Первичные черные дыры гибридной инфляции

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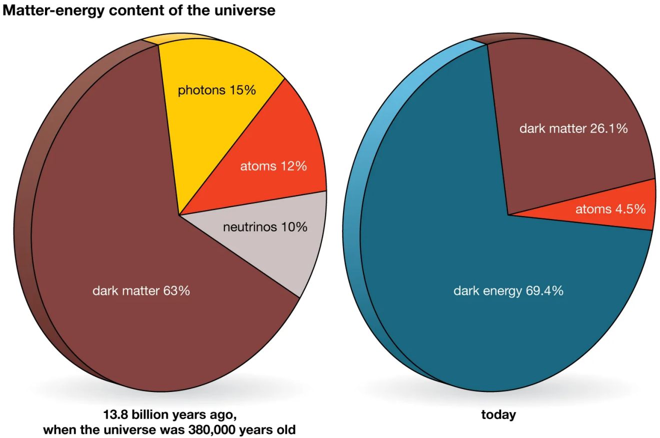
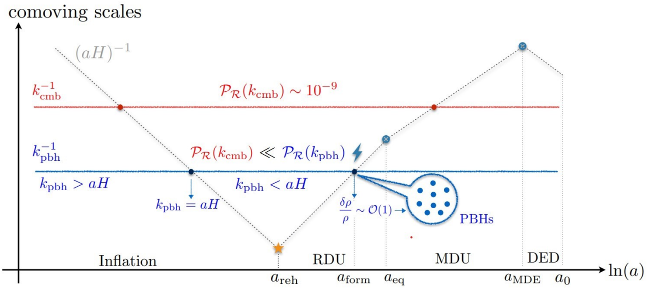
[1] based on E.O. Pozdeeva, S.Yu. Vernov, arXiv:2401.12040

https://www.britannica.com/science/dark-matter



• A black hole is called primordial (PBH) if it has been formed before the matter dominance epoch of the Universe evolution.
Large peaks in the amplitude of perturbations during inflation can lead to PBH formation in early post-inflation stage of the Universe evolution.
• The PBH could form by the gravitational collapse of over-dense inhomogeneities in the early universe
Zel'dovich, Ya. B. ; Novikov, I. D., Soviet Astron. AJ ***10*** *(1967) 602. S. Hawking, Mon. Not. Roy. Astron. Soc.* ***152*** *(1971) 75*
• The PBHs are the DM candidates. The hypothesis that a part of the dark matter consists of PBH has been proposed in
Dolgov A., Silk J., Baryon isocurvature fluctuations at small scales and baryonic dark matter, Phys. Rev. D, 1993, ***47****, p. 4244.*
Ivanov P., Naselsky P., Novikov I., Inflation and primordial black holes as dark matter, Phys. Rev. D, 1994, ***50****, p. 7173*



O. Özsoy and G. Tasinato, Inflation and Primordial Black Holes, Universe **9** (2023) 203 [arXiv:2301.03600].

If PBH mass belongs to the interval $10^{-17} \leq M_{pbh}[M_{\odot}] \leq 10^{-12}$, ($M_{\odot} \simeq 1.98 \cdot 10^{33}$ gr), then PBH can a part of DM.

It corresponds to $34 < N_{PBH} - N_{CMB} < 40$.

Let us consider the modified gravity model, described by the following action:

$$S_R = \int d^4x\sqrt{-\tilde{g}}\left[F(\tilde{R},\xi) - \frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\xi\partial_\nu\xi\right], \qquad (1)$$

where F is a differentiable function, ξ is a scalar field.
The model (1) is equivalent to the following two-field model

$$S_J = \int d^4x\sqrt{-\tilde{g}}\left[F_{,\sigma}\tilde{R} + \left(F - F'_{,\sigma}\sigma\right) - \frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\xi\partial_\nu\xi\right], \qquad (2)$$

under condition $F_{,\sigma\sigma} \neq 0$. We use the notation $F_{,\sigma} = \frac{dF}{d\sigma}$.
PBHs have been studied both in $F(R)$ gravity
*S. Saburov and S. V. Ketov, Universe **9** (2023) 323 [arXiv:2306.06597]*
and $F(R,\xi)$ gravity
*A. Gundhi, S. V. Ketov and C. F. Steinwachs, Phys. Rev. D **103** (2021) 083518 [arXiv:2011.05999].*

