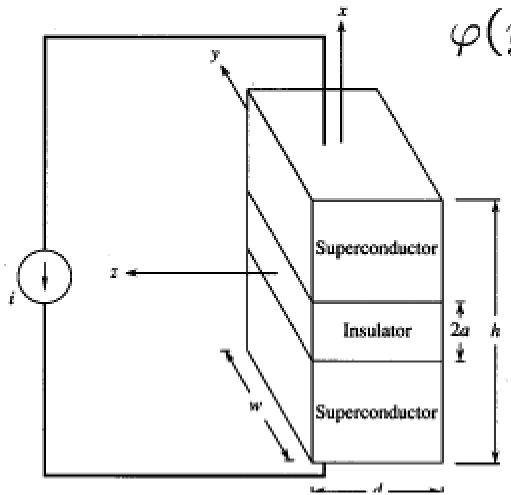
$$\mathbf{J}_S(\mathbf{r}, t) = \mathbf{J}_C(y, z, t) \sin \varphi(y, z, t)$$

where the *gauge-invariant phase* is defined as

$$\varphi(y, z, t) = \theta_1(y, z, t) - \theta_2(y, z, t) - \frac{2\pi}{\Phi_o} \int_1^2 \mathbf{A}(\mathbf{r}, t) \cdot d\mathbf{l}$$

Which is invariant under $\mathbf{A}' = \mathbf{A} + \nabla\chi$ , $\theta' = \theta + \frac{q^\star}{\hbar}\chi$ , $\phi' \equiv \phi - \frac{\partial \chi}{\partial t}$

# Josephson Voltage-Phase relation




The *gauge-invariant phase* is

$$\varphi(y,z,t) = \theta_1(y,z,t) - \theta_2(y,z,t) - \frac{2\pi}{\Phi_o}\int_1^2 \mathbf{A}(\mathbf{r},t)\cdot d\mathbf{l}$$

The rate of change of the *gauge-invariant phase* is

$$\frac{\partial\varphi}{\partial t} = \frac{\partial\theta_1}{\partial t} - \frac{\partial\theta_2}{\partial t} - \frac{2\pi}{\Phi_o}\frac{\partial}{\partial t}\int_1^2 \mathbf{A}(\mathbf{r},t)\cdot d\mathbf{l}$$

At the boundary in the electrodes,

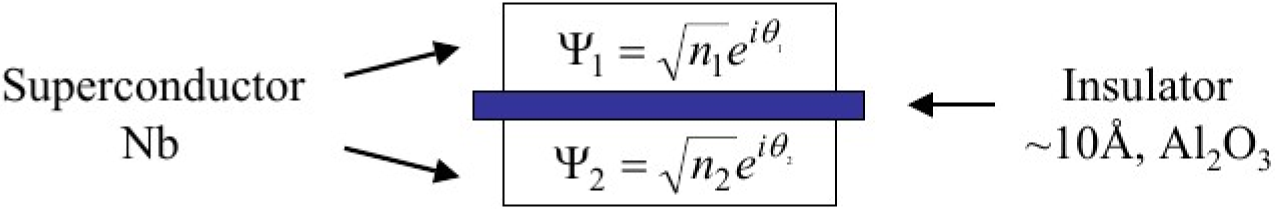
$$\frac{\partial}{\partial t}\theta(\mathbf{r},t) = -\frac{1}{\hbar}\left(\frac{\Lambda \mathbf{J}_\mathrm{s}^2}{2n^\star} + q^\star\phi(\mathbf{r},t)\right) \quad \text{so that}$$

$$\frac{\partial\varphi}{\partial t} = -\frac{1}{\hbar}\left(\frac{\Lambda}{2n^\star}\underbrace{\left[\mathbf{J}_\mathrm{s}^2(-a) - \mathbf{J}_\mathrm{s}^2(a)\right]}_{0} + q^\star\underbrace{\left[\phi(-a) - \phi(a)\right]}_{\int_1^2 -\nabla\phi\cdot d\mathbf{l}}\right) - \frac{2\pi}{\Phi_o}\frac{\partial}{\partial t}\int_1^2 \mathbf{A}(\mathbf{r},t)\cdot d\mathbf{l}$$

Therefore, $\dfrac{\partial\varphi}{\partial t} = \dfrac{2\pi}{\Phi_o}\displaystyle\int_1^2\left(-\nabla\phi - \frac{\partial\mathbf{A}}{\partial t}\right)\cdot d\mathbf{l}$ or

$$\boxed{\frac{\partial\varphi(y,z,t)}{\partial t} = \frac{2\pi}{\Phi_o}\int_1^2 \mathbf{E}(\mathbf{r},t)\cdot d\mathbf{l}}$$

# Summary: Basic Josephson Junction ($I<I_c$)




- Josephson relations:

$$I = I_c \sin\varphi$$

$$V = \frac{\Phi_0}{2\pi}\frac{d\varphi}{dt}$$

$$\varphi = \theta_2 - \theta_1 - \frac{2\pi}{\Phi_0}\int_2^1 \vec{A}(r,t)\cdot d\vec{l}$$

