Lectures 3&4: Longitudinal Motion and IR Focusing Limitations

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US Particle Accelerator School
February 2022

Objectives

- Beam acceleration and bunching are based on the dependence of particle revolution frequency on its energy
- Longitudinal OSC and CEC use dependence of particle longitudinal displacement on particle momentum
  while transverse OSC and CEC use dependence of particle longitudinal displacement on particle betatron motion
- In this lecture we consider
  - basics of linear optics for longitudinal degree of freedom
  - longitudinal motion in a harmonic RF
  - the perturbation theory for symplectic motion
  - and limitations on beam focusing in the design of interaction region

S-X Coupled Motion and Beam Acceleration in Harmonic RF

Transfer Matrix for X-S Coupled Motion

- Parameterization of transfer matrix in the absence of RF

$$\mathbf{x}_2 = \mathbf{M}\mathbf{x}_1 \quad , \quad \mathbf{M} = \begin{bmatrix} M_{11} & M_{12} & 0 & M_{16} \\ M_{21} & M_{22} & 0 & M_{26} \\ M_{51} & M_{52} & 1 & M_{56} \\ 0 & 0 & 0 & 1 \end{bmatrix} , \quad \mathbf{x} \equiv \begin{bmatrix} x \\ \theta_x \\ s \\ \theta_s \end{bmatrix} , \quad \theta_s = \frac{\Delta p}{p}$$

Longitudinal displacements are counted relative to the reference particle

- Elements M_{16} and M_{26} are directly related to dispersion

$$\begin{bmatrix} D_2 \\ D_2' \\ \dots \\ 1 \end{bmatrix}^T = \begin{bmatrix} M_{11} & M_{12} & 0 & M_{16} \\ M_{21} & M_{22} & 0 & M_{26} \\ M_{51} & M_{52} & 1 & M_{56} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} D_1 \\ D_1' \\ \dots \\ 1 \end{bmatrix} , \quad \begin{cases} D_2 = M_{11}D_1 + M_{12}D_1' + M_{16} \\ D_2' = M_{21}D_1 + M_{22}D_1' + M_{26} \end{cases} \xrightarrow[D_2=D_1,\ D_2'=D_1']{\text{For a ring}} \begin{cases} M_{16} = D(1-M_{11}) - D'M_{12} \\ M_{26} = -M_{21}D + D'(1-M_{22}) \end{cases}$$

- Elements M_{51} and M_{52} are bound to others by symplecticity condition

$$\hat{\mathbf{M}}^T \mathbf{U} \hat{\mathbf{M}} = \mathbf{U} \;\Rightarrow\; \begin{cases} M_{51} = M_{21}M_{16} - M_{11}M_{26} \\ M_{52} = M_{22}M_{16} - M_{12}M_{26} \end{cases} \xrightarrow[D_2=D_1,\ D_2'=D_1']{\text{For ring}} \begin{cases} M_{51} = DM_{21} + D'\left(1-M_{11}\right) \\ M_{52} = -D(1-M_{22}) - D'M_{12} \end{cases}$$

where we accounted that $M_{11}M_{22} - M_{12}M_{21} = 1$

- i.e. for a ring without RF M_{16}, M_{26}, M_{51} and M_{52} can be expressed through dispersion and its derivative. M_{56} is independent on other elements

Equations of Longitudinal Motion (no acceleration)

- Orbit lengthening

$$\begin{bmatrix} \dots \\ \dots \\ \Delta s \\ \dots \end{bmatrix}^T = \begin{bmatrix} M_{11} & M_{12} & 0 & M_{16} \\ M_{21} & M_{22} & 0 & M_{26} \\ M_{51} & M_{52} & 1 & M_{56} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} D \\ D' \\ \dots \\ 1 \end{bmatrix} \frac{\Delta p}{p} \quad \Rightarrow \quad \Delta s = \left(M_{51} D + M_{52} D' + M_{56} \right) \frac{\Delta p}{p} = \alpha C \frac{\Delta p}{p}$$

- Momentum compaction: $\dfrac{\Delta C}{C} = \alpha \dfrac{\Delta p}{p}\,, \quad \alpha = \dfrac{M_{51} D + M_{52} D' + M_{56}}{C}$

- Slip-factor: $\dfrac{\Delta T}{T} = \dfrac{\Delta C}{C} - \dfrac{\Delta \mathrm{v}}{\mathrm{v}} = \eta \dfrac{\Delta p}{p} \xrightarrow[\frac{\Delta C}{C} = \alpha \frac{\Delta p}{p}]{\frac{\Delta \mathrm{v}}{\mathrm{v}} = \frac{1}{\gamma^2} \frac{\Delta p}{p}} \left(\alpha - \dfrac{1}{\gamma^2} \right) \dfrac{\Delta p}{p} \quad \Rightarrow \quad \eta = \alpha - \dfrac{1}{\gamma^2}$

- Equations for the longitudinal motion

$$\begin{cases} \dfrac{d\varphi}{dt} = q \omega_0 \eta \dfrac{\Delta p}{p} \\ \dfrac{d}{dt} \dfrac{\Delta p}{p} = \dfrac{1}{\beta^2} \dfrac{d}{dt} \dfrac{\Delta E}{E} = -\dfrac{1}{\beta^2 E} e V_0 \dfrac{\omega_0}{2\pi} \sin \varphi \end{cases} \quad \Rightarrow \quad \frac{d^2 \varphi}{dt^2} = -{\Omega_s}^2 \sin \varphi\,, \quad \Omega_s = \omega_0 \sqrt{\frac{e V_0 q \eta}{2\pi m c^2 \beta^2 \gamma}}$$