Assuming $F_{,\sigma} > 0$ and using the conformal transformation of the metric

$$g_{\mu\nu} = \frac{2F_{,\sigma}}{M_{Pl}^2}\tilde{g}_{\mu\nu},$$

we obtain a chiral cosmological model with two scalar fields, described by the following action

$$S_E = \int d^4x\sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}R - \frac{g^{\mu\nu}}{2}\partial_\mu\phi\partial_\nu\phi - \frac{M_{\rm Pl}^2}{4F_{,\sigma}}g^{\mu\nu}\partial_\mu\xi\partial_\nu\xi - V_E\right],$$

where

$$\phi = \sqrt{\frac{3}{2}}\,M_{\rm Pl}\ln\left(\frac{2F_{,\sigma}}{M_{\rm Pl}^2}\right), \qquad (3)$$

$$V_E = \frac{M_{\rm Pl}^4}{4F_{,\sigma}^2}\left(F_{,\sigma}\sigma - F\right). \qquad (4)$$

We assume that the function $F(\sigma,\xi)$ has the following form:

$$F(\sigma,\xi) = \frac{M_{\rm Pl}^2}{2}\left[X_0(\xi)F_0(\sigma) + X_1(\xi)\sigma - U(\xi)\right], \qquad (5)$$

where $F_0(\sigma)$, $X_0(\xi)$, $X_1(\xi)$ and $U(\xi)$ are differentiable functions.
The corresponding potential $V_E(\sigma,\xi)$ has the following form:

$$V_E(\sigma,\xi) = \frac{M_{\rm Pl}^2\left(\left[\sigma F_{0,\sigma}(\sigma) - F_0(\sigma)\right]X_0(\xi) + U(\xi)\right)}{2\left(F_{0,\sigma}(\sigma)X_0(\xi) + X_1(\xi)\right)^2}\,. \qquad (6)$$

To get the potential $V_E(\phi,\xi)$ in the explicit form we should find the function $\sigma(\phi,\xi)$.

Let

$$F_0 = F_{Star} = R + \frac{R^2}{6m^2}. \tag{7}$$

Solving equation

$$y = \frac{M_{Pl}^2}{2F_{,\sigma}}, \quad \text{where} \quad y \equiv \exp\left(-\sqrt{\frac{2}{3}}\frac{\phi}{M_{Pl}}\right) \tag{8}$$

we get

$$\sigma = -\frac{3\,m^2\left(X_0(\xi)y + X_1(\xi)y - 1\right)}{X_0(\xi)y} \tag{9}$$

and $F(y,\xi)$

$$F(y,\xi) = \frac{3\,m^2\,M_{\rm Pl}^2\left(y^{-2} - (X_0 + X_1)^2\right)}{4X_0} \tag{10}$$

The corresponding expression for potential is:

$$V_E(y,\xi) = \frac{1}{4X_0(\xi)}\left[3M_{\rm Pl}^2 m^2 y^2\left(X_0(\xi)\right)^2 + 3\,M_{\rm Pl}^2 m^2\left(X_1(\xi)y - 1\right)^2 \right. \\ \left. + \left(6\,M_{\rm Pl}^2 m^2 y^2 X_1(\xi) - 6\,M_{\rm Pl}^2 m^2 y + 4y^2 V_J(\xi)\right) X_0(\xi)\right]. \tag{11}$$