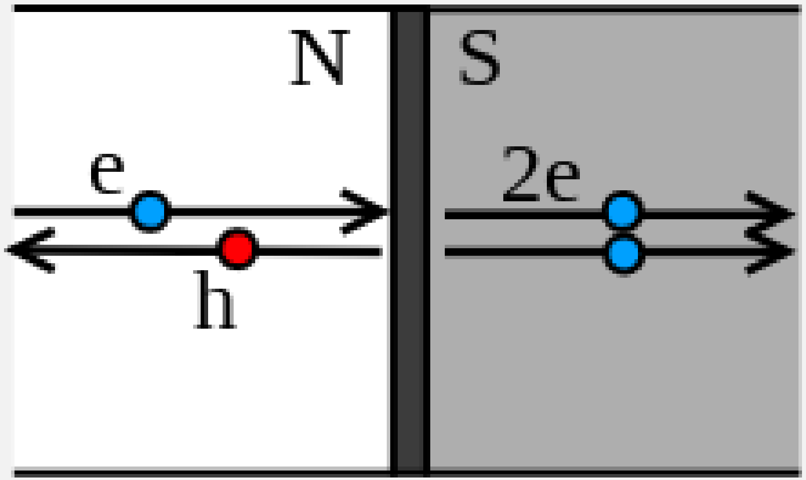
- Behaves as a nonlinear inductor:

