Energy Calibration updates

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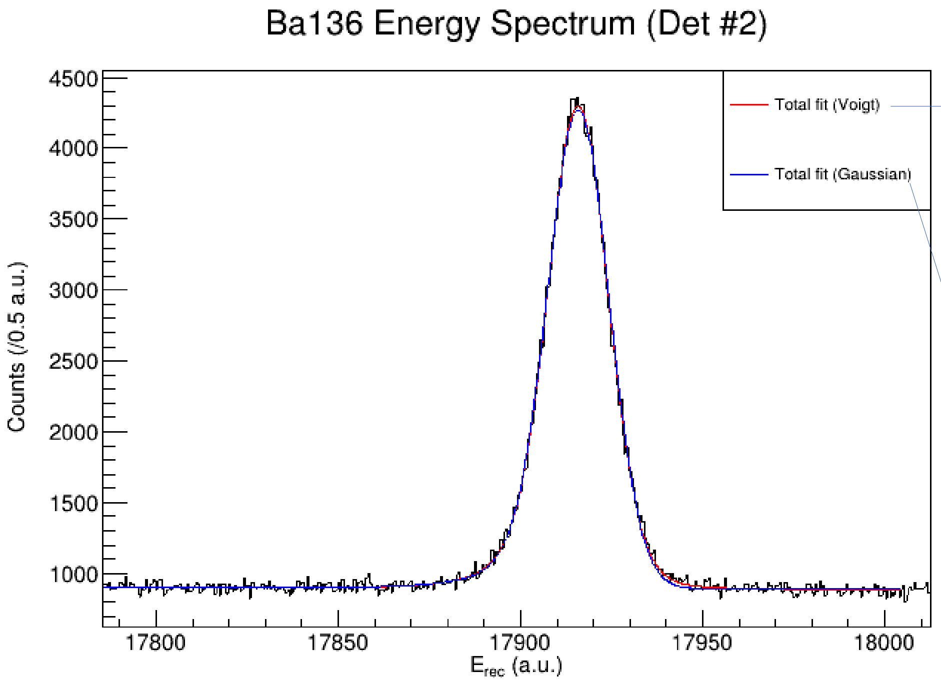
Energy Calibration using Eu+Ba+Tl gamma lines

- Used all Oct Ba data in this study
- Improved the fit function of Ba muonic X-rays by replacing peak fit function earlier described by Gaussian to Voigt function
- Voigt describes the natural broadening of the muonic X-ray peaks arising due to uncertainty in lifetime of these peaks
- The component for natural broadening in ROOT's Voigt function is lg = 2*γ (γ = natural spread)
- The natural broadening for the peaks in consideration (~ 4 MeV) are of the order of 0.23 keV
- Using an iterative approach, first obtained the calibration parameters of the detector which give an estimate of lg in a.u.
- Next, fixed the lg value in the fitting function

```
Double_t TMath::Voigt ( Double_t  xx,
                        Double_t  sigma,
                        Double_t  lg,
                        Int_t     r = 4
                      )
```

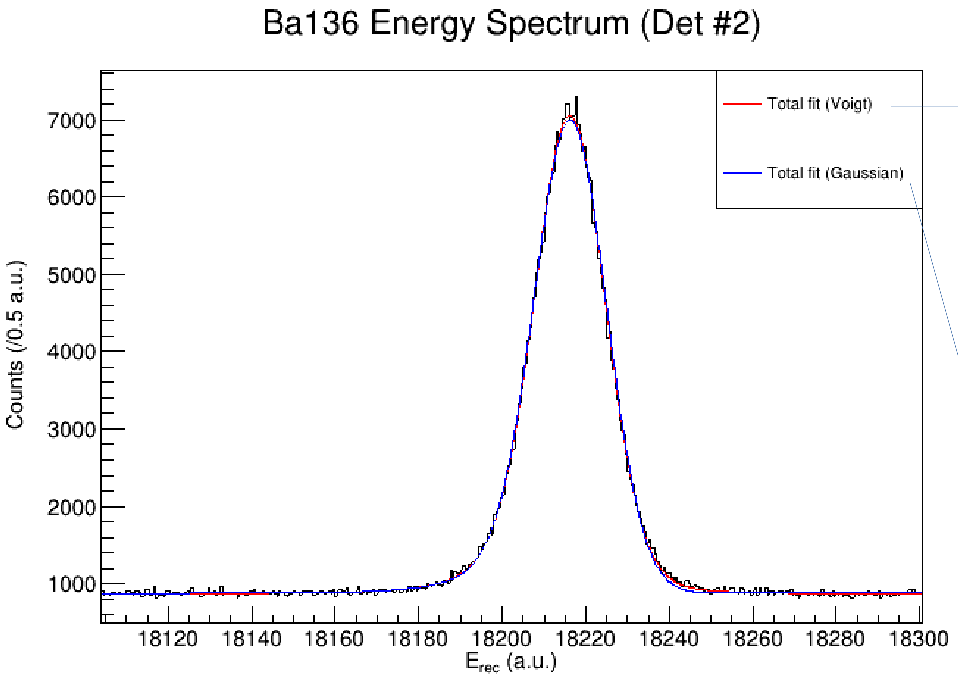
Ba-136 3924.3 keV fit




```
 FCN=474.515 FROM MIGRAD    STATUS=CONVERGED     130 CALLS         131 TOTAL
                     EDM=3.02319e-07    STRATEGY= 1      ERROR MATRIX ACCURATE
  EXT PARAMETER                                   STEP         FIRST
  NO.   NAME      VALUE            ERROR          SIZE      DERIVATIVE
   1  Amplitude of peak   7.70594e+04   3.30471e+02   2.56434e+00  -1.10755e-06
   2  Mean of peak   1.79162e+04   1.18029e-01   8.54312e-03  -8.34960e-03
   3  Sigma of peak   7.82969e+00   5.47412e-02   3.30848e-04   2.97302e-03
   4  Err fun height   3.39819e-08   2.58975e+00   1.73948e-03** at limit **
   5  Slope- left tail   1.00731e+01   1.42432e+00   1.81294e-04   8.70977e-03
   6  intercept-lin bkg   3.20081e+03   3.63766e+02   1.54720e-02  -6.15749e-05
   7  SLope-lin bkg  -1.29123e-01   2.03495e-02   8.65282e-07  -1.09683e+00
   8  Frac. of Voigt peak   8.94564e-01   2.14273e-02   3.27829e-04  -4.64085e-03
chi2: 474.515
NDF: 472
reduced chi2: 1.00533
```

```
 FCN=554.474 FROM MIGRAD    STATUS=CONVERGED     132 CALLS         133 TOTAL
                     EDM=9.80316e-09    STRATEGY= 1      ERROR MATRIX ACCURATE
  EXT PARAMETER                                   STEP         FIRST
  NO.   NAME      VALUE            ERROR          SIZE      DERIVATIVE
   1  Amplitude of peak   7.48371e+04   3.15965e+02   2.70611e+00  -2.54420e-07
   2  Mean of peak   1.79163e+04   9.56211e-02   8.54315e-03  -9.47709e-04
   3  Sigma of peak   8.36686e+00   5.37787e-02   3.43310e-04   3.18835e-03
   4  Err fun height   3.49853e-11   1.19587e+00   1.27541e-03** at limit **
   5  Slope-left tail   1.10707e+01   1.27124e+00   1.76770e-04   3.99757e-03
   6  intercept-lin bkg   2.23326e+03   3.63406e+02   1.67223e-02  -3.29793e-05
   7  Slope-lin bkg  -7.46103e-02   2.03270e-02   9.35206e-07  -5.88294e-01
   8  Frac. of Gauss peak   8.71017e-01   1.76217e-02   3.22300e-04  -2.46804e-03
chi2: 554.474
NDF: 472
reduced chi2: 1.17473
```