Effect of Deceleration due to SR on Longit. Motion

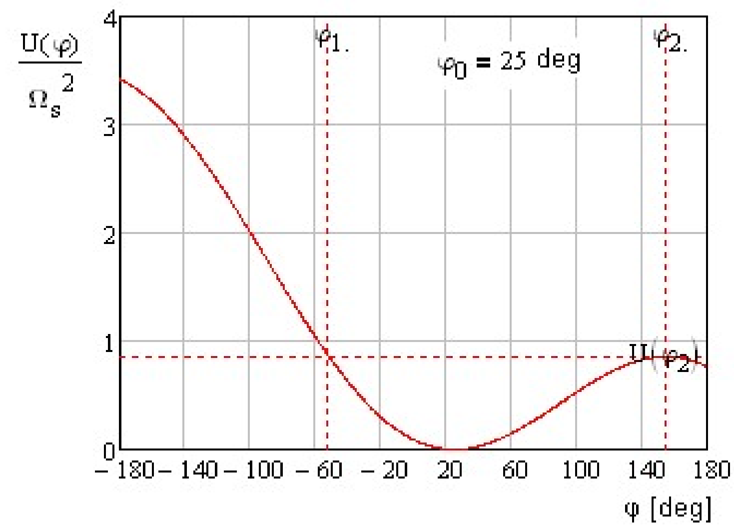
- Motion equation: $\dfrac{d^2\varphi}{dt^2} = -\Omega_s^{\,2}\left(\sin\varphi - \sin\varphi_0\right), \quad \sin\varphi_0 = \dfrac{V_{SR}}{V_0}$
- Its solution

$$\xrightarrow[\frac{d^2\varphi}{dt^2}=\frac{d\varphi}{dt}\frac{d}{d\varphi}\frac{d\varphi}{dt}=\hat{p}\frac{d\hat{p}}{d\varphi}]{d\varphi/dt=\hat{p}} \quad \frac{d}{d\varphi}\left(\frac{\hat{p}^2}{2}\right) = -\Omega_s^{\,2}\left(\sin\varphi - \sin\varphi_0\right) \Rightarrow \frac{\hat{p}^2}{2} = \Omega_s^{\,2}\left(\cos\varphi + \varphi\sin\varphi_0\right) + C$$

- Introduce Hamiltonian and potential energy

$$H = \frac{p^2}{2} + U(\varphi),$$

$$U(\varphi) = \Omega_s^{\,2}\left(\cos\varphi_0 - \cos\varphi - (\varphi - \varphi_0)\sin\varphi_0\right)$$



- Separatrix boundaries

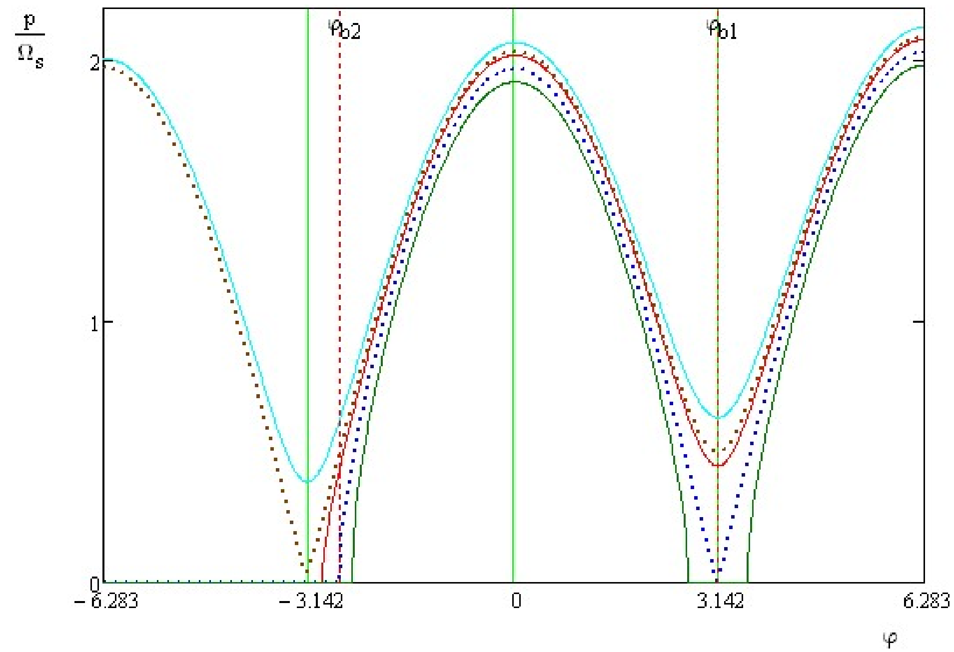
$$\sin\varphi_2 = \sin\varphi_0 \Rightarrow \varphi_2 = \pi - \varphi_0$$

- For small φ_0 the transcendent equation for finding the second boundary, φ_1, can space be using perturbation theory, numerical solution is required for large φ_0
- Accounting of acceleration one needs to account vortex electric field due to changing magnetic field!!! -> conservation of the phase

For details see "Theory of Cyclic Accelerators" by Lebedev, Kolomensky

Particle Migration due to Scattering out & in RF Bucket

- In the above consideration we neglected damping due to SR
- In its absence a particle which was knocked out of the RF bucket will never drift back
- Particles which have energy above the bucket separatrix will be decelerated, and then they will penetrate to lower energy through the gap between two RF buckets and will continue deceleration to the momentum acceptance



- Probability of a particle to be accepted back to the RF bucket (due to scattering) is inversely proportional to the ratio of damping time to the synchrotron period.
- For a proton collider it is very big number.
- Consequently, the probability to jump to another bucket after Touschek scattering is strongly suppressed

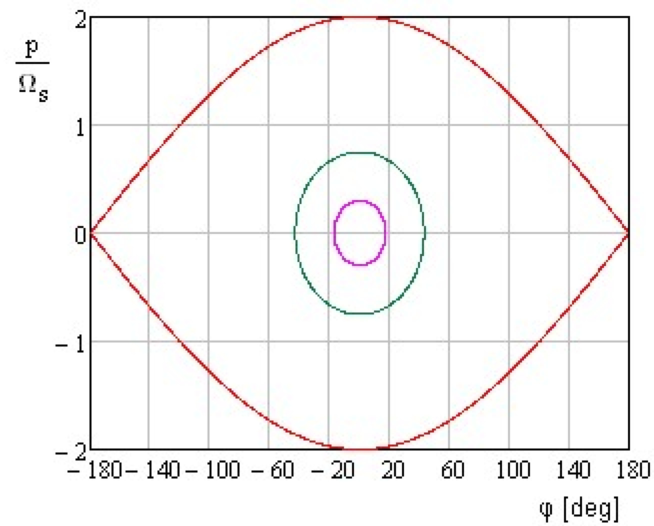
RF Bucket Acceptance in the absence of Acceleration

$$H = \frac{\hat{p}^2}{2} + \Omega_s^{\ 2}\left(1 - \cos\varphi\right)$$

$$\xrightarrow[\text{boundary}]{\text{At the RF bucket}} 2\Omega_s^{\ 2} = \frac{\hat{p}^2}{2} + \Omega_s^{\ 2}\left(1 - \cos\varphi\right)$$

