Dear Mikhail and Vadim,

Sorry for a delay with my comments on the pdt paper from June 27 - I spent a time with inspecting literature on the coalescence problem and making some calculations.

The resulting text appears quite long and perhaps difficult to read, so please start with Comments, questions and suggestions at the end and inspect the rest for details.

I would say that the main step in the coalescence problem was done in paper [sat81], where the expressions for A-nucleon cluster (d, t and α) formation rate $\mathcal{A}_A$ multiplied by A^3 (due to the momentum per nucleon used instead of the A-cluster momentum) are given on page 156; note the relation between formation rate $\mathcal{A}_A$ and the coalescence parameter B_A [mro20]:

$$B_A = (A\, m/m^A)\mathcal{A}_A,$$

where m is the nucleon mass.

These expressions were obtained with the help of Jacobi variables, assuming (some more details and further development can be found, e.g., in [bel18, blu19, baz20, bel21]):

- independent nucleon emission, i.e., approximating the A-nucleon emission function D_A by a product of the single-nucleon ones: $D_A(r_1,p_1;..;r_A,p_A) = D(r_1,p_1)\,..\,D(r_A,p_A)$; note that D_A is closely related with the Wigner phase space density used in [21], the latter collecting all the D_A contributions up to a given time [akk02];
- no position-momentum (x-p) correlation in the A-cluster rest frame: $D(r_\alpha,p_\alpha) = D_r(r_\alpha)D_p(p_\alpha)$, $\alpha = 1,..,A$; the x-p correlation can however enter in $D_r(r_\alpha)$ through its dependence on the nucleon momentum (y, p_t, φ) in a global (laboratory or nucleon-nucleon c.m.s.) frame;
- a spherically symmetrical Gaussian nucleon $\boldsymbol{r}_\alpha$-distribution around the source center in the A-cluster rest frame, $\mathbf{x}^* = \boldsymbol{r}^*_\alpha - \langle \boldsymbol{r}^*_\alpha \rangle$,
$$\exp[-\nu\, \mathbf{x}^{*2}] \equiv \exp[-\nu \textstyle\sum_i x^{*2}_i],$$
characterized by a universal Gaussian $\mathbf{x}^{*2}$ slope $\nu = 1/(2R^*_{ii})$, same in all the three spatial directions i, corresponding to a universal squared source radius $R^{*2} = 1/(2\nu) \equiv R^*_{ii}$ in the assumed case of vanishing cross terms: $R^*_{ij} = 0$ at $i \neq j$;
- a Gaussian parametrization of the A-cluster wave function squared, $|\psi_A(\boldsymbol{r}^*_1,..,\boldsymbol{r}^*_A)|^2$, characterized by a universal $\boldsymbol{r}^{*2}_{\alpha\alpha'}$ slope $\nu_A/2$ ($\boldsymbol{r}^*_{\alpha\alpha'} = \boldsymbol{r}^*_\alpha - \boldsymbol{r}^*_{\alpha'}$ is the spatial separation of the nucleons α and α' in the A-cluster rest frame), related to the cluster r.m.s. radius R_A (also, see Eq. (3) in [bel18] and note that the quantities λ_A and r_A, entering in this equation, coincide with R_A and $(2/\nu_A)^{1/2}$, respectively):
$$1/\nu_A = 2n_A R_A^2,\ n_A = A/[(3(A-1)];$$
- a weak momentum dependence of the single-nucleon spectra; for a Gaussian momentum spectrum $\sim \exp(-\beta \boldsymbol{p}^2)$, with the slope $\beta \approx 1/(2mT_0)$ inversely related to the effective temperature T_0, the correction factor to the calculated formation rate composes $(1+\beta\nu_A)^{-(A-1)/2}$ and is usually close to unity (also, see Eq. (3.21) in [21]); this, so called Kopylov-Podgoretsky "smoothness assumption", is widely used in momentum correlation femtoscopy, see, e.g., [pod89]; in the context of a close connection between correlation and coalescence femtoscopy, it was used also in [lyu88, 20, led09];

- equal emission times in the A-cluster rest frame (allowing one to substitute the time-dependent Bethe-Salpeter amplitudes by the usual wave functions), justified on the condition of a small time-separation $t_{\alpha\alpha'}$ between nucleons α and α':
$$| t_{\alpha\alpha'}^{*} | << m\, r_{\alpha\alpha'}^{*2}$$
and usually leading to a percent correction only (due to a small inverse nucleon mass as compared with a typical space-time separation of the emission points) [lyu88, led09, kac23].

The results of [sat81] have been generalized (see, e.g., [21], [bel18], [blu19], [bel21]), introducing different Gaussian slopes $\nu_i = 1/(2R^{*}_{ii})$ in the directions i = o (out), s (side), l (longitudinal), where out-direction is chosen along the transverse cluster velocity.

Generally, to take into account also the cross terms, one should substitute $\nu\sum_i x_i^{*2}$ by the quadratic form $\sum_{ij} \{R^{*-1}\}_{ij} x^{*}_i x^{*}_j/2$, so that the x^{*}_i - distribution becomes
$$\exp[-\sum_{ij} \{R^{*-1}\}_{ij} x^{*}_i x^{*}_j/2],$$
where $\{R^{*-1}\}_{ij}$ are the elements of the inverse squared radii matrix and
$$R^{*}_{ij} = \langle x^{*}_i x^{*}_j \rangle;$$
obviously, the diagonal quadratic radii $R^{*}_{ii} = R^{*2}_i$ are positive definite, while the non-diagonal ones R^{*}_{ij} at $i \neq j$ can be negative.

In principle, the quadratic radii matrices can be measured with the help of two-particle momentum correlation femtoscopy based on the two-particle final state interaction (FSI) and quantum statistics (QS):
the two-particle correlation function at the relative momentum $\mathbf{q}^{*} = \mathbf{p}_1^{*} - \mathbf{p}_2^{*}$ in the pair rest frame (PRF) is given, in the equal-time approximation, by a sum of the scattering wave functions squared, averaged over spatial separations $\mathbf{r}^{*} = \mathbf{r}_1^{*} - \mathbf{r}_2^{*} \equiv \mathbf{x}_1^{*} - \mathbf{x}_2^{*}$ and spin projections of the detected particles; the sum is done over the contributing intermediate channels and corresponding spin projections (the opposite sign of the vector $\mathbf{q}^{*}$ in the scattering wave functions appears since the scattering due to FSI is viewed in the opposite time direction; also, the detected channel should be considered as the entrance one; see [led82, led98, led09] and references therein):
$$\Re(\mathbf{q}^{*}) = \sum\langle |\psi_{-\mathbf{q}^{*}}(\mathbf{r}^{*})|^2 \rangle;$$
the spin projection and channel indices, affecting the summation and averaging, are omitted for simplicity. For identical particles, in accordance with the Bose-Einstein or Fermi-Dirac QS, the non-symmetrized wave function should be substituted by the symmetrized one:
$$\psi^{S}_{-\mathbf{q}^{*}}(\mathbf{r}^{*}) \rightarrow [\psi^{S}_{-\mathbf{q}^{*}}(\mathbf{r}^{*}) + (-1)^{S} \psi^{S}_{\mathbf{q}^{*}}(\mathbf{r}^{*})]/\sqrt{2},$$
where S is the total pair spin.

In the considered case of independent particle emission and Gaussian distributions of $\mathbf{x}_1^{*}$ and $\mathbf{x}_2^{*}$, the $\mathbf{r}^{*}$ – distribution is also Gaussian:
$$\exp[-\sum_{ij} \{R^{(12)*-1}\}_{ij} r^{*}_i r^{*}_j/2],$$
where
$$R^{(12)*}_{ij} = \langle r^{*}_i r^{*}_j \rangle = R^{(1)*}_{ij} + R^{(2)*}_{ij}.$$

Thus, studying correlations of various particle species, one can extract the quadratic femtoscopic radii matrices for separate species, including nucleons. Doing this, one should take into account that non-Gaussian tails of the emission functions (e.g., due to particles from long-lived emitters) and possible admixtures of misidentified particles lead to the correlation

suppression factor $\Lambda \approx \Lambda_1\Lambda_2$, where Λ_1 and Λ_2 are the fractions of correlated particles [led79, akk02, led09]. As a result:

$$\Re\,(\mathbf{q}^*) = (1-\Lambda) + \Lambda \sum\langle\, |\psi_{-\mathbf{q}^*}(\mathbf{r}^*)|^2\rangle.$$

However, the multidimensional correlation analysis requires very large statistics and up to now, it has been practically realized for identical charged pions only. In this case, after a correction for the Coulomb FSI [sin98], the correlation function is dominated by the effect of QS, reflected in the symmetrization of the plane waves describing "non-interacting" pions:

$$\psi_{-\mathbf{q}^*}(\mathbf{r}^*) = [\exp(-i\,\mathbf{q}^*\mathbf{r}^*/2) + \exp(i\,\mathbf{q}^*\mathbf{r}^*/2)]/\sqrt{2}.$$

Taking into account the suppression factor Λ, the corresponding two-pion correlation function

$$\Re\,(\mathbf{q}^*) = 1+\Lambda\langle\, \cos(\mathbf{q}^*\mathbf{r}^*)\rangle = 1+\Lambda\, \exp[-\textstyle\sum_{ij} R^*_{ij}\, q^*_i\, q^*_j],$$

where

$$R^*_{ij} = \tfrac{1}{2}\,\langle\, r^*_i\, r^*_j\rangle = \langle\, x^*_i\, x^*_j\rangle$$

are the elements of the matrix of the femtoscopic pion squared radii in PRF.

In fact, the 3-vector scalar product in the plane wave in PRF can be written in any frame as a 4-vector scalar product: $-\,(\mathbf{q}^*\mathbf{r}^*) = (q\,\Delta x) = q_0\,t - \mathbf{q}\,\mathbf{r}$, where $t = t_1 - t_2$, $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$ and $q_0 = E_1 - E_2$ is the difference of particle energies (note that $q^*_0 = 0$ for equal-mass particles).

Taking into account vanishing of the scalar product of the 4-vectors $q = \{q_0, \mathbf{q}\}$ and $P = p_1+p_2$ for equal-mass particles: $Pq \equiv P_0q_0 - \mathbf{Pq} = m_1^2 - m_2^2 = 0$ and noting that $\boldsymbol{\beta} = \mathbf{P}/P_0$ is the pair 3-velocity, one can rewrite the 4-vector scalar product $(q\,\Delta x)$ as

$$(q\,\Delta x) = -\textstyle\sum_i q_i\,(r_i - \beta_i\,t).$$

As a result, the correlation function of two non-interacting identical pions can be written in any frame as [pod83, cha95]:

$$\Re\,(\mathbf{q}) = 1+\Lambda\langle\, \cos(\mathbf{q}\,\mathbf{r})\rangle = 1+\Lambda\, \exp[-\textstyle\sum_{ij} R_{ij}\, q_i\, q_j],$$

where

$$R_{ij} = \tfrac{1}{2}\,\langle\, (r_i - \beta_i\,t)\,(r_j - \beta_j\,t)\rangle = \langle\, [x_i - \beta_i\,(t-\langle t\rangle)]\,[r_j - \beta_j\,(t-\langle t\rangle)]\,\rangle.$$

Note that the out-longitudinal cross term is the only one possible in case of azimuthally symmetric analysis [pod83, cha95]; the other two, out-side and side-long, terms are possible in the analysis with respect to the reaction plane.