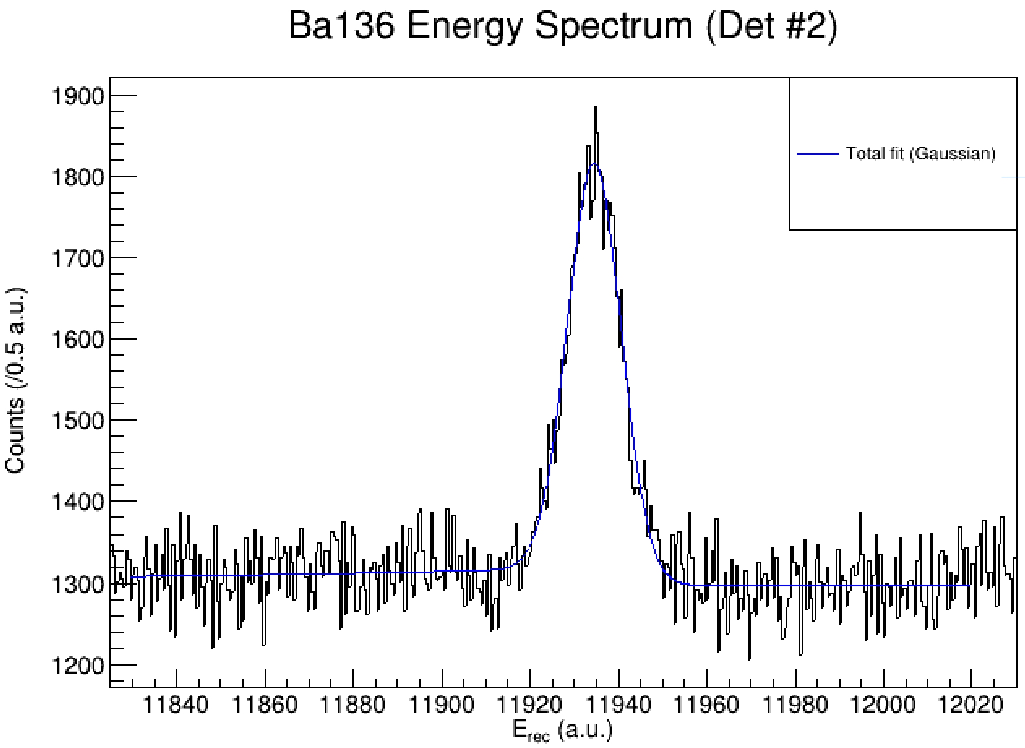
Ba-136 3990.0 keV fit



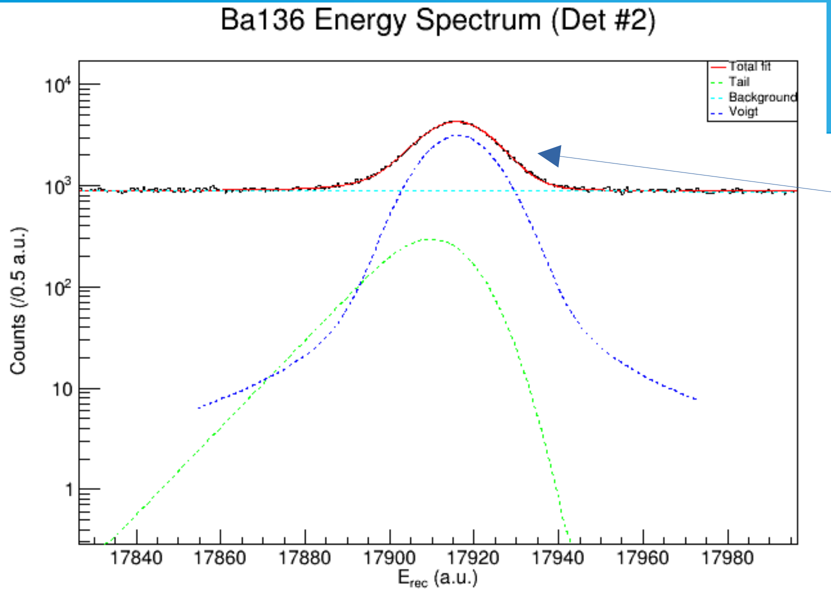

```
 FCN=562.356 FROM MIGRAD    STATUS=CONVERGED     108 CALLS         109 TOTAL
                     EDM=8.87735e-09    STRATEGY= 1      ERROR MATRIX ACCURATE
  EXT PARAMETER                                   STEP         FIRST
  NO.   NAME      VALUE            ERROR          SIZE      DERIVATIVE
   1  Amplitude of peak   1.42773e+05   3.72678e+02   3.52732e+00  -6.80002e-08
   2  Mean of peak   1.82166e+04   8.33799e-02   8.68637e-03   7.82580e-04
   3  Sigma of peak   8.02853e+00   3.58833e-02   2.42191e-04  -2.36934e-03
   4  Err fun height   0.00000e+00   1.63174e+00   4.01175e-03** at limit **
   5  Slope- left tail   9.13415e+00   7.15458e-01   1.05228e-04  -5.59963e-03
   6  intercept-lin bkg   2.00006e+03   3.60154e+02   4.18130e-02  -8.18831e-05
   7  SLope-lin bkg  -6.23269e-02   1.97948e-02   2.30263e-06  -1.49104e+00
   8  Frac. of Voigt peak   8.85403e-01   1.54138e-02   2.37897e-04   2.43872e-03
chi2: 562.356
NDF: 472
reduced chi2: 1.19143
```

