

Collective Flow in Heavy-Ion Collisions

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NICA Days 2023, Belgrade, Serbia, October 2-3, 2023

Relativistic Heavy-Ion Collisions and Quark Gluon Plasma (QGP)

HADRON

QGP

Bevalac
~1 GeV



AGS
~5 GeV



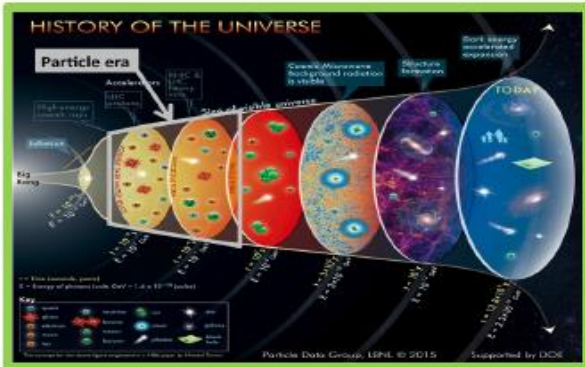
SPS
~20 GeV



RHIC
~100 GeV

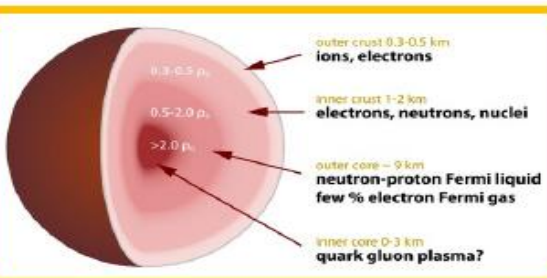


LHC
~5000 GeV



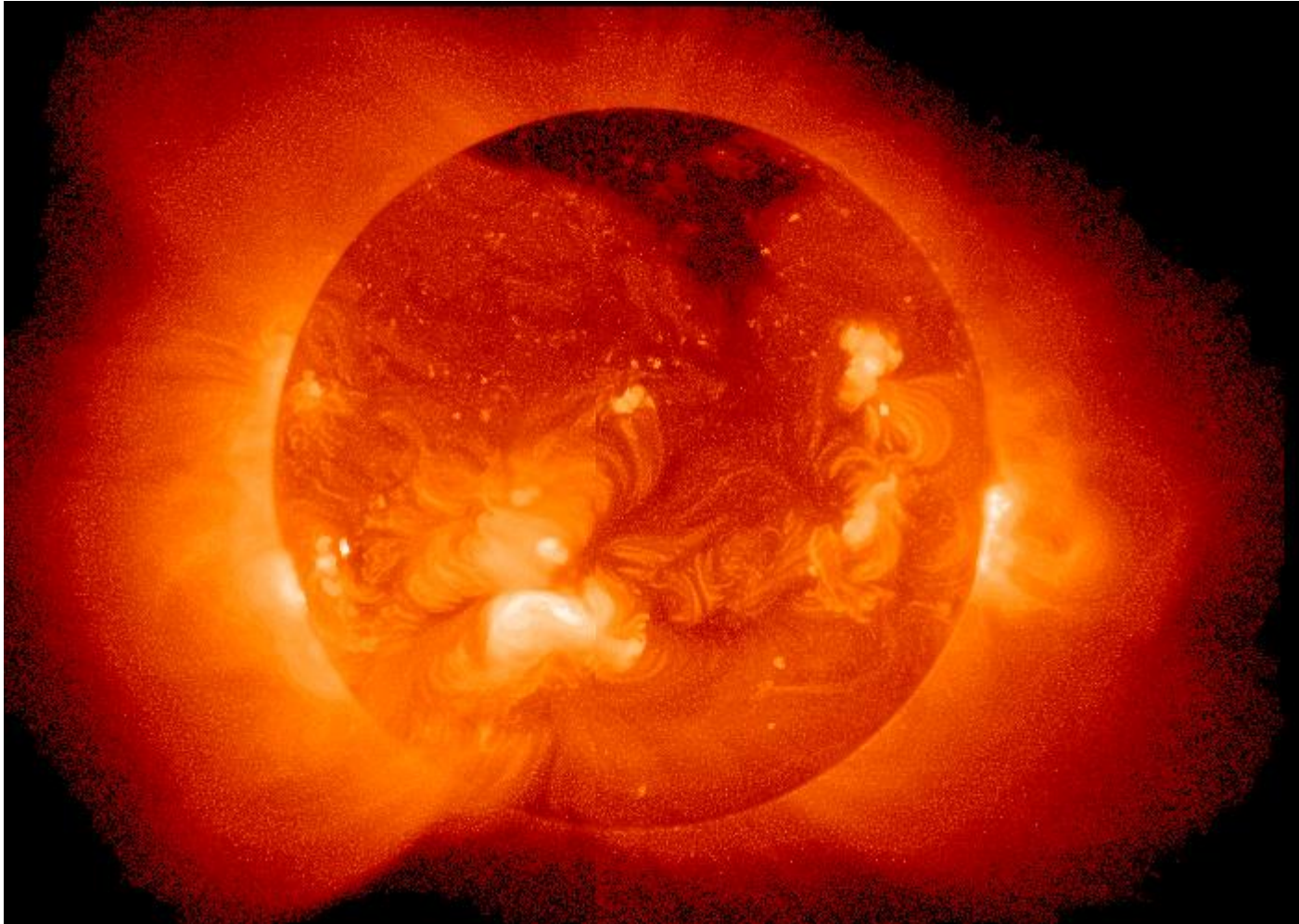
High temperature:
Early Universe evolution

High baryon density:
Inner structure of
compact stars



1. *It is the primordial form of QCD matter at high temperature or baryon density.*
2. *It was present during the first few microseconds of the Big Bang.*
3. *It provides an example of phase transitions which may occur at a variety of higher temperature scales in the early universe.*
4. *It can provide important insights on the origin of mass for matter, and how quarks are confined into hadrons.*

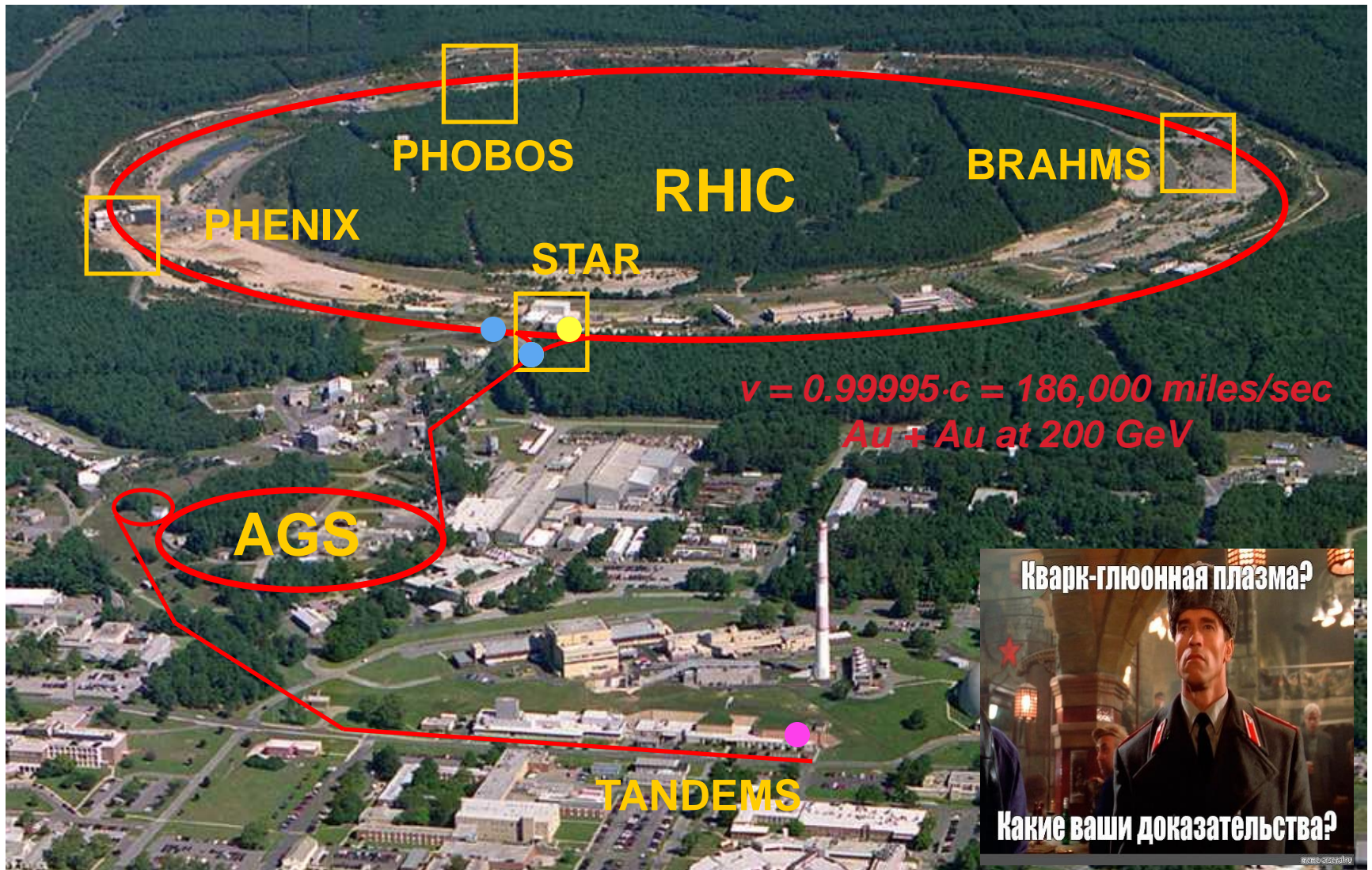
Temperature at the centre of the Sun ~ 15 000 000 K



A medium of 150-170 MeV is more than 100 000 times hotter !!!

2005: Quark-Gluon Plasma is a “perfect liquid”

Relativistic Heavy-Ion Collider (BNL), Upton, NY (USA)



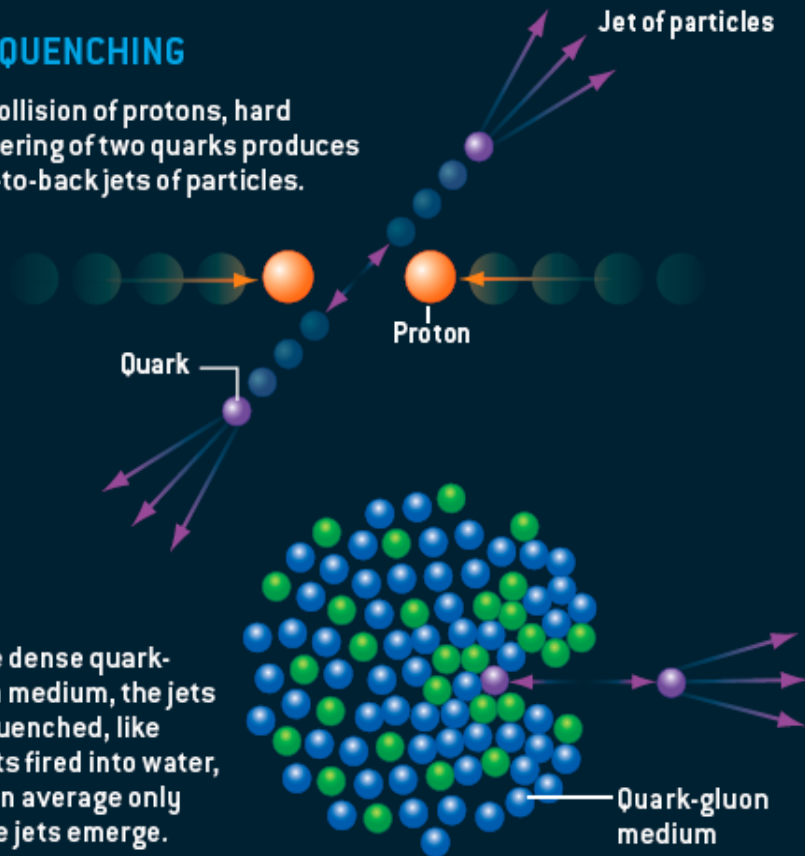
The sQGP Discovered at RHIC: 2005

EVIDENCE FOR A DENSE LIQUID

Two phenomena in particular point to the quark-gluon medium being a dense liquid state of matter: jet quenching and elliptic flow. Jet quenching implies the quarks and gluons are closely packed, and elliptic flow would not occur if the medium were a gas.

JET QUENCHING

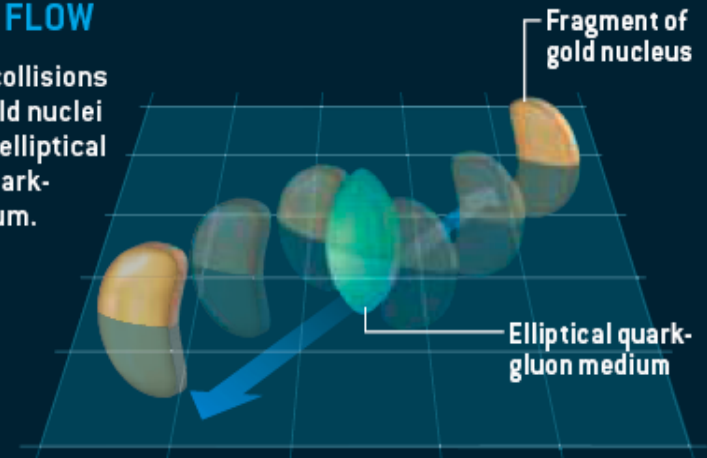
In a collision of protons, hard scattering of two quarks produces back-to-back jets of particles.



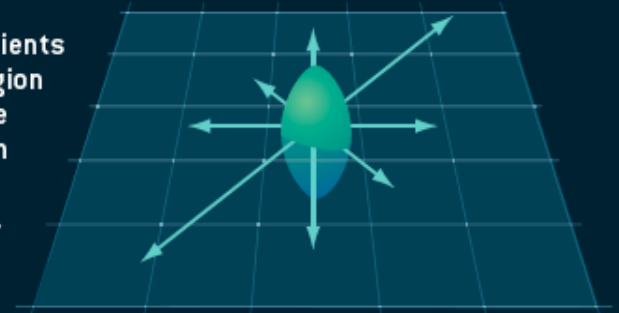
In the dense quark-gluon medium, the jets are quenched, like bullets fired into water, and on average only single jets emerge.

ELLIPTIC FLOW

Off-center collisions between gold nuclei produce an elliptical region of quark-gluon medium.

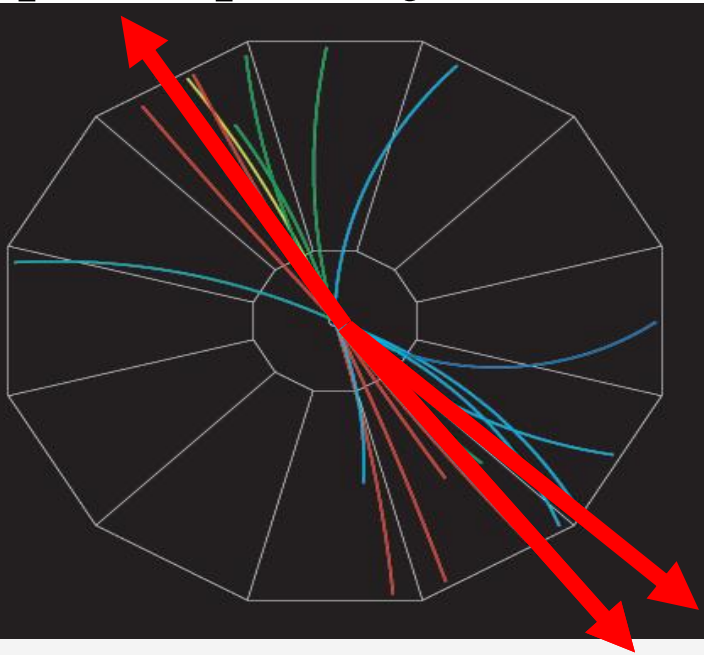


The pressure gradients in the elliptical region cause it to explode outward, mostly in the plane of the collision (arrows).

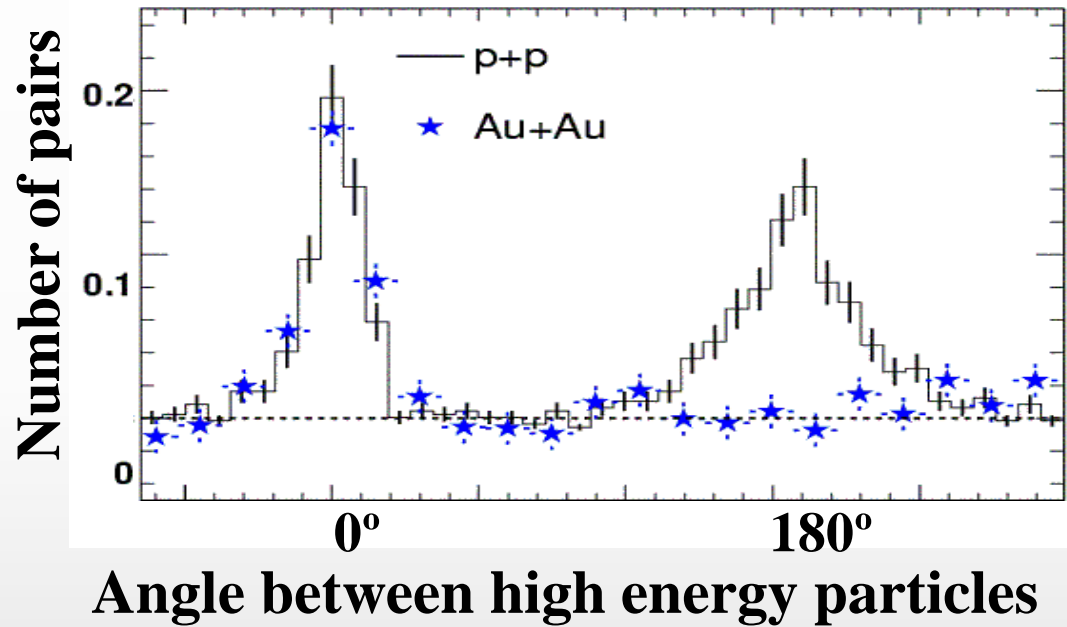


RHIC Experiment: “Jet quenching”

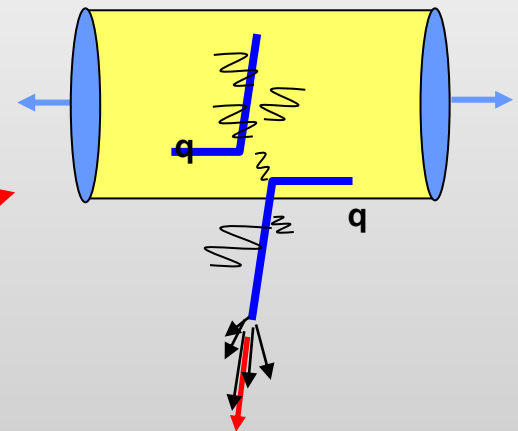
proton-proton jet event



Analyze by measuring (azimuthal) angle between pairs of particles



- In Au-Au collisions we see **only one “jet” at a time !**
- How can this happen ?
- Jet quenching!