In longitudinally comoving system (LCMS), the pair moves in the out-direction and the time dependence affects this direction only. Also, the Lorentz boost from LCMS to PRF acts along the out-direction only ($\gamma_t = m_t/m$, $\beta_t = p_t/m_t$):

$$x^*_o = \gamma_t\,[x_o - \beta_t\,(t-\langle t\rangle)],\; x^*_s = x_s,\; x^*_l = x_l,$$

enhancing the effective space-time separation $[x_o - \beta_t\,(t-\langle t\rangle)]$ by the transverse Lorentz factor γ_t. E.g., the PRF radius in the out-direction (see also footnote 5 in [sin98]):

$$R^*_o = \gamma_t\,R_o.$$

Sometimes, the opposite relation between PRF and LCMS out-separation is mistakenly derived due to the incorrect and unnecessary assumption of equal nucleon emission times in PRF, see, e.g., Eqs. (26-27) in [zha22]; in [21], the same assumption led to the incorrect statement on page 12: "Deuterons with non-zero transverse velocity see the fireball Lorentz-contracted in their direction of motion; this decreases the corresponding length of homogeneity and thus $\langle C_d\rangle$".

Similarly, the Lorentz boost from laboratory system to the LCMS enhances the longitudinal LCMS radius R_l by the longitudinal Lorentz factor $\gamma_l = \cosh(y)$: $R_l = \gamma_l R^{lab}_l$, leading to a bell-shape y -dependence of the laboratory longitudinal radius in case of a constant R_l in LCMS [mak88]: $R^{lab}_l = R_l/\cosh(y)$.

In the presence of the out-longitudinal cross term (a non-zero out-longitudinal squared radius R_{ol}), the Gaussian approximations of the emission function and the A-cluster wave function squared allow one to express the coalescence parameter B_A for the A-cluster of spin-S in a simple analytical form:

$$B_A = g_S\, A\, [(2\pi)^3/m)]^{A-1} \int d^3\boldsymbol{r}_1 \,..\, d^3\boldsymbol{r}_A\, D_A(\boldsymbol{r}_1,\boldsymbol{p}_1;..;\boldsymbol{r}_A,\boldsymbol{p}_A)|\psi_A(\boldsymbol{r}_1,..,\boldsymbol{r}_A)|^2$$

$$= g_S\, \Lambda_A\, A\, [(2\pi)^3/m)]^{A-1} \int d^3\boldsymbol{r}_1 \,..\, d^3\boldsymbol{r}_A\, \widetilde{D}_A(\boldsymbol{r}_1,\boldsymbol{p}_1;..;\boldsymbol{r}_A,\boldsymbol{p}_A)|\psi_A(\boldsymbol{r}_1,..,\boldsymbol{r}_A)|^2$$

$$= g_S\, \Lambda_A\, A^{-1/2}\{(2\pi)^{3/2}/[m([(R^{*}_{oo} + n_A R_A^2)(R^{*}_{ll} + n_A R_A^2) - R^{*2}_{ol}] \times (R^{*}_{ss} + n_A R_A^2))^{1/2}]\}^{A-1}$$

$$\equiv g_S\, \Lambda_A\, A^{-1/2}\{(2\pi)^{3/2}/[m([(R^{*2}_{o} + d_A^2)(R^{*2}_{l} + d_A^2) - R^{*2}_{ol}] \times (R^{*2}_{s} + d_A^2))^{1/2}]\}^{A-1}, \qquad (1)$$

where $g_S = (2S+1)/2^A$ is the spin factor in case of unpolarized nucleons and $\widetilde{D}_A$ is the A-nucleon emission function, describing nucleons from short-lived emitters (characterized by finite PRF radii) and thus contributing to the production of the A-cluster. The corresponding fraction of such A nucleons is the A-cluster suppression parameter $\Lambda_A \approx \Lambda_N^A$, similar to the two-particle correlation suppression parameter $\Lambda \approx \Lambda_1\Lambda_2$ (except for the admixture of misidentified particles, usually corrected for in the production cross sections); the fraction Λ_N of correlated nucleons is less than unity due to non-Gaussian tails of the nucleon emission functions, e.g., due to nucleons from hyperon decays [led79, bel21].

In the last row, we have introduced a modified A-cluster radius squared d_A^2 with the absorbed factor $n_A = A/[(3(A-1)$:

$$d_A^2 = n_A R_A^2 .$$

One finds [ang13]:

	deuteron	triton	^{3}He	^{4}He	^{6}Li	^{9}Be
R_A , fm	2.13	1.76	1.97	1.68	2.59	2.52
d_A , fm	1.74	1.24	1.39	1.12	1.64	1.54

Neglecting the out-longitudinal squared radius R^{*}_{ol}, Eq. (1) at $R^{*}_{oo} = R^{*}_{ss}$ coincides with Eq. (31) in [blu19] (if correcting Eq. (B3) for a missed factor 1/2) and, for A = 2 and 3, - with respective Eqs. (49) and (50) in [bel21], which take into account also different out- and side-radii.

Introducing the homogeneity volumes $V^{*}_{hom} = R^{*}_{o}R^{*}_{s}R^{*}_{l}$ in PRF and $V_{hom} = R_oR_sR_l$ in LCMS (defined as products of the Gaussian PRF and LCMS femtoscopic radii in the out-, side- and longitudinal- directions) and taking into account that $R^{*}_{o} = \gamma_t R_o$, $R^{*}_{s} = R_s$, $R^{*}_{l} = R_l$ and

$$mV^{*}_{hom} = m_t V_{hom} ,$$

one may express B_A through the homogeneity volumes as:

$$B_A = g_S\,\Lambda_A\,A^{-1/2}\{[[1+(d_A/R^*_o)^2]\,[1+(d_A/R^*_l)^2]-(R^*_{ol}/(R^*_o R^*_l))^2]\times [1+(d_A/R^*_s)^2]\}^{-(A-1)/2}\,[(2\pi)^{3/2}/(mV^*_{hom})]^{(A-1)}$$

$$\equiv g_S\,\Lambda_A\,A^{-1/2}\{[[1+(d_A/(\gamma_t R_o))^2]\,[1+(d_A/R_l)^2]-(R_{ol}/(R_o R_l))^2]\times [1+(d_A/R_s)^2]\}^{-(A-1)/2}\,[(2\pi)^{3/2}/(m_t V_{hom})]^{(A-1)}. \qquad (1')$$

The obvious generalization of the Eq. (1') to azimuthally (φ) dependent analysis with non-zero mixed squared radii R_{os} and R_{sl} is achieved by the substitution

$$[1+(d_A/R_s)^2] \to \{[1+(d_A/(\gamma_t R_o))^2]\,[1+(d_A/R_s)^2]-(R_{os}/(R_o R_s))^2\}\times \{[1+(d_A/R_l)^2]\,[1+(d_A/R_s)^2]-(R_{sl}/(R_s R_l))^2]\}\,/\, \{[1+(d_A/(\gamma_t R_o))^2]\,[1+(d_A/R_s)^2]\,[1+(d_A/R_l)^2]\}. \qquad (1'')$$

The above equations for the coalescence parameter do not rely on a concrete production model. They allow one to calculate $B_A(y,m_t)$ or $B_A(y,m_t,\varphi)$ provided that the femtoscopic quadratic radii $R_{ij}(y,m_t)$ or $R_{ij}(y,m_t,\varphi)$ are known (e.g., from pp femtoscopy) and compare B_A with the data. The eventual discrepancy may be related with the assumptions of:

- Gaussian approximations of both, the wave function squared and the emission function;
  the former assumption leads to a decrease of deuteron coalescence parameter (this decrease composes ~ 4% at $R_o = R_s = 5.1$ fm and $R_l = 3.2$ fm [21] and ~ 30% at $R_o = R_s = R_l = 2$ fm (see Fig. 1 in [bel21]);
  as for the latter assumption, in the expanding fireball model with a linear transverse flow rapidity profile [21], it leads to a weak power-like $\sim m_t^{\,b}$ ($b \lesssim 0.5$) increase of B_A; this increase may become nearly exponential in m_t, when substituting the Gaussian transverse density profile by a box-like one (see Fig. 2 in [21]);
  an additional B_A increase with m_t can be introduced by the increasing suppression parameter Λ_A [blu19];
- independent nucleon emission;
  this assumption can be checked as a flat behavior of the A-nucleon correlation function outside the region of small relative nucleon momenta, which is free of the effects of nucleon final state interaction (FSI) and quantum statistics (QS); such a correlation plateau is usually observed in heavy ion collisions (see, e.g., [zbr11]).

The coalescence parameter B_A in Eq. (1) can be rewritten in a form similar to Eq. (6.2) of [21]:

$$B_A = g_S\,\Lambda_A\,A^{-1/2}\,C_A\,[(2\pi)^{3/2}/(m_t V_{hom})]^{A-1}, \qquad (2)$$

where C_A is the so-called quantum correction factor:

$$C_A = A^{3/2}\,[(2\pi)^{3/2}V^*_{hom}]^{A-1}\int d^3\boldsymbol{r}_1 \,..\, d^3\boldsymbol{r}_A\,\widetilde{D}_A(\boldsymbol{r}_1,\boldsymbol{p}_1;..;\boldsymbol{r}_A,\boldsymbol{p}_A)|\psi_A(\boldsymbol{r}_1,..,\boldsymbol{r}_A)|^2$$

$$= \{[[1+(d_A/R^*_o)^2]\,[1+(d_A/R^*_s)^2]-(R^*_{ol}/(R^*_o R^*_l))^2]\,[1+(d_A/R^*_l)^2]\}^{-(A-1)/2}$$

$$= \{[[1+(d_A/(\gamma_t R_o))^2]\,[1+(d_A/R_s)^2]-(R_{ol}/(R_o R_l))^2]\,[1+(d_A/R_l)^2]\}^{-(A-1)/2}; \qquad (3)$$

$C_A = 1$ for a production of A-cluster, which can be considered "point-like" as compared with the PRF-size of the nucleon source ($d_A \ll R^*_i$).

In case of azimuthally dependent analysis, the substitution (1'') should be done in Eq. (3).