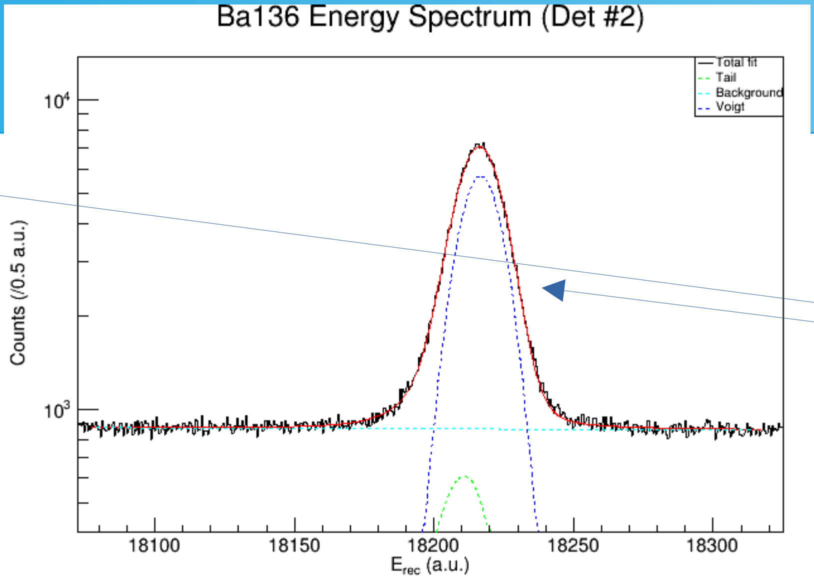
```
 FCN=804.8 FROM MIGRAD    STATUS=CONVERGED     108 CALLS         109 TOTAL
                     EDM=5.93114e-08    STRATEGY= 1      ERROR MATRIX ACCURATE
  EXT PARAMETER                                   STEP         FIRST
  NO.   NAME      VALUE            ERROR          SIZE      DERIVATIVE
   1  Amplitude of peak   1.38912e+05   3.58195e+02   4.12556e+00  -3.08887e-07
   2  Mean of peak   1.82167e+04   6.68363e-02   8.68640e-03  -6.14339e-03
   3  Sigma of peak   8.58559e+00   3.57633e-02   2.78188e-04   5.26281e-03
   4  Err fun height   9.15934e-15   6.79562e-01   3.08774e-03** at limit **
   5  Slope-left tail   1.03280e+01   7.05969e-01   1.18955e-04   2.22626e-02
   6  intercept-lin bkg   8.00531e+02   3.61527e+02   4.99870e-02   5.01721e-05
   7  Slope-lin bkg   4.28197e-03   1.98677e-02   2.75242e-06   9.14954e-01
   8  Frac. of Gauss peak   8.66248e-01   1.23972e-02   2.55288e-04  -8.41275e-03
chi2: 804.8
NDF: 472
reduced chi2: 1.70509
```

Tl-208 2614.51 keV fit

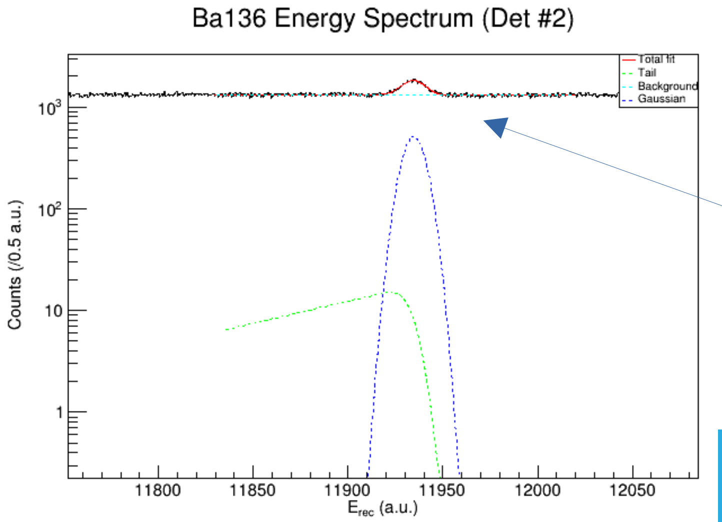



```
 FCN=374.769 FROM MIGRAD    STATUS=CONVERGED     112 CALLS         113 TOTAL
                     EDM=1.5217e-07    STRATEGY= 1      ERROR MATRIX ACCURATE
  EXT PARAMETER                                   STEP         FIRST
  NO.   NAME      VALUE            ERROR          SIZE      DERIVATIVE
   1  Amplitude of peak   9.47796e+03   9.42139e+02   1.53753e+00   1.40911e-06
   2  Mean of peak   1.19346e+04   1.66151e-01   5.69086e-03  -1.57977e-04
   3  Sigma of peak   6.09055e+00   1.55289e-01   1.18094e-03   1.82997e-03
   4  Err fun height   5.00000e+00   3.36707e+00   4.70986e-02** at limit **
   5  Slope-left tail   1.00000e+02   9.55061e+01   2.39627e-02** at limit **
   6  intercept-lin bkg   1.26221e+03   6.03572e+02   1.77662e-02   1.24741e-04
   7  Slope-lin bkg   2.93288e-03   5.04301e-02   1.48987e-06   1.48933e+00
   8  Frac. of Gauss peak   8.16309e-01   8.14221e-02   9.78939e-04   1.06245e-03
chi2: 374.769
NDF: 372
reduced chi2: 1.00744
```