$$\Rightarrow \hat{p}^2 = 2\Omega_s^{\ 2}\left(1 + \cos\varphi\right) = 4\Omega_s^{\ 2}\cos^2\left(\frac{\varphi}{2}\right)$$

$$\boxed{\hat{p} = \pm 2\Omega_s^{\ 2}\cos\left(\frac{\varphi}{2}\right)}$$



- RF bucket acceptance: $\hat{\varepsilon}_s = 2\int_{-\pi}^{\pi} \hat{p}\, d\varphi = 4\Omega_s \int_{-\pi}^{\pi} \cos\left(\frac{\varphi}{2}\right) d\varphi = 16\Omega_s$
  - Returning to the dimensional variables, for one RF bucket we obtain

$$\boxed{\varepsilon_{s0} = 2\int_{-\pi}^{\pi} \Delta p\, dx = \frac{8}{\pi}\frac{m_p c}{q}\sqrt{\frac{ZeV_0\gamma}{2\pi A m_p c^2 q\,|\eta|}}}$$

!!!Here Δp is taken for one nucleon!!!

- since all energies we account per one nucleon!!!

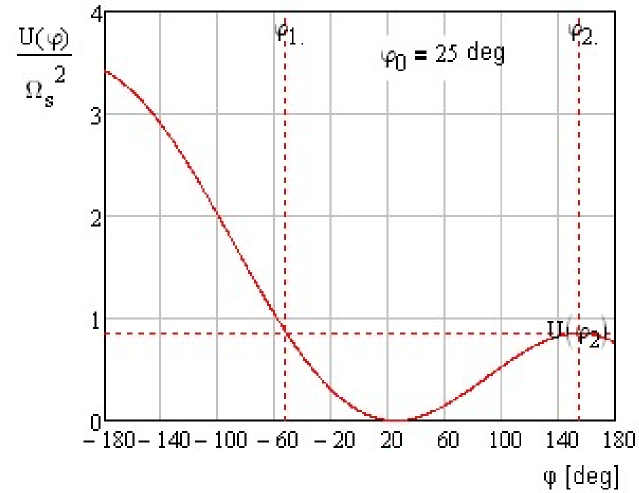
Units for the longitudinal emittance are “eV s/a”

Effect of Accelerating Phase on RF Bucket Acceptance

$$H = \frac{\hat{p}^2}{2} + \Omega_s^{\,2}\left(\cos\varphi_0 - \cos\varphi - (\varphi-\varphi_0)\sin\varphi_0\right)$$

$$\xrightarrow[\varphi=\varphi_2=\pi-\varphi_0,\ \hat{p}=0]{\text{At the RF bucket boundary}} \Omega_s^{\,2}\left(2\cos\varphi_0 - (\pi-2\varphi_0)\sin\varphi_0\right) = \frac{\hat{p}^2}{2} + \Omega_s^{\,2}\left(\cos\varphi_0 - \cos\varphi - (\varphi-\varphi_0)\sin\varphi_0\right)$$

$$\Rightarrow \hat{p}^2 = 2\Omega_s^{\,2}\left(\cos\varphi_0 + \cos\varphi - (\pi-\varphi_0-\varphi)\sin\varphi_0\right)$$



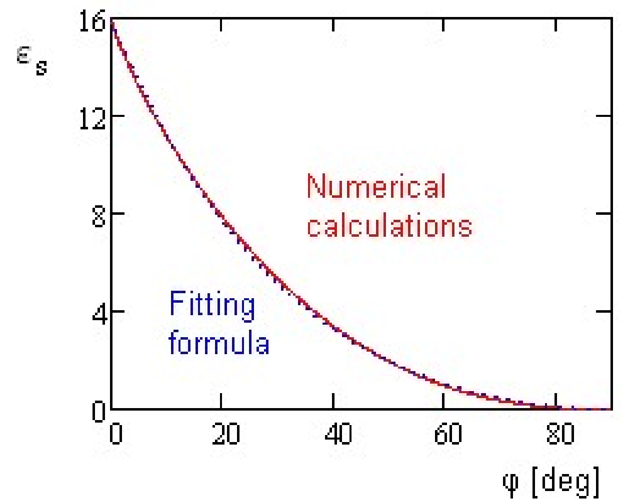
- RF bucket acceptance:

$$\hat{\varepsilon}_s = 2\int_{-\pi}^{\pi} \hat{p}\,d\varphi = 2\sqrt{2}\,\Omega_s \int_{\varphi_1}^{\varphi_2} \sqrt{\cos\varphi_0 + \cos\varphi - (\pi-\varphi_0-\varphi)\sin\varphi_0}\,d\varphi$$

  - The integral cannot be expressed in elementary functions. For practical applications it can be approximated as follows:

$$\varepsilon_s(\varphi_0) \approx \varepsilon_{s0}\frac{1-\sin\varphi_0}{\left(1+\dfrac{\sin\varphi_0}{2}\right)^2}$$

This fitting has ±4% accuracy for φ_0<55 deg.



Perturbation Theory for Symplectic Motion

Perturbation Theory

- The symplecticity enables to build an effective perturbation theory for the case of coupled motion.
- For the perturbed motion one can write: $\left(\mathbf{I}+\Delta\mathbf{M}\right)\mathbf{M}\tilde{\mathbf{v}}_j=\left(\lambda_j+\Delta\lambda_j\right)\tilde{\mathbf{v}}_j$
  - $\mathbf{M}$ – symplectic
  - transfer matrix, $\left(\mathbf{I}+\Delta\mathbf{M}\right)\mathbf{M}$, is not required to be symplectic
- Express the eigenvectors of perturbed motion as a sum of the unperturbed ones $\tilde{\mathbf{v}}_j=\mathbf{v}_j+\sum_{i=1}^{4}\varepsilon_{ij}\mathbf{v}_i\,,\quad \varepsilon_{ij}<<1\,,$
  - without limitation of generality one can consider that $\varepsilon_{ii}=0$ for every i.
  - Substituting and using properties of eigen-vectors one obtains

$$\left(\mathbf{I}+\Delta\mathbf{M}\right)\mathbf{M}\left(\mathbf{v}_j+\sum_{i=1}^{4}\varepsilon_{ij}\mathbf{v}_i\right)=\left(\lambda_j+\Delta\lambda_j\right)\left(\mathbf{v}_j+\sum_{i=1}^{4}\varepsilon_{ij}\mathbf{v}_i\right)$$