Note that Eq. (2) was obtained in [21] within the hydrodynamic motivated parametrization of the expanding locally equilibrated fireball, characterized by a local temperature T at the freeze-out longitudinal proper time $\tau = (t^2-z^2)^{1/2}$ with a Gaussian distribution of a width $\Delta\tau$ around a mean evolution time τ_0, a Gaussian width $\Delta\rho$ of the transverse radius $\rho = (x^2+y^2)^{1/2}$ distribution and a Gaussian width $\Delta\eta$ of the longitudinal space-time rapidity $\eta = \mathrm{arctanh}(z/t)$ distribution of the expanding matter, assuming so called Bjorken scaling of the longitudinal flow with the matter longitudinal rapidity: $\eta_l = \eta$ and power-law transverse flow profile with the matter transverse rapidity: $\eta_t = \eta_f (\rho/\Delta\rho)^n$, usually chosen linear: $n = 1$. The transverse characteristics are assumed to be decoupled from the longitudinal ones, i.e., independent of τ and η; also, see [ame06, ame08] for more details, including consideration of Hubble like flow and azimuthally dependent non-central collisions.

Within this model, one may generalize the approximate expressions (2.17) in [21] for the LCMS femtoscopic radii at $n = 1$, transverse velocity $\beta_t \to 0$ and rapidity $y \to 0$ in the fireball rest frame (usually coinciding with the nucleon-nucleon c.m.s.), to moderate β_t and y as:

$$R_o^2 = R_s^2 + (\beta_t \Delta\tau)^2[1 + \Delta\eta^2/(\Delta\eta^2 m_t/T+1) + y^2/(\Delta\eta^2 m_t/T+1)^2],$$

$$R_s^2 = \Delta\rho^2/(\eta_f^2 m_t/T+1),$$

$$R_l^2 = \tau_0^2\{\Delta\eta^2/(\Delta\eta^2 m_t/T+1) + y^2[(\Delta\tau/\tau_0)/(\Delta\eta^2 m_t/T+1)]^2\},$$

$$R_{ol} = y\beta_t\{\Delta\tau^2[2(\Delta\eta^2 m_t/T+1) + \Delta\eta^2 + y^2/(\Delta\eta^2 m_t/T+1)]/2 + \tau_0^2\Delta\eta^2\} / (\Delta\eta^2 m_t/T+1)^2; \quad (4)$$

to simplify the notation, we put here $\tau_0^2 = \langle\tau^2\rangle = \langle\tau\rangle^2 + \Delta\tau^2$, which leads to τ_0 slightly (~1%) higher than $\tau_0 = \langle\tau\rangle$ used in Eq. (2.11) of [21]. Eqs. (4) indicate the so called m_t scaling of the side and longitudinal LCMS femtoscopic radii and a decrease of the measured radii with increasing m_t, roughly as $m_t^{-\alpha}$ with $\alpha \lesssim 0.5$, in agreement with experimental observations.

Note that the momentum (p_t, y)-dependence of the femtoscopic radii in the considered model is generated by the x-p correlation due to the transverse and longitudinal fireball expansion. It leads to a suppression of the radii with increasing transverse momentum and a strong decrease of the longitudinal radius with the rapidity $|y|$ in the fireball rest frame. The latter decrease is compensated in the LCMS by the longitudinal Lorentz factor $\cosh(y)$, while a decrease of the LCMS out radius with p_t is overcompensated in the pair rest frame by the transverse Lorentz factor γ_t, leading to R_o^* increasing with p_t.

In [21], the rapidity dependence of the out and longitudinal radii, as well as the second term in the expression for out radius, were omitted due to assumed small $|y|$ and β_t. As a result, $\gamma_t \to 1$, $R_o^2 \to R_s^2$, $R_{ol}^2 \to 0$, and the deuteron coalescence parameter B_2 obtained within this model (see Eq. (4.12) and (6.3) in [21]) coincides, at $\Lambda_A = 1$, with the general one in Eqs. (1) or (2), (3').

Note that the predicted linear rapidity rise and sign change at $y = 0$ of the mixed out-longitudinal squared radius R_{ol} and parabolic in y rise of out squared radius R_{oo} has been recently confirmed by the pion LCMS radii measured by STAR in Au+Au collisions at 3 GeV c.m.s. nucleon-nucleon energy [kra23].

The side radius appears to be rapidity independent (as expected in the case of decoupled transverse and longitudinal matter characteristics) in non-central collisions, while at 0-10% and 10-30% centralities a parabolic decrease with rapidity y is observed, thus indicating a decrease of the Gaussian transverse radius $\Delta\rho$ with the increasing longitudinal space-time rapidity $|\eta|$ in the collisions at moderate energies.

Contrary to the parabolic rapidity increase of the longitudinal squared radius R_{ll} in Eq. (4), the measured y-dependence of the R_{ll} indicates a more complicated behavior (pointing to slight enhancements at edge and middle rapidities), thus again indicating a violation of the decoupling of the transverse and longitudinal matter characteristics at moderate energies.

As for the size of R_{ol}, it appears to be less than a quoter of the squared diagonal radii R_{ii} at midrapidities $|y| < 0.5$ and $p_t = 0.15$-0.6 GeV/c ($\beta_t > 0.73$). Using the above limit, the account of $(R_{ol}/(R_oR_l))^2$ in Eq. (1') would lead to an increase of B_A for deuterons (tritons) less than 3% (6%). In fact, due to much smaller $|y|\beta_t$ for nucleons, this increase will be substantially smaller. Therefore, at small nucleon momenta in fireball rest frame, one can safely neglect $(R_{ol}/(R_oR_l))^2$ in Eq. (1') or (3) and rewrite the quantum correction factor C_A in Eq. (3) as:

$$C_A \cong \{[1+ (d_A/(\gamma_t R_o))^2] [1+ (d_A/R_s)^2] [1+ (d_A/R_l)^2]\}^{-(A-1)/2}. \qquad (3')$$

To account for a more realistic transverse density profile as compared with the Gaussian one, Eq. (3') can be amended by an exponential factor $\exp[b A (m_t - m)]$, multiplying C_A in Eq. (3'):

$$C_A \cong \exp[b A (m_t - m)]\{[1+ (d_A/(\gamma_t R_o))^2] [1+ (d_A/R_s)^2] [1+ (d_A/R_l)^2]\}^{-(A-1)/2}, \quad (3'')$$

as suggested in Eq. (6.4) of [21] with $b = (1/T_p - 1/T_A)$ for a box-like transverse density profile (leading to different proton and A-cluster m_t-slopes $1/T_p$ and $1/T_A$). This intuitive factor retains Eq. (3') at small p_t and hopefully extends its validity to higher transverse momenta.

Using the m_t-scaling of the side and longitudinal LCMS radii, one may rescale the measured pion radii at 0-10% centrality and $m_t \approx 0.4$ GeV/c^2 ($R_s \approx 5$ fm, $R_l \approx 4$ fm) to $m_t \approx 1$ GeV/c^2 (using the same parameters as in [21], except for $\Delta\rho$ and τ_0, decreased to $\Delta\rho = 5.9$ fm, $\tau_0 = 7.7$ fm/c in order to describe the measured pion radii) and roughly estimate nucleon LCMS radii $R_o \approx R_s \approx 4.2$ fm, $R_l \approx 2.7$ fm in central Au+Au collisions at $\sqrt{s_{NN}} = 3$ GeV, 0-10% centrality and $m_t \approx 1$ GeV/c^2. Taking $d_d = 1.74$ fm ($d_t = 1.24$ fm), one then gets from Eq. (3') the corresponding deuteron (triton) coalescence parameter $C_A = 0.72$ (0.70).

As for the considered expanding fireball model, using parameters adjusted to higher (SPS) energies: $\Delta\eta = 1.3$, $\eta_f = 0.35$, $\Delta\rho = 7$ fm, $\tau_0 = 9$ fm/c, $\Delta\tau = 1.5$ fm/c, i.e., $R_o = R_s = 5.1$ fm, $R_l = 3.2$ fm at $\beta_t = y = 0$ [21], one finds somewhat higher deuteron (triton) coalescence parameter $C_A = 0.79$ (0.77), close to the result of [21]: 0.81+0.03-0.05 (0.78+0.05-0.06).

The measurement of the out, side and longitudinal nucleon radii from two-nucleon momentum correlations is not an easy task. Up to now, only one-dimensional analysis of two-nucleon correlation functions has been done, assuming spherically symmetric nucleon emission functions in PRF: $R_o^* = R_s = R_l = R_{inv}$.

The so-called invariant radii R_{inv}, extracted by STAR collaboration from two-proton momentum correlations in the beam energy scan of Au+Au collisions at proton p_t = 0.4-0.8 GeV/c and |y| < 0.5 are about 2.5 fm, 3.5 fm and 4.5 fm for 30-80%, 10-30% and 0-10% centralities, respectively, slightly increasing with energy [zbr11]. Unfortunately, the residual correlations have been taken into account for 62 GeV and 200 GeV data only, so that the results at lower energies (7.7, 11.5 and 39 GeV) are still preliminary.

Using the empirical formula [kis14]

$$R_{inv} \approx [(\sqrt{\gamma_t}\, R_o^2 + R_s^2 + R_l^2)/3]^{1/2}, \qquad (5)$$

one can calculate the proton R_{inv} in central collisions at small p_t and |y| from the proton LCMS out, side and longitudinal radii, estimated with the help of the expanding fireball model (Eq. (4)) from the measured pion radii for central Au+Au collisions at 3 GeV [kra23] and central Pb+Pb collisions at SPS energies [21] as $R_o \approx R_s \approx 4.2$ fm, $R_l \approx 2.7$ fm and $R_o = R_s = 5.1$ fm, $R_l = 3.2$ fm, respectively. The corresponding values of R_{inv} are 3.8 and 4.6 fm. They are in reasonable agreement with R_{inv} of ~ 4.5 fm from the two-proton femtoscopy of the most central Au+Au collisions at 62 and 200 GeV [zbr11], as well as, - with the expected slight decrease of R_{inv} with the energy $\sqrt{s_{NN}}$ going down to 3 GeV.

At $\gamma_t = 1$, $R_o = R_s$ and $R_l = [3R_{inv}^2 - 2R_s^2]^{1/2}$ (calculated according to Eq. (5)), Eqs. (3') at a given R_{inv} leads to a wide maximum of the quantum correction factor C_A at $R_s = R_l = R_{inv}$. One can thus use the measured R_{inv} to calculate C_A with a reasonable accuracy at moderate transverse momenta. E.g., putting $R_o \approx R_s \approx R_l \approx R_{inv}$ = 2-3-4 fm, Eq. (3') at $\gamma_t = 1$ yields C_A = 0.43-0.65-0.77 (0.38-0.62-0.76) for deuterons (tritons). From the measured coalescence parameter B_A one can then extract the homogeneity volume $V_{hom} = R_o R_s R_l$ using Eq. (2) and check the expected relation: $V_{hom} \lesssim R_{inv}^3$.