Ba muonic X-ray peaks



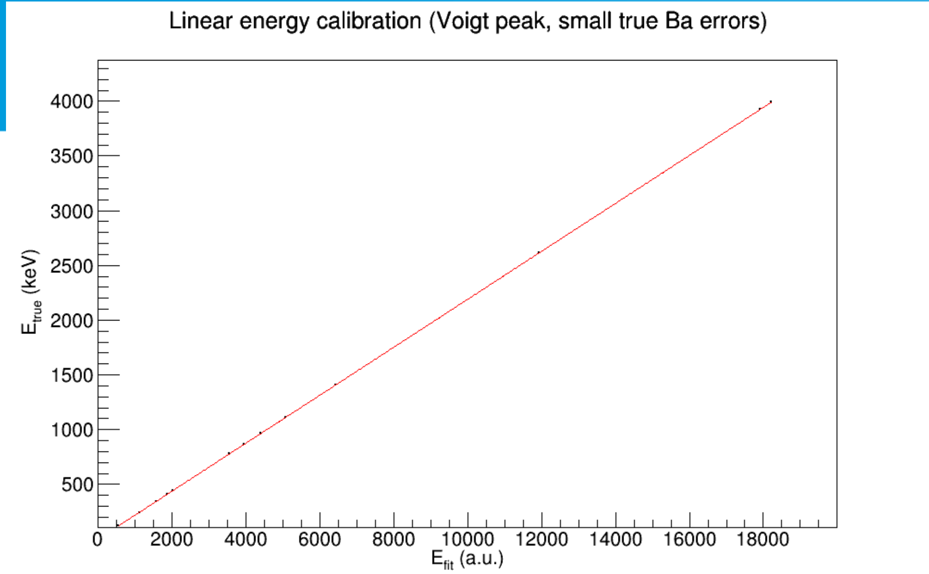
Tl-208 2614.51 peak

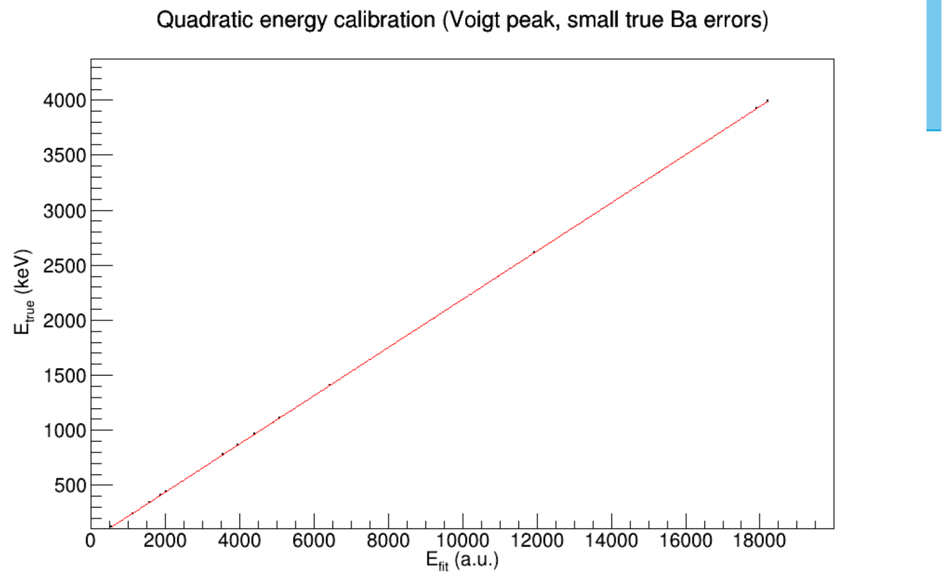
- Total fit with individual components for Ba muonic X-rays and Tl-208 peak

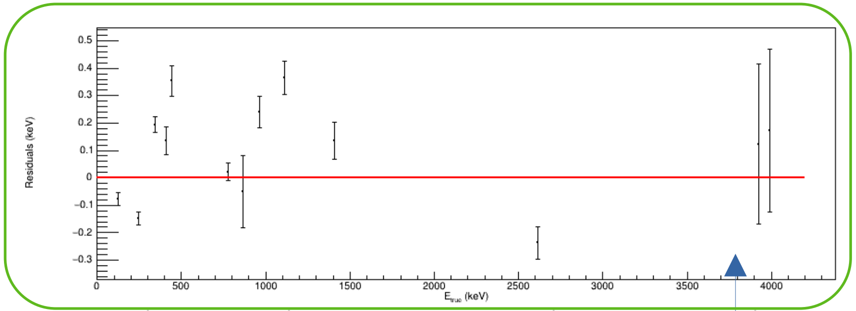
Energy calibration

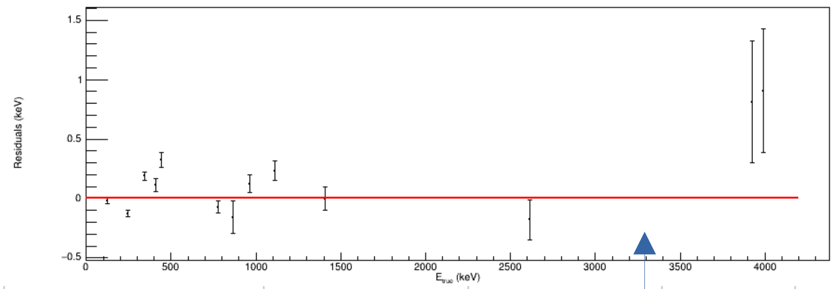
- We obtained true Ba K-shell muonic X-rays using mudirac code and found a discrepancy of ~1.1 keV with Schopper data
- Earlier we used an error of 0.2 keV for true Ba energies but here we compare the calibration constants taking two different errors (0.2 keV-small and 1.1 keV-large)
- Calibration constants are also compared by taking Gaussian/Voigt peak functions