Dropping 2nd order terms: $\mathbf{M}\mathbf{v}_j+\Delta\mathbf{M}\mathbf{M}\mathbf{v}_j+\mathbf{M}\sum_{i=1}^{4}\varepsilon_{ij}\mathbf{v}_i\simeq\lambda_j\mathbf{v}_j+\Delta\lambda_j\mathbf{v}_j+\lambda_j\sum_{i=1}^{4}\varepsilon_{ij}\mathbf{v}_i$

$\Delta\mathbf{M}\mathbf{M}\mathbf{v}_j+\sum_{i=1}^{4}\lambda_i\varepsilon_{ij}\mathbf{v}_i\simeq\Delta\lambda_j\mathbf{v}_j+\lambda_j\sum_{i=1}^{4}\varepsilon_{ij}\mathbf{v}_i$ => $\sum_{i=1}^{4}\left(\lambda_i-\lambda_j\right)\varepsilon_{ij}\mathbf{v}_i\simeq\left(\Delta\lambda_j\mathbf{I}-\Delta\mathbf{M}\mathbf{M}\right)\mathbf{v}_j$

Perturbation Theory (2)

- introducing matrix $\mathbf{V}_p = \begin{bmatrix} \mathbf{v}_1 & \mathbf{v}_1^* & \mathbf{v}_2 & \mathbf{v}_2^* \end{bmatrix}$ we rewrite it as 2 matrix eq.

$$\mathbf{V}_p \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \lambda_1 - \lambda_1^* & 0 & 0 \\ 0 & 0 & \lambda_1 - \lambda_2 & 0 \\ 0 & 0 & 0 & \lambda_1 - \lambda_2^* \end{bmatrix} \begin{bmatrix} \Delta\lambda_1 \\ \varepsilon_{21} \\ \varepsilon_{31} \\ \varepsilon_{41} \end{bmatrix} = \mathbf{\Delta M\, M\, v}_1 \,,$$

$$\sum_{i=1}^{4} \left(\lambda_j - \lambda_i\right) \varepsilon_{ij} \mathbf{v}_i + \Delta\lambda_j \mathbf{v}_j = \mathbf{\Delta M M v}_j$$ =>

$$\mathbf{V}_p \begin{bmatrix} \lambda_2 - \lambda_1 & 0 & 0 & 0 \\ 0 & \lambda_2 - \lambda_1^* & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \lambda_2 - \lambda_2^* \end{bmatrix} \begin{bmatrix} \varepsilon_{12} \\ \varepsilon_{22} \\ \Delta\lambda_2 \\ \varepsilon_{42} \end{bmatrix} = \mathbf{\Delta M\, M\, v}_2 \,.$$

- Matrix $\mathbf{V}_p$ is built from symplectic vectors and its inverse is:

$$\mathbf{V}_p^{-1} = -\frac{1}{2i} \mathbf{U} \mathbf{V}_p^{\ T} \mathbf{U}$$

⇨ $$\begin{bmatrix} \Delta\lambda_1 \\ \left(\lambda_1 - \lambda_1^*\right)\varepsilon_{21} \\ \left(\lambda_1 - \lambda_2\right)\varepsilon_{31} \\ \left(\lambda_1 - \lambda_2^*\right)\varepsilon_{41} \end{bmatrix} = -\frac{1}{2i} \mathbf{U} \mathbf{V}_p^{\ T} \mathbf{U \Delta M\, M\, v}_1$$ => $$\begin{cases} \Delta\lambda_1 = -\dfrac{\lambda_1}{2i} \; \mathbf{v}_1^{\ +} \mathbf{U\, \Delta M\, v}_1 \\ \Delta\lambda_2 = -\dfrac{\lambda_2}{2i} \; \mathbf{v}_2^{\ +} \mathbf{U\, \Delta M\, v}_2 \end{cases}$$

Perturbation Theory (3)

- Account relationship between eigenvalue corrections and tune shifts

$$\Delta Q_n = i/(4\pi)\left(\Delta\lambda_n/\lambda_n\right)$$

- That finally yields

$$\begin{cases} \Delta Q_1 = -\dfrac{1}{4\pi}\,\mathbf{v}_1^{+}\mathbf{U}\,\mathbf{\Delta M}\,\mathbf{v}_1 \\ \Delta Q_2 = -\dfrac{1}{4\pi}\,\mathbf{v}_2^{+}\mathbf{U}\,\mathbf{\Delta M}\,\mathbf{v}_2 \end{cases}$$

Linear Tune Shifts in Strongly Coupled Lattice

- Let us find tune shifts in strongly coupled lattice for a general case local focusing perturbation.
- Corresponding addition to Hamiltonian: $\Phi_x x^2 + 2\Phi_s xy + \Phi_y y^2$

$$\Rightarrow \quad \mathbf{\Delta M} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ -\Phi_x & 0 & -\Phi_s & 0 \\ 0 & 0 & 0 & 0 \\ -\Phi_s & 0 & -\Phi_y & 0 \end{bmatrix}$$

- Substitution Δ**M** to the tune shift equation and using the eigen-vector parameterization yields:

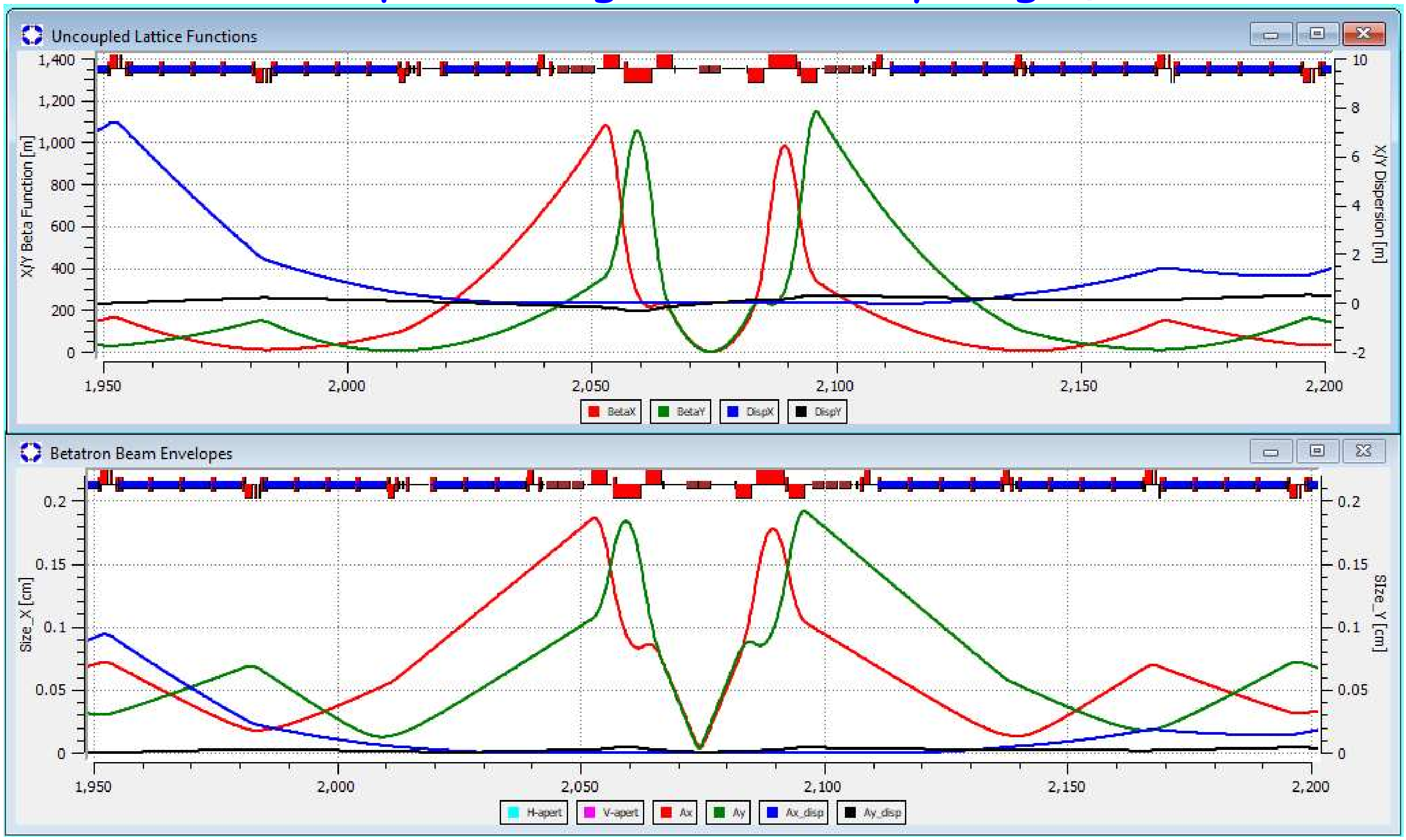
$$\Delta Q_1 = \frac{1}{4\pi}\left(\Phi_x\beta_{1x} + 2\Phi_s\sqrt{\beta_{1x}\beta_{1y}}\cos\nu_1 + \Phi_y\beta_{1y}\right),$$

$$\Delta Q_2 = \frac{1}{4\pi}\left(\Phi_x\beta_{2x} + 2\Phi_s\sqrt{\beta_{2x}\beta_{2y}}\cos\nu_2 + \Phi_y\beta_{2y}\right).$$

Limitations on the Focusing of Interaction Region Quads

Collider Type Optics

- High luminosity -> small beta in IP $\beta(s) = \beta^* + \frac{s^2}{\beta^*}$
- Detector requires long drift => very large β-function in IR quads



- Therefore, the IR quads introduce major limitation on ring focusing

Changes of Tune and β-function at Perturbation Location

- Consider a lattice with one local perturbation =>

$$M = \begin{bmatrix} 1 & 0 \\ -\Phi/2 & 1 \end{bmatrix} \begin{bmatrix} c & s\beta \\ -s/\beta & c \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\Phi/2 & 1 \end{bmatrix} = \begin{bmatrix} c - \Phi\beta s/2 & \beta s \\ -\Phi c - \left(1 - \Phi^2\beta^2/4\right) s/\beta & c - \Phi\beta s/2 \end{bmatrix}$$

On other hand: $M = \begin{bmatrix} c' & s'\beta' \\ -s'/\beta' & c' \end{bmatrix}, \quad c' = \cos(\mu_0 + \Delta\mu), \quad s' = \sin(\mu_0 + \Delta\mu)$

- Equalizing we obtain

$$\cos(\mu_0 + \Delta\mu) = \cos\mu_0 - \frac{\Phi\beta \sin\mu_0}{2}, \quad \beta' = \frac{\beta}{\sqrt{1 + \Phi\beta/\tan\mu_0 - (\Phi\beta/2)^2}}$$

- For $\Phi > 0$ the stability is not lost above the half-integer resonance

It is used in KEKB

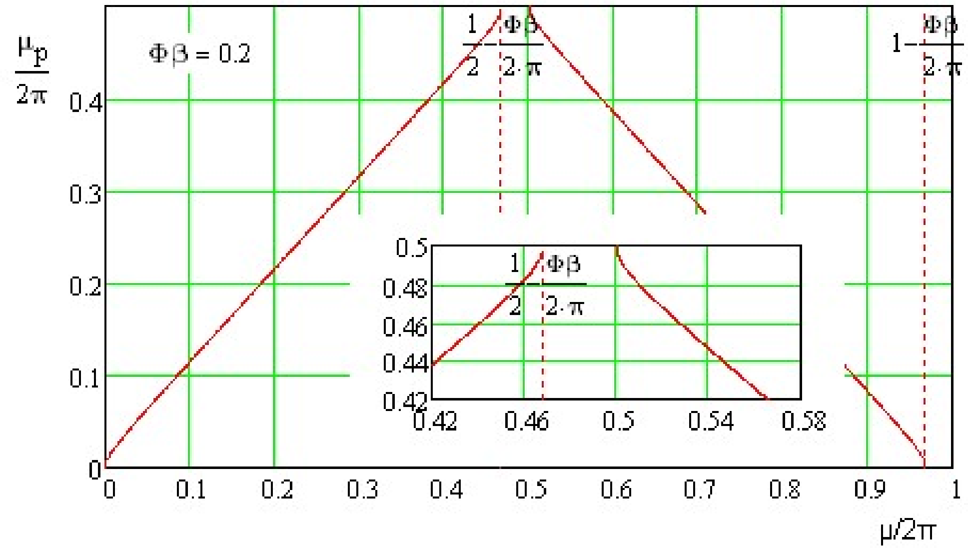
- Stability is lost when

$$\cos\mu_0 - \frac{\Phi\beta\sin\mu_0}{2} = 1 \quad \Rightarrow$$

$$\Phi\beta = 2\frac{\cos\mu_0 - 1}{\sin\mu_0} = -2\frac{2\sin^2(\mu_0/2)}{2\sin(\mu_0/2)\cos(\mu_0/2)}$$