It should be stressed that, in the presence of a strong transverse flow, the substitution of the Gaussian transverse density profile by a box-like one leads to different out and side radii at non-zero p_t ($R_o < R_s$ at zero emission duration $\Delta\tau$) and different effective nucleon and A-cluster temperatures (inverse slopes) T_p and T_A [21], linearly increasing with A in agreement with experiment; for a Gaussian transverse density profile, these slopes are A-independent.

Though the box-like profile makes the analytical B_A calculation practically impossible, one may try to account for the violation of the Gaussian ansatz by the intuitive factor exp[b A (m_t - m)] introduced in Eq. (3''), similar to Eq. (6.4) of [21].

In fact, the measured coalescence factors show a faster increase with m_t as compared with the moderate one ($m_t^{b(A-1)}$, b $\lesssim$ 0.5), predicted by Eqs. (2) and (3'). I already mentioned the attempt in [zha22] to explain this fact by using the incorrect relation $R^*_o = R_o/\gamma_t$ instead of the true one $R^*_o = \gamma_t R_o$.

One should be however careful in the interpolation to $p_t \to 0$ and rather use the observed m_t - dependence of the coalescence parameter B_A than the introduced exponential factor, which may fail at higher transverse momenta.

To be more quantitative, instead of Eq. (7) in the paper, I would present coalescence parameter B_A in Eq. (2) and C_A given in Eq. (3'') with $b = (1/T_p - 1/T_A)$ for a box-like transverse density profile [21], making however a comment on its possible modification at higher transverse momenta and using b as a free parameter.

As the theoretical coalescence parameter B_A in (1) relates the produced numbers of A nucleons in continuous (at $p_i = P/A$) and discrete spectrum A-cluster (with momentum P), one should multiply B_A in (1) or (2) (or divide the experimental B_A) by the residual correlation function [lyu88, 20]

$$\Re_{res} = d^{3A}N/(d^3\boldsymbol{p}_1 .. d^3\boldsymbol{p}_A)/[(d^3N/d^3\boldsymbol{p}_1) .. (d^3N/d^3\boldsymbol{p}_A)],$$

measured outside the region of small relative velocities (affected by the FSI and QS correlations) and interpolated to zero relative velocities, responsible for the A-cluster formation. In heavy-ion collisions, one may expect independent nucleon production, so that $\Re_{res} \approx 1$.

Since B_A is determined based on proton spectra only, its value can be affected by a possible difference between proton and neutron production. On the assumption of independent nucleon production, the experimental B_A value should then be divided by the neutron to proton ratio R_{np} to the power N, where N is equal to the number of neutrons in the A-cluster. Assuming further the same neutron and proton spectra, R_{np} equals to the fugacity ratio $\lambda_n/\lambda_p = \exp[(\mu_n-\mu_p)/T]$, where μ_i are possibly different neutron and proton chemical potentials at the fireball freeze-out. The correction factor $(\lambda_n/\lambda_p)^N = \exp[N(\mu_n-\mu_p)/T]$ then coincides with the one in Eq. (6.5) of [21].

Taking into account that only a part of the neutron excess in the colliding nuclei is transferred to the fireball, one may estimate the upper limits of the fugacity ratio λ_n/λ_p for Ar+A collisions as 1.11, 1.15, 1.22, 1.30 and 1.37 for A = C, Al, Cu, Sn and Pb, respectively. Obviously, one may expect a noticeable neutron excess in the fireballs excited in collisions with heavy nuclei.

As for the transfer of the initial neutron excess to the fireball, it decreases with the collision energy and R_{np} approaches the initial value of ~1.5 in collisions of heavy ions at AGS energies only (see Fig. 25 of [41]).

The neglect of the correction factor $(\lambda_n/\lambda_p)^N$ leads to B_A overestimation and corresponding underestimation of $(V_{hom})^{A-1}$. The extracted homogeneity volume V_{hom} is underestimated by a factor of $(\lambda_n/\lambda_p)^{N/(A-1)}$. This factor is equal to λ_n/λ_p for deuterons or tritons; it is closer to unity, being $(\lambda_n/\lambda_p)^c$ with $½ \le c < 1$, for other clusters; e.g., $c = ½$ for He-3.
Note that for the considered Ar+A collisions, the underestimation of the deuteron and triton R_{coal} radii due to the neglected correction factor $(\lambda_n/\lambda_p)^{1/3}$ is expected less than 10%.

Since UrQMD estimate of R_{np} is already used to calculate the proton phase-space density according to paper Eq. (10), one could do the same to correct the measured coalescence parameters B_A (see below).

As for the suppression factor $\Lambda_A \approx \Lambda_N^A$, caused by a non-Gaussian tail of the nucleon emission function, it is substantially lower than unity at high energies due to a significant fraction of the nucleons from hyperon decays ($\Lambda_N \approx 0.5$ in Au+Au collisions at top RHIC energies [zbr11, ada06] and 0.7-0.8 in p+Pb and S+Pb collisions at top SPS energies [bog99]). At our energy, Λ_N is closer to unity and its neglect, leading to overestimation of V_{hom} by a factor $\Lambda_N^{-A/(A-1)}$, may be justified. The corresponding overestimation of R_{coal} is maximal for deuterons, composing $(\Lambda_N^{-2/3} - 1)$ = 7-16-27 % for Λ_N = 0.9-0.8-0.7.

Note that the correction factors $(\lambda_n/\lambda_p)^N > 1$ and $\Lambda_N^A < 1$ partly compensate each other.

Finally, since the homogeneity volume $V_{hom} = R_o R_s R_l$, extracted from the coalescence parameter B_A, is close to R_{inv}^3 and represents its lower estimate, it is quite natural (instead of multiplying V_{hom} by rather arbitrary factor 3/2) to introduce the coalescence radius

$$R_{coal} = V_{hom}^{1/3},$$

allowing one (with such a definition of R_{coal}) to directly check the relation $R_{coal} \lesssim R_{inv}$.

The above definition of R_{coal} was used in Eq. (B.5) of [bel21] with a misleading notation for it (R_{inv} instead of R_{coal}).

Note that such R_{coal} values, recalculated from Fig. 15 (divided by the factor $1.145 = (3/2)^{1/3}$) lead to R_{coal} range of 2.6-3.8 fm.

For heavy systems, the R_{coal} values agree with R_{inv} from two-proton femtoscopy of central heavy-ion collisions. Unfortunately, there is no femtoscopy information about Ar+A collisions to be compared with R_{coal} = 2.6-2.8 fm.

Assuming a volume A-scaling and scaling down by a factor $(A_{Ar}/A_{Au})^{1/3} = 0.59$ the radii R_{coal} (R_{inv}) = 3.6 (3.8) fm, estimated from the two-pion femtoscopy of central Au+Au collisions at 3 GeV [kra23], one arrives at R_{coal} (R_{inv}) = 2.1 (2.2) fm.
These numbers are close to Ar r.m.s. radius divided by $\sqrt{3}$ (composing 2.0 fm) and thus look too small for Ar collisions with heavier nuclei, indicating that the volume A-scaling of V_{hom} leads to underestimation of R_{coal}.

A more appropriate may be surface $A^{2/3}$ scaling of V_{hom}, retaining the original longitudinal radius; indeed, multiplying R_{coal} by a factor $(A_{Ar}/A_{Au})^{2/9} = 0.70$, leads to R_{coal} (R_{inv}) = 2.5 (2.7) fm in agreement with the extracted R_{coal} values.
However, neglecting the expansion of the collision zone, the geometrical volume and surface scalings should be taken with a caution.

Comments, questions and suggestions

l. 317:
it would be interesting to compare T_0 -values with T* from Eq. (5);
however, reference [37] is not available;

note that the single-exponential fit is justified at $m_t >> m$ only and that T_0 should then coincide with the blue shifted temperature in Eq. (5) (Eq. (19) in [sch93]).

Answer:
According to (3)
⟨E_T ⟩ = ⟨m_T ⟩ − m = T0 + T0^2 /(T0 + m) --> <E_T> = 2 T0 for m = 0, i.e. corresponding to 3/2T* in formula (4)
T0 is the m_t inverse slope measured for a particle with mass m (p,d,t), whereas T* is the inverse slope extrapolated to m = 0, i.e. T0 and T* values have different meaning.

Comment:
Eq. (8) below shows that < Et > in BW model is linear in the cluster mass m at sufficiently high m ($m \gtrsim m_p$) only, so that the linear m-dependence cannot be extrapolated to m=0.
As for Eq. (4) in the paper, see below the extended comment about its incorrectness.

Note that BW model predicts the effective temperature T0 decreasing with mt, i.e., depending on the fit Et-interval. From Fig. 7, one may see that for protons, deuterons and tritons, the fit intervals are 0-0.5 GeV, 0-0.4 GeV and 0-0.35 GeV, respectively.

The respective Et-variations of T0 in BW model (see Eq. (6) below) with T=100 MeV and β_s = 0.4 (<β> = 0.267) are: 157-151 MeV, 230-183 MeV and 345-227 MeV, thus pointing to a possible problem with the <Et> extraction based on the single-exponential fits for deuterons and tritons.

l. 357:
perhaps a comment about the accuracy of Eq. (3) (based on a single-exponential fit) could be given;
note that in [41] the measured m_t points were combined with a two-exponential fit to determine $<m_t>$.

Answer:
A single-exponential fit gives reasonable description of the m_t spectra in the BMN pt range, see Fig.7. We see no reason to use 2-exponential fit. We can add a sentence on that point.

Comment:
A reason might be a systematical error in the extracted temperature T due to neglected mt-dependence of the effective temperature T0.

Eqs. (4), (5):
a successful use in [41] of these equations to determine T and <β> in agreement with BW model is likely accidental, as Eq. (4) combines non-relativistic (3/2T*) and relativistic (γ) terms and Eq. (5) is valid in relativistic limit only (see Eq. (19) in [sch93]).
It would be more correct to use BW model to extract T and <β> (as done in [41]).

Answer:
Attempts to use the BW model for the fit do not give stable results. A parameterization with the T* and beta terms gives a robust result.

Comment:
Below, within the BW model, a dependence of <Et> on the temperature T and the transverse flow velocity β is given in Eq. (8) and its difference from the paper Eq. (4) points to the incorrectness of the latter.

One can use Eqs. (6) and (7) from [sch93] (implemented as Eq. (8) in [41]) in a combined fit of the m_t spectra of protons, deuterons and tritons to determine the temperature T and fireball-surface velocity β_s at a given flow profile power n
(a linear profile n=1 is often used; parabolic profile n=2 is used in [sch93];
note that $<\beta_r> = \beta_s\, 2/(2+n)$).