Ch 1



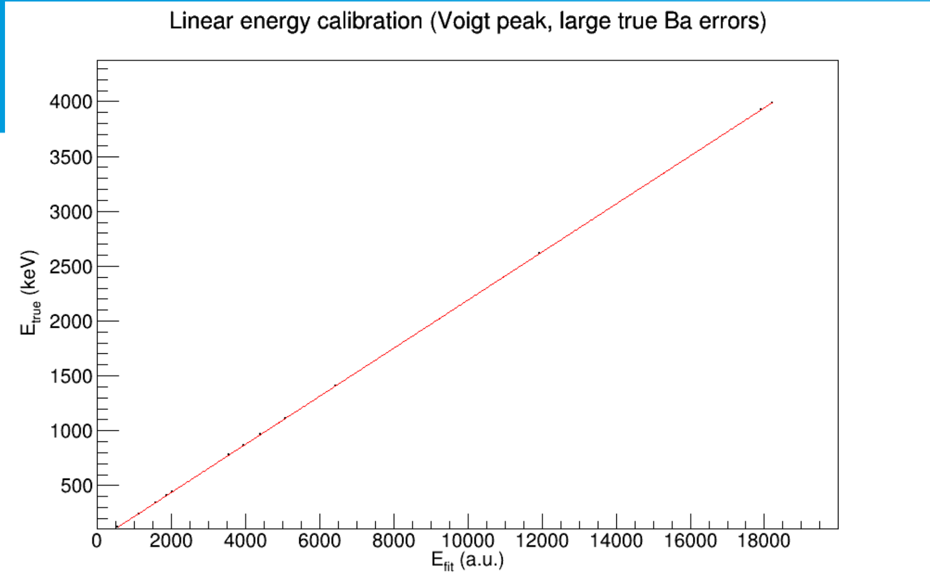


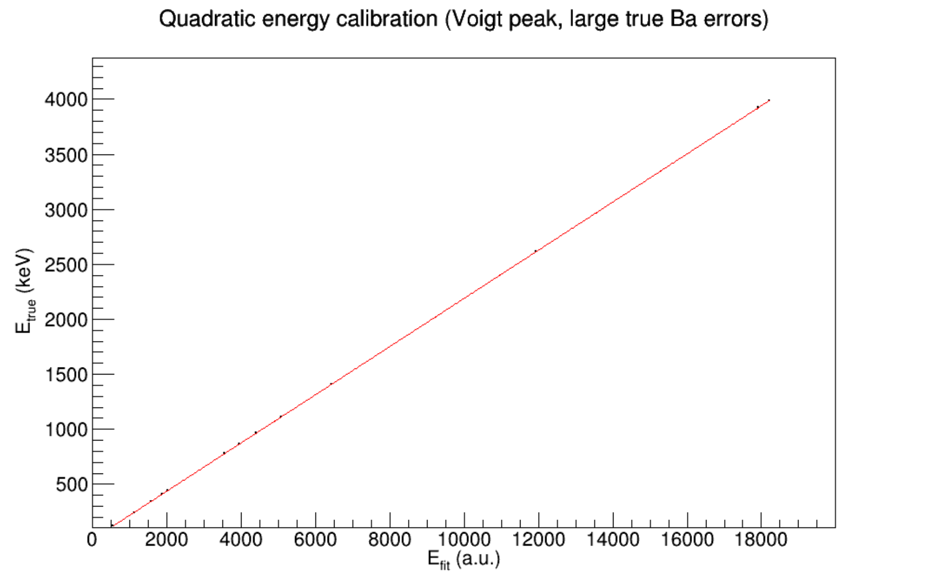


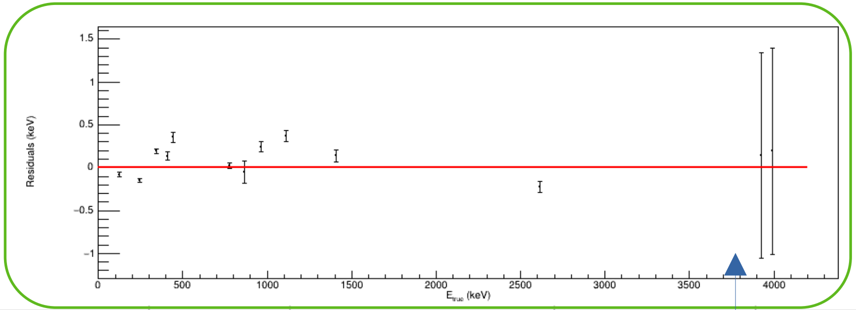


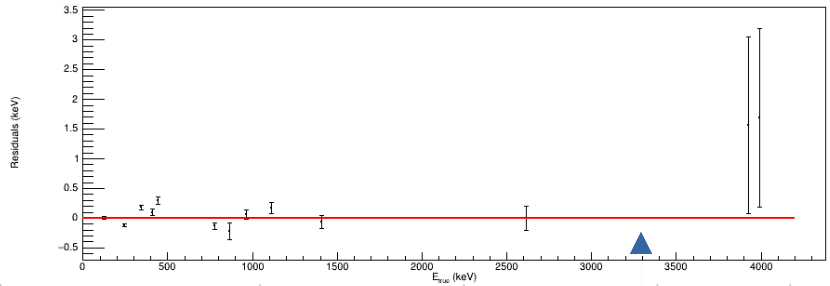
Channel 1	Energy calibration Fit function y = a+b*x a (keV)	b (keV/a.u.)	Residuals E_{true} - E_{fit} (keV)	RSqS	Red χ^2	Energy calibration Fit function y = a+b*x+c*x^2 a (keV)	b (keV/a.u.)	c (keV/a.u.^2)	Residuals E_{true} - E_{fit} (keV)	RSqS	Red χ^2
Voigt											
1(0.2 err)	0.29±0.01	0.219065±0.000003	(-0.24)- (0.36)	0.52	26.2	0.20±0.02	0.21914±0.00001	-6.1(±0.8)e-9	(-0.2)- (0.9)	1.78	22.6