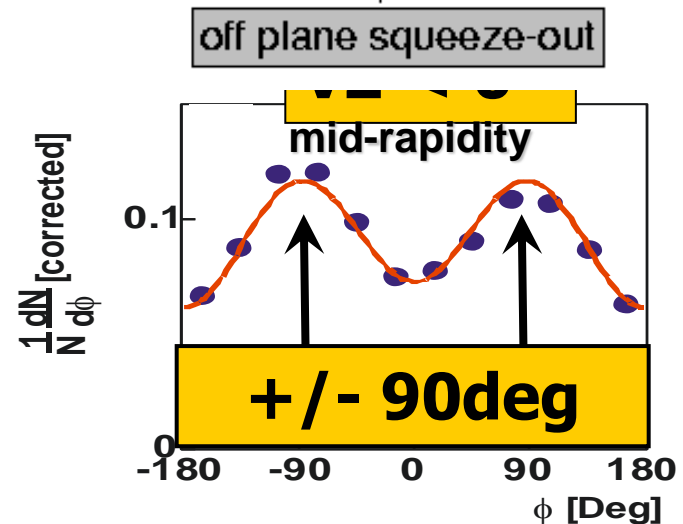
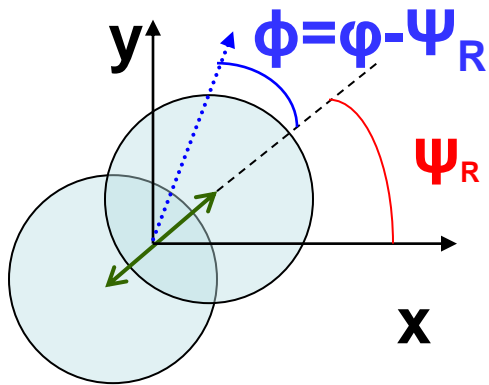
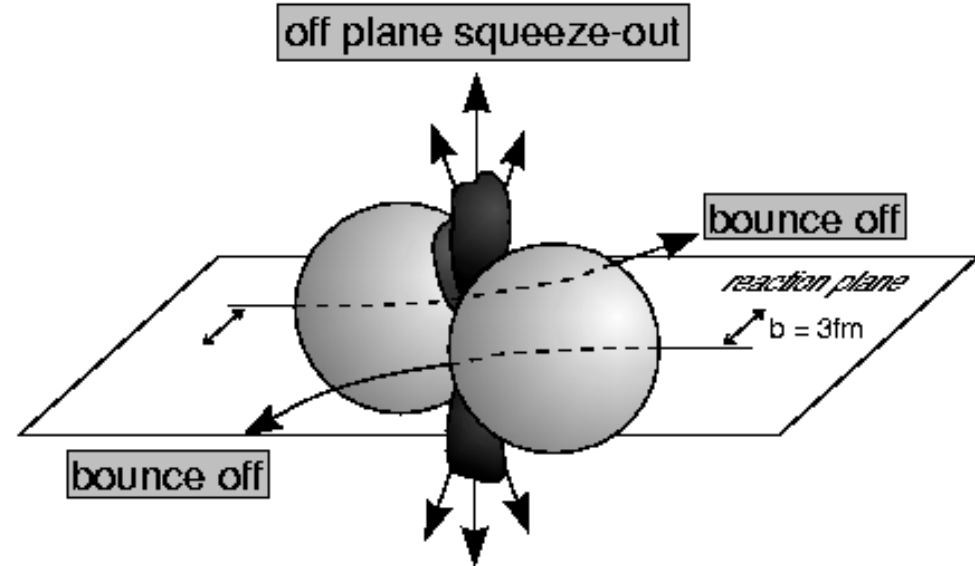
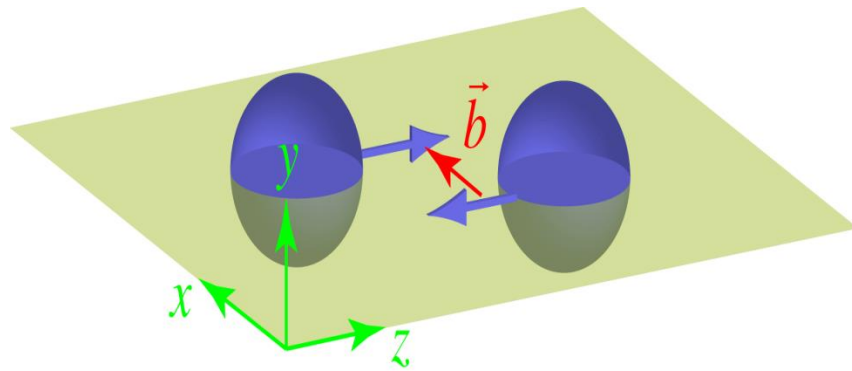
“Squeeze-Out” - First Elliptic Flow Signal in HIC

Diogene, M. Demoulin et al., Phys. Lett. B241, 476 (1990)

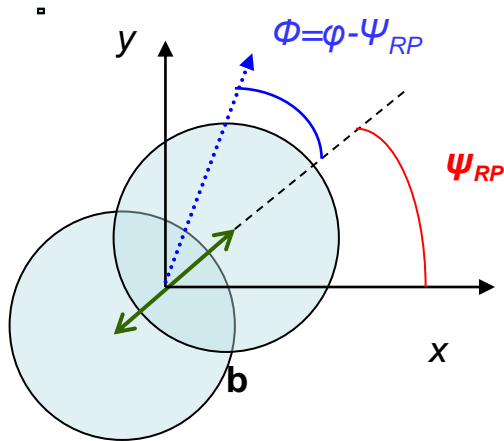
Plastic Ball, H.H. Gutbrod et al., Phys. Lett. B216, 267 (1989)

1989

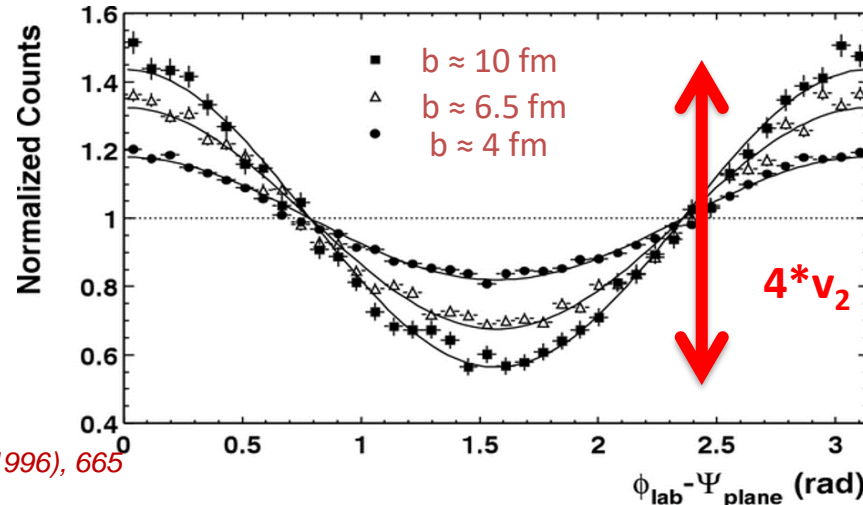
Reaction plane



Azimuthal anisotropy of particles at HIC



STAR, PRL90 032301 (2003)



Sergei Voloshin, Y. Zhang, Z. Phys. C70,(1996), 665

$$\frac{dN}{d(\varphi - \Psi_{RP})} = \frac{N_0}{2\pi} (1 + 2v_1 \cos(\varphi - \Psi_{RP}) + 2v_2 \cos(2(\varphi - \Psi_{RP})) + \dots)$$

- The sinus terms are skipped by symmetry arguments
- From the properties of Fourier's series one has

$$v_n = \langle \cos[n(\varphi - \Psi_{RP})] \rangle$$

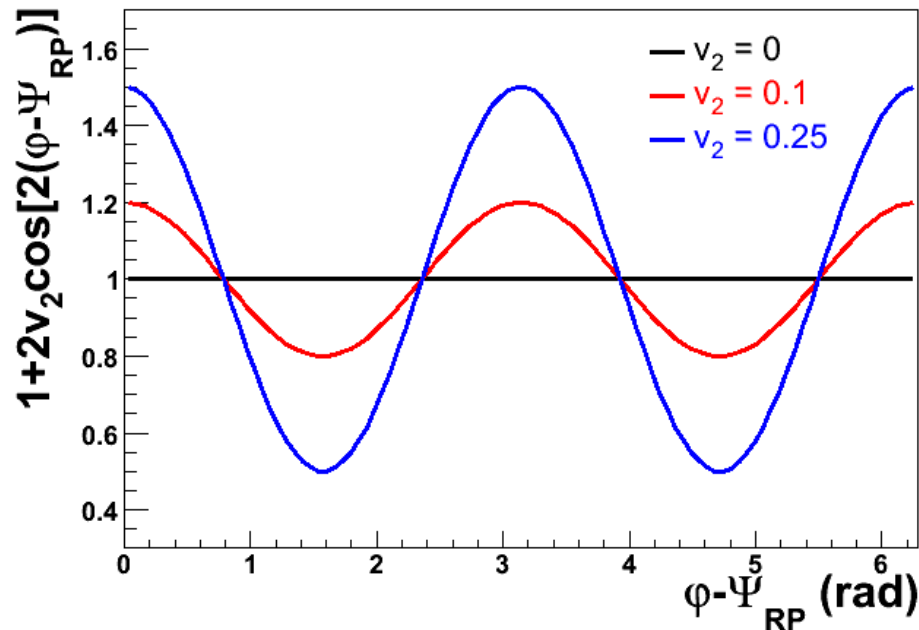
- Fourier coefficients v_n quantify anisotropic flow:
 - v_1 is **directed flow**, v_2 is **elliptic flow**, v_3 is **triangular flow**, etc.

Term “flow” does not mean necessarily “hydro” flow – used only to emphasize the collective behavior of particles in event or multiparticle azimuthal correlation

Elliptic Flow V_2

$$\frac{dN}{d(\varphi - \Psi_{RP})} = \frac{N_0}{2\pi} (1 + 2v_1 \cos(\varphi - \Psi_{RP}) + 2v_2 \cos(2(\varphi - \Psi_{RP})) + \dots)$$

$$v_2 = \langle \cos[2(\varphi - \Psi_{RP})] \rangle$$

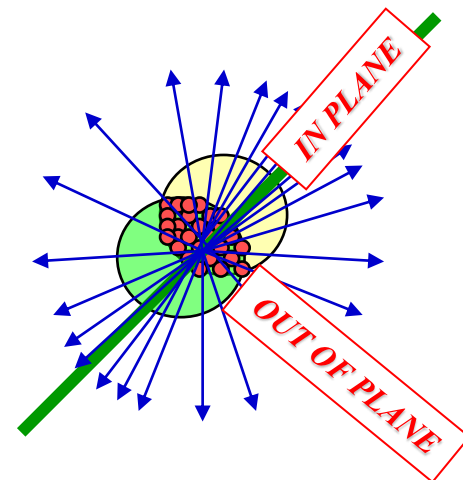


$v_2 > 0 \rightarrow$ in-plane flow,
 $v_2 < 0$ out-of-plane flow

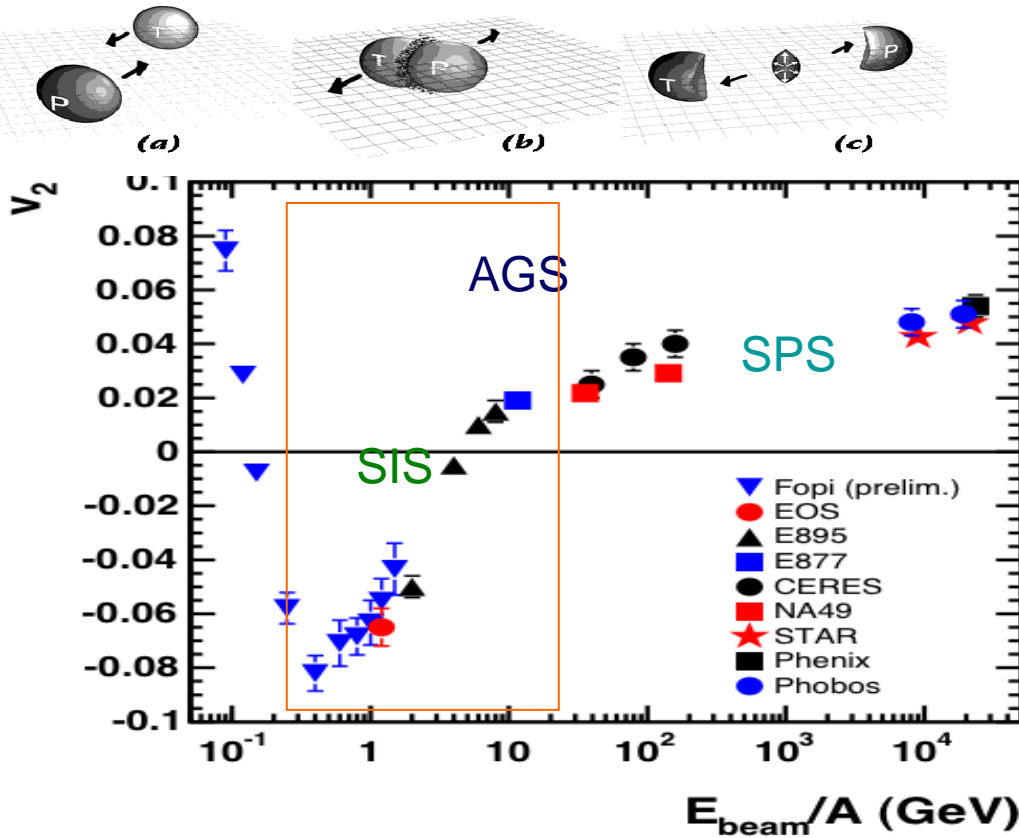
□ $v_2 \neq 0$ - difference between the
 (||) [$N(0^\circ) + N(180^\circ)$] and
 (\perp) [$N(90^\circ) + N(270^\circ)$] to
 the impact parameter (**b**)

□ $V_2 = 0.25 \rightarrow R_{in/out} = 3$ (3 times more
 particles emitted in-plane than
 out-of- the plane)

$$R_{in/out} = \frac{N(0^\circ) + N(180^\circ)}{N(90^\circ) + N(270^\circ)} \sim \frac{1 + 2V_2}{1 - 2V_2}$$



Excitation function of elliptic flow – 0.4-10 GeV (SIS/AGS) energies



Passage time:

$$2R/(\beta_{\text{cm}}\gamma_{\text{cm}})$$

Expansion time: R/c_s

$c_s = c\sqrt{dp/d\varepsilon}$ - speed of sound

(time for the development of expansion perpendicular to the reaction plane)

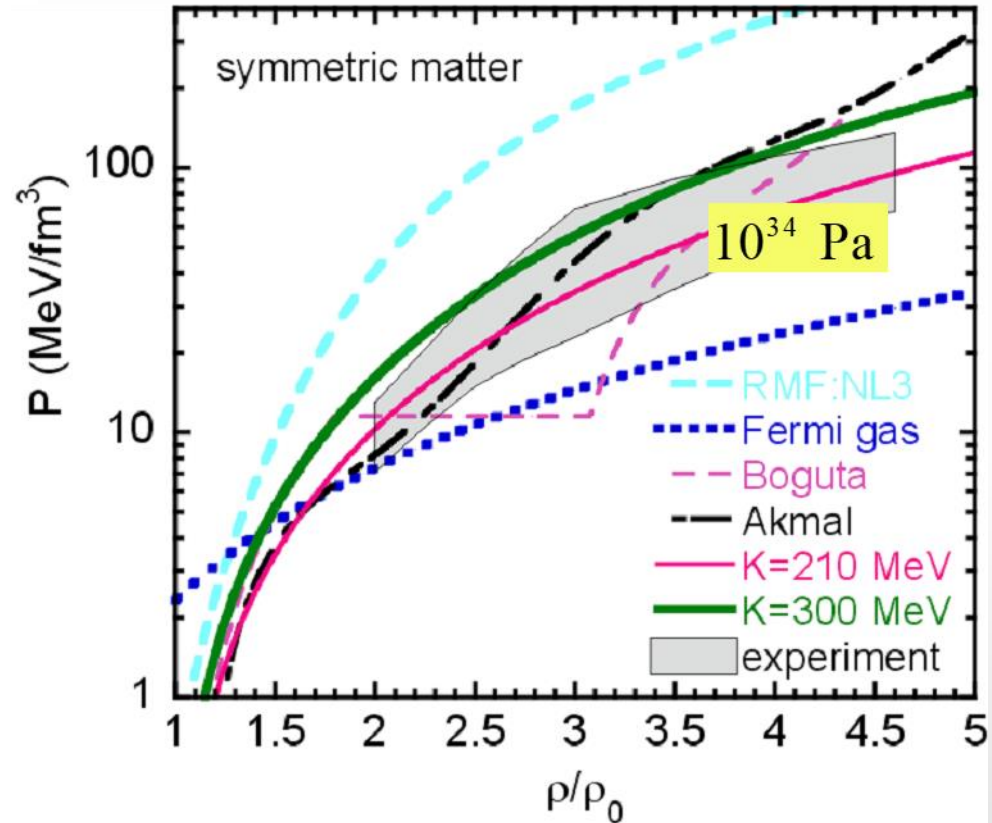
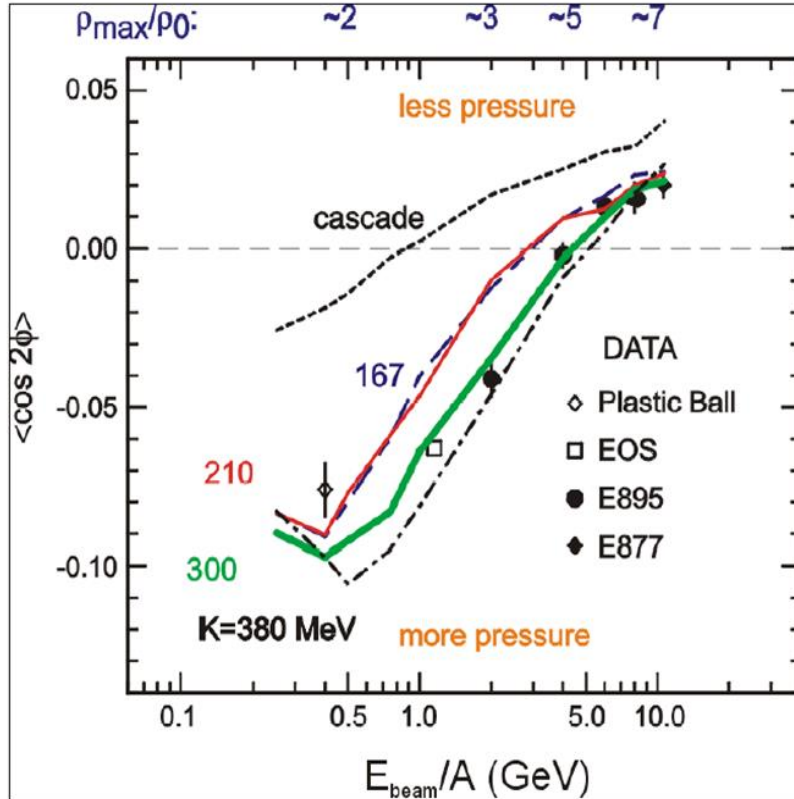
Delicate balance between:

1) **Ability of pressure** developed early in the reaction zone to effect a rapid transverse expansion of nuclear matter

2) **Passage time** for removal of the **shadowing** of participant hadrons by projectile and target spectators