Another possibility is a combined fit of the "measured" mean transverse kinetic energies

$$<E_t> = < m_t> - m,$$

using Eq. (7) of [sch93] (Eq. (8) in [41]) to calculate $<E_t>$ on the assumption of a box-like transverse density profile:

$$<E_t> = \int_m^\infty \mathrm{dm_t} \int_0^R dr\ \mathrm{r\ E_t\ m_t\ BesselK[1,\ mt\ Cosh[\eta/T]\ BesselI[0,\ p_t\ Sinh[\eta/T]/}$$
$$\int_m^\infty dm_t \int_0^R dr\ \mathrm{r\ m_t\ BesselK[1,\ mt\ Cosh[\eta/T]\ BesselI[0,\ p_t\ Sinh[\eta/T],} \qquad (6)$$

where $\eta = \mathrm{ArcTanh}[\beta_s\,(r/R)^n]$ is the collective transverse flow rapidity.

Such a fit of the integrated data, though less informative and suffering from the approximate $<E_t>$ determination based on an exponential interpolation to the unmeasured m_t-region, may appear more robust. In the present paper, it was done using paper Eqs. (4) and (5). In fact, instead of these incorrect equations, one should fit the parameters T and β_s, using Eq. (6).

To simplify the extraction of these parameters, one may consider the limit of a large mass-to-temperature ratio $z = m/T >> 1$ and a small squared surface velocity $\beta_s^2 << 1$ and calculate <Et> up to terms $O(1/z^2)$ and $O(\beta_s^4)$ as:

$$<E_t> = T\{[1+3/(2z)] + \beta_s^2 z(1+1/z)(1+3/z)/[2(n+1)]\}. \qquad (7)$$

Note that Eq. (7) slightly (negligibly) differs from previous Eq. (8) due to the account of the higher order T/m_t-terms in the m_t-integration, leading to the substitution $(15+16z+4z^2) \rightarrow (12+16z+4z^2) = 4(1+z)(3+z)$.

For $m = m_p$, m_d and m_t, the relative differences of Eq. (7) from the exact values calculated according to Eq. (6) at T = 100 MeV and $\beta_s = 0.4$ ($\beta_s = 0$) compose:
-2.23% (+1.12%), -4.27% (+0.30%) and -5.27% (+0.14%), respectively.

A better approximation,

$$<E_t> = T\{[1+3/(2z)-9/(8z^2)] +$$
$$\beta_s^2 z[(1+1/z)(1+3/z)-9/(2z^3)]/[2(n+1)]+$$
$$\beta_s^4 z[(3+n(6+5n))+(9+n(18+17n))/z+3(3+n(6+7n))/(8z^2)-$$
$$9(1+n(2+9n))/(8z^3)]/[8(1+n)^2(1+2n)]\}, \qquad (8)$$

which is valid up to terms $O(1/z^3)$ and $O(\beta_s^6)$, allows one to decrease the above relative differences below a percent level; they compose -0.36% (+0.00%), -0.51% (+0.00%) and -0.61% (+0.00%), respectively. At typical temperatures of a hundred MeV, one can thus use Eq. (8) for practical calculations at small or moderate surface velocities β_s.

Note that, at temperatures T of a hundred MeV, the β_s^2-terms in Eqs. (7) and (8) are nearly linear in the cluster mass m down to proton mass and that they approximately coincide with $m(\gamma-1) \approx m<\beta>^2/2$ in paper Eq. (4) for a constant (n=0) collective transverse velocity only. Also, the m-independent and <β>-dependent first term in paper Eq. (4) differs from the first terms in square brackets in Eqs. (7) and (8), which are m-dependent and <β>-independent.

Using the linear fits from Fig. 12, one can calculate $<E_t>$ at deuteron and triton masses and solve the corresponding two equations (7) for two variables:
T and $b = \beta_s(n)/(n+1)^{1/2} = \beta_s(0) = <\beta>_0$.
See the results below and note that $<\beta>_n = \beta_s(n)\ 2/(n+2) = b\ 2(n+1)^{1/2}/(n+2)$:

	$<E_t(m_d)>$ MeV	$<E_t(m_t)>$ MeV	T MeV	$<\beta>_0$	$<\beta>_1$	$<\beta>_2$
Ar+Al	209	227	134	0.222	0.210	0.193
Ar+Cu	227	258	124	0.276	0.260	0.239
Ar+Sn	237	286	101	0.334	0.315	0.289
Ar+Pb	223	258	114	0.289	0.272	0.250

Compared with paper Table 3, one may see that T extracted from the BW model is about 30 MeV higher, while the mean velocity is about the same (quite the same for n=1 flow profile).

The lower collective transverse velocity <β>, as compared with the results at SPS energies [41, alt08], is in agreement with the longer re-scattering phase and thus more thermal energy transferred to the collective flow at higher collision energies.

A problem represents the temperature parameter T, which is expected to decrease with decreasing collision energy and, at our energy, one may expect $T \lesssim 100$ MeV. The extracted larger temperature parameters may thus indicate systematic errors of the experimental $<E_t>$-values, extracted from the single-exponential Et-fits.

One can estimate possible systematic errors in the parameters β_s and T, extracted from the <Et> values with the help of Eq. (8), comparing them with those obtained from combined PDT fits of the proton, deuteron and triton m_t-spectra.

The <Et> values, reported in paper Fig. 12 at $y_{CMS}=0$ (y=1.08), were obtained from single-exponential fits by averaging the data at y= 1.0, 1.2 and 1.4. To avoid a problem of averaging y-dependent data with different p_t-acceptance, I have used BMN $dN/dm_t dy$ data at y=1.4 with the largest p_t-acceptance, allowing for the most reliable m_t-fits.
In principle, the data at y= 1.0 and 1.2 could be included in the combined m_t-fits on the assumption of y-independent parameters β_s and T – the results are added at the end.

The fit results for Al, Cu, Sn and Pb targets using the single-exponential in m_t and BWM with a linear transverse velocity profile (n=1) are given in tables below;

the three <Et> values for the combined PDT fits correspond to protons, deuterons and tritons, respectively; the latest data from October 2024 and total (stat+syst) errors for m_t -spectra are used; the 2nd (blue) rows show T and <Et> with total errors from Mikhail's fits using statistical errors for m_t -spectra:

Ar+Al y=1.4

		χ^2/ndf	β_s	T MeV	<Et> MeV
Exp[-m_t/T]	P	9.4/ 8	-	159.6± 4.3	182.8 ± 5.5
				160.8± 3.9	184.3 ± 5.0
	D	2.8/ 8	-	192.6±18.6	210.5 ± 21.9
				191.5±17.4	209.2 ± 20.5
	T	4.9/ 4	-	127.9±37.0	133.5 ± 40.2
				129.5±36.9	135.2 ± 40.1
	PDT	21.3/22	-	161.8± 4.2	185.6 174.7 170.6
BWMn=1	P	9.3/ 8	.0	150.2± 3.8	182.0 ± 5.6
		9.5/ 8	.3	125.7± 3.8	183.5
	D	2.8/ 8	.0	184.6±17.2	209.8 ± 22.3
		2.9/ 8	.3	136.9±17.4	209.1
	T	4.9/ 4	.0	125.3±35.5	133.4 ± 40.3
		5.3/ 4	.3	58.2±34.1	132.3
	PDT	22.2/22	.0	152.5± 3.7	185.2 170.0 164.4
		19.8/22	.3	126.0± 3.7	
	PDT	19.8/21	.279±.081	129.7±14.3	184.3 191.7 206.5
Eq. (8) & <Et> BWMn=1 PDT		0.00/1	.279±.094	129.7±16.1	
Eq. (8) & <Et> Exp[-m_t/T] P,D,T		3.59/1	.206±.131	139.3±16.7	

Ar+Cu y=1.4

		χ^2/ndf	β_s	T MeV	<Et> MeV
Exp[-m_t/T]	P	4.8/8	-	170.1± 3.9	196.2 ± 5.0
				171.5± 3.7	198.0 ± 4.8
	D	3.0/8	-	200.4±16.9	219.8 ±20.0
				201.6±16.4	221.1 ±19.4
	T	1.6/4	-	190.8±34.2	202.9 ±38.4
				184.4±31.3	195.8 ±35.1
	PDT	13.2/22	-	184.4± 3.8	214.7 200.9 195.8
BWMn=1	P	4.9/8	.0	159.5± 3.5	195.1 ± 5.3
		4.8/8	.3	135.0± 3.5	197.1
	D	3.0/8	.0	191.7±15.5	218.8 ±20.3
		3.1/8	.3	143.8±15.9	218.2

	T	1.6/4	.0	185.2±32.3	202.6 ±38.7	
		1.4/4	.3	107.3±31.3	190.4	
	PDT	14.8/22	.0	161.9± 3.4	198.5	181.5 175.3
	PDT	10.2/21	.290±.061	137.1±11.5	197.7	205.4 221.3
Eq. (8) & <Et> BWMn=1 PDT		0.00/1	.290±.084	137.2±15.0		
Eq. (8) & <Et> Exp[-m_t/T] P,D,T		0.75/1	.308±.078	133.4±14.8		

y=1.4 Ar+Sn

		χ^2/ndf	β_s	T MeV	<Et> MeV
Exp[-m_t/T]	P	2.5/8	-	168.7± 4.5	194.4 ± 5.8
				171.3± 4.7	197.7 ±6.1
	D	2.6/8	-	225.2±16.0	249.3 ±19.3
				216.5±14.3	238.9 ±17.3
	T	3.7/4	-	180.5±40.5	191.4 ±45.2
				203.4±41.7	217.1 ±46.6
	PDT	23.9/22	-	177.9± 5.1	206.3 193.3 188.5
BWMn=1	P	2.5/8	.0	158.5± 4.6	193.6 ± 6.9
		2.6/8	.3	133.6± 4.6	195.1
	D	2.6/8	.0	214.3±14.6	247.9 ±19.6
		2.6/8	.3	166.3±16.6	248.0
	T	3.7/4	.0	175.4±38.3	191.1 ±45.5
		4.1/4	.3	107.9±41.8	191.1
	PDT	26.7/22	.0	167.5± 4.6	206.4 188.4 181.8
	PDT	11.7/21	.385±.036	116.9±10.4	194.9 223.0 257.3
Eq. (8) & <Et> BWMn=1 PDT		0.00/1	.387±.064	116.9±15.2		
Eq. (8) & <Et> Exp[-m_t/T] P,D,T		3.97/1	.393±.063	115.8±15.1		

Ar+Pb y=1.4

		χ^2/ndf	β_s	T MeV	<Et> MeV
Exp[-m_t/T]	P	4.1/8	-	177.6± 7.1	205.9 ± 9.2
				169.2± 6.8	195.0 ± 8.8
	D	.46/8	-	188.9±29.6	206.2 ±34.8
				190.9±27.7	208.5 ±32.5