Ch 1



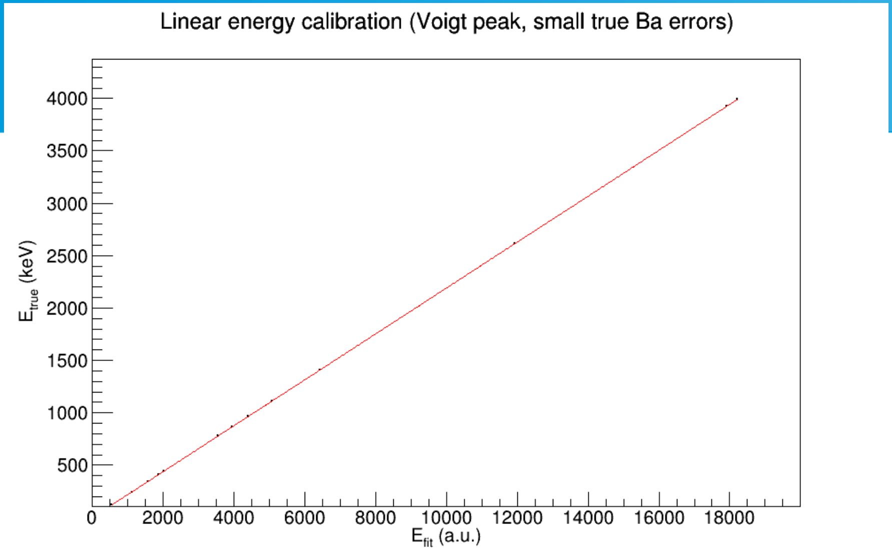


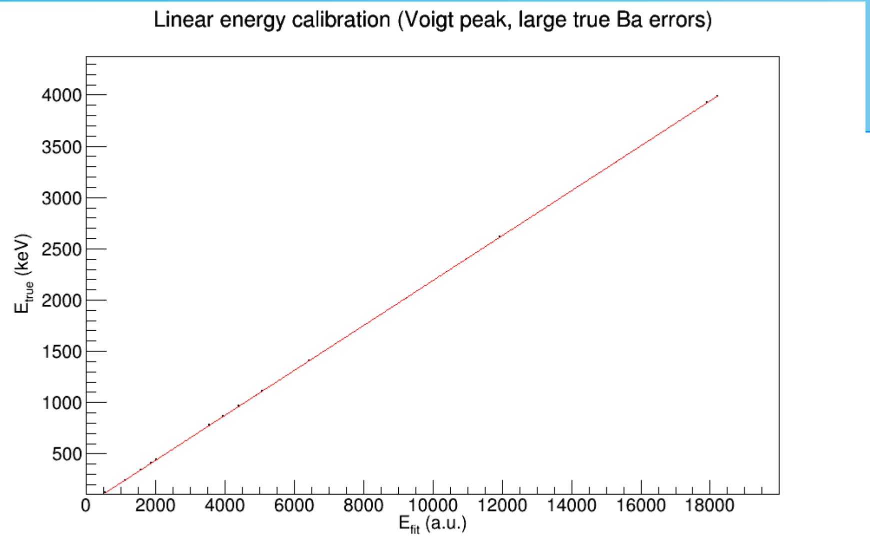


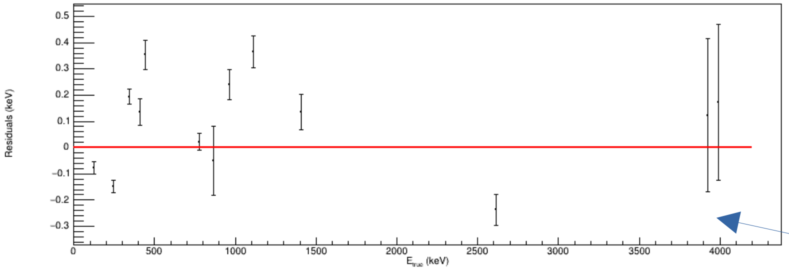


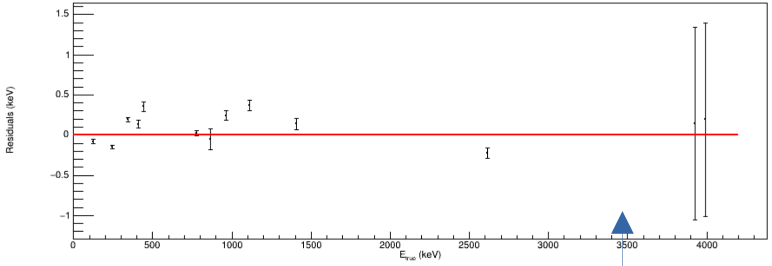
Channel 1	Energy calibration, Fit function y = a+b*x		Residuals	RSqS	Red χ^2	Energy calibration, Fit function y = a+b*x+c*x^2			Residuals	RSqS	Red χ^2
	a (keV)	b (keV/a.u.)	E_{true} - E_{fit} (keV)			a (keV)	b (keV/a.u.)	c (keV/a.u.^2)	E_{true} - E_{fit}(keV)		
Voigt											
1(1.1 err)	0.30±0.01	0.219064±0.000004	(-0.22)- (0.37)	0.54	26.1	0.14±0.02	0.21919±0.00001	-1.1(±0.1)e-8	(-0.2)- (1.7)	5.55	16.7

Ch 1









- In Ch 1, on comparing small/large errors on true Ba energy, there is very minute change in residual range

Channel 1	Energy calibration Fit function y = a+b*x a (keV)	 b (keV/a.u.)	Residuals E_{true} - E_{fit} (keV)	RSqS	Red χ^2
Voigt					
1(0.2 err)	0.29±0.01	0.219065±0.000003	(-0.24)- (0.36)	0.52	26.2
1(1.1 err)	0.30±0.01	0.219064±0.000004	(-0.22)- (0.37)	0.54	26.1

Comparison of residuals in Ch 1 (Detector 2)

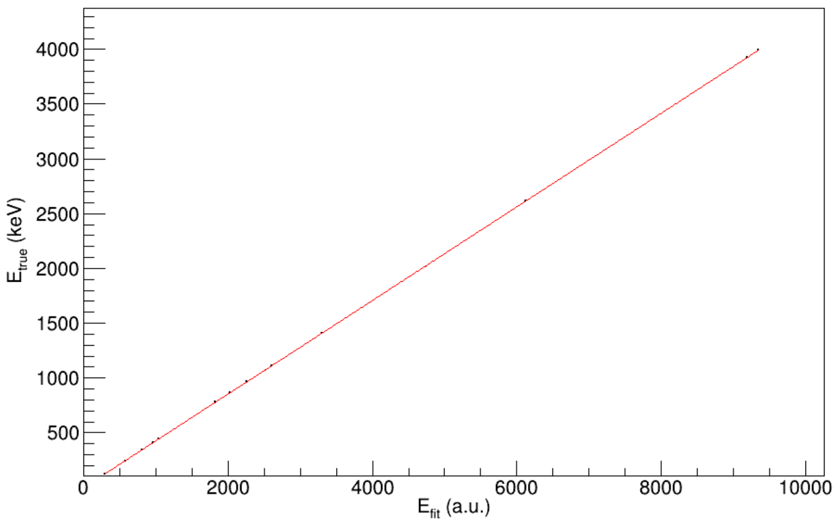
Channel 1	Energy calibration Fit function y = a+b*x		Residuals	RSqS	Red χ^2	Energy calibration Fit function y = a+b*x+c*x^2			Residuals	RSqS	Red χ^2
	a (keV)	b (keV/a.u.)	E_{true} - E_{fit} (keV)			a (keV)	b (keV/a.u.)	c (keV/a.u.^2)	E_{true} - E_{fit}(keV)		
Voigt											
1(0.2 err)	0.29±0.01	0.219065±0.000003	(-0.24)- (0.36)	0.52	26.2	0.20±0.02	0.21914±0.00001	-6.1(±0.8)e-9	(-0.2)- (0.9)	1.78	22.6
1(1.1 err)	0.30±0.01	0.219064±0.000004	(-0.22)- (0.37)	0.54	26.1	0.14±0.02	0.21919±0.00001	-1.1(±0.1)e-8	(-0.2)- (1.7)	5.55	16.7
Gaussian											
1(0.2 err)	0.29±0.01	0.219065±0.000003	(-0.24)- (0.36)	0.52	26.2	0.19±0.02	0.21914±0.00001	-6.2(±0.8)e-9	(-0.2)- (0.9)	1.78	22.5
1(1.1 err)	0.30±0.01	0.219064±0.000004	(-0.22)- (0.37)	0.53	26.1	0.14±0.02	0.21919±0.00001	-10.9(±0.9)e-9	(-0.2)- (1.7)	5.46	16.6