Flow at AGS: Constraints for the Hadronic EOS

Danielewicz, Lacey, Lynch, Science 298 (2002) 1592-1596



Passage time: $2R/(\beta_{\text{cm}}\gamma_{\text{cm}})$

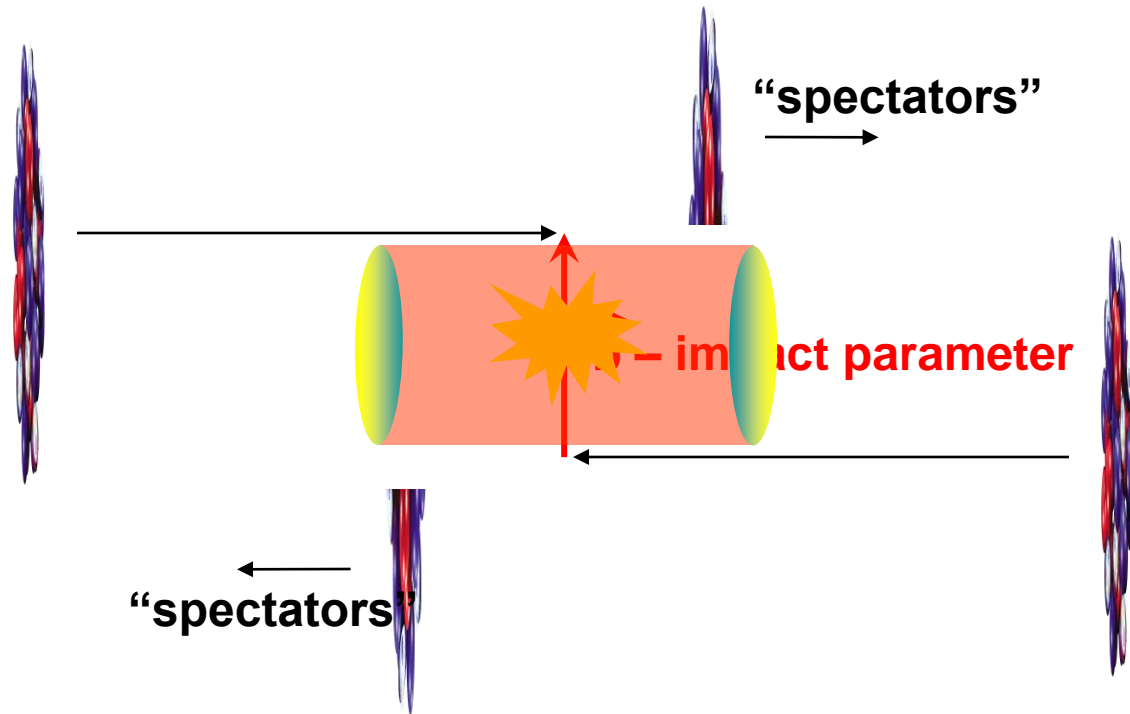
Expansion time: R/c_s

$c_s = c\sqrt{dp/d\varepsilon}$ - speed of sound

$$c_s = \sqrt{\frac{K}{9m_N}} \approx 0.15c, 0.21c$$

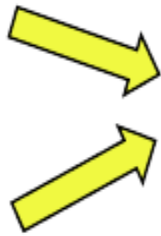
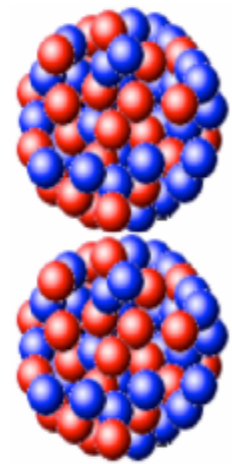
Flow at AGS/Nuclotron = Interplay of passage/expansion times

Elliptic flow at RHIC/LHC



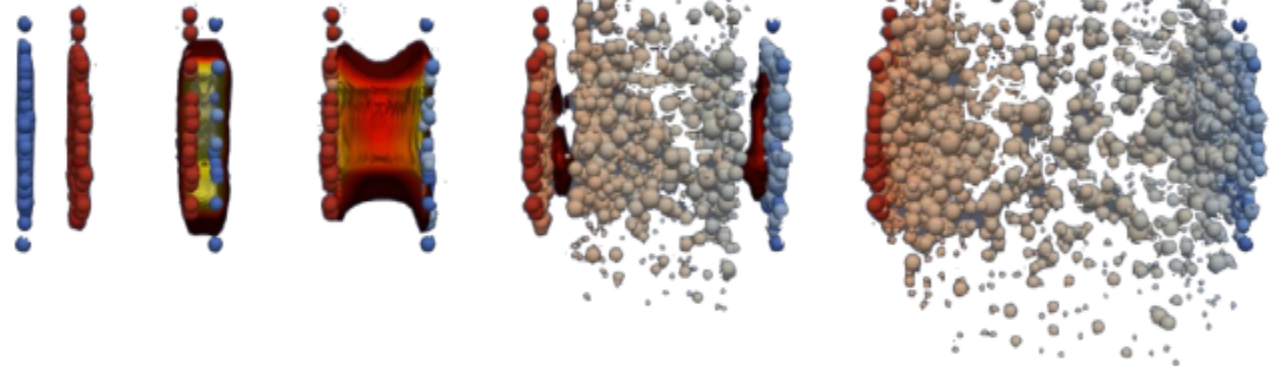
Passage time: < 0.15 fm/c

**Longitudinal and transverse expansion =>
no influence of spectator matter at midrapidity**



$$\tau < \frac{R}{\gamma} \leq 0.1 \text{ fm}/c$$

$$\frac{\sqrt{s_{\text{NN}}}}{2} = 100 - 2500 \text{ GeV}$$



initial stages

QGP medium expansion

freeze-out

0

$\tau_{\text{hydro}} \sim 1 \text{ fm}/c$

$\tau_{\text{fo}} \sim 10 \text{ fm}/c$

400 nucleons



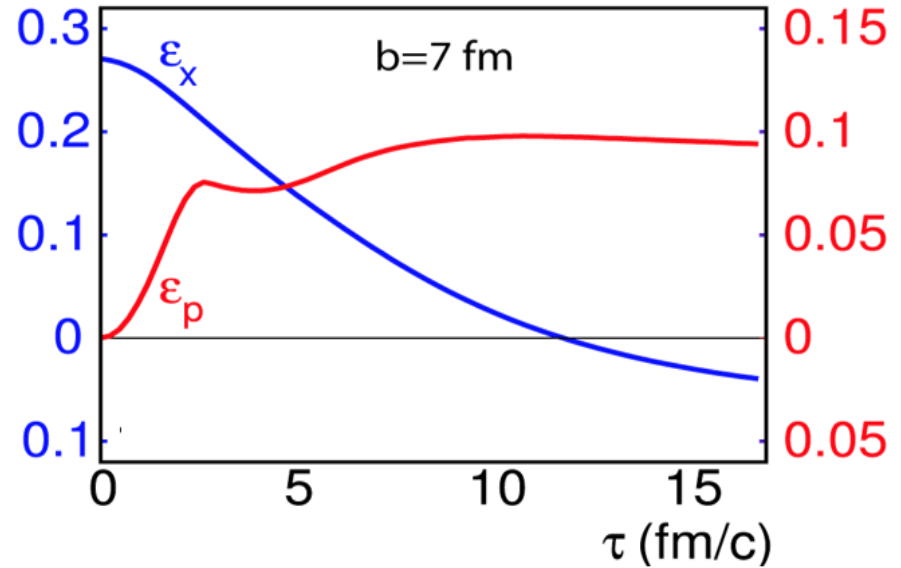
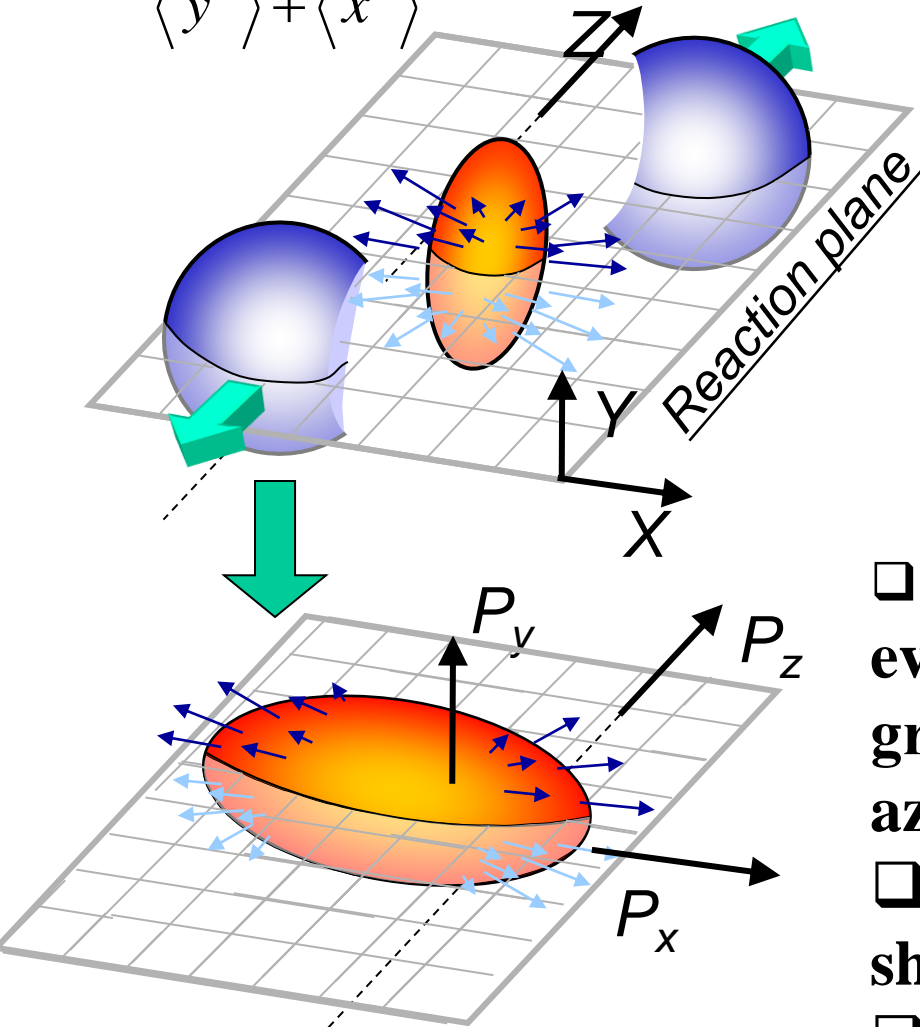
30000 hadrons

in 10^{-23} seconds

Key features facilitating the connection to nuclear structure 1) **Extremely short passing time** means that collision takes a snap-shot of the nuclear and nucleon wavefunction in the two nuclei. 2) **Large particle production** in overlap region means the produced QGP expands hydrodynamically in each event

Elliptic Flow at RHIC

$$\varepsilon = \frac{\langle y^2 \rangle - \langle x^2 \rangle}{\langle y^2 \rangle + \langle x^2 \rangle}$$



- The initial spatial anisotropy ε_x evolves (via interactions and density gradients) → Momentum-space azimuthal anisotropy $\varepsilon_p = v_2$
- Pressure gradient is largest in the shortest direction of the ellipsoid.
- Signal is self-quenching with time

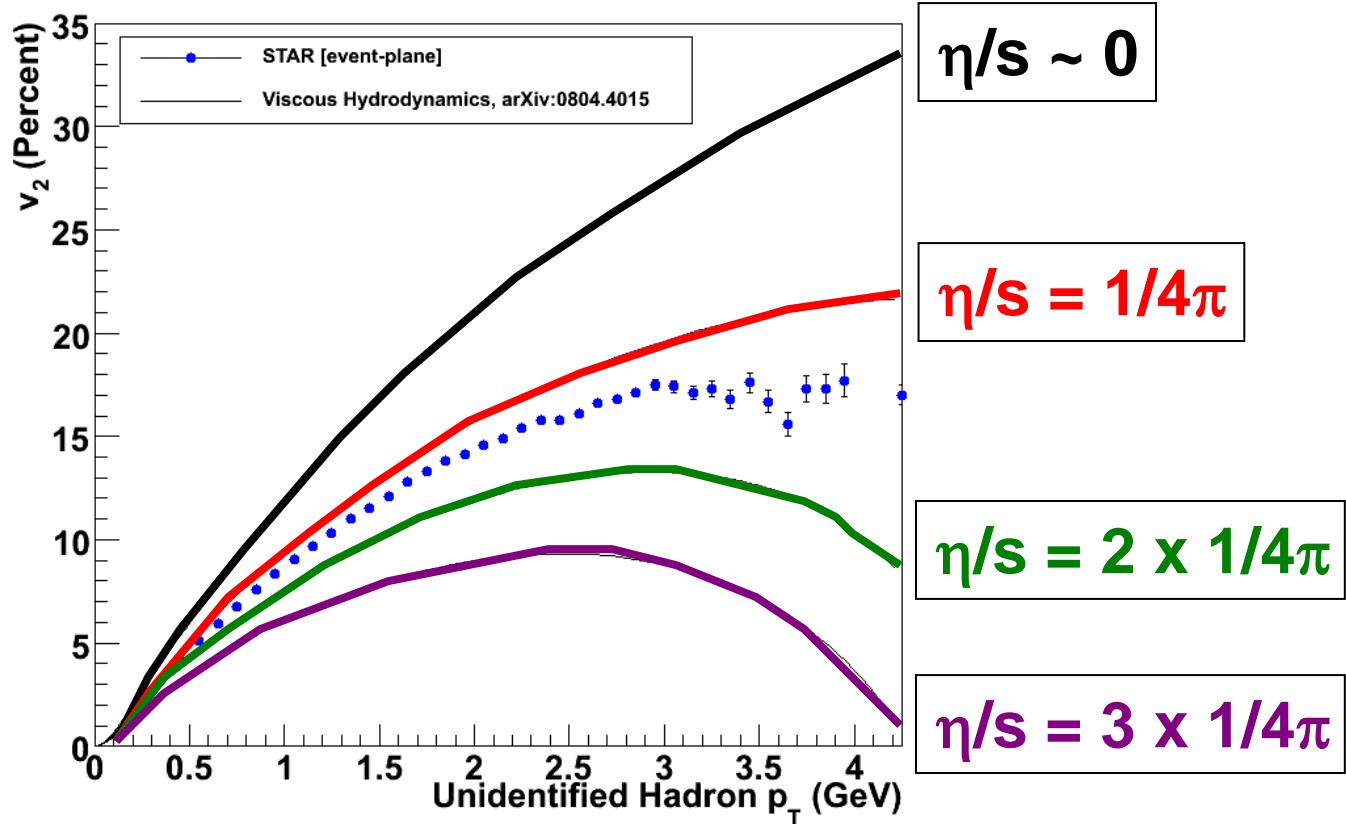
$$v_2 = \frac{\langle p_x^2 \rangle - \langle p_y^2 \rangle}{\langle p_x^2 \rangle + \langle p_y^2 \rangle}$$

$$v_2 = \langle \cos(2[\varphi - \Psi_R]) \rangle$$

Relativistic viscous hydrodynamics compared to data

Luzum, Romatschke, Phys. Rev. C78, 034915 (2008)

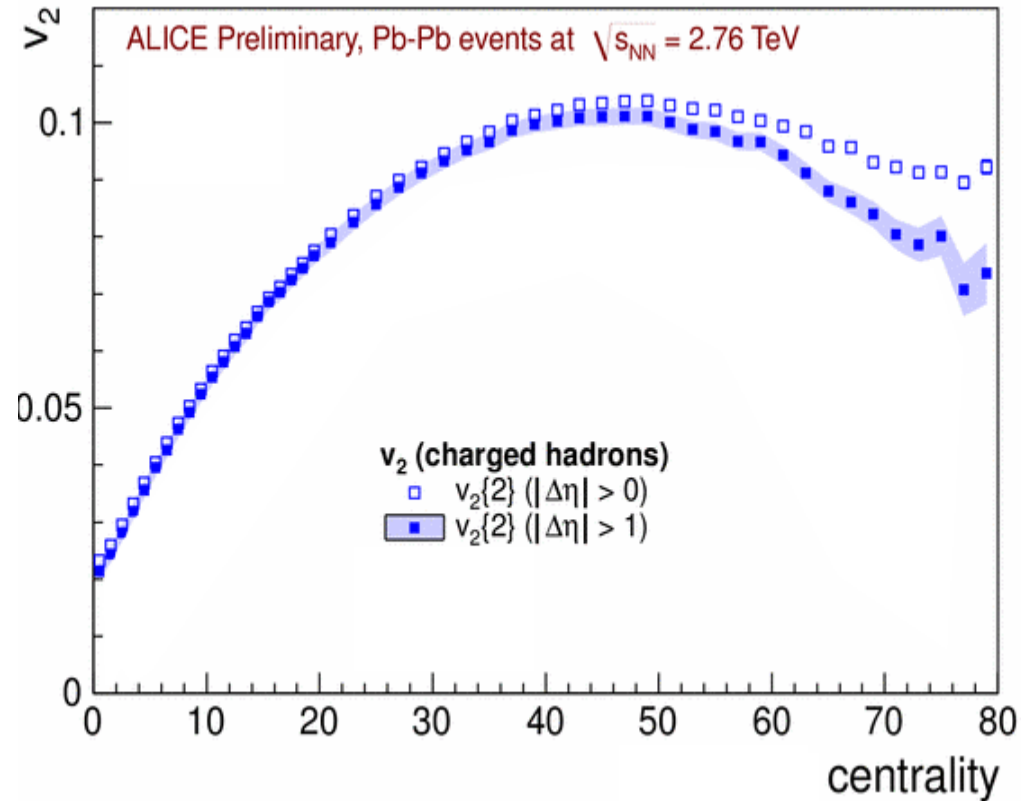
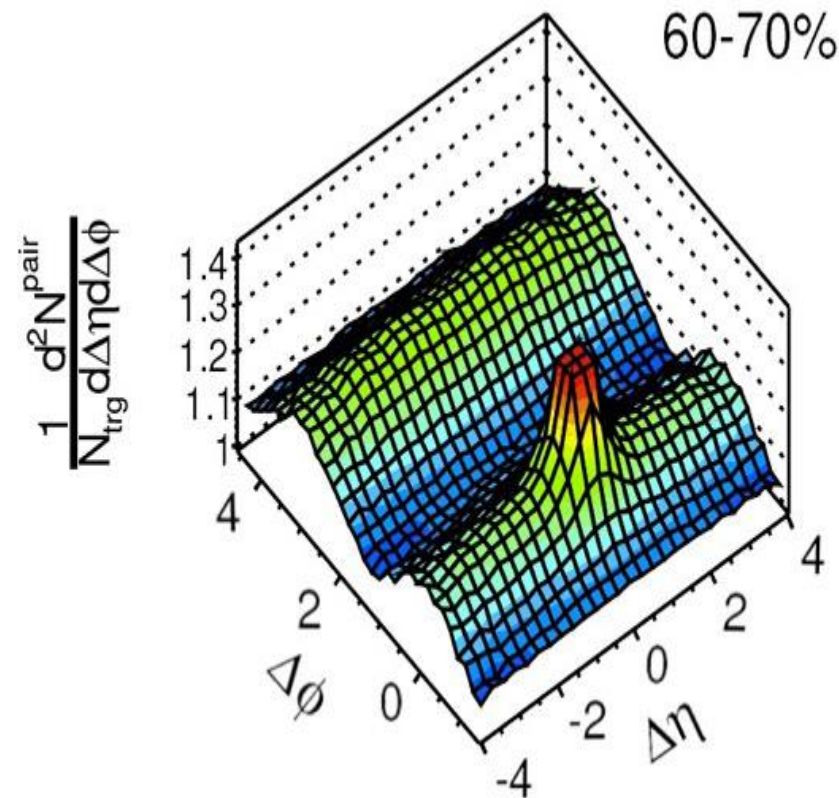
Lower Viscosity
↓
Less Dissipation
↓
More Flow
↓
Larger v_2



$$\left(\frac{\eta}{s}\right) / \left(\frac{1}{4\pi}\right) = 1.3 \pm 1.3 \text{ (theory)} \pm 1.0 \text{ (experiment)}$$

Two-particle correlations

$$v_2\{2\} > v_2\{2, |\Delta\eta|\}$$



$$\begin{aligned} \langle\langle e^{in(\phi_1 - \phi_2)} \rangle\rangle &= \langle\langle e^{in(\phi_1 - \Psi_{RP} - (\phi_2 - \Psi_{RP}))} \rangle\rangle \\ &= \langle\langle e^{in(\phi_1 - \Psi_{RP})} \rangle \langle e^{-in(\phi_2 - \Psi_{RP})} \rangle \rangle = \langle v_n^2 \rangle \end{aligned}$$

Requires the suppression of non-flow contributions
(HBT, Jets & di-jets, Res. Decay, Mom. Consrv.)