	T	4.0/4	-	209.5±46.4	224.0 ±52.6	
				193.9±42.6	206.4 ±48.3	
	PDT	9.2/22	-	179.4± 6.9	208.2 195.1 190.2	
BWMn=1	P	4.4/8	.0	166.4± 6.3	204.9 ± 9.7	
		3.9/8	.3	141.5± 6.3	206.7	
	D	.47/8	.0	181.0±27.3	205.3 ±35.2	
		.49/8	.3	132.7±27.9	203.7	
	T	4.0/4	.0	202.7±43.6	223.4 ±53.0	
		3.9/4	.3	124.3±43.9	211.1	
	PDT	10.0/22	.0	168.4± 6.1	207.7 189.5 182.9	
	PDT	8.4/21	.267±.105	146.7±18.5	206.4 209.2 221.2	
Eq. (8) & <Et> BWMn=1 PDT		0.00/1	.267±.140	146.8±23.9		
Eq. (8) & <Et> Exp[-m_t/T] P,D,T		0.01/1	.268±.140	146.3±23.9		

One may conclude:

- the results of single-exponential fits from Mikhail (blue) differ because of used statistical errors in fitting the m_t -spectra; if doing the same, the fitted T $\pm$ σT values coincide;
- the single-exponential fits, as well as the similar BWM fits with β_s = 0 and 0.3, yield the expected <Et> increase with the cluster mass for Ar+Pb data only;
- a more complex BWM m_t-distribution with β_s = 0.3 (which may be considered as a combination of exponential distributions) yields the same <Et>-values within the errors, thus demonstrating that the single-exponential parametrization is sufficient to estimate <Et> from the present BMN data;
- both the <Et>-fits according to Eq. (8) and the combined PDT m_t-fits appear to be quite robust; taking into account systematic errors, they yield χ^2/ndf $\lesssim$ 1, except for the <Et>-fits of Al and Cu data, when χ^2/ndf $\approx$ 4/1;
- a sufficient accuracy of Eq. (8) is confirmed by coinciding β_s and T parameters from the combined PDT m_t-fits and the <Et>-fits with the <Et> values determined in the combined PDT m_t-fits;
- the <Et>-fits with the <Et>-values from exponential m_t-fits recover within the errors the β_s and T parameters from the combined PDT m_t-fits; the parameter errors in the <Et>-fits are 20-70% higher due to integrated out a part of the information;
- typically, $\beta_s \approx 0.3$ and T $\approx$ 120-150 MeV in the combined PDT m_t-fits, with 10-40% and $\sim$10% errors, respectively.

Some more remarks:

- for proton spectra from Al and Cu targets, the exponential fits were also done with the centers of the m_t-bins shifted to those of the p_t-bins, leading to a negligible T-increase by 0.2 MeV; also, a negligible T-change of -0.2 (+0.5) MeV for Al (Cu) targets is obtained by removing the last m_t-point;
- there is almost 100% anticorrelation of the β_s and T parameters in the separate proton, deuteron and triton m_t-fits, thus making impossible the determination of

these parameters with reliable errors in such fits; this problem is cured in the combined PDT m_t-fits;

- one may use the <Et> fits as an alternative to the combined PDT m_t-fits; I would however prefer combined PDT fits, which are more informative, substantially robust and avoid the assumption of exponential form of the m_t-spectra, used to extract <Et>;
- there was a problem in Mathematica with the fitting function represented by a numerical integral; a huge time was required, since Mathematica automatically increased precession to achieve a reliable calculation of the derivatives and covariance matrix; the problem is solved by constructing a sufficiently accurate interpolating analytical function for $dN/(m_t dm_t dy)$ from BWM, using the expansion in powers of the surface velocity β_s; the expansion up to β_s^{16} was required to interpolate the BWM function with sufficient accuracy up to $\beta_s \sim 0.6$, guaranteeing the correct fit parameters and their covariance matrix at typical $\beta_s \sim 0.3$;
- Vadim's results point to an accuracy problem with the Minuit (which is less elaborated than the Mathematica fit procedure) due to too small errors in β_s and T parameters in separate proton, deuteron and triton fits and absent anticorrelation of these parameters in the covariance matrix; this problem can likely be overcome by using the interpolating function F for $dN/(m_t dm_t dy)$ from BWM;
- in Mathematica notation, F reads as
(bs = β_s, mt = m_t, m is the cluster mass and N is the normalization parameter; N is multiplied by Exp[m/T] to compensate Exp[-m/T] from the Bessel functions):

```
 F[mt_, m_, N_, bs_, T_, n_] :=
(zt = mt/T; vt = Sqrt[mt^2 - m^2]/mt;

N Exp[m/T] T zt (BesselK[1, zt]/2
+
    bs^2 ((2 + vt^2 zt^2) BesselK[1, zt] -
     2 zt BesselK[2, zt])/(8 (1 + n))
+
    bs^4 ((24 + 8 (1 + 3 vt^2) zt^2 +
       vt^4 zt^4) BesselK[1, zt] -
     8 zt (2 + vt^2 zt^2) BesselK[2, zt])/(128 (1 + 2 n))
+
    bs^6 ((720 + 216 (2 + 5 vt^2) zt^2 +
       18 vt^2 (4 + 5 vt^2) zt^4 + vt^6 zt^6) BesselK[1, zt] –
       6 zt (72 + 8 (1 + 9 vt^2) zt^2 +
       3 vt^4 zt^4) BesselK[2, zt])/(4608 (1 + 3 n))
 +
    bs^8 ((40320 + 11520 (3 + 7 vt^2) zt^2 +
       96 (4 + 15 vt^2 (8 + 7 vt^2)) zt^4 +
       32 vt^4 (9 + 7 vt^2) zt^6 + vt^8 zt^8) BesselK[1, zt] –
       32 zt (720 + 24 (8 + 45 vt^2) zt^2 +
       6 vt^2 (4 + 15 vt^2) zt^4 + vt^6 zt^6) BesselK[2, zt])
+
    bs^10 ((3628800 + 1008000 (4 + 9 vt^2) zt^2 +
```

```
      960 (116 + 525 vt^2 (4 + 3 vt^2)) zt^4 +
      2400 vt^2 (4 + 21 vt^2 (2 + vt^2)) zt^6 +
      50 vt^6 (16 + 9 vt^2) zt^8 + vt^10 zt^10)
      BesselK[1, zt] –
      10 zt (201600 + 2304 (39 + 175 vt^2) zt^2 +
      96 (4 + 240 vt^2 + 525 vt^4) zt^4 +
      160 vt^4 (3 + 7 vt^2) zt^6 + 5 vt^8 zt^8)
      BesselK[2, zt])/(29491200 (1 + 5 n))
+
   bs^12 ((479001600 + 130636800 (5 + 11 vt^2) zt^2 +
      17280 (1784 + 1575 vt^2 (16 + 11 vt^2)) zt^4 +
      11520 (4 + 468 vt^2 + 2835 vt^4 + 1155 vt^6) zt^6 +
      5400 vt^4 (16 + 96 vt^2 + 33 vt^4) zt^8 +
      72 vt^8 (25 + 11 vt^2) zt^10 + vt^12 zt^12)
      BesselK[1, zt] -
      24 zt (10886400 + 5760 (1208 + 4725 vt^2) zt^2 +
      2880 (28 + 948 vt^2 + 1575 vt^4) zt^4 +
      1440 vt^2 (4 + 80 vt^2 + 105 vt^4) zt^6 +
      50 vt^6 (16 + 27 vt^2) zt^8 + 3 vt^10 zt^10)
      BesselK[2, zt])/(4246732800 (1 + 6 n))
+
   bs^14 ((87178291200 + 23471078400 (6 + 13 vt^2) zt^2 +
      241920 (39992 + 24255 vt^2 (20 + 13 vt^2)) zt^4 +
      40320 (976 + 63672 vt^2 + 8085 vt^4 (36 + 13 vt^2)) zt^6 +
      35280 vt^2 (64 + 2352 vt^2 + 7920 vt^4 + 2145 vt^6) zt^8 +
      1176 vt^6 (400 + 1650 vt^2 + 429 vt^4) zt^10 +
      98 vt^10 (36 + 13 vt^2) zt^12 + vt^14 zt^14)
      BesselK[1, zt] -
      14 zt (3353011200 + 138240 (20429 + 72765 vt^2) zt^2 +
      17280 (3464 + 86912 vt^2 + 121275 vt^4) zt^4 +
      11520 (4 + 756 vt^2 + 8337 vt^4 + 8085 vt^6) zt^6 +
      840 vt^4 (144 + 1600 vt^2 + 1485 vt^4) zt^8 +
      168 vt^8 (25 + 33 vt^2) zt^10 + 7 vt^12 zt^12)
      BesselK[2, zt])/(832359628800 (1 + 7 n))
+
    bs^16 ((20922789888000 + 5579410636800 (7 + 15 vt^2) zt^2 +
      232243200 (15388 + 21021 vt^2 (8 + 5 vt^2)) zt^4 +
      77414400 (344 + 3 vt^2 (5496 + 7007 vt^2 (3 + vt^2))) zt^6 +
      161280 (64 + 15 vt^2 (1280 + 7 vt^2 (3680 + 9152 vt^2 +
                                                  2145 vt^4))) zt^8 +
      752640 vt^4 (48 + 944 vt^2 + 429 vt^4 (5 + vt^2)) zt^10 +
      18816 vt^8 (100 + 312 vt^2 + 65 vt^4) zt^12 +
      128 vt^12 (49 + 15 vt^2) zt^14 + vt^16 zt^16)
      BesselK[1, zt] –
      128 zt (87178291200 + 14515200 (6320 + 21021 vt^2) zt^2 +
      3628800 (824 + 17036 vt^2 + 21021 vt^4) zt^4 +
      201600 (32 + 3288 vt^2 + 26376 vt^4 + 21021 vt^6) zt^6 +
```

```
5040 vt^2 (64 + 7 vt^2 (528 + 3184 vt^2 + 2145 vt^4)) zt^8 +
1176 vt^6 (80 + 600 vt^2 + 429 vt^4) zt^10 +
98 vt^10 (12 + 13 vt^2) zt^12 + vt^14 zt^14)
BesselK[2, zt])/(213084064972800 (1 + 8 n)))
 )
```

Fits of the combined m_t-spectra in "central" rapidities y = 1.0, 1.2 and 1.4 (1.1 and 1.4 for tritons), assuming y-independent parameters β_s and T, but different normalization parameters at each rapidity:

Ar+C y=1.0 (0.9 for deuterons), 1.2, 1.4 (1.1 and 1.4 for tritons)