i.e. the stop-band width

$$\Phi\beta = -2\tan\left(\frac{\mu_0}{2}\right)$$



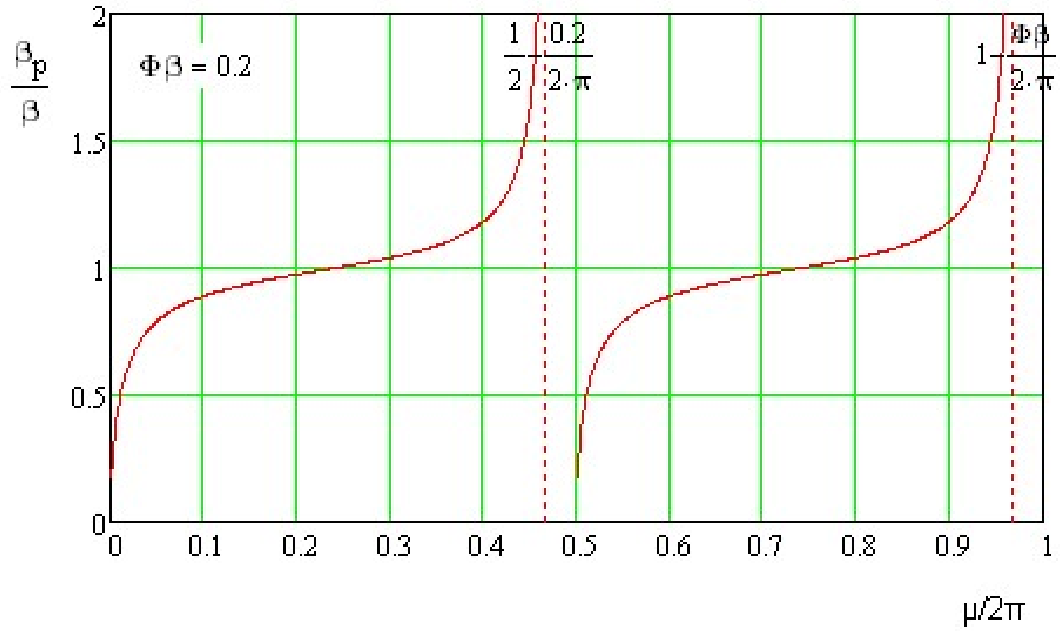
Changes of Tune and β-function in Linear Approximation

- In linear approximation

$$\Delta\mu = \frac{1}{2}\Phi\beta \qquad \frac{\beta'}{\beta} = 1 + \frac{\Phi\beta}{\tan\mu_0}$$

- Let's find the β-function perturbation for the rest of the ring

$$\hat{\beta}(\mu) \equiv \frac{\beta'}{\beta} = 1 + \Delta\beta\cos 2\mu$$



- Account that there is discontinuity at the perturbation location

$$\hat{\beta}(\mu) \equiv \frac{\beta'}{\beta} = 1 + \Delta\beta\cos(\mu_0 - 2\mu) \quad \Rightarrow \quad \hat{\beta}(0) = 1 + \Delta\beta\cos(\mu_0)$$

$$\hat{\beta}(\mu) = 1 + \frac{\Phi\beta}{\tan\mu_0}\frac{\cos(\mu_0 - 2\mu)}{\cos\mu_0}$$

$$\hat{\beta}(\mu) = 1 + \frac{\Phi\beta}{\sin\mu_0}\cos(\mu_0 - 2\mu)$$

Tune and β-function Chromaticities

- Change in momentum changes focusing $\Phi \equiv \frac{1}{F} = \frac{eGL}{pc}$ => $\frac{\Delta\Phi}{\Phi} \equiv -\frac{\Delta p}{p}$
- Chromaticity for point-like single perturbation

$$\Delta\nu = \frac{\Delta\mu}{2\pi} = \frac{1}{2\pi}\left(\frac{1}{2}\Delta\Phi\beta\right) = -\frac{\Phi\beta}{4\pi}\frac{\Delta p}{p}$$

- Summing for all perturbation sources we have: $\xi \equiv p\frac{d\nu}{dp} = -\frac{1}{4\pi}\sum_k \Phi_p \beta_k$
- Estimate for Tevatron collider

$$\xi = -\frac{1}{4\pi}\sum_k \Phi_p \beta_k \xrightarrow[\beta = L^2/\beta^*,\ 2\,\mathrm{IP\,quads}]{\Phi = 1/F = 1/L} -\frac{1}{4\pi}2\frac{1}{L}\frac{L^2}{\beta^*} = -\frac{1}{2\pi}\frac{L}{\beta^*} \xrightarrow{\text{Tevatron}} -\frac{1}{2\pi}\frac{30m}{30cm} \approx -15$$

Contribution of 2 IPs exceeds the ring natural chromaticity of ~20

- What can be more important is the chromaticity of β-functions
- For single quad: $\left.\frac{\Delta\beta}{\beta}\right|_{\max} = \frac{2\Delta\mu}{\sin\mu_0} = \frac{2\xi}{\sin\mu_0}\frac{\Delta p}{p}$

=> chromatic β: $\left. p\frac{d}{dp}\left(\frac{\Delta\beta}{\beta}\right)\right|_{\max} = \frac{2\xi}{\sin\mu_0} \xrightarrow{Tevatron} \approx \frac{2\cdot 7.5}{0.2} = 75$

- The chromaticity of β-functions is closely related to the 2nd order chromaticity. Affects beam-beam. Has to be suppressed.