Non-flow correlations

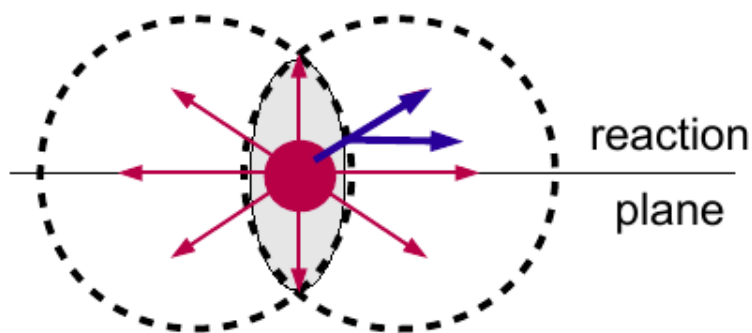
Non-flow: correlations among the particles unrelated to the reaction plane

In case of two particle correlations: $\langle \cos[n(\phi_i - \phi_j)] \rangle = \langle v_n^2 \rangle + \delta_{2,n}$

Sources of non-flow correlations:

- Resonance decay
- Jet production
- In general - any cluster production

Example: 2-particle decay



Probability to be correlated for one particle with another out of M -particles is $1/(M-1)$:

$$\delta_2 \sim \frac{1}{M-1}$$

To measure flow with 2-particle correlations:

$$v_n \gg 1/\sqrt{M}$$

Collective flow:
correlations between particles through
the common plane of symmetry

$$M = 200 \rightarrow v_n \gg 0.07$$

For RHIC/LHC: $v_n \approx 0.04 - 0.07$

Suppressing non-flow with multi-particle correlations

Two particle correlations:

$$\delta_2 \sim \frac{1}{M}$$

Measurement requirement:

$$v_n \gg \frac{1}{M^{1/2}}$$

$$M = 200 \rightarrow v_n \gg 0.07$$

Four-particle correlations:

$$\delta_4 \sim \frac{1}{M^3}$$

$$v_n \gg \frac{1}{M^{3/4}}$$

$$M = 200 \rightarrow v_n \gg 0.019$$

k -particle correlations:

$$\delta_k \sim \frac{1}{M^{k-1}}$$

$$v_n \gg \frac{1}{M^{(k-1)/k}}$$

Large k ($k \rightarrow \infty$)

$$v_n \gg \frac{1}{M}$$

$$M = 200 \rightarrow v_n \gg 0.005$$

Elliptic flow from cumulants

How fluctuations affect the measured values of v_n : $\sigma_{v_n}^2 = \langle v_n^2 \rangle - \langle v_n \rangle^2$ - magnitude of flow fluctuations
 The effect of the fluctuations on v_n estimates can be obtained from

$$\begin{aligned}\langle v_n^2 \rangle &= \bar{v}_n^2 + \sigma_{v_n}^2, \\ \langle v_n^4 \rangle &= \bar{v}_n^4 + 6\sigma_{v_n}^2 \bar{v}_n^2, \\ \langle v_n^6 \rangle &= \bar{v}_n^6 + 15\sigma_{v_n}^2 \bar{v}_n^4,\end{aligned}\quad (1)$$

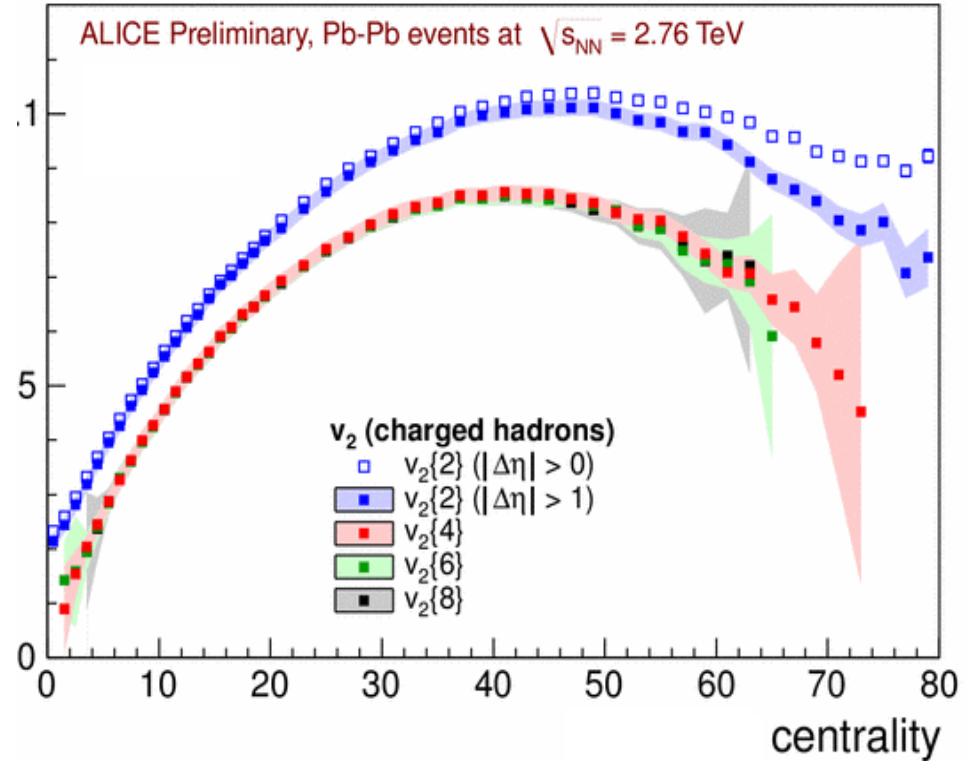
$$\begin{aligned}v_n\{2\} &= \sqrt{\langle v_n^2 \rangle}, \\ v_n\{4\} &= \sqrt[4]{2\langle v_n^2 \rangle^2 - \langle v_n^4 \rangle}, \\ v_n\{6\} &= \sqrt[6]{\frac{1}{4}(\langle v_n^6 \rangle - 9\langle v_n^2 \rangle \langle v_n^4 \rangle + 12\langle v_n^2 \rangle^3)}.\end{aligned}\quad (2)$$

Here we have introduced the notation $v_n\{k\}$ as the flow estimate from the cumulant $c_n\{k\}$. In case that $\sigma_{v_n} \ll \bar{v}_n$ we obtain, up to order $\sigma_{v_n}^2$:

$$\begin{aligned}v_n\{2\} &= \bar{v}_n + \frac{1}{2} \frac{\sigma_{v_n}^2}{\bar{v}_n}, \\ v_n\{4\} &= \bar{v}_n - \frac{1}{2} \frac{\sigma_{v_n}^2}{\bar{v}_n}, \\ v_n\{6\} &= \bar{v}_n - \frac{1}{2} \frac{\sigma_{v_n}^2}{\bar{v}_n}.\end{aligned}\quad (3)$$

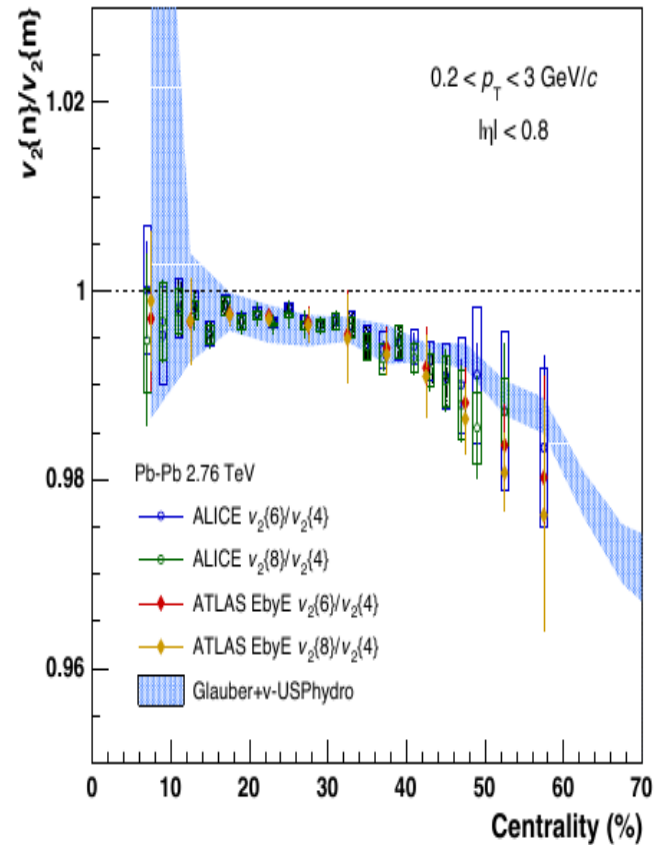
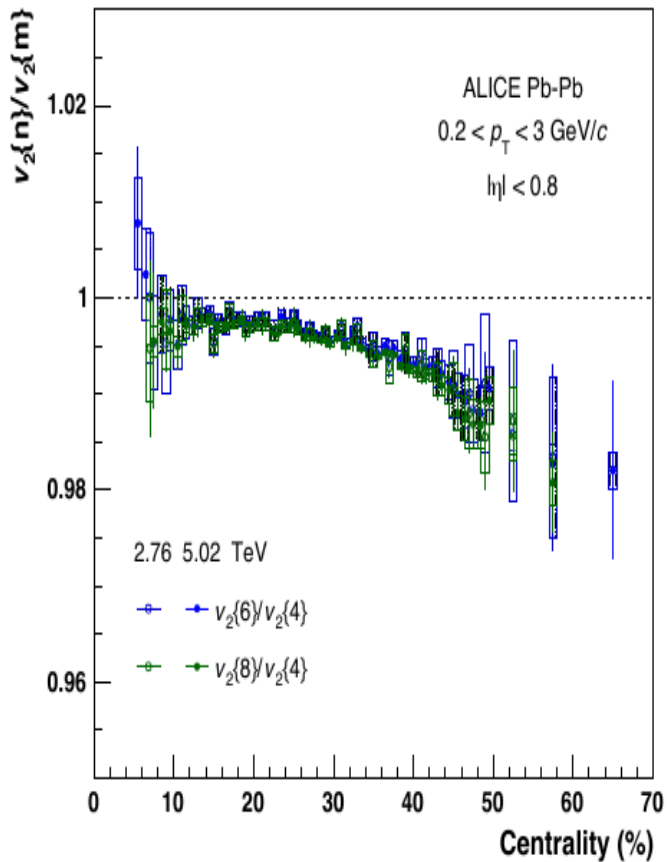
The difference between $v_n\{2\}$ and $v_n\{4\}$ is sensitive to not only nonflow but also to the event-by-event v_n fluctuations.

$$v_2\{4\} \approx v_2\{6\} \approx v_2\{8\}$$



The difference between $v_2\{2\}$ with and without η gap is driven by the contribution from nonflow

The difference between 2- and multi-particle estimates is due to fluctuations in the initial geometry

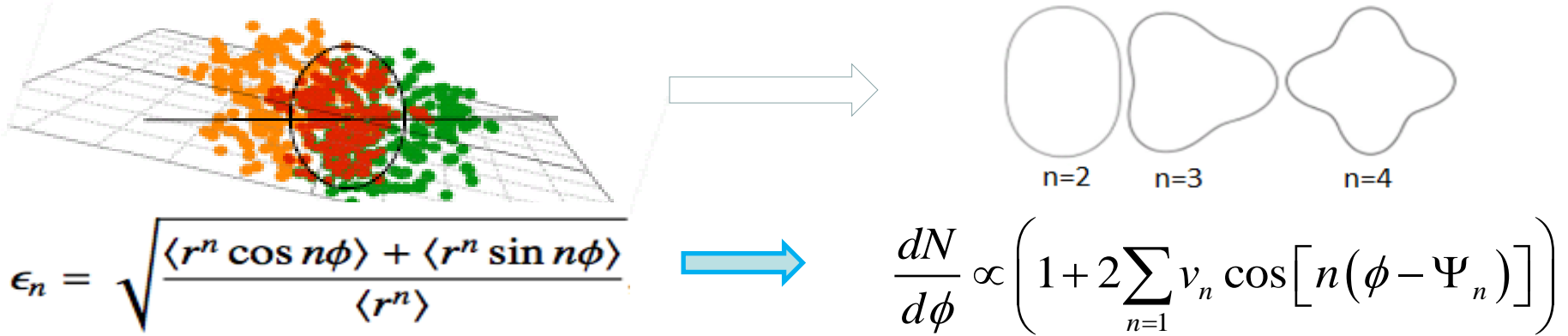


A fine splitting is observed which is centrality dependent showing the non Bessel Gaussian contribution

The splitting does not depend on the p_t range used and collision energy

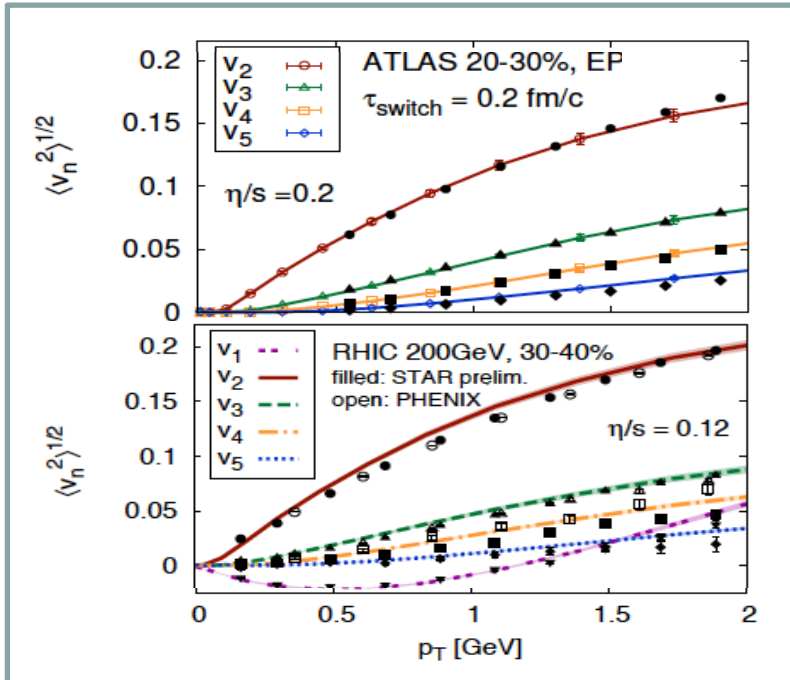
The results agree well with model calculations as well as with ATLAS results based on a different technique

Anisotropic Flow at RHIC-LHC

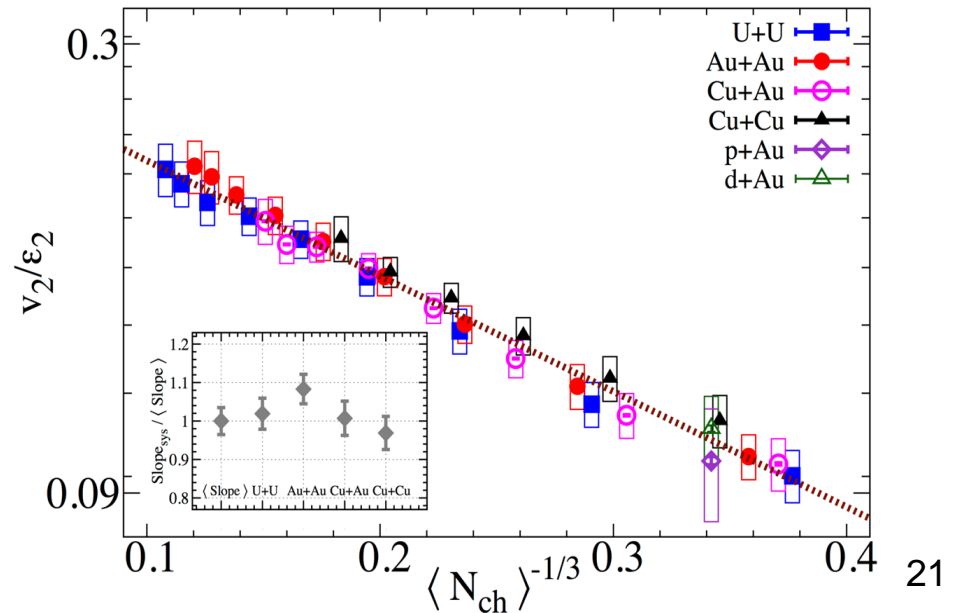


Initial eccentricity (and its attendant fluctuations) ϵ_n drive momentum anisotropy v_n with specific viscous modulation

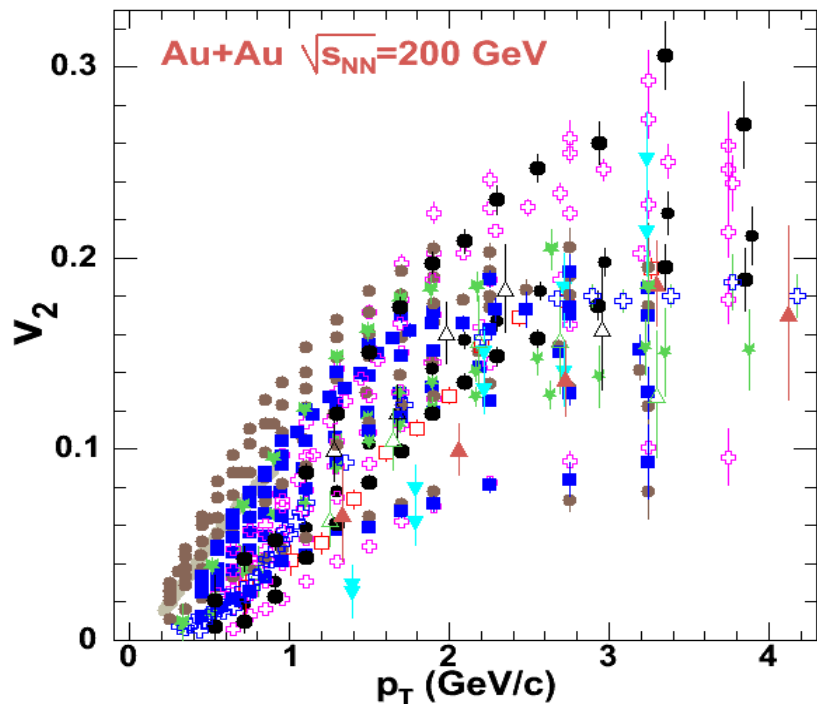
Gale, Jeon, et al., *Phys. Rev. Lett.* 110, 012302



Phys. Rev. Lett. 122 (2019) 172301



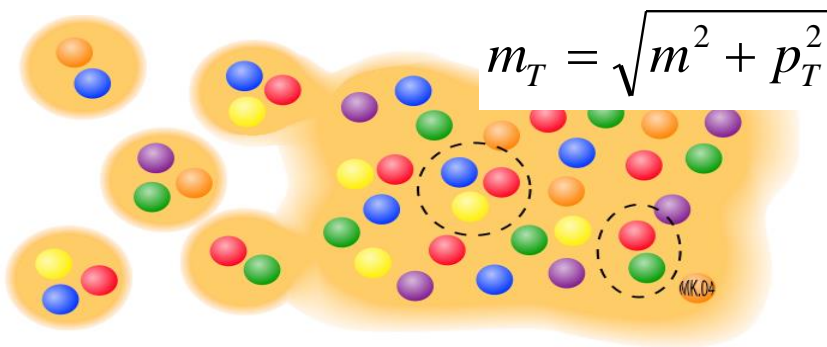
Anisotropic Flow at RHIC – scaling relations