		χ^2/ndf	β_s	T MeV	<Et> MeV
Exp[-m_t/T]	P	21.0/20	-	150.7± 5.8	171.6 ± 7.3
	D	17.6/22	-	128.0±14.3	136.2 ± 16.1
	T	4.6/6	-	159.4±53.2	168.0 ± 58.8
	PDT	44.8/50	-	148.7± 5.3	169.0 159.6 156.2
BWMn=1	P	20.9/20	.0	141.9± 5.1	170.5 ± 7.4
		21.0/20	.3	117.9± 5.2	172.3
	D	17.6/22	.0	124.2±13.5	135.9 ± 16.2
		18.3/22	.3	77.3±13.8	134.0
	T	4.6/6	.0	155.4±50.6	167.8 ± 59.0
		4.4/6	.3	74.2±44.6	150.9
	PDT	44.3/50	.0	140.5± 4.8	168.5 155.4 150.6
	PDT	44.3/49	.001±44.0	140.5±16.6	168.5 155.4 150.6
Eq. (8) & <Et> BWMn=1 PDT		0.00/1	.004±6.30	140.5±18.3	
Eq. (8) & <Et> Exp[-m_t/T] P,D,T		1.7/1	.000±$8\cdot10^7$	140.4±18.3	

Ar+Al y=1.0, 1.2, 1.4 (1.1 and 1.4 for tritons)

		χ^2/ndf	β_s	T MeV	<Et> MeV
Exp[-m_t/T]	P	105.6/20	-	160.5± 3.2	184.0 ± 4.1
	D	11.5/25	-	193.4±13.8	211.5 ± 16.3
	T	7.5/9	-	163.1±29.5	172.1 ± 32.7
	PDT	131.2/56	-	163.0± 3.1	187.1 176.0 171.9
BWMn=1	P	106.0/20	.0	150.7± 2.8	182.7 ± 4.2
		105.3/20	.3	126.7± 2.9	185.0
	D	11.6/25	.0	185.2±12.7	210.6 ± 16.5
		11.7/25	.3	137.2±13.0	209.5
	T	7.5/9	.0	158.9±28.0	171.8 ± 32.8

		7.9/9	.3	87.1±29.5	166.2		
	PDT	134.0/56	.0	153.3± 2.8	186.3	170.9	165.3
	PDT	126.8/55	.291±.053	128.6± 9.8	185.5	194.8	211.4
Eq. (8) & <Et> BWMn=1 PDT		0.00/1	.291±.070	128.6±12.4			
Eq. (8) & <Et> Exp[-m_t/T] P,D,T		2.6/1	.289±.070	128.3±12.4			

Ar+Cu y=1.0, 1.2, 1.4 (1.1 and 1.4 for tritons)

		χ^2/ndf	β_s	T MeV	<Et> MeV		
Exp[-m_t/T]	P	99.4/20	-	169.4± 2.5	195.3 ± 3.2		
	D	6.5/25	-	202.6±12.3	222.4 ±14.6		
	T	6.9/9	-	208.5±30.9	222.9 ±35.0		
	PDT	122.8/56	-	171.6± 2.5	198.1	186.0	181.5
BWMn=1	P	100.6/20	.0	158.4± 2.2	193.5 ± 3.3		
		98.8/20	.3	134.7± 2.3	196.7		
	D	6.6/25	.0	193.6±11.3	221.2 ±14.8		
		6.7/25	.3	145.7±11.6	220.7		
	T	6.7/9	.0	201.8±29.4	222.4 ±35.7		
		6.6/9	.3	124.3±30.1	211.1		
	PDT	191.5/56	.0	160.7± 2.2	196.8	180.0	173.9
	PDT	112.9/55	.316±.040	132.2± 8.1	197.1	209.7	230.0
Eq. (8) & <Et> BWMn=1 PDT		0.00/1	.317±.061	132.2±11.5			
Eq. (8) & <Et> Exp[-m_t/T] P,D,T		.54/1	.353±.053	124.2±11.2			

Ar+Sn y=1.0, 1.2, 1.4 (1.1 and 1.4 for tritons)

		χ^2/ndf	β_s	T MeV	<Et> MeV		
Exp[-m_t/T]	P	54.8/20	-	169.6± 3.3	195.6 ± 4.2		
	D	19.7/25	-	238.2±11.2	265.0 ± 13.6		
	T	8.5/9	-	230.1±33.1	247.5 ± 37.9		
	PDT	135.9/56	-	181.8± 3.3	211.3	197.9	192.9
BWMn=1	P	55.5/20	.0	158.7± 3.0	193.9 ± 4.5		
		54.4/20	.3	134.9± 3.0	197.0		
	D	19.6/25	.0	226.0±10.1	263.1 ± 13.8		
		19.7/25	.3	178.2±10.4	264.1		
	T	8.5/9	.0	221.9±30.9	246.6 ± 38.2		

	8.6/9	.3	147.0±33.4	239.1		
PDT	147.4/56	.0	170.4± 3.0	210.6	192.0	185.2
PDT	86.3/55	.405±.020	112.8± 6.7	195.5	228.7	267.7

Eq. (8) & <Et>	0.00/1	.407±.045	112.8±11.2
BWMn=1 PDT			
Eq. (8) & <Et>	3.8/1	.479±.037	95.9±10.6
Exp[-m_t/T] P,D,T			

Ar+Pb y=1.0, 1.2, 1.4 (1.1 and 1.4 for tritons)

		χ^2/ndf	β_s	T MeV	<Et> MeV		
Exp[-m_t/T]	P	144.1/20	-	167.1± 2.7	192.4 ± 3.5		
	D	9.4/25	-	242.8±19.6	270.6 ±23.8		
	T	8.6/9	-	179.7±26.1	190.5 ±29.1		
	PDT	187.1/56	-	170.8± 2.7	197.1	185.1	180.6
BWMn=1	P	146.5/20	.0	156.3± 2.4	190.5 ± 3.6		
		142.7/20	.3	132.7± 2.4	193.7		
	D	9.5/25	.0	230.2±17.7	268.7 ±24.2		
		9.4/25	.3	182.9±18.2	270.5		
	T	8.6/9	.0	174.7±24.7	190.2 ±29.3		
		9.1/9	.3	100.0±26.4	181.6		
	PDT	194.4/56	.0	159.9± 2.4	195.6	179.0	173.0
	PDT	171.6/55	.342±.039	125.7± 8.9	194.8	212.9	238.2

Eq. (8) & <Et>	0.00/1	.343±.062	125.8±12.5
BWMn=1 PDT			
Eq. (8) & <Et>	8.7/1	.343±.062	124.5±12.5
Exp[-m_t/T] P,D,T			

Summary of the BWM fits of the m_t-data at y = 1.0 (0.9), 1.2, 1.4 (1.1 and 1.4 for tritons)
1st row – results of the combined PDT m_t-fit,
2nd row (blue) – results of the <Et>-fit, using Eq. (8) with <Et> calculated from the exponential m_t-fit

PDT	126.8/55	.291±.053	128.6± 9.8	185.5	194.8	211.4

Eq. (8) & <Et>	2.6/1	.289±.070	128.3±12.4

	Ar+C	Ar+Al	Ar+Cu	Ar+Sn	Ar+Pb
β_s	.001±44.0	.291±.053	.316±.040	.405±.020	.342±.039

	$.000\pm8\cdot10^{7}$	$.289\pm.070$	$.353\pm.053$	$.479\pm.037$	$.343\pm.062$
T, MeV	140.5 ± 16.6	128.6 ± 9.8	132.2 ± 8.1	112.8 ± 6.7	125.7 ± 8.9
	140.4 ± 18.3	127.0 ± 10.6	124.2 ± 11.2	95.9 ± 10.6	124.5 ± 12.5
χ^2/ndf	44.3/49	126.8/55	112.9/55	86.3/55	171.6/55
	1.7/1	2.6/1	.54/1	3.8/1	8.7/1

One may conclude:

- the single-exponential fits, as well as the similar BWM fits with β_s = 0 and 0.3, yield the expected <Et> increase for deuterons, giving however <Et> for tritons smaller than for deuterons, thus leading to a large χ^2/ndf of 3-9/1 except for the Cu target;
- the <Et> fits according to Eq. (8) with the <Et>-values from exponential m_t-fits recover within one or two standard deviations the β_s and T parameters obtained in the combined PDT m_t-fits;
- the parameter errors in the <Et> fits are 10-80% higher than in the combined PDT m_t-fits;
- nearly the same parameters from the <Et> and combined PDT m_t-fits confirm sufficient stability of the latter, so that the use of the approximate robust <Et> procedure (which integrates out a part of the available information and leads to higher errors) is not required;
- typically, in the combined PDT m_t-fits, $\beta_s \approx$ 0.3-0.4 and T ≈ 110-130 MeV with about 5-18% and 6-8% errors, respectively;
- the χ^2/ndf of 2- 3 in the combined PDT m_t-fits indicates an underestimation of the systematical errors and may thus call for an increase of the parameter errors by a factor $(\chi^2/ndf)^{1/2}$ of 1.4-1.7;
- the increased χ^2/ndf as compared with the fits at y=1.4 may indicate a violation of rapidity independence of the parameters β_s and T, when doing combined fits of the m_t-spectra at “central” rapidities y = 1.0, 1.2, 1.4 (1.1 and 1.4 for tritons).

Compared with Vadim’s fit results from August 26, one may see that he got:

- essentially smaller parameter errors;
  since a strong β_s - T anticorrelation should lead to huge parameter errors in separate fits of proton, deuteron and triton spectra, the small Minuit errors point to a problem, as also noticed by Vadim;
  the problem can be solved with the help of the analytical interpolating BWM function;
- reasonable χ^2/ndf -values, in contrast to large values in my fits (a difference may be due to Vadim’s fits of averaged spectra at y=1.0, 1.2, 1.4);
- about the same β_s (= 1.5 <β> at n=1) and by ~ 10 MeV smaller T;

l. 424:
$B_A \propto V_{eff}^{\,1-A}$ in any coalescence model (thermodynamic approach is not applicable to direct A-cluster production due to a small binding energy); in [21], hydrodynamic motivated parametrization of expanding fireball is used to describe the initial nucleon system participating in the coalescence process.

Answer:
change in the draft: thermodynamic approach -> coalescence models

Eq. (7):
make this equation more quantitative (similar to Eq. (9) in [48]), substituting it by Eq. (2) (with C_A given in Eq. (3')), multiplied by the neutron-to-proton correction factor $(\lambda_n/\lambda_p)^N$ and a phenomenological factor exp[b A (m_t -m)], the latter suggested in the context of the box-like density profile with $b = 1/T_p - 1/T_A$ [21], making a comment on its possible modification at higher transverse momenta and using b as a free parameter.

Answer:
it could be done, i.e., the equation could be written more precisely

Comment:
I have simplified the notation of Eq. (2) and (3'), introducing, instead of the cluster radii R_A, the radii $d_A = \sqrt{n_A}\, R_A$.
Also, the suppression factor Λ_A was introduced, related to the nucleon "purity" factor Λ_N: $\Lambda_A \approx \Lambda_N^A$.
Also, C_A is now given in Eq. (3''), taking into account the phenomenological factor exp[b A (m_t -m)] due to a violation of the Gaussian transverse density profile [21].