To get

$$V_E = \frac{\lambda}{4}(\xi^2 - \xi_0^2)^2 + (C_0 + C_1\xi^2)(1-y)^2 + \tilde{d}\,\xi\,, \qquad (12)$$

proposed in Ref. [*M. Braglia, A. Linde, R. Kallosh and F. Finelli, JCAP* ***04*** *(2023) 033 [arXiv:2211.14262]*], we introduce the following set of parameters

$$\lambda = \frac{M^2}{\xi_0^2}, \quad C_0 = 3\alpha M^2\tilde{m}^2, \quad \tilde{d} = M^2 d, \quad C_1 = M^2\tilde{g}^2, \qquad (13)$$

and choose the following functions:

$$U = \frac{2\left(\lambda\left(\xi^2 - \xi_0^2\right)^2 + 4\,d\,\xi\right)\left(C_1\,\xi^2 + C_0\right)}{M_{\rm Pl}^2\left(\lambda\left(\xi^2 - \xi_0^2\right)^2 + 4\,C_1\,\xi^2 + 4\,d\,\xi + 4\,C_0\right)}\,, \qquad (14)$$

$$X_0 = \frac{3\,m^2 M_{\rm Pl}^2}{\lambda\left(\xi^2 - \xi_0^2\right)^2 + 4\,C_1\,\xi^2 + 4\,d\,\xi + 4\,C_0}\,, \qquad (15)$$

$$X_1 = \frac{4\,C_1\,\xi^2 + 4\,C_0 - 3\,m^2 M_{\rm Pl}^2}{\lambda\left(\xi^2 - \xi_0^2\right)^2 + 4\,C_1\xi^2 + 4\,d\,\xi + 4\,C_0}\,. \qquad (16)$$

We get the following chiral cosmological model in the Einstein frame

$$S_E = \int d^4x\sqrt{-g}\left[\frac{M_{\mathrm{Pl}}^2}{2}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{y}{2}g^{\mu\nu}\partial_\mu\xi\partial_\nu\xi - V_E\right]. \quad (17)$$

In the spatially flat FLRW metric with the interval

$$ds^2 = -\,dt^2 + a^2(t)\left(dx_1^2 + dx_2^2 + dx_3^2\right), \quad (18)$$

the Einstein equations have the following form:

$$H^2 = \frac{1}{6M_{\mathrm{Pl}}^2}\left(\dot\sigma^2 + 2V_E\right), \quad (19)$$

$$\dot H = -\,\frac{\dot\sigma^2}{2M_{\mathrm{Pl}}^2}, \quad (20)$$

where

$$\dot\sigma \equiv \sqrt{\dot\phi^2 + y\,\dot\xi^2}, \quad (21)$$

the dots denote the time derivative, and the Hubble parameter $H(t)$ is the logarithmic derivative of the scale factor: $H = \dot a/a$.

The field equations are as follows:

$$\ddot{\phi} + 3H\dot{\phi} + \frac{y}{\sqrt{6}M_{\rm Pl}}\dot{\xi}^2 + \frac{\partial V_E}{\partial \phi} = 0\,, \qquad (22)$$

$$\ddot{\xi} + 3H\dot{\xi} - \sqrt{\frac{2}{3}}\frac{\dot{\xi}\dot{\phi}}{M_{\rm Pl}} + \frac{1}{y}\frac{\partial V_E}{\partial \xi} = 0\,. \qquad (23)$$

The following equations

$$\ddot{\sigma} + 3H\dot{\sigma} + \frac{\dot{\phi}}{\dot{\sigma}}\frac{\partial V_E}{\partial \phi} + \frac{\dot{\xi}}{\dot{\sigma}}\frac{\partial V_E}{\partial \xi} = 0\,, \qquad (24)$$

$$\frac{d}{dt}\dot{\sigma}^2 + 6H\dot{\sigma}^2 + 2\dot{V}_E = 0\,, \qquad (25)$$

are consequence of Eqs. (22) and (23).