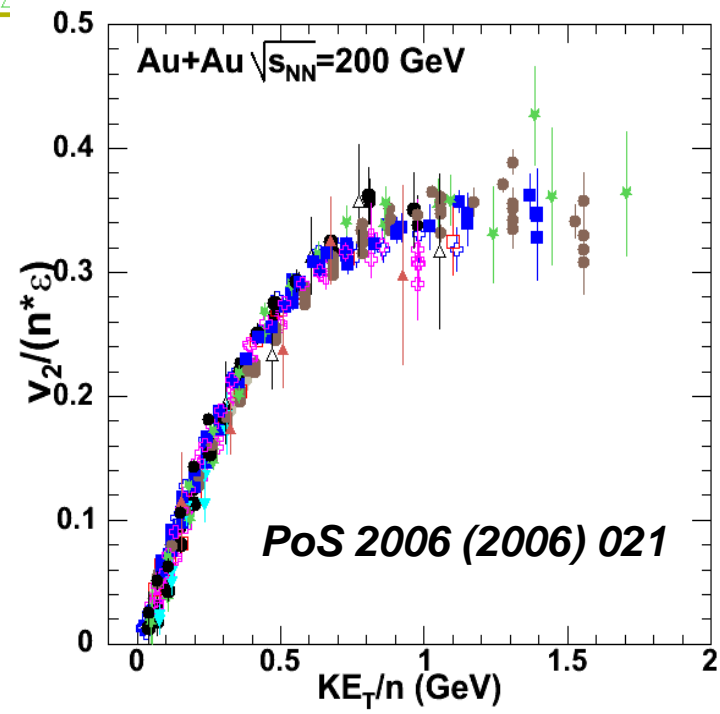
- PHENIX** (Phys.Rev.Lett.91, Preliminary: QM05, GRC 06)
- - $\pi^+ + \pi^-$: min.bias, 0-10%,10-20%,20-30%,30-40%,40-50%,20-60%
 - - $K^+ + K^-$: min.bias, 0-10%,10-20%,20-30%,30-40%,40-50%,20-60%
 - ⊕ - $p + \bar{p}$: min.bias, 0-10%,10-20%,20-30%,30-40%,40-50%,20-60%
 - ▼ - d : min.bias, 10-50%
 - △ - ϕ : 20-60%
- STAR** (Phys. Rev. Lett. 92, Phys. Rev. C 72 (2005), Preliminary QM05, SQM06)
- - $\pi^+ + \pi^-$: min.bias
 - ★ - K_S^0 : min.bias, 5-30%,30-70%
 - ⊕ - $p + \bar{p}$: min.bias
 - - $\Lambda + \bar{\Lambda}$: min.bias, 5-30%,30-70%
 - - $\Xi + \bar{\Xi}$: min.bias
 - ▲ - $\Omega + \bar{\Omega}$: min.bias

$$KE_T = m (\gamma_T - 1)$$

$$= m_T - m$$

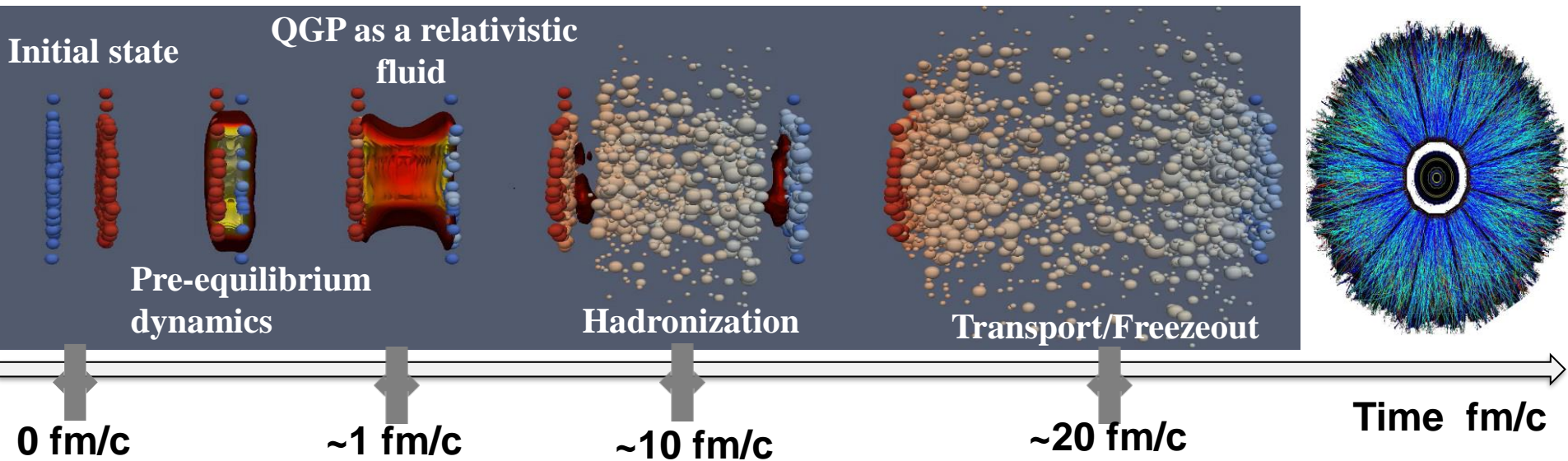


n=2 for mesons and n=3 for baryons



Evolution of the system created in RHIC/LHC

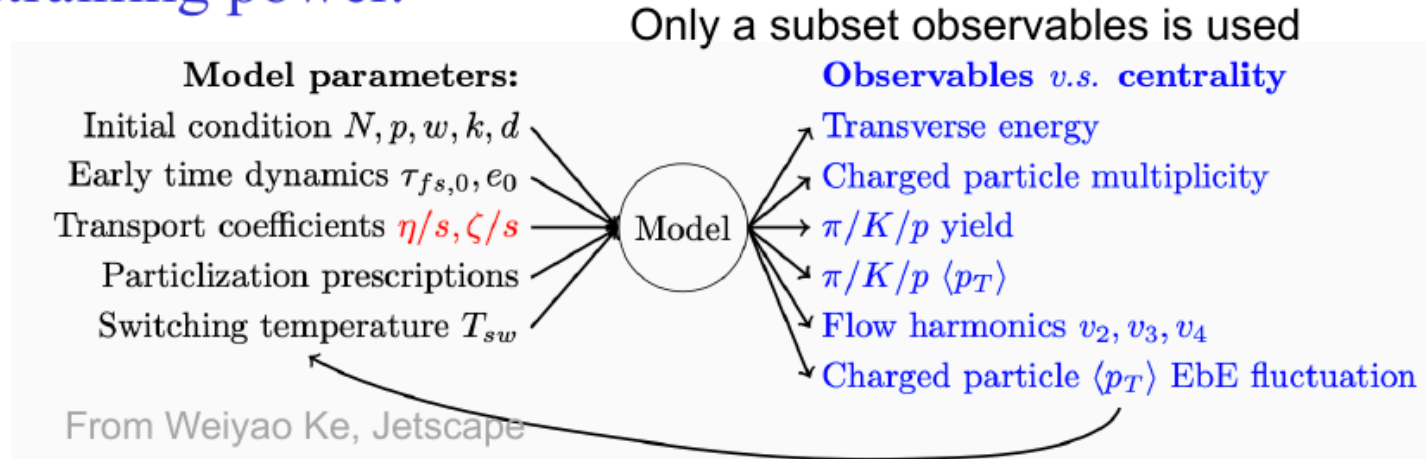
Fireball is $\sim 10^{-15}$ meters across and lives for 5×10^{-23} seconds



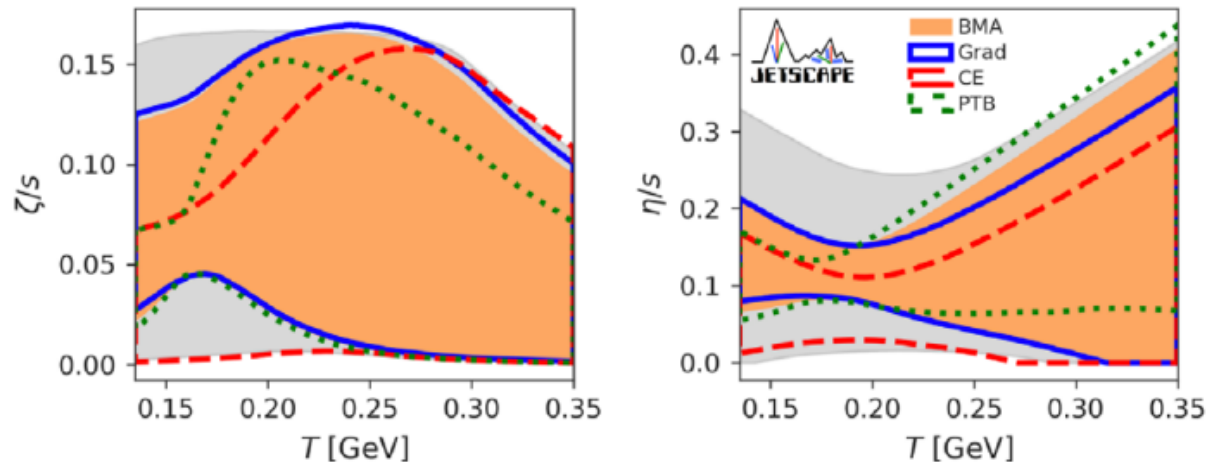
- ❑ **Initial state** (heavy Au+Au or Pb+Pb nuclei Lorentz contracted by $\gamma \sim 100$ – RHIC, $\gamma \sim 1400$ – LHC)
- ❑ **Pre-equilibrium state:** hard parton scattering & jet production
- ❑ **Quark-gluon plasma formation:** thermalization (viscous hydrodynamics)
- ❑ **Hadronization**
- ❑ **Transport/Freeze-out:** Rescattering & chemical freeze-out, Kinetic freeze-out (stop interacting)

State-of-the-art modeling of HI collisions

- Data-model comparison via Bayesian inference to optimize constraining power.

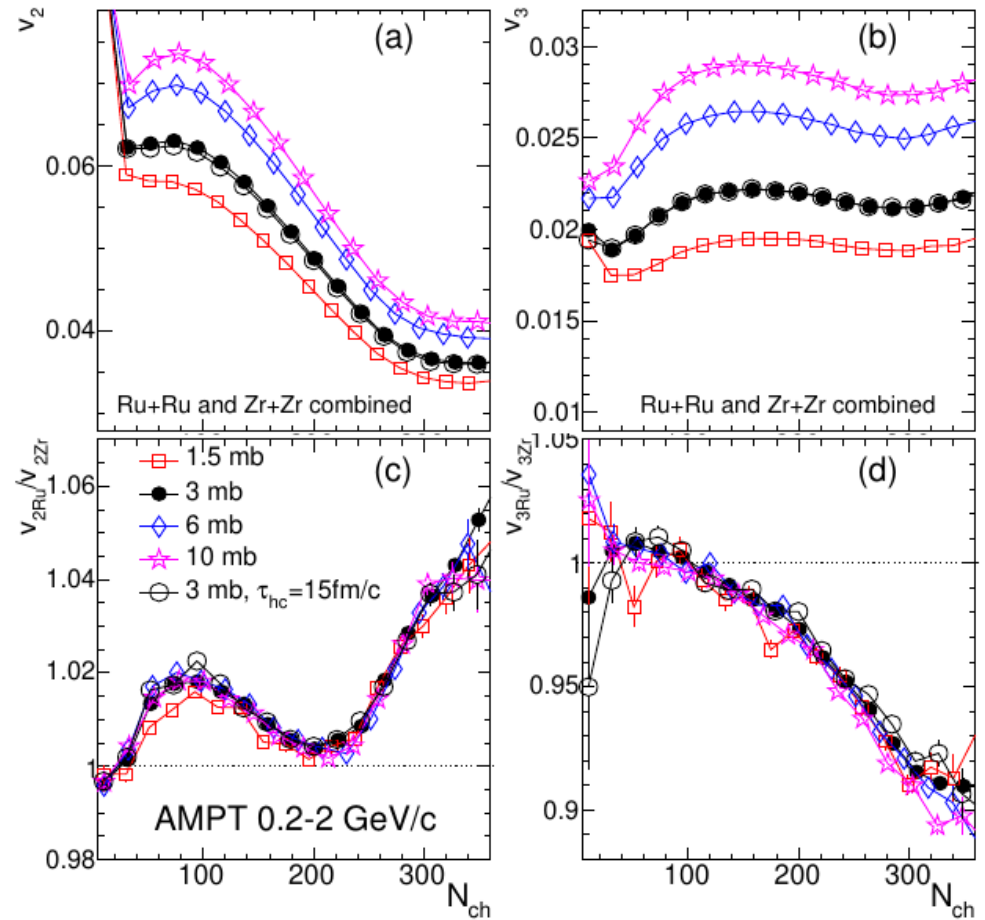
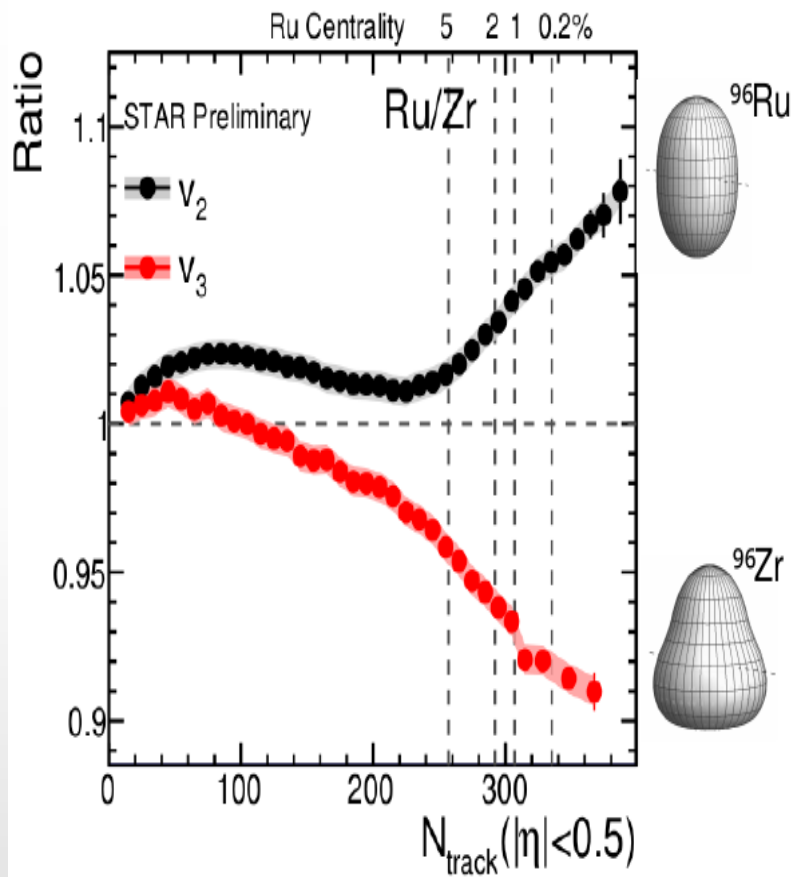


- Detailed temperature dependence of viscosity!



Jetscape PRL.126.242301
Trjactum PRL.126.202301

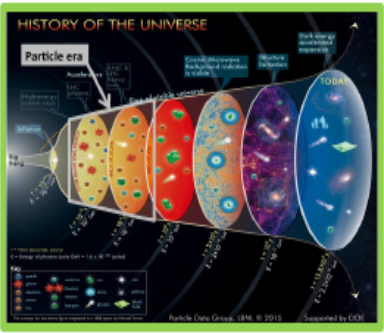
Major uncertainty: initial condition and pre-hydro phase



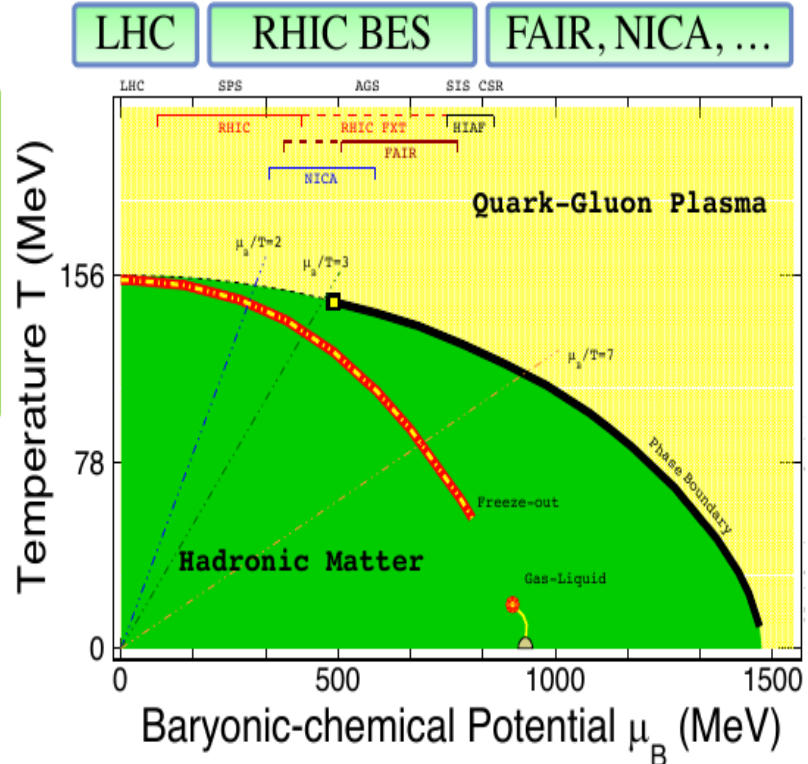
Phys.Rev.C 105 (2022) 1, 014901 • e-Print: 2109.00131

The V_n ratio for isobars – not affected by final state – is a good tool for precision studies of nuclear shapes.