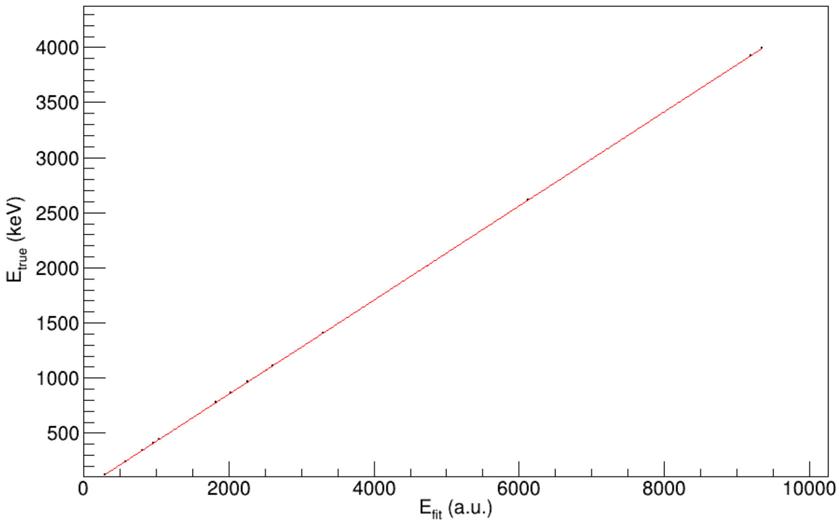
No difference in in preferred calibration constants using small or large errors on true Ba energies

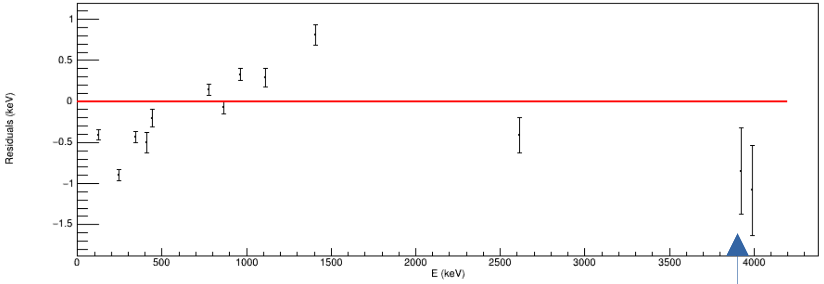
No difference in residuals on using V/G fit function to describe the peak

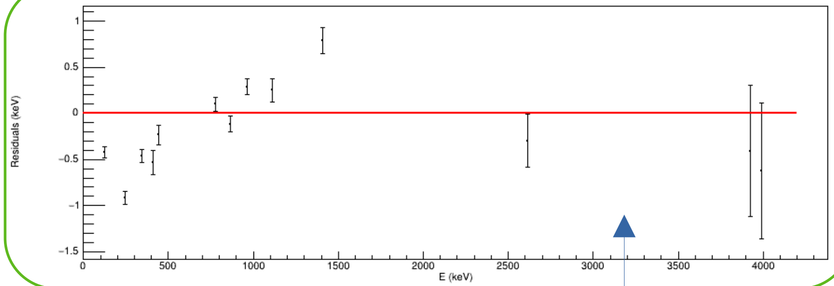
There is a difference in residuals on using lin/quad calibration functions