Note a different notation for masses and transverse masses: here ($m = m_p$) and in the paper ($m = m_A$). In Eqs. (2) and (3'), one should therefore make the substitutions:

$$m_t \rightarrow m_{pt}$$
$$A\,(m_t - m) \rightarrow (m_t - m)$$

l. 430:
transverse momentum slopes → transverse mass inverse slopes

Answer: OK

l. 443-444:
Eq. (7) predicts exponential fit in $m_t - m \approx p_t^2/(2m)$ at small p_t (not exponential in p_t). Also, the power-like in (m_t/m) of B_A due to its denominator leads to p_t^2 (or (m_t-m)) dependence at small p_t. Therefore, the interpolation of the experimental B_A to $p_t = 0$ should be done by a polynomial or an exponential in (m_t-m).

Answer:
replacement of a*exp(b*pt) to a*exp(b*(mt-m)) increased the resulting parameter "a" (B_A) values by a factor of 1.5; 1/mt^(A-1) term before exp change "a" value only little, but effect the slope b value.

Comment:
the 50% B_A increase due to exp[b (m_t-m)] fit is larger than the expected effects of the neutron-to-proton ratio and the nucleon "purity" and should likely be included in the systematic error if there are arguments for a linear in p_t dependence of B_A at small p_t.

Also note, that the factor $1/m_t^{b(A-1)}$ actually becomes $m_t^{b(A-1)}$ (where $b \lesssim 0.5$) due to m_t - dependence of the LCMS femtoscopic radii (decreasing as$1/m_t^b$).

Fig.13(b):
the scale for Ar+Sn $\rightarrow$ t+X should be multiplied by 10^3.

Answer: done

l. 456:
As explained in the text, it is natural to determine $R_{coal} = (V_{hom})^{1/3}$, where $V_{hom} = R_s^2 R_l$ at $p_t = 0$ (when $R_o = R_s$ by definition) and becomes maximal at $R_o = R_s = R_l = R_{inv}$ and equal to R_{inv}^3. One can thus directly compare R_{coal} and R_{inv} and check the inequality $R_{coal} \leq R_{inv}$.
At the same time, it will serve as a consistency check of the C_A calculation (see a discussion after Eq. (5) and the next comment).

Answer: We used equations (11) and (14) from [48] to calculate R_t^2*R_l values from B_A. They do not include $(\lambda_n/\lambda_p)^{(A-1)}$ factors.
We could apply $(\lambda_n/\lambda_p)^{(A-1)}$ factors in equation (7) if you can give an estimation for these values and a reference to the source. You gave upper values in your note, but the factors depend on the difference in μn and μp which is uncertain. We can use your estimation of the maximal $(\lambda_n/\lambda_p)^{(A-1)}$ values as a source of systematics of Rcoal.

To get estimation of Rcoal we assumed 4/3 π R_coal^3 = πR_s^2*2R_l, i.e. make transformation from a cylinder with radius R_t and 0.5 length of R_l to a sphere with R_coal of the same Volume. We can use $R_{coal} = (V_{hom})^{1/3}$ as you propose, but in Fig.15 other experiments probably used different transformation from V_{hom} -> R_{coal}

Comment:
As for the neutron-to-proton ratio, I have already suggested to use the UrQMD estimates, as it was already done to calculate the proton phase-space density according to paper Eq. (10).

As for the coalescence volume, its relation to the Gaussian femtoscopic radii is rather arbitrary and has no real physical meaning. In fact, it is a question of agreement, which quantity to be considered as a volume and V_{hom} is one of them. As for the other experimental points in Fig. 15, they can simply be divided by the factor $1.145 = (3/2)^{1/3}$.

l.461:
One may recover the values C_d (C_t) = 0.55-0.61 (0.48-0.53), using (3') and R_{inv} = 2.5-2.8 fm, giving C_d (C_t) = 0.55-0.61 (0.52-0.58).
Could you specify how the factors C_d and C_t have been "scaled according to the mass of the colliding system"?
It looks like (see a paragraph above "Comments, questions and suggestions") that the volume A -scaling is too strong and that the surface $A^{2/3}$ scaling of R^3 is more appropriate.

Answer:

Cd and Ct values are taken from table 3 of [48]: Cd = 0.55 for S+S and 0.61 for S+Pb. Using scaling by A1+A2, we got Cd =0.55 for Ar+C , Cd=0.61 for Ar+Pb and intermediate values for Ar+Al, Cu, Sn.
To get Ct values for Ar+A from Cd values we scaled them by the ratio Ct/Cd for Pb+Pb from table 3 of [48], i.e 0.7/0.8.
If you can provide more precise predictions for the Cd and Ct values, we would use them. But in the paper, we need a reference to the source/method.

Comment:
It is not clear what is scaled by A1+A2. A linear scaling of C_d with A1+A2 looks oversimplified. To see this, consider for simplicity pt $\to$ 0 and all the radii equal to a universal radius $R \approx R_{inv}$, i.e.,

$$C_A \cong [1+ (d_A/R)^2]^{3(A-1)/2}.$$

Assuming a reasonable, linear in A1+A2, scaling of the volume R^3, there is generally no A1+A2 scaling of C_A. For deuterons, a linear scaling of C_A takes place in the unrealistic case of $(d_A/R)^2$ only. In a more realistic opposite limit of $(d_A/R)^2 \ll 1$,

$$C_A \cong 1+ (d_A/R)^2 (3(A-1)/2),$$

so that C_A - 1 would scale as $(d_A/R)^2 \sim 1/(A1+A2)^{2/3}$.

Instead, a linear in (A1+A2) volume scaling could be used and the factors C_A calculated from Eq. (3') with the help of the rescaled radii.

In fact, in [48], the C_d factors for S+S and S+Pb were obtained from Eq. (4.12) of [21] (which coincides with Eq. (3') at pt $\to$ 0) rescaling the LCMS femtoscopic nucleon radii at pt $\to$ 0 for Pb+Pb (Rs=Ro=5.1 fm, Rl=3.2 fm) based on the measured pion radii from [kai97].

From the reported values [48], C_d(S+S) = 0.55 and C_d(S+Pb) = 0.61, one can deduce the scaling factors R(S+S)/R(Pb+Pb) = 0.59 and R(S+Pb)/R(Pb+Pb) = 0.66 and the corresponding rescaled radii and C_A factors (calculated according to Eq. (3')):

	Pb+Pb	S+Pb	S+S
Rs=Ro, Rl [fm]	5.1, 3.2	3.4, 2.1	3.0, 1.9
C_d, C_t	.79, .77	.61, .58	.55, .51

The C_d and C_t factors are of similar size due to smaller r.m.s. triton radius, which almost compensates the additional power.
Note that the C_A factors for Pb+Pb agree with those obtained in [21] and that the underestimated C_t value of 0.7 was used in [48] (calculated in [21] with different fireball parameters compared to those giving the C_d factor of 0.8).

Assuming a linear in (A1+A2) volume scaling, $V_{hom} \sim$ (A1+A1), for heavy ion symmetric (Pb+Pb) collision, one finds a slight violation of the linearity for other collisions: V_{hom}(S+S) $\sim (A1+A1)^{1.07}$ and V_{hom}(S+Pb) $\sim (A1+A1)^{0.87}$.

Neglecting the violation of linear volume scaling, as well as, a decrease of the radii with decreasing collision energy, one can use the radii for S+S and S+Pb collisions to obtain the radii estimates interpolating between cubes of the S+S and S+Pb radii and calculate corresponding C_A factors:

Ar+C	Ar+Al	Ar+Cu	Ar+Sn	Ar+Pb

Rs=Ro, Rl [fm]	3.0, 1.9	3.0, 1.9	3.1, 2.0	3.2, 2.0	3.4, 2.1
C_d, C_t	.55, .51	.55, .51	.57, .53	.58, .54	.61, .58

General comment:
Clearly, the determination of the coalescence (homogeneity) volume and the corresponding coalescence radius is affected by the uncertainties in the neutron-to-proton ratio λ_n/λ_p and - in the quantum correction factor C_A, the latter related with the uncertainties in the LCMS femtoscopic nucleon radii and in the suppression Λ_A (due to a non-Gaussian tail of the nucleon emission function). A discussion (including estimates of upper limits) would be therefore appropriate.

Answer:
we can add systematic uncertainty in R_coal due to uncertainty of λ_n/λ_p and C_A.
If you provide sentences in the discussion part it would be nice.

Comment on B_A uncertainties due to Λ_A and λ_n/λ_p and corresponding uncertainties in R_{coal}:

- $\Lambda_A = \Lambda_N^A$: at our collision energy, one may expect [ada06, bog99] $\Lambda_N \gtrsim 0.9$, so that putting $\Lambda_N = 1$ would lead to B_d (B_t) underestimation $\lesssim$19% (27%);
- λ_n/λ_p: for our collision systems, one may expect λ_n/λ_p essentially smaller than the intrinsic values, ranging from 1.11 to 1.37 from the lightest (C) to heaviest (Pb) target (actual values can be estimated from UrQMD simulations); e.g., assuming $\lambda_n/\lambda_p = 1.2$ and putting $\lambda_n/\lambda_p = 1$ would lead to B_d (B_t) overestimation $\lesssim$ 20% (44%);
- the overestimations (due to Λ_A) and underestimations (due to λ_n/λ_p) of $R_{coal} = (V_{hom})^{1/3} \sim B_A^{-(A-1)/3}$ compose $\lesssim$7% (11%) and $\lesssim$11% (22%), respectively.

Comments, using Vadim's Rnp estimates from UrQMD (1.09, 1.09, 1.11, 1.14 and 1.18 for Ar+C, Ar+Al, Ar+Cu, Ar+Sn and Ar+Pb):

- their neglect leads to B_d (B_t) overestimation by
  $\sim$ 9% (19%), 9% (19%), 11% (23%), 14% (30%) and 18% (39%), respectively;
  the corresponding underestimations of $R_{coal} = (V_{hom})^{1/3} \sim B_A^{-(A-1)/3}$ compose
  $\sim$3% (6%), 3% (6%), 3% (7%), 4% (8%) and 5% (11%) respectively;
- as for the net effect of neglected differences of $\Lambda_A = \Lambda_N^A$ and $R_{np} = \lambda_n/\lambda_p$ from unity, assuming $\Lambda_N \approx 0.9$ and multiplying the corresponding enhancement and suppression factors, one arrives at net B_d (B_t) underestimations of
  $\sim$ 12% (13%), 12% (13%), 10% (10%), 8% (5%) and 4% (-1.5%), respectively;
  the corresponding overestimations of $R_{coal} = (V_{hom})^{1/3} \sim B_A^{-(A-1)/3}$ compose
  $\sim$ 4% (10%), 4% (10%), 4% (7%), 3% (4%) and 1.5% (-1%), respectively.
- the estimated net B_d (B_t) underestimations somewhat differ from Vadim's numbers; his conclusion about the additional 10% uncertainty looks OK except for Sn and Pb targets, where the additional 5% uncertainty seems sufficient.

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