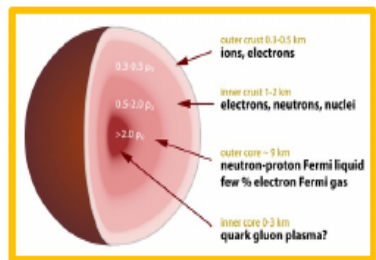
Relativistic Heavy-Ion Collisions and QCD Phase Diagram



High temperature:
Early Universe evolution



High baryon density:
Inner structure of compact stars

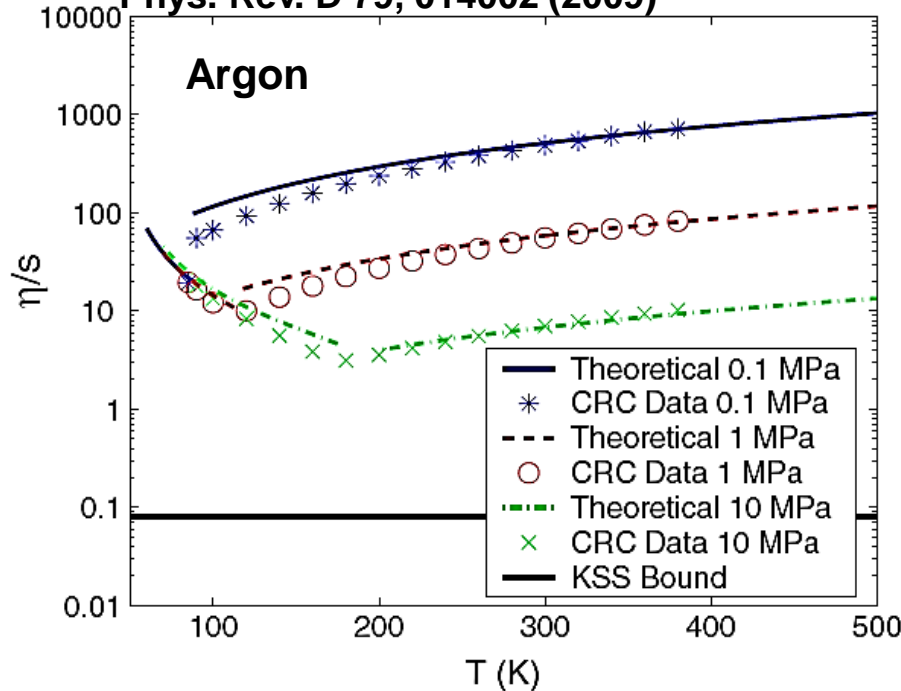


- 1) At $\mu_B = 0$, smooth crossover (LGT + data) ;
- 2) Large μ_B , 1st order phase transition → **QCD critical point**

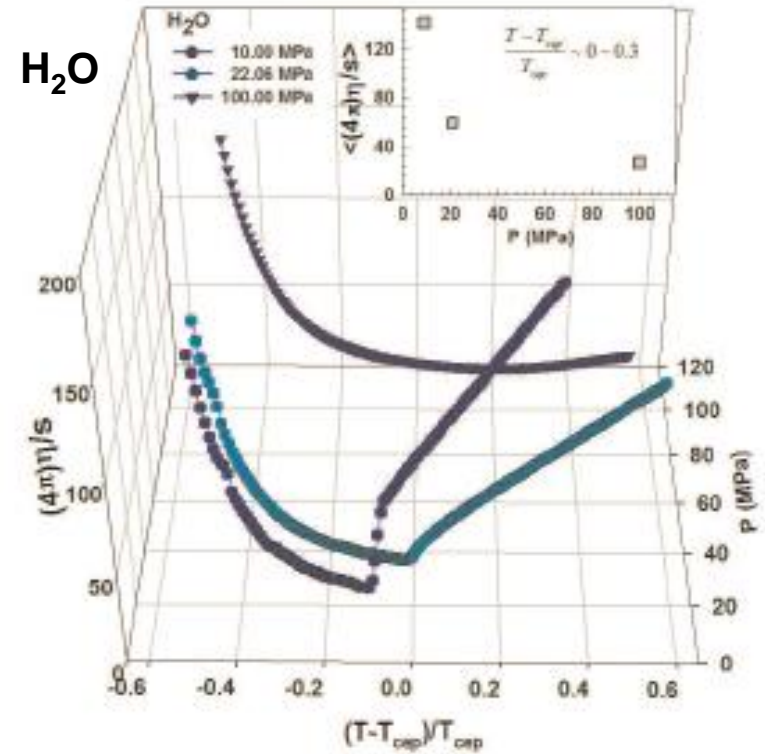
Possible signals for the CEP

Csernai et. al,
Phys.Rev.Lett. 97 (2006) 152303

A. Dobado et. al,
Phys. Rev. D 79, 014002 (2009)



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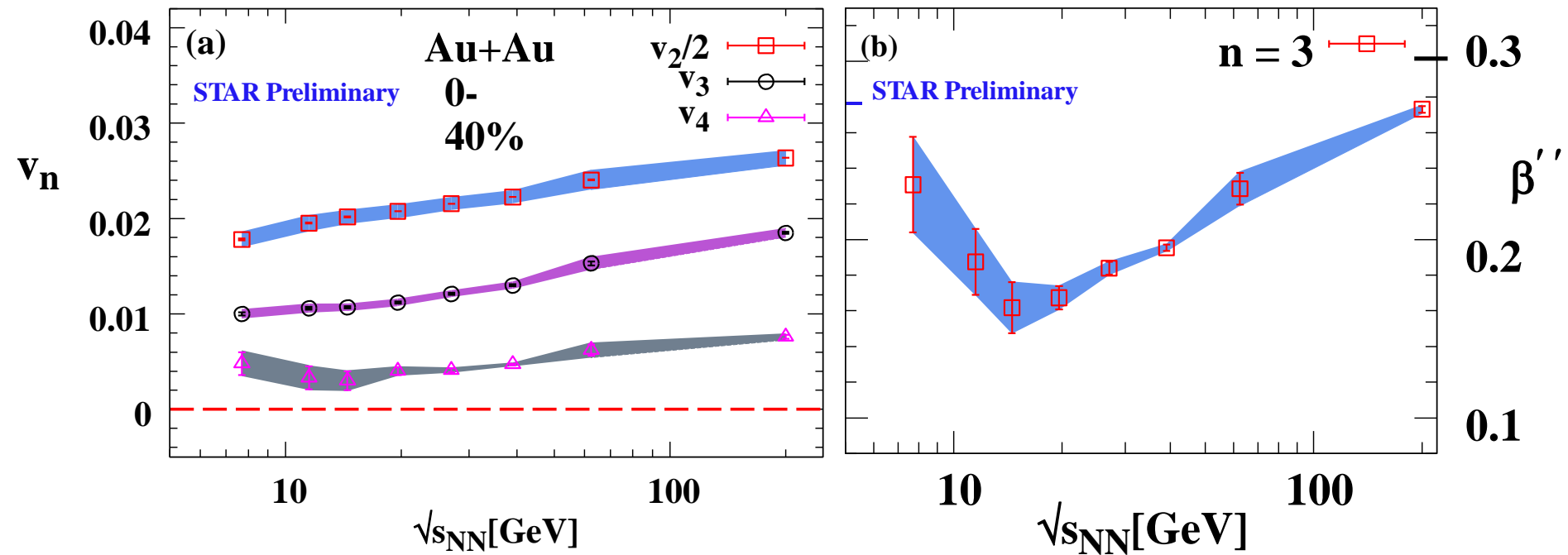


At the CEP or close to it, anomalies in the dynamic properties of the medium can drive abrupt changes in transport coefficients

Anisotropic flow (v_n) measurements are an invaluable probe

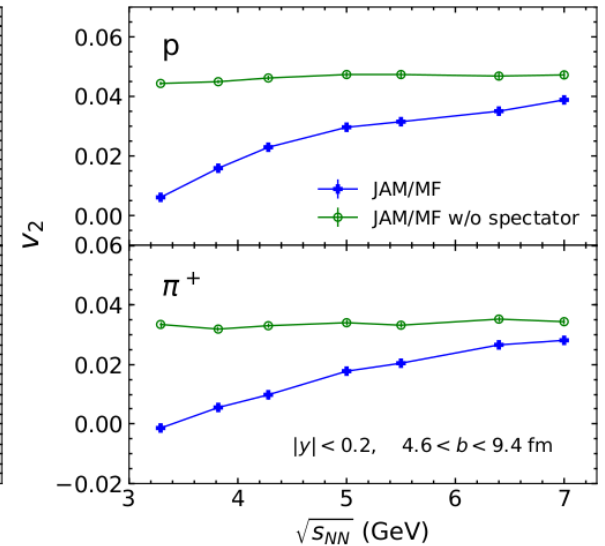
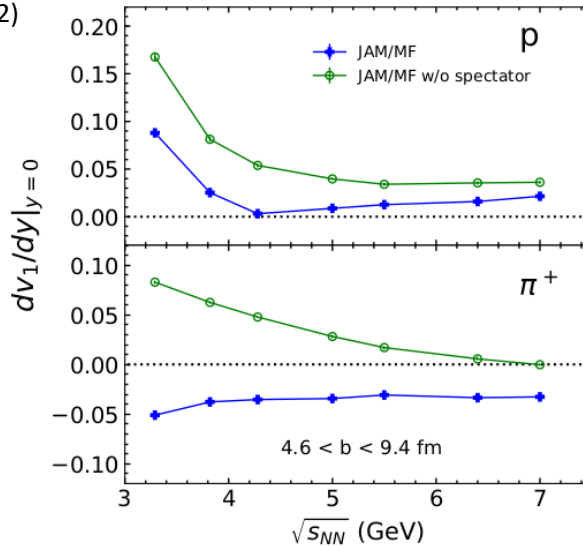
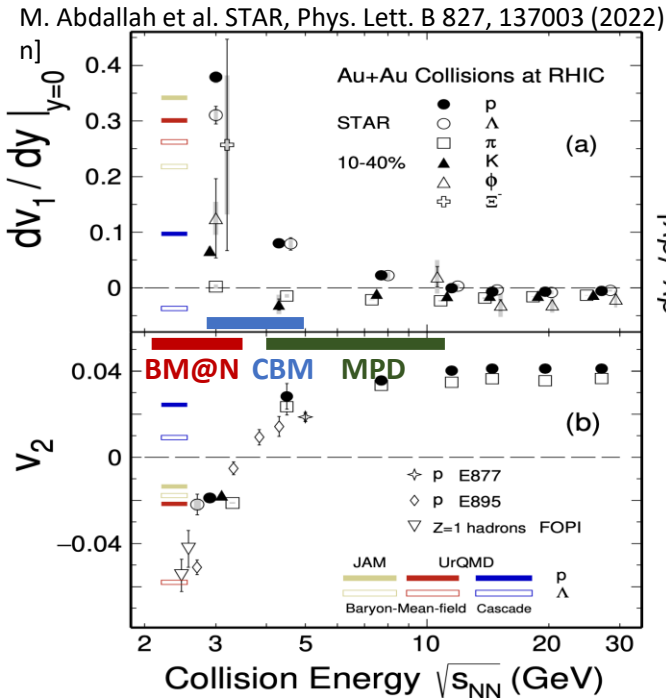
$$VC = \ln \left(\frac{(v_n)^{\frac{1}{n}}}{(v_2)^{\frac{1}{2}}} \right) \left(\frac{dN}{d\eta} \right)^{\frac{1}{3}}$$

$$VC \propto \frac{\eta}{s}$$



V_n shows a monotonic increase with beam energy. The viscous coefficient, which encodes the transport coefficient (η/s), indicates a non-monotonic behavior as a function of beam energy.

Anisotropic flow in Au+Au collisions at Nuclotron-NICA energies

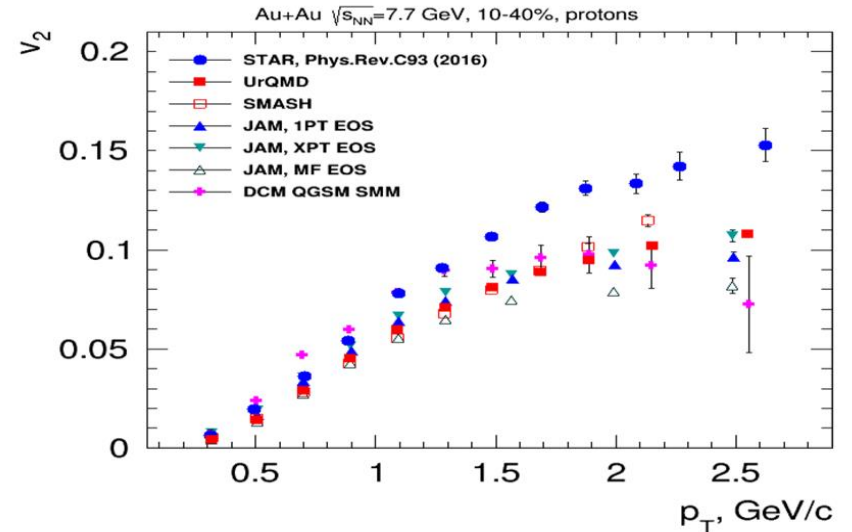
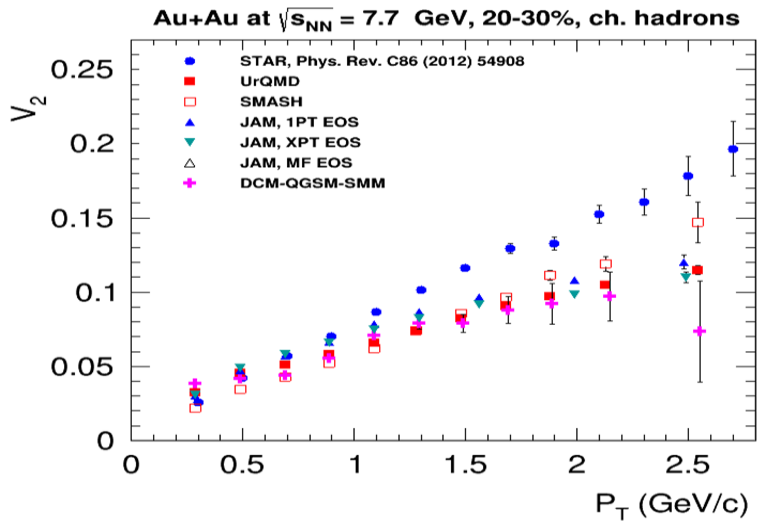
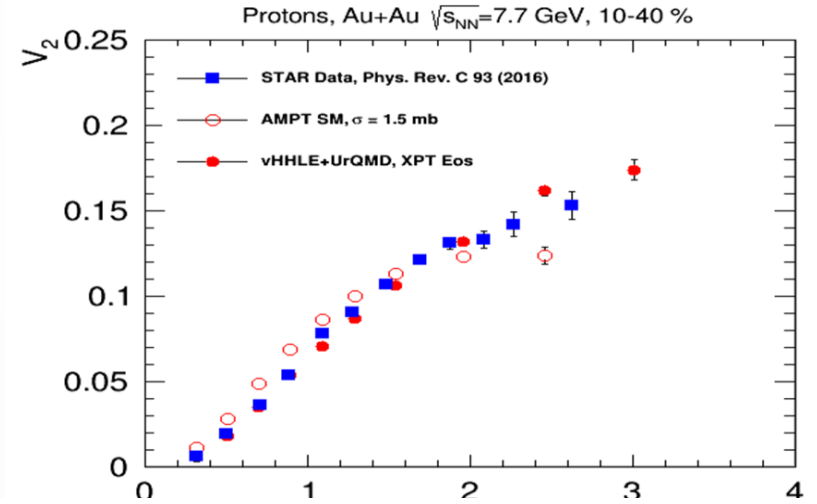
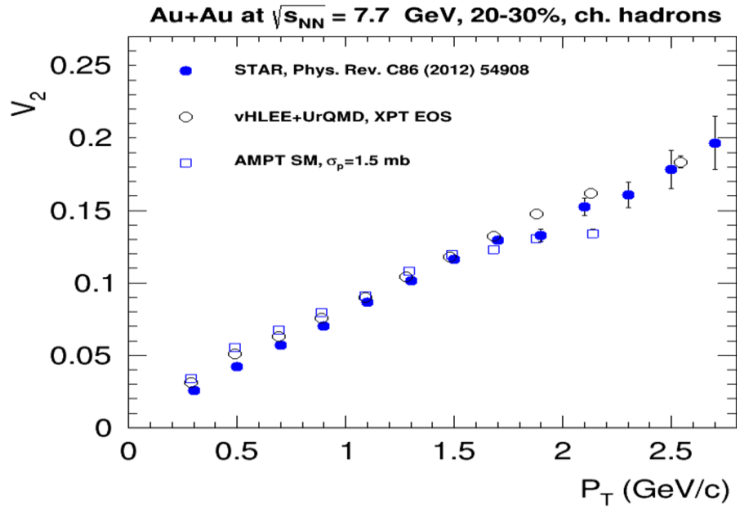


Phys. Rev. C 97, 064913 (2018)

Anisotropic flow at FAIR/NICA energies is a delicate balance between:

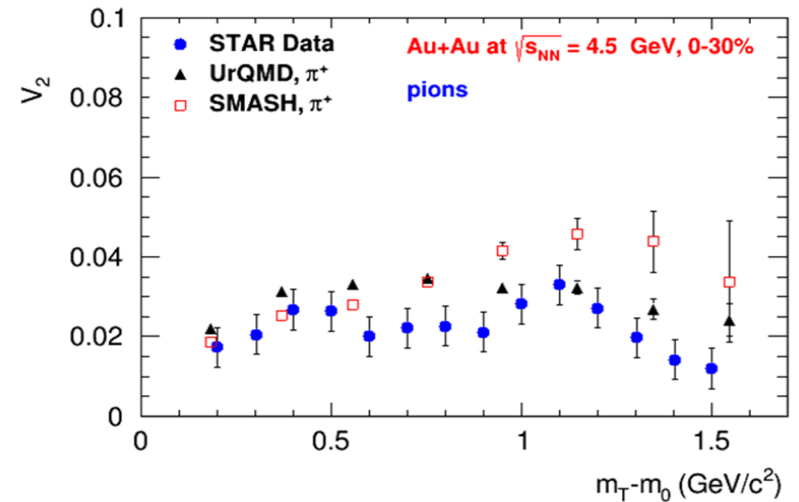
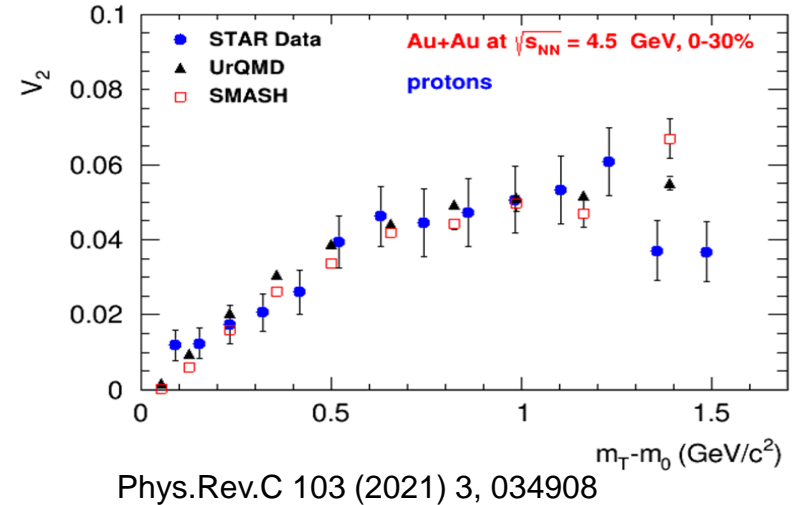
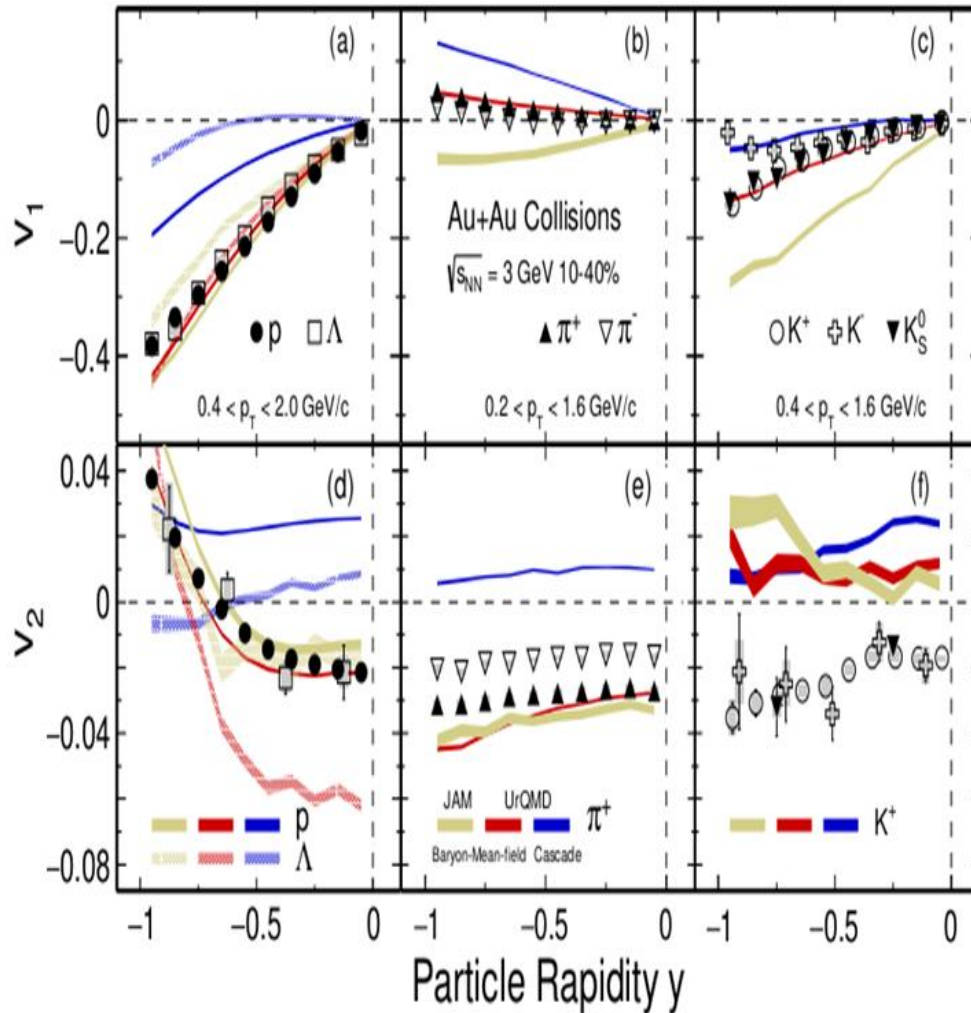
- I. The ability of pressure developed early in the reaction zone ($t_{exp} = R/c_s, c_s = c\sqrt{dp/d\varepsilon}$) and
- II. The passage time for removal of the shadowing by spectators ($t_{pass} = 2R/\gamma_{CM}\beta_{CM}$)

Elliptic Flow (v_2) at NICA energies: Models vs Data



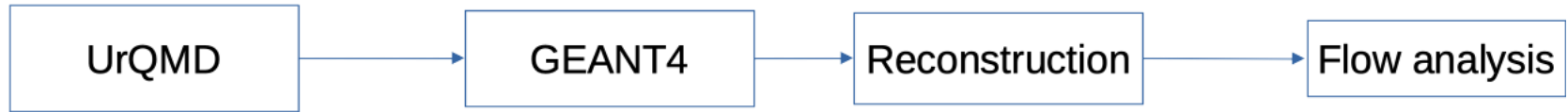
at $\sqrt{s_{NN}} \geq 7.7$ GeV pure string/hadronic cascade models underestimate v_2 – need hybrid models with QGP phase (vHLEE+UrQMD, AMPT with string melting,...)