As usually for inflationary model construction, the e-folding number $N = \ln(a/a_e)$, where a_e is a constant, is considered as a measure of time during inflation:

$$H^2 \equiv Q = \frac{2V_E}{6M_{\rm Pl}^2 - (\sigma')^2}\,, \tag{26}$$

$$Q' = -\frac{Q}{M_{\rm Pl}^2}(\sigma')^2\,, \tag{27}$$

where

$$(\sigma')^2 \equiv (\phi')^2 + y\,(\xi')^2 \tag{28}$$

$$Q\phi'' + \frac{1}{2}Q'\phi' + 3Q\phi' + \frac{yQ}{\sqrt{6}\,M_{\rm Pl}}\xi'^2 + \frac{\partial V_E}{\partial \phi} = 0\,, \tag{29}$$

$$Q\xi'' + \frac{1}{2}Q'\xi' + 3Q\xi' - \frac{2Q}{\sqrt{6}\,M_{\rm Pl}}\xi'\phi' + \frac{1}{y}\frac{\partial V_E}{\partial \xi} = 0\,, \tag{30}$$

where primes denote derivatives with respect to N.

Using Eqs. (26) and (27) to eliminate Q and Q' from the field equations, we get the following dynamical system:

$$\begin{aligned}
\phi' =& \psi\,,\\
\psi' =& \frac{1}{2M_{\rm Pl}^2}\left(y\chi^2+\psi^2\right)\psi - 3\psi - \frac{y}{\sqrt{6}M_{\rm Pl}}\chi^2 - \frac{6M_{\rm Pl}^2 - y\chi^2 - \chi^2}{2V_E}\frac{\partial V_E}{\partial\phi}\,,\\
\xi' =& \chi\,,\\
\chi' =& \frac{1}{2M_{\rm Pl}^2}\left(y\chi^2+\psi^2\right)\chi - 3\chi + \frac{2}{\sqrt{6}M_{\rm Pl}}\chi\psi - \frac{6M_{\rm Pl}^2 - \psi^2 - y\chi^2}{2yV_E}\frac{\partial V_E}{\partial\xi}\,.
\end{aligned}$$

Slow-roll parameters

The standard slow-roll parameters ϵ_1 and ϵ_2 are:

$$\epsilon_1 = -\frac{Q'}{2Q} = \frac{(\sigma')^2}{2M_{Pl}^2} \quad (31)$$

$$\epsilon_2 = \frac{\epsilon_1'}{\epsilon_1} \quad (32)$$

Inflation corresponds to

$$\epsilon_1 < 1. \quad (33)$$

If $|\epsilon_2| < 1$, then we get a slow-roll inflation. A slow-roll inflation is not suitable for BH production.
We need some period of the ultra slow roll regime, when $\epsilon_1 < 1$ and $|\epsilon_2| > 1$ or $\eta = 2\epsilon_1 - \epsilon_2/2 \approx 3$.

Inflationary parameters

To check inflationary scenario we use inflationary parameters: the spectral index

$$n_s \approx 1 - 2\epsilon_1 - \epsilon_2, \qquad (34)$$

the tensor-to-scalar ratio

$$r \approx 16\epsilon_1 \qquad (35)$$

and the amplitude of the scalar perturbations

$$\mathcal{A}_s = \frac{Q}{\pi^2 U_0\, r}\,. \qquad (36)$$

The main inflationary parameters are constrained by the combined analysis of Planck, BICEP/Keck and other observations as follows:

$$A_s = (2.10 \pm 0.03)\times 10^{-9}, \qquad n_s = 0.9654 \pm 0.0040 \qquad \text{and} \qquad r < 0.028\,. \qquad (37)$$

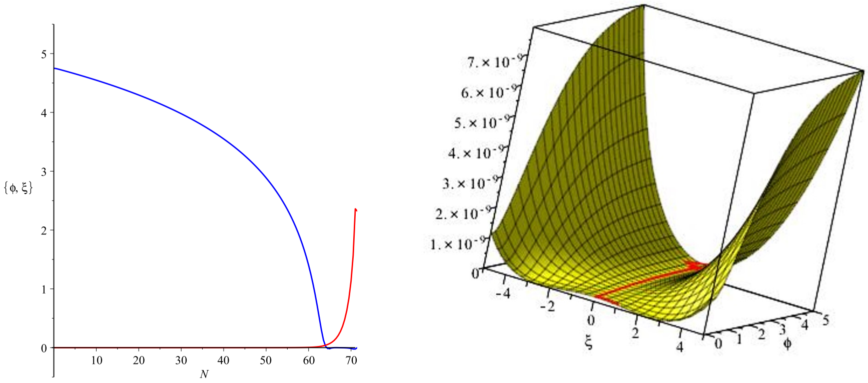


Рис.: Fields $\xi(N)$, $\phi(N)$ and potential $V_E(\phi, \xi)$

where

$$M = 0.55 \times 10^{-5},\ \alpha = 0.7,\ \tilde{g} = 1,\ \tilde{m} = 1,\ \xi_0 = 2.5,\ d = -10^{-3}$$