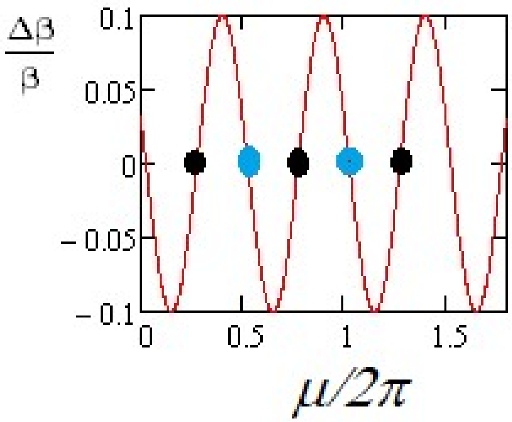
Correction Tune and β-function Chromaticities

- Sextupoles are used for the correction: $B = \frac{1}{2} S x^2 \quad \Rightarrow \quad G_S(x_0) = S x_0$

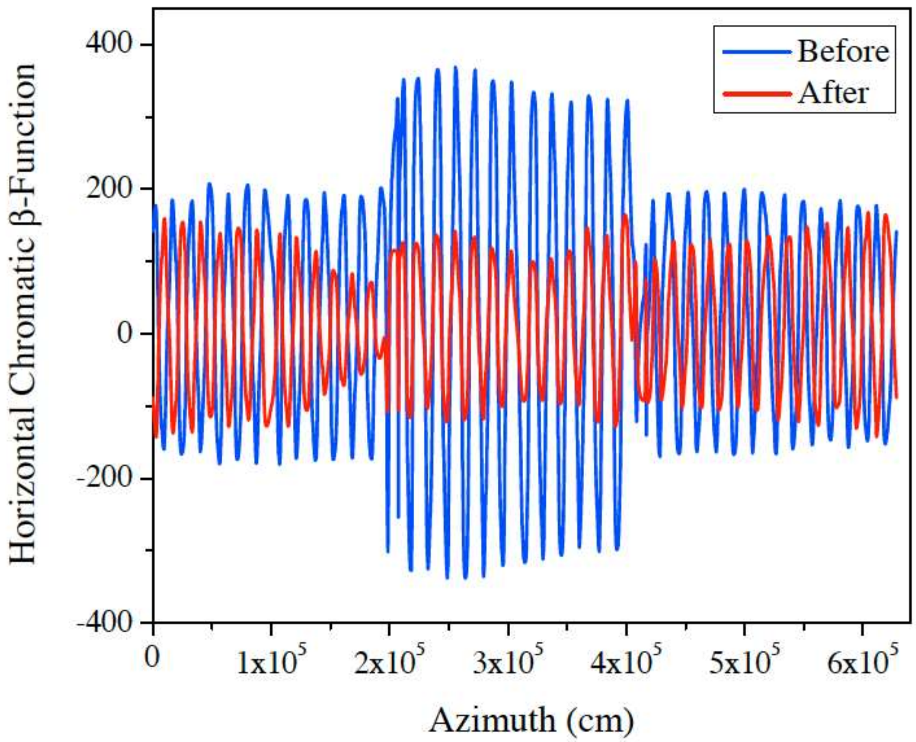
$$\Delta\nu = \frac{\Delta\Phi\beta}{4\pi} = -\frac{1}{4\pi}\sum_k \beta_k \frac{eSL}{pc}\left(D\frac{\Delta p}{p}\right)$$

$$\Rightarrow \qquad \xi = -\frac{e}{4\pi pc}\sum_k \beta_k D_k (SL)_k$$

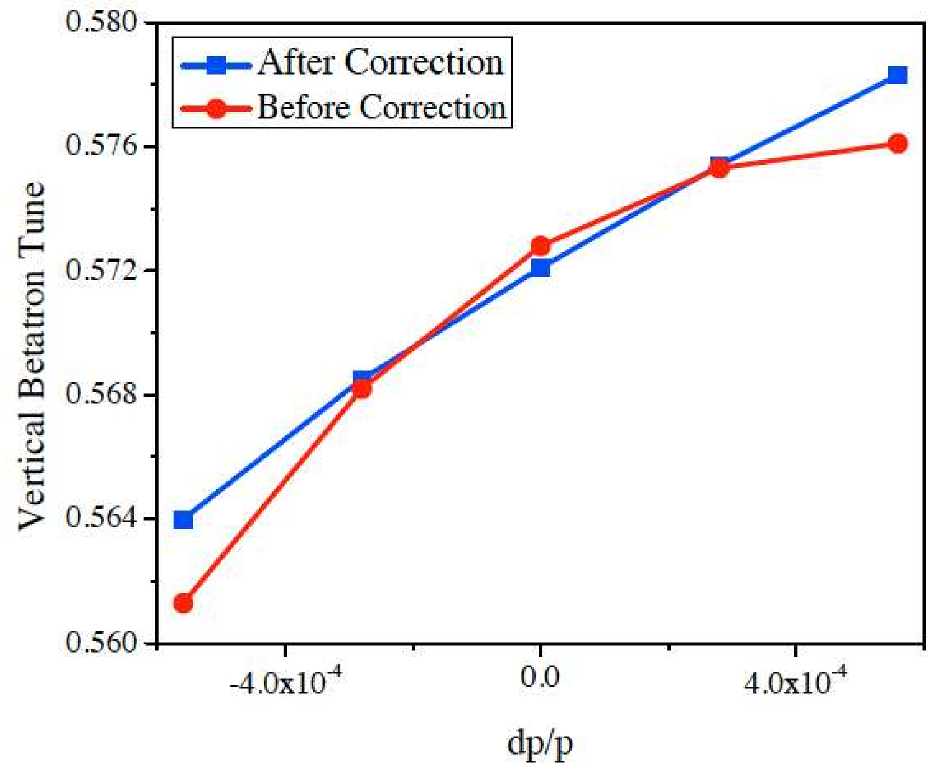
- Location of F and D sextupoles near F and D quads enables chromaticity correction for both planes
- For correction of chromatic β-function sextupoles located at "right" phases are used