Anisotropic Flow at Nuclotron/NICA energies: Models vs Data

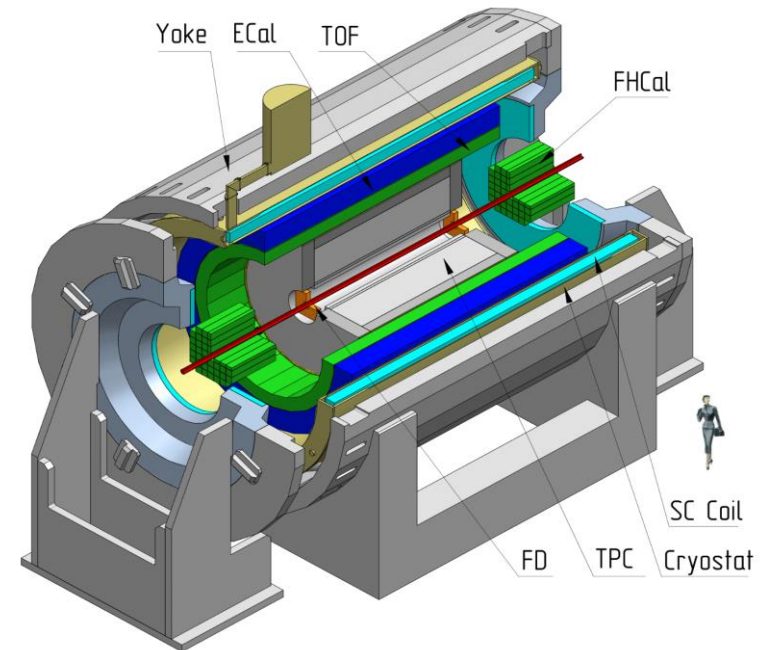
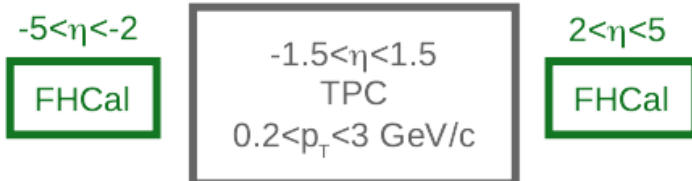


at $\sqrt{s_{NN}} \geq 3-4.5 \text{ GeV}$ pure hadronic models give similar v_2 signal compared to STAR data

MPD Experiment at NICA



- Bi+Bi: 50M at $\sqrt{s_{NN}} = 9.2$ GeV (prod. 25)
- Centrality determination: Bayesian inversion method and MC-Glauber
- Event plane determination: TPC, FHCaI
- Track selection:
 - ▶ Primary tracks
 - ▶ $N_{\text{TPC hits}} \geq 16$
 - ▶ $0.2 < p_T < 3.0$ GeV/c
 - ▶ $|\eta| < 1.5$
 - ▶ PID - ToF + dE/dx

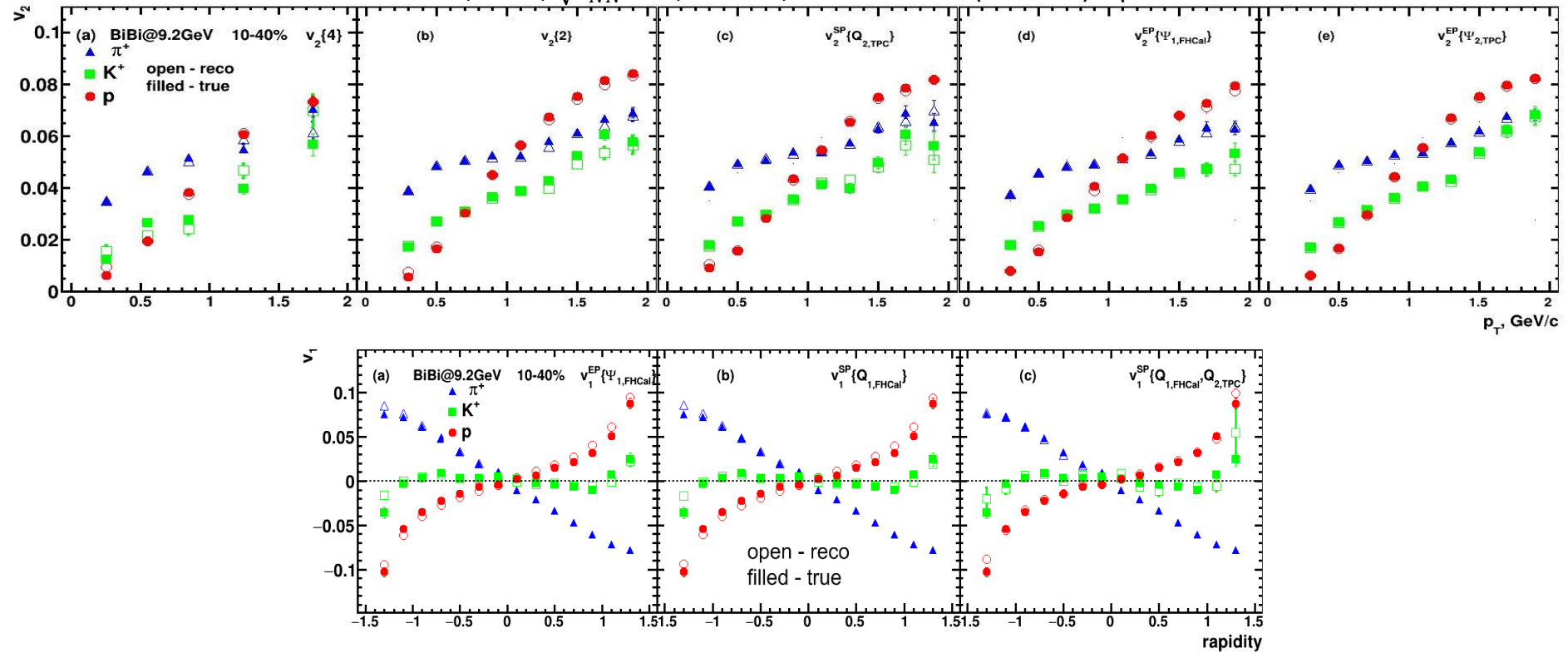


Multi-Purpose Detector (MPD) Stage 1

See Peter Parfenov presentation at MPD Collaboration meeting

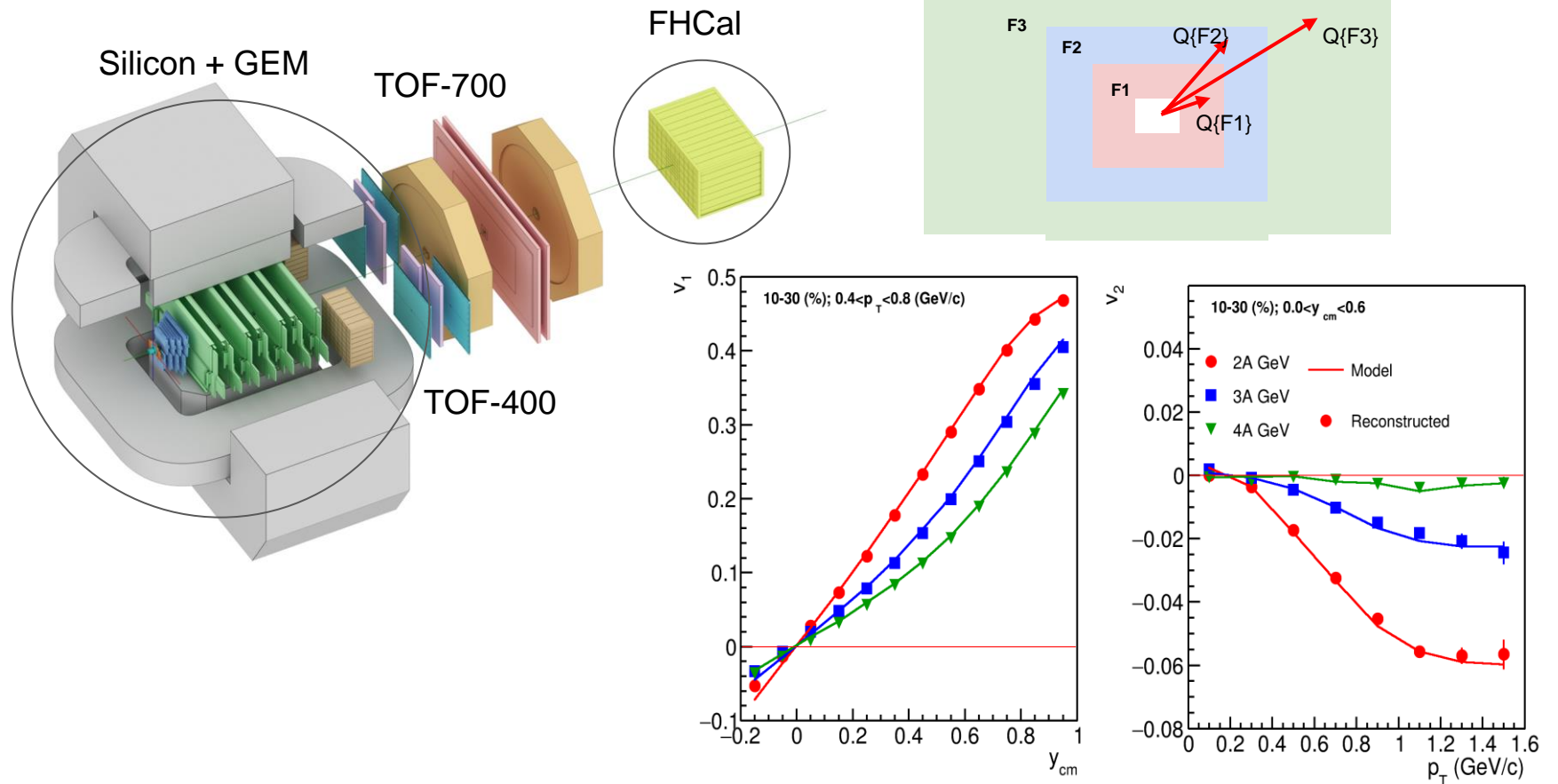
Performance of $v_{1,2}$ of identified hadrons in MPD

UrQMD, Bi+Bi, $\sqrt{s_{NN}}=9.2$, 10-40%, reconstructed (GEANT4) – production 25



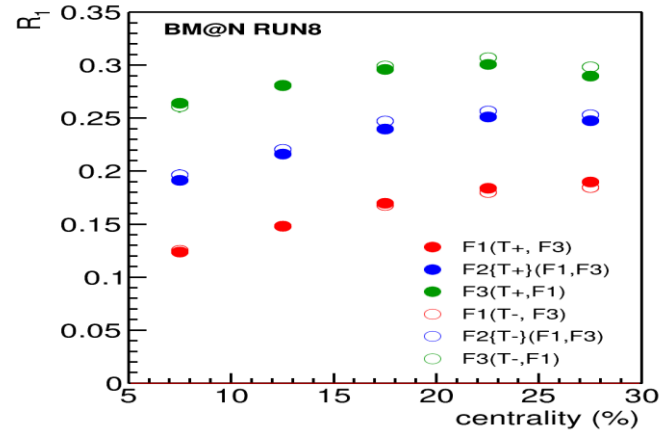
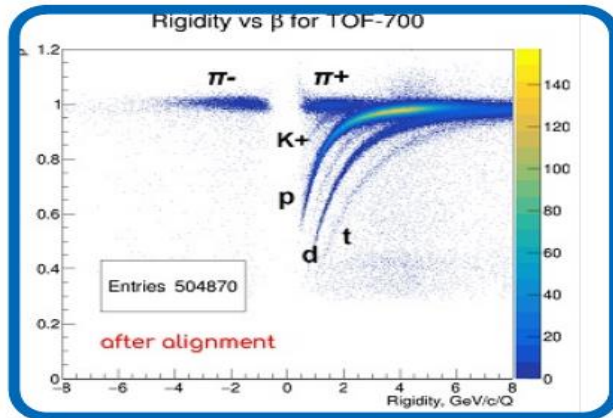
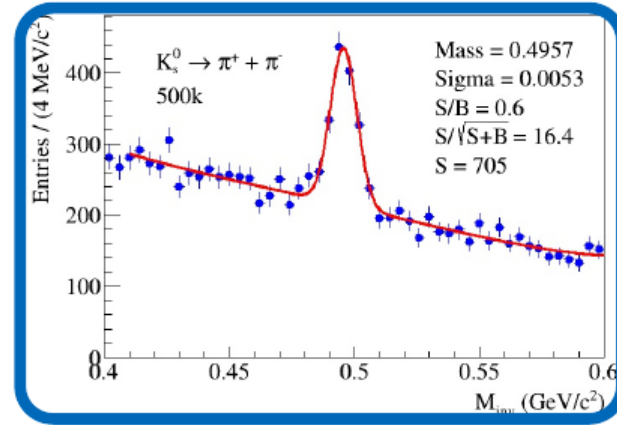
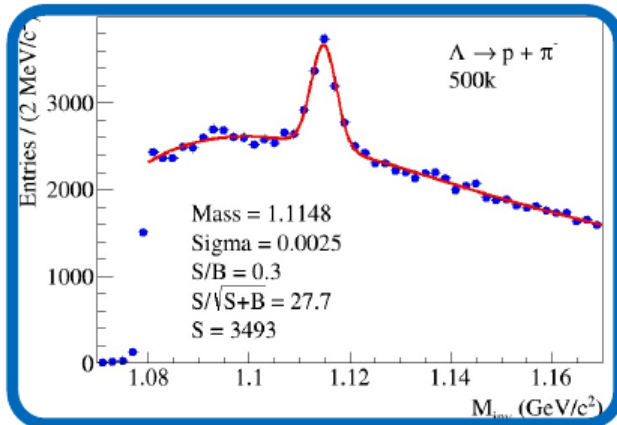
Reconstructed and generated $v_{1,2}$ of identified hadrons have a good agreement for all methods

The BM@N experiment (GEANT4 simulations for Xe+Cs(I) run)



Good agreement between reconstructed directed and elliptic flow of protons and pure model data

BM@N (Baryonic Matter @ Nuclotron)



December 2022 – February 2023: first physics run with Xe+Cs(I) (3.0 AGeV (50 M events) и 3.8 AGeV (500 M events))

Collective Flow in Heavy-Ion Collisions: Summary

Anisotropic Collective Flow is an important experimental observable to study the evolution of heavy-ion collision and properties of the strongly interacting matter.

It provides access to:

- Equation of State of the produced matter
- Transport properties: shear/bulk viscosity, speed of sound
- Initial conditions in heavy-ion collisions
- Mechanism of hadronization
- Origin of the correlations between produced particles

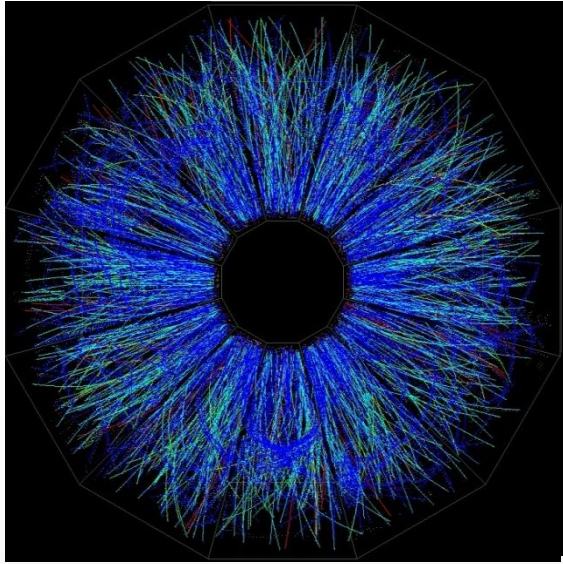
Feasibility study: MPD/MPD FXT/BM@N: will provide the detailed differential measurements of anisotropic collective flow with high efficiency.