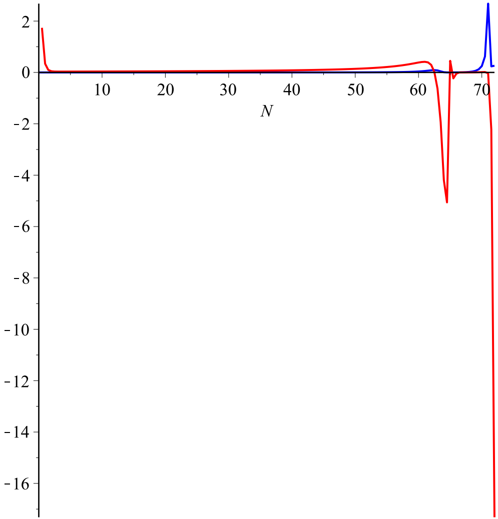


Рис.: Values of slow-roll parameters during inflation.

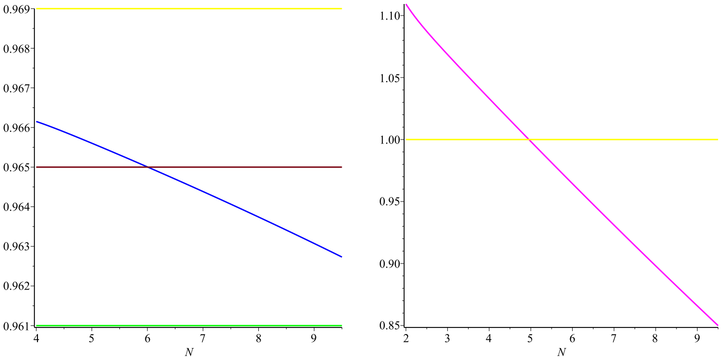


Рис.: Values of n_s and $(A_s/2.1) \times 10^9$ in the beginning of inflation. $N_0 = 5.845$.

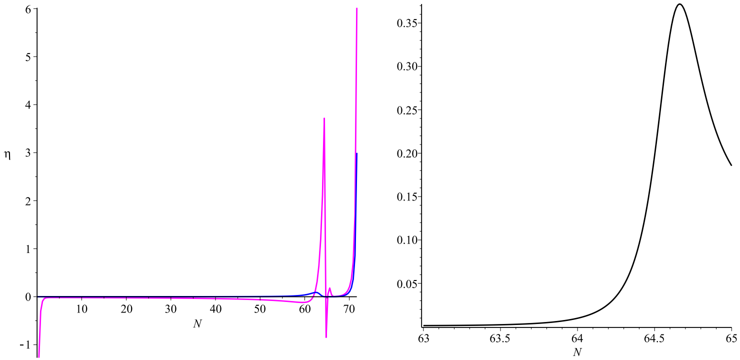


Рис.: Slow-roll parameters ϵ, η, amplitude of perturbations $(\mathcal{P}_{\mathcal{R}}/2.1) \times 10^9$

Conclusions

1. We suggest the generalization of the Starobinsky inflationary model. The generalization includes $R + R^2/(6m)$, linear term R multiplied on different functions of the scalar field ξ and the potential $U(\xi)$.
2. Due to the conformal transformation we get the chiral cosmological model with two scalar fields.
3. We analyze trajectories and slow-roll parameters during inflation.
4. We get parameters of the model that is suitable for BH formation, but the mass of BH is too small to be a part of DM.

This study was conducted within the scientific program of the National Center for Physics and Mathematics, section 5 'Particle Physics and Cosmology'. Stage 2023-2025.

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Thank for your attention