Correction Tune and β-function Chromaticities at Tevatron



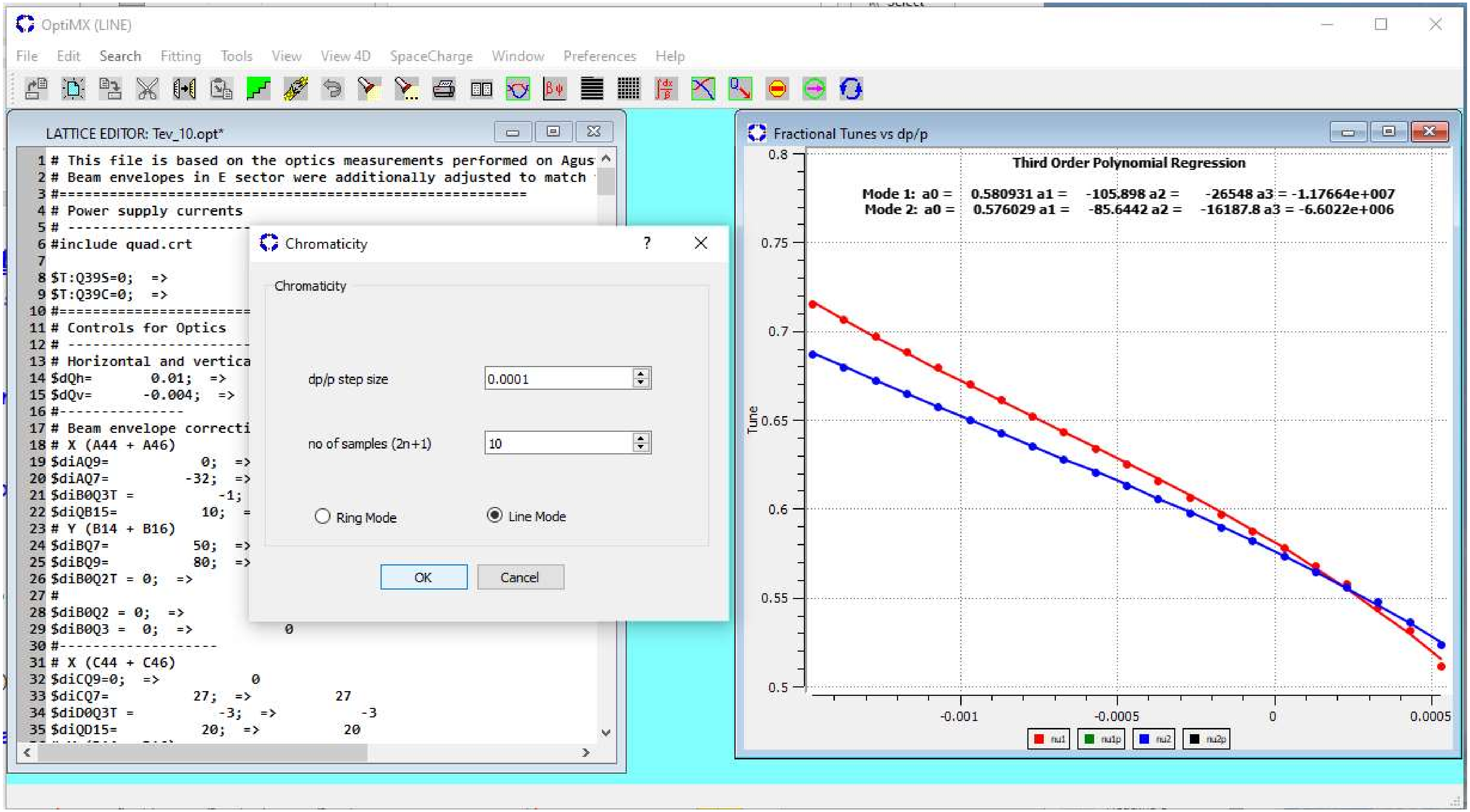
Horizontal chromatic beta-function at the injection energy. Blue line is for the original sextupole configuration, red - for the proposed correction



Dependence of the vertical betatron tune on particle momentum in the collider mode.

How OptiMX Computes Betatron Tune Shifts

- All nonlinearities are described by zero length multipoles
- The closed orbit can be excited by dipole correctors
- In Reference Orbit mode program finds new CO by iterations with accounting all non-linearities. Then it builds new lattice where feeddown from high order multipoles are accounted.
  - Consequently, in linear optics calculations all corrections to optics are correctly accounted.
- In View4D|Chromaticity this procedure is produced automatically on a number of momentum offsets. That yields dependence of mode tunes on momentum
  ⇨Linear and non-linear chromaticities


OptiMX (LINE)
File Edit Search Fitting Tools View View 4D SpaceCharge Window Preferences Help
LATTICE EDITOR: Tev_10.opt*
Chromaticity
dp/p step size
0.0001
no of samples (2n+1)
10
Ring Mode
Line Mode
OK
Cancel
Fractional Tunes vs dp/p
Third Order Polynomial Regression
Mode 1: a0 = 0.580931 a1 = -105.898 a2 = -26548 a3 = -1.17664e+007
Mode 2: a0 = 0.576029 a1 = -85.6442 a2 = -16187.8 a3 = -6.6022e+006
Tune

References

- "Accelerator Physics at the Tevatron Collider", edited by V. Lebedev and V. Shiltsev, Springer, 2014.

Problems

1. Using symplecticity condition prove that for the 4x4 matrix written through 2x2 matrices as $\begin{bmatrix} P & p \\ q & Q \end{bmatrix}$ the following is correct: $\det(P)+\det(p)=\det(Q)+\det(q)=1$ and $\det(P)=\det(Q), \det(p)=\det(q)$
2. Using software for analytical computations prove that for a ring without RF
$$\begin{cases} M_{16}=D(1-M_{11})-D'M_{12} \\ M_{26}=-M_{21}D+D'(1-M_{22}) \end{cases} \quad \begin{cases} M_{51}=DM_{21}+D'\left(1-M_{11}\right) \\ M_{52}=-D(1-M_{22})-D'M_{12} \end{cases}$$
3. Prove that for matrix built from symplectic normalized eigen-vectors, $\mathbf{V}_p=\left[\mathbf{v}_1 \quad \mathbf{v}_1^* \quad \mathbf{v}_2 \quad \mathbf{v}_2^*\right]$, the following is correct:
$$\mathbf{V}_p^{-1}=-\frac{1}{2i}\mathbf{U}\mathbf{V}_p^{T}\mathbf{U}$$
4. Find dependence of synchrotron frequency on the particle amplitude/action for the beam motion in a harmonic RF voltage. Obtain asymptotic dependence for small amplitudes.
5. Restore missed calculations in computation of tune shifts in strongly coupled optics
$$\Delta Q_1=\frac{1}{4\pi}\left(\Phi_x\beta_{1x}+2\Phi_s\sqrt{\beta_{1x}\beta_{1y}}\cos\nu_1+\Phi_y\beta_{1y}\right),$$
$$\Delta Q_2=\frac{1}{4\pi}\left(\Phi_x\beta_{2x}+2\Phi_s\sqrt{\beta_{2x}\beta_{2y}}\cos\nu_2+\Phi_y\beta_{2y}\right).$$