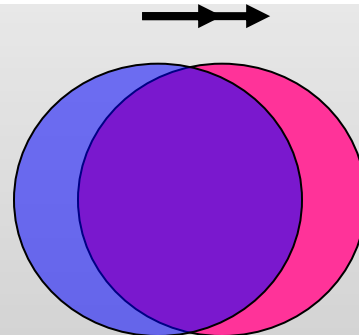
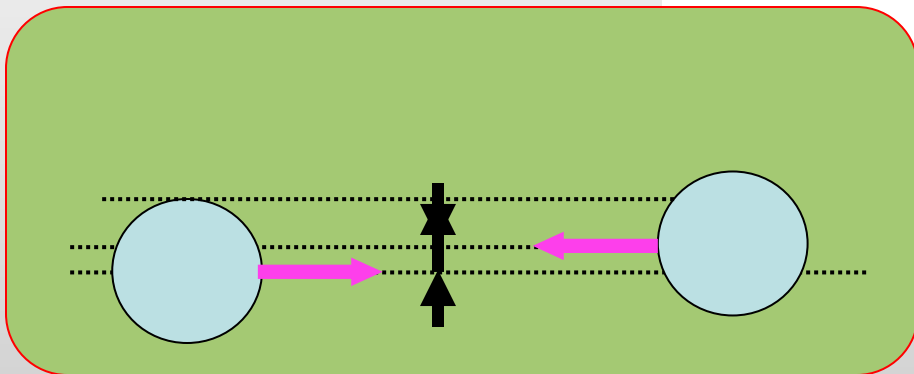
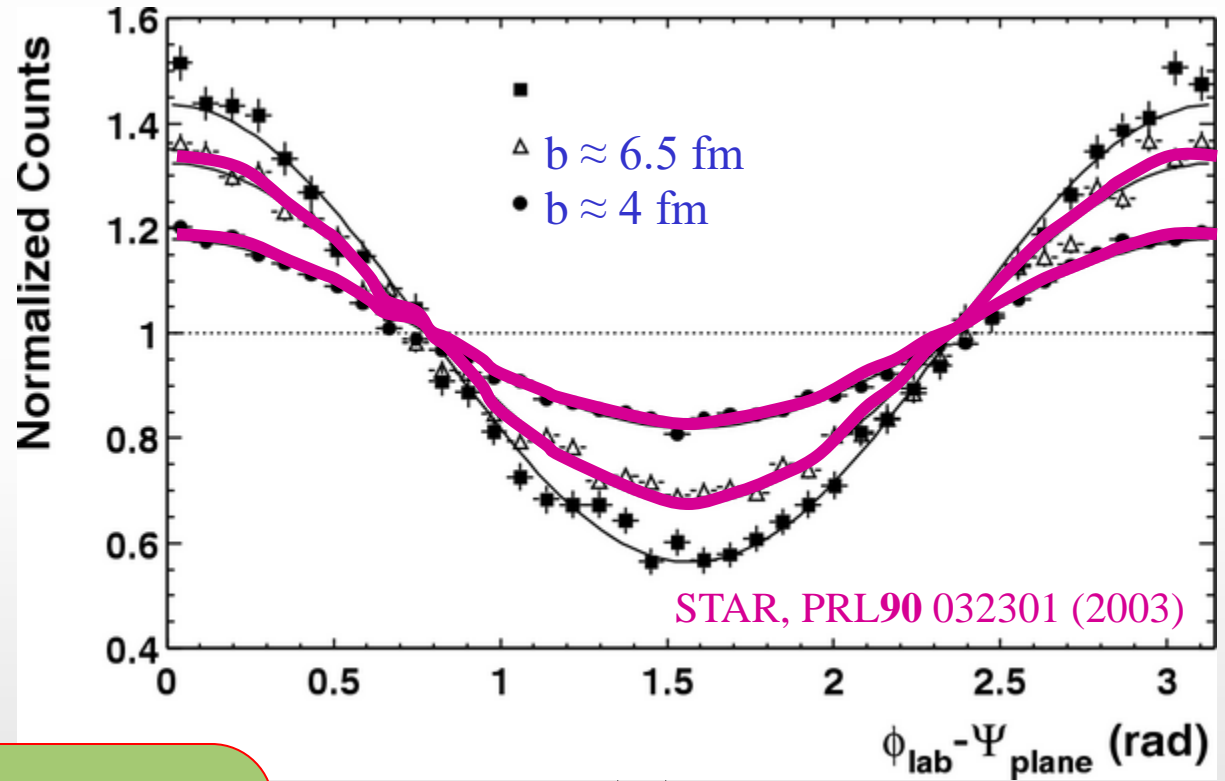
Backup slides

Azimuthal distributions at RHIC

$$\frac{dN}{d\phi} \propto \left(1 + 2 \sum_{n=1} v_n \cos [n(\phi - \Psi_n)] \right)$$

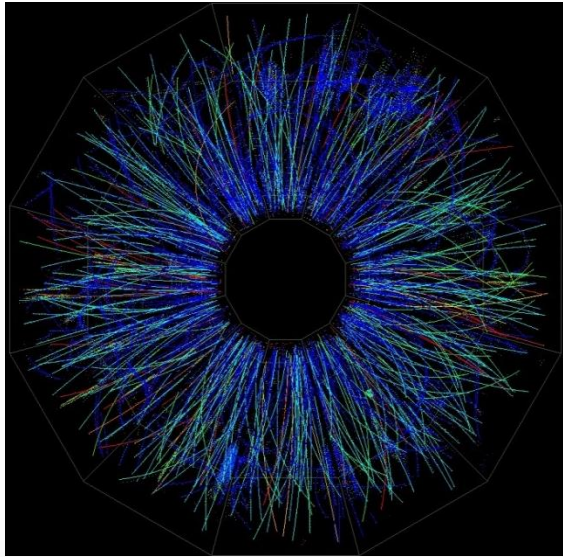


midcentral collisions

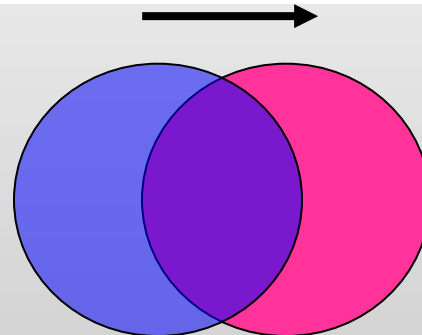
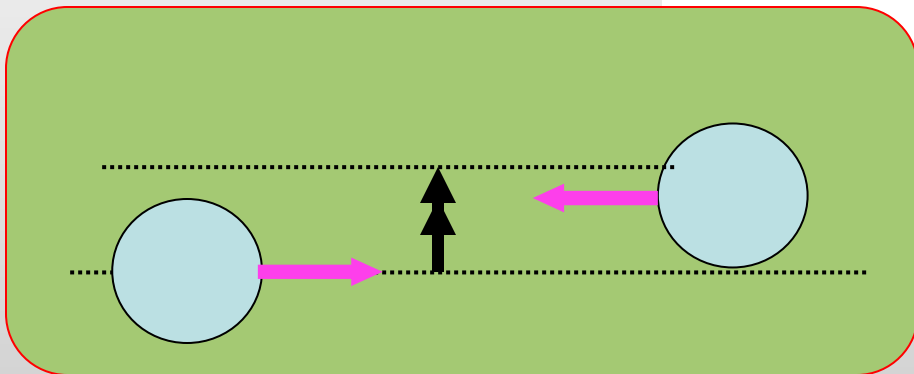
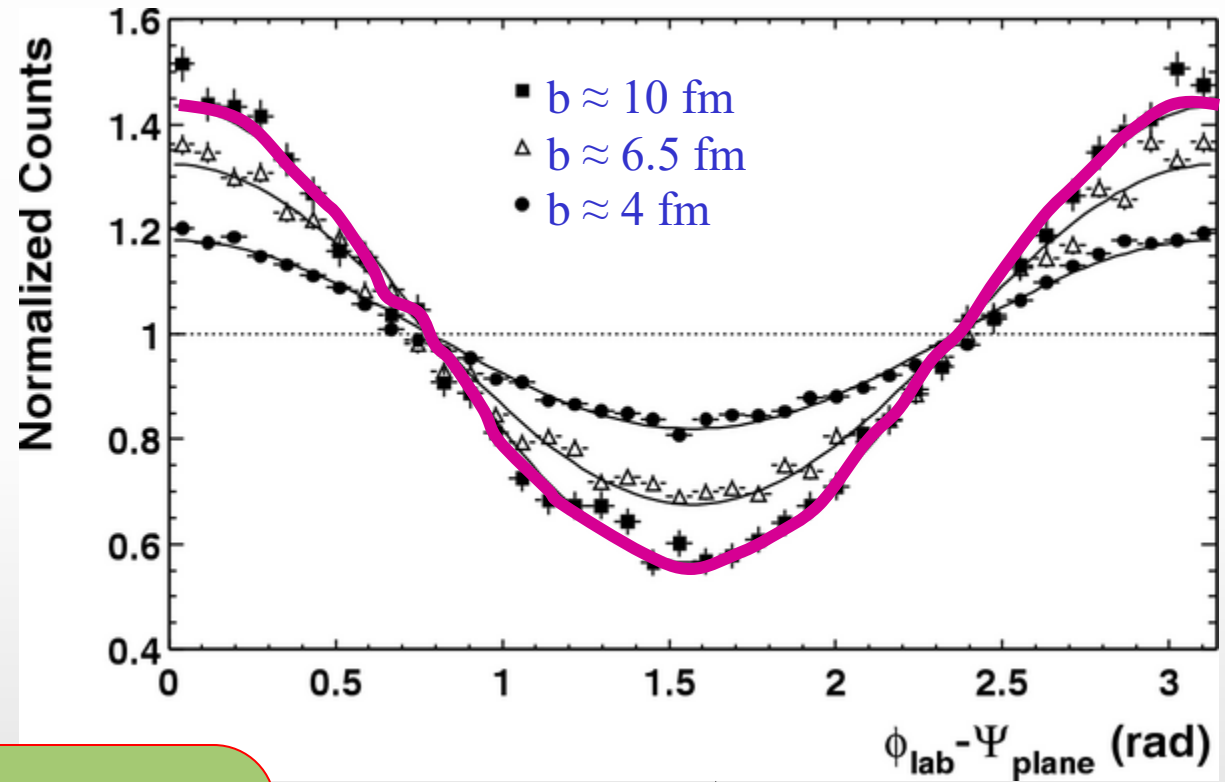


Azimuthal distributions at RHIC

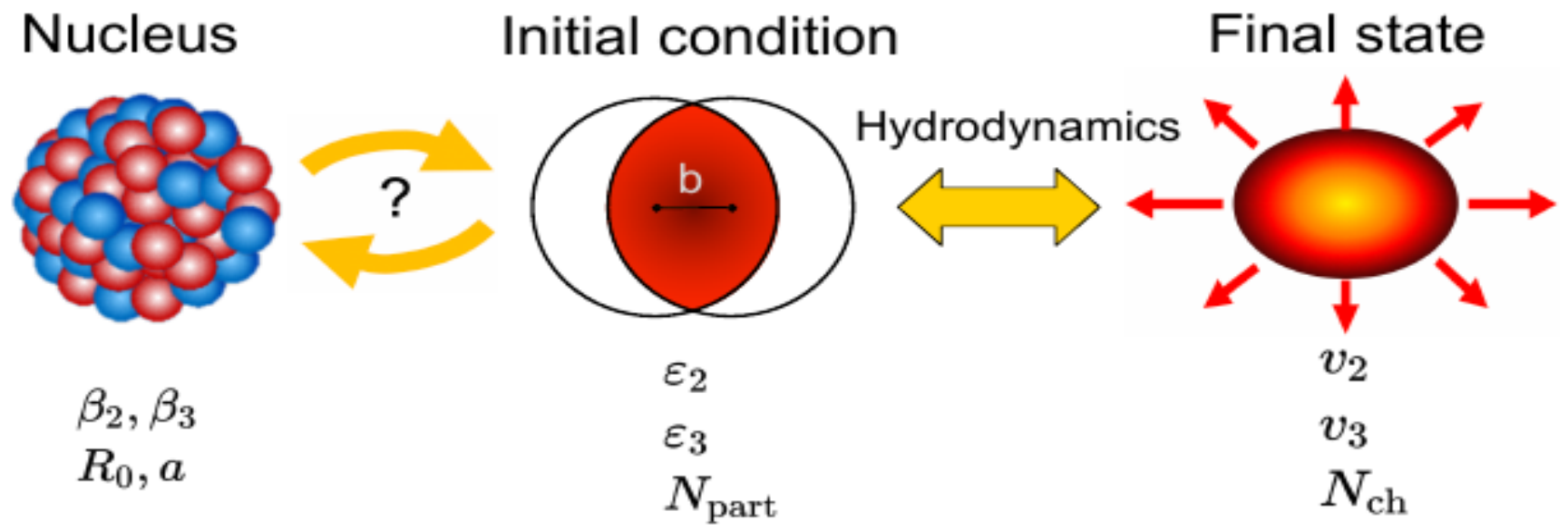
$$\frac{dN}{d\phi} \propto \left(1 + 2 \sum_{n=1} v_n \cos[n(\phi - \Psi_n)] \right)$$



peripheral collisions



2022: Imaging the initial condition of heavy-ion collisions and nuclear structure across the nuclide chart



$$\rho(r, \theta, \phi) \propto \frac{1}{1 + e^{[r - R_0(1 + \beta_2 Y_2^0(\theta, \phi) + \beta_3 Y_3^0(\theta, \phi))]/a}}$$

The extraction of the properties of the QGP, is currently limited by our poor knowledge of the initial condition, in particular how it is shaped from the colliding nuclei

To exploit collisions of selected species to precisely assess how the initial condition changes under variations of the structure of the colliding ions.

(<https://arxiv.org/abs/2209.11042>)

STAR BES-I and BES-II Data Sets

Au+Au Collisions at RHIC

Collider Runs						Fixed-Target Runs					
	$\sqrt{s_{NN}}$ (GeV)	#Events	μ_B	y_{beam}	run		$\sqrt{s_{NN}}$ (GeV)	#Events	μ_B	y_{beam}	run
1	200	380 M	25 MeV	5.3	Run-10, 19	1	13.7 (100)	50 M	280 MeV	-2.69	Run-21
2	62.4	46 M	75 MeV		Run-10	2	11.5 (70)	50 M	320 MeV	-2.51	Run-21
3	54.4	1200 M	85 MeV		Run-17	3	9.2 (44.5)	50 M	370 MeV	-2.28	Run-21
4	39	86 M	112 MeV		Run-10	4	7.7 (31.2)	260 M	420 MeV	-2.1	Run-18, 19, 20
5	27	585 M	156 MeV	3.36	Run-11, 18	5	7.2 (26.5)	470 M	440 MeV	-2.02	Run-18, 20
6	19.6	595 M	206 MeV	3.1	Run-11, 19	6	6.2 (19.5)	120 M	490 MeV	1.87	Run-20
7	17.3	256 M	230 MeV		Run-21	7	5.2 (13.5)	100 M	540 MeV	-1.68	Run-20
8	14.6	340 M	262 MeV		Run-14, 19	8	4.5 (9.8)	110 M	590 MeV	-1.52	Run-20
9	11.5	157 M	316 MeV		Run-10, 20	9	3.9 (7.3)	120 M	633 MeV	-1.37	Run-20
10	9.2	160 M	372 MeV		Run-10, 20	10	3.5 (5.75)	120 M	670 MeV	-1.2	Run-20
11	7.7	104 M	420 MeV		Run-21	11	3.2 (4.59)	200 M	699 MeV	-1.13	Run-19
						12	3.0 (3.85)	2000 M	750 MeV	-1.05	Run-18, 21

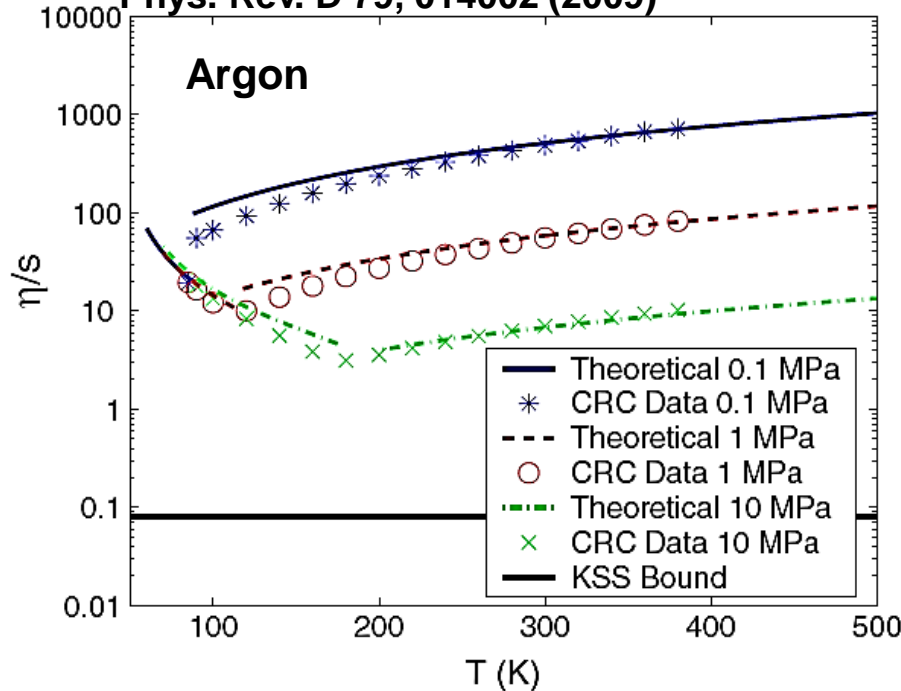
Precision data to map the QCD phase diagram

$$3 < \sqrt{s_{NN}} < 200 \text{ GeV}; \quad 750 < \mu_B < 25 \text{ MeV}$$

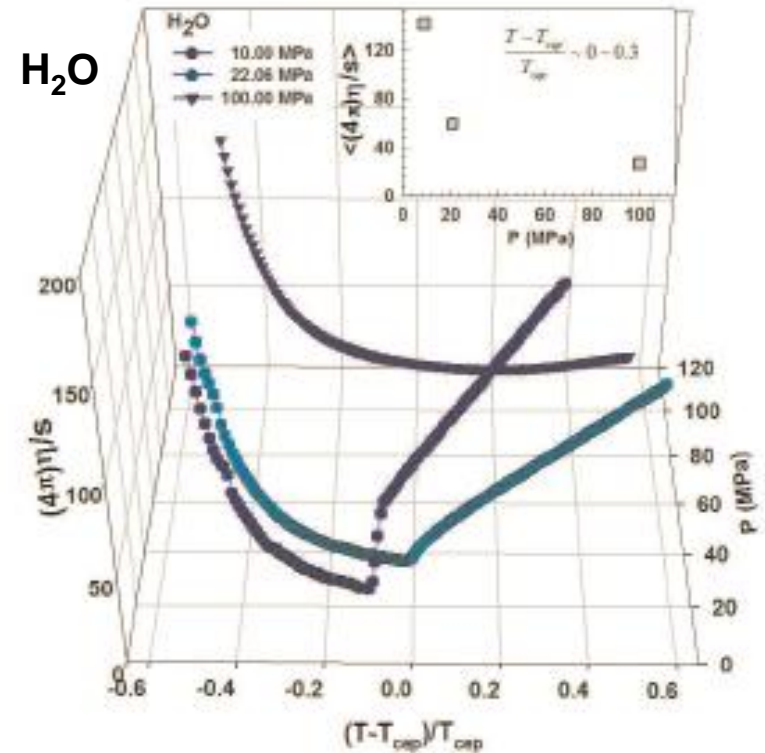
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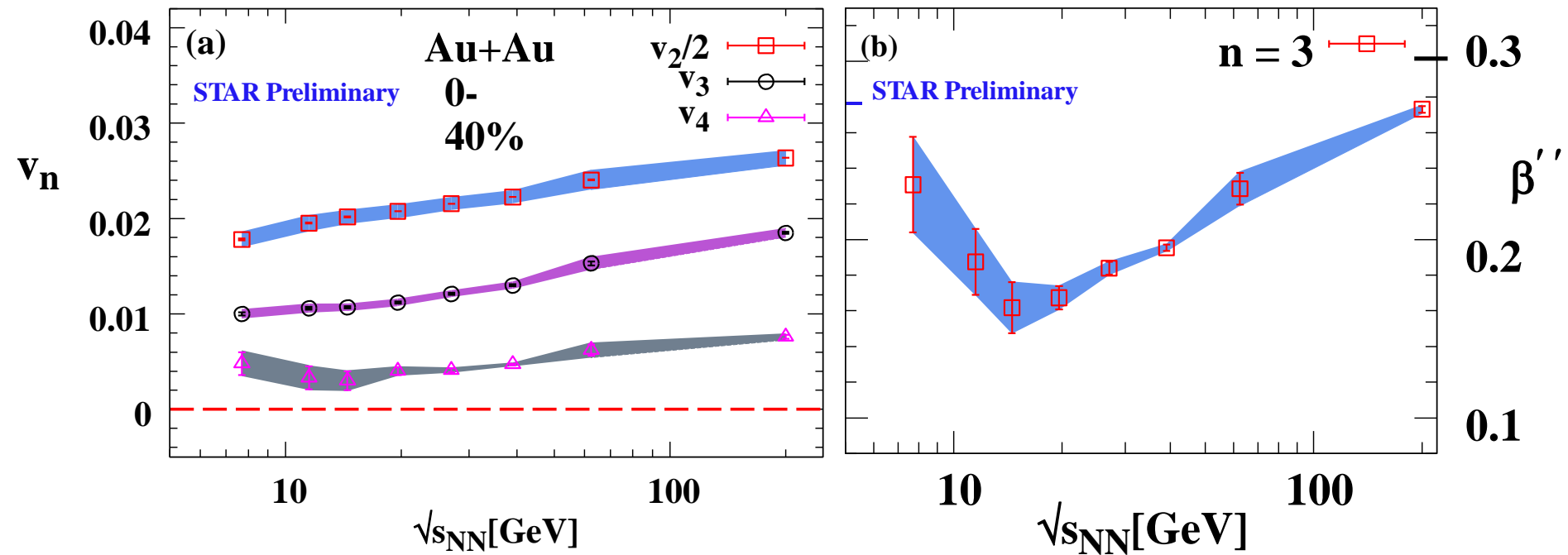


At the CEP or close to it, anomalies in the dynamic properties of the medium can drive abrupt changes in transport coefficients

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