Ch 3



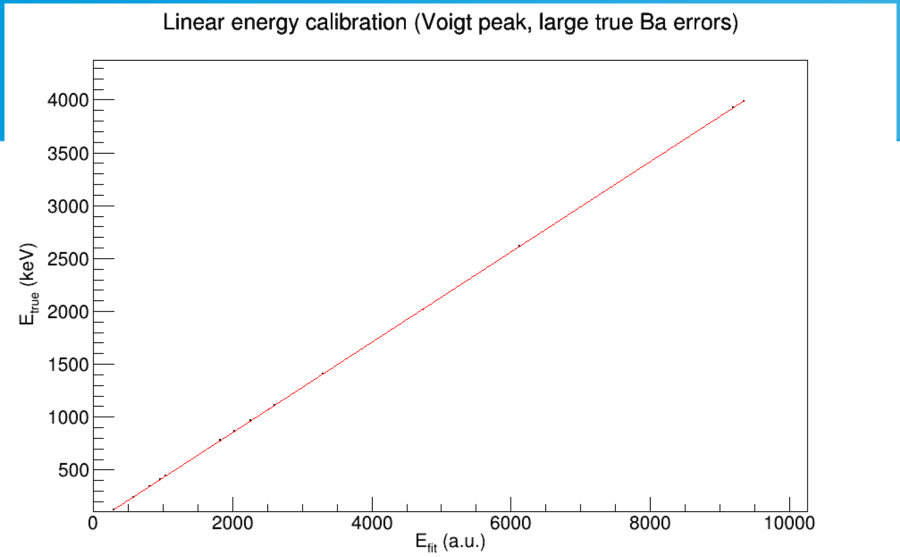


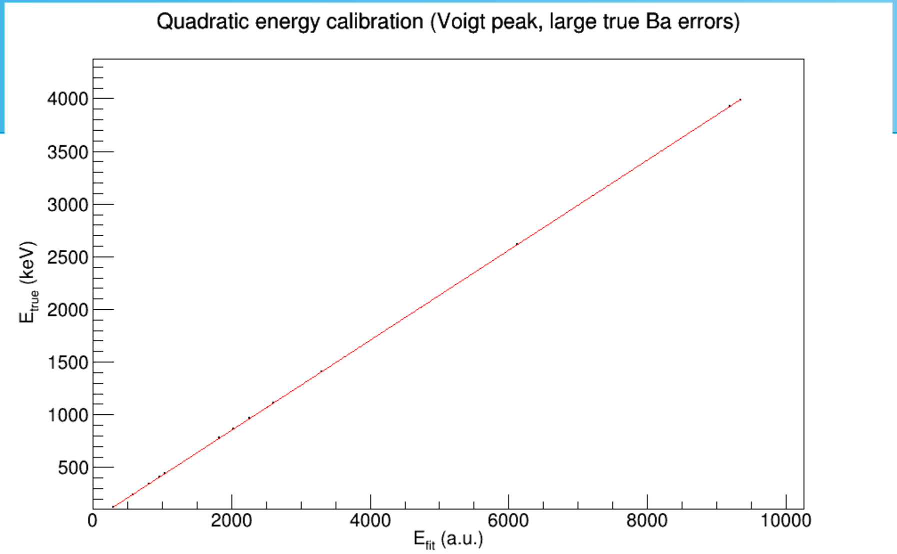


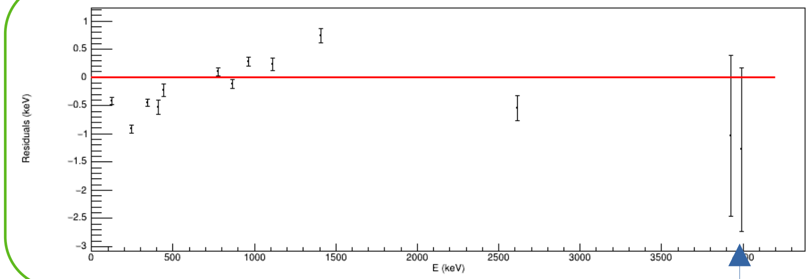


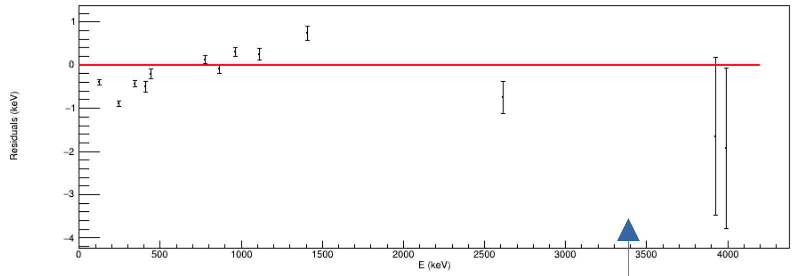
Channel 3	Energy calibration: Fit function y = a+b*x		Residuals	RSqS	Red χ^2	Energy calibration: Fit function y = a+b*x+c*x^2			Residuals	RSqS	Red χ^2
	a (keV)	b (keV/a.u.)	E_{true} - E_{fit} (keV)			a (keV)	b (keV/a.u.)	c (keV/a.u.^2)	E_{true} - E_{fit}(keV)		
Voigt											
1(0.2 err)	0.000±0.002	0.42692±0.00001	(-1.0)- (0.8)	4.25	34.9	0.000±0.001	0.42696±0.00002	-0.9(±0.4)e-8	(-0.92)- (0.8)	2.99	37.9

Ch 3









Channel 3	Energy calibration: Fit function y = a+b*x		Residuals	RSqS	Red χ^2	Energy calibration: Fit function y = a+b*x+c*x^2			Residuals	RSqS	Red χ^2
	a (keV)	b (keV/a.u.)	E_{true} - E_{fit} (keV)			a (keV)	b (keV/a.u.)	c (keV/a.u.^2)	E_{true} - E_{fit}(keV)		
Voigt											
1(1.1 err)	0.000±0.001	0.42694±0.00001	(-1.2)- (0.7)	5.24	33.6	0.000±0.002	0.42691±0.00003	1.1(±0.7)e-8	(-1.9)- (0.7)	9.12	36.8

Comparison of residuals in Ch 3 (Detector 2)

Channel 3	Energy calibration Fit function y = a+b*x		Residuals	RSqS	Red χ^2	Energy calibration Fit function y = a+b*x+c*x^2			Residuals	RSqS	Red χ^2
	a (keV)	b (keV/a.u.)	E_{true} - E_{fit} (keV)			a (keV)	b (keV/a.u.)	c (keV/a.u.^2)	E_{true} - E_{fit} (keV)		
Voigt											
1(0.2 err)	0.000±0.002	0.42692±0.00001	(-1.0)- (0.8)	4.25	34.9	0.000±0.001	0.42696±0.00002	-0.9(±0.4)e-8	(-0.92)- (0.8)	2.99	37.9
1(1.1 err)	0.000±0.001	0.42694±0.00001	(-1.2)- (0.7)	5.24	33.6	0.000±0.002	0.42691±0.00003	1.1(±0.7)e-8	(-1.9)- (0.7)	9.12	36.8
Gaussian											
1(0.2 err)	0.000±0.002	0.42692±0.00001	(-1.0)- (0.8)	4.13	34.9	0.000±0.001	0.42696±0.00002	-0.9(±0.4)e-8	(-0.92)- (0.8)	2.89	37.9
1(1.1 err)	0.000±0.001	0.42694±0.00001	(-1.2)- (0.7)	4.96	33.6	0.000±0.002	0.42691±0.00003	1.1(±0.7)e-8	(-1.9)- (0.7)	8.76	36.7

- There is a difference in preferred calibration constants using small or large errors on true Ba energies
- Using small errors prefer quadratic calibration and large errors prefer linear one
- No difference in residuals on using V/G fit function to describe the peak

Outline and next steps

- Voigt peak function gives better reduced χ^2 compared to Gaussian peak function
- Introduced Tl-208 peak in the calibration set
- Calibration constants give smaller residual range depending on the error (small/large) we use for the true Ba muonic X-rays and the channel in consideration
- Continuing to produce my own files for time stability study of Ch 1
- Question:
  - Are we fixed to use large errors on true Ba lines?
  - Are the right tails fitted with Voigt actually from lifetime of the peak and not something else