$$V = L_J \frac{dI}{dt},$$

where $L_J = \dfrac{\Phi_0}{2\pi\, I_c \cos\varphi}$

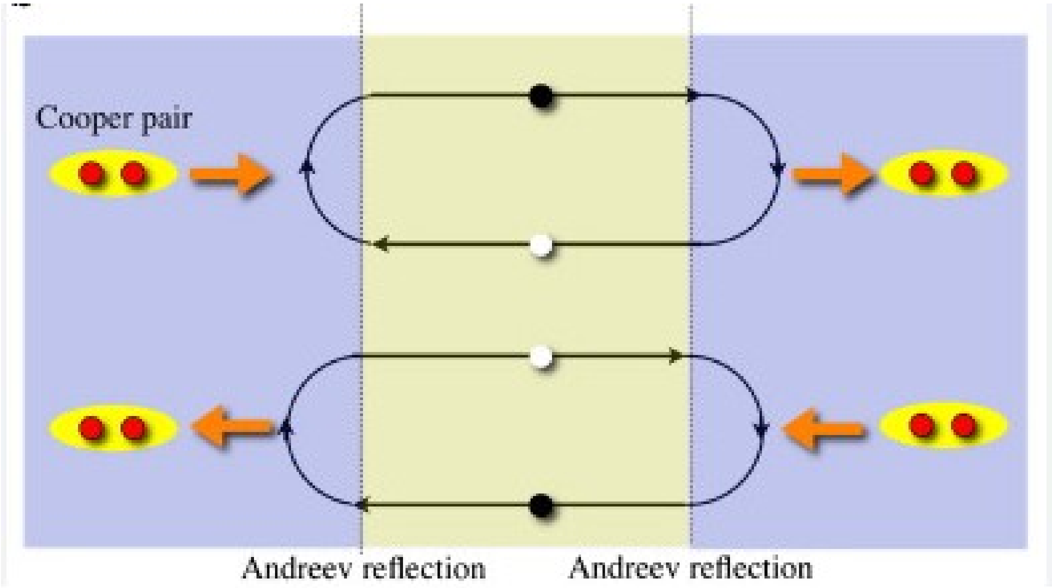
$\Phi_0$ = flux quantum

483.6 GHz / mV




Andreev reflection

# Andreev Bound States



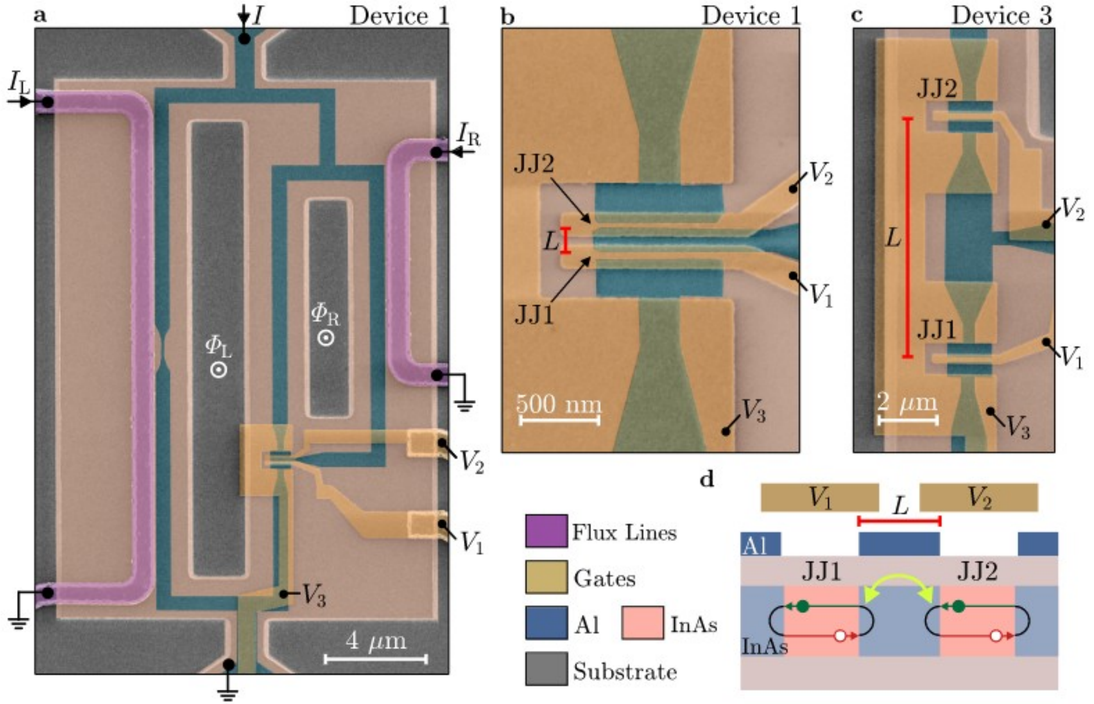



Figure 1: Devices under study and measurement setup. (a) False-colored scanning electron micrograph of a sample identical to Device 1, together with a measurement schematics. The substrate is shown in gray, the exposed III-V semiconductor in pink, the epitaxial Al in blue, gates in yellow and flux lines in purple. (b) Zoom-in of (a) around the Josephson junctions (JJs). The distance between the junctions is $L = 150$ nm. (c) Similar to (b), but for Device 3, which has $L = 4\ \mu$m. (d) Schematic cross-section (not to scale) of the JJs sharing a common electrode of length $L$. Andreev bound states originating from the two JJs, spatially extended over distances in excess of $L$, overlap, and hybridize forming an Andreev molecule.

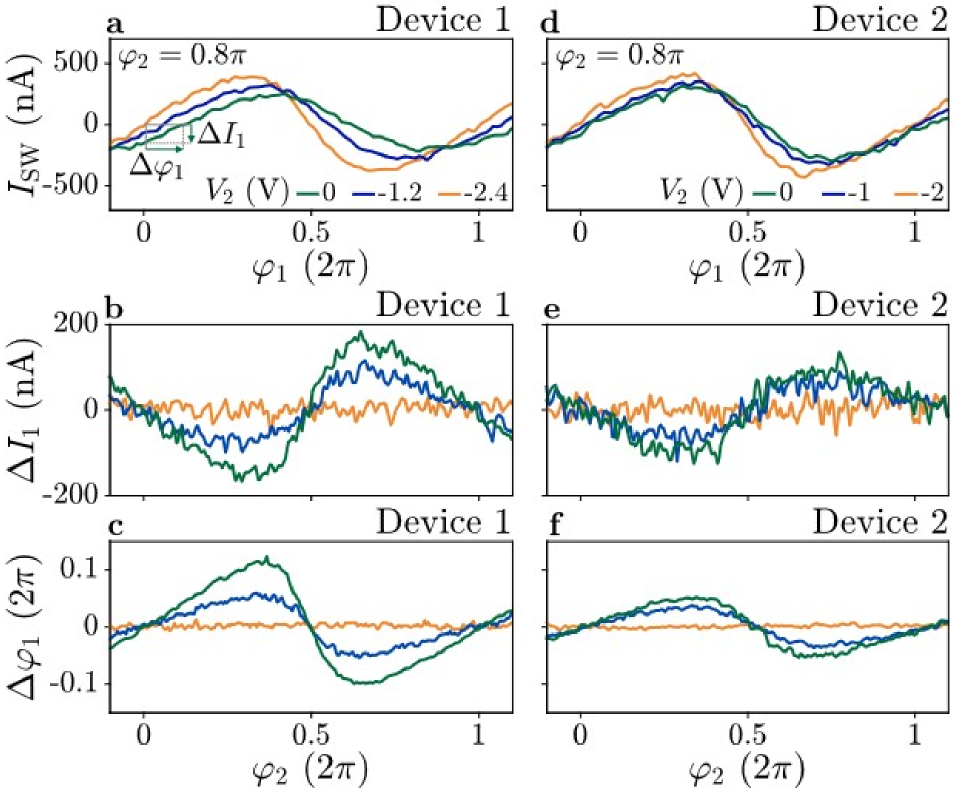



Figure 4: Anomalous supercurrent and anomalous phase shift. (a) Switching current $I_{SW}$ in Device 1 as a function of $\varphi_1$ measured for $\varphi_2 = 0.8\pi$ at three values of $V_2$. Quantities $\Delta I_1$ and $\Delta\varphi_1$ are defined. More details on data analysis required to produce this plot are presented in the Supplementary Information. (b) Anomalous supercurrent $\Delta I_1$ as a function of $\varphi_2$ for three values of $V_2$ [see legend in (a)]. (c) Anomalous phase shift $\Delta\varphi_1$ as a function of $\varphi_2$ for three values of $V_2$ [see legend in (a)]. (d-f) As (a-c), but for Device 2.