Pair charmonia production in the Parton Reggeization Approach in the LHCb experiment[1]

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Outline

Introduction: motivation

Heavy **quarkonia** production in QCD

=

perturbative part (hard scattering) + *non–perturbative* part (hadronization)

Studying hadronization:

- Being a *bound non–relativistic state* consist of heavy quarks, quarkonium is the best way to understand *hadronization* in QCD;

Quarkonia as tools:

- **Gluon content of the proton:**
  - Relatively small $M_{Q[1S]} \simeq 3.10$–9.46 GeV is access to small $x \sim Me^{-y}/\sqrt{s}$, at LHC: $x \sim 10^{-4} - 10^{-6}$;
  - Clean experimental reconstruction;

  (*Important for future SPD NICA experiment*)

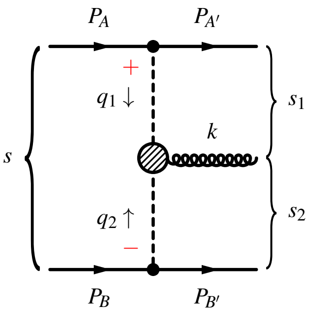
- **Multipartonic interactions:**
  - In the pair J/ψ production since pioneer works of NA3 Collaboration in pA and $\pi^- A$ collisions at $\sqrt{s} \simeq 17$ GeV to ALICE, ATLAS CMS, and LHCb Collaborations works at $\sqrt{s} = 7 - 13$ TeV;
  - In associated production with Z/W bosons, D mesons, $\ldots$;

- **...**

In this talk we discuss $p+p \to \psi[1S]+\psi[1S]+X$ and $p+p \to \psi[1S]+\psi[2S]+X$ pair charmonia productions. Our work is motivated by recent LHCb Collaboration measurements[Aaij et.al. '23].

Multi–Regge kinematics

At high energies $s \to \infty$ scattering $2 \to n+2$ can be considered in Multi–Regge kinematics (MRK).[3]



Double Regge limit:

$$\boxed{\forall i = 1,2: \quad s \gg s_i \gg t_i = -q_i^2 \simeq \mathbf{q}_{T_i}^2,}$$

momentum fractions: $z_1 = q_1^+/P_A^+$ and $z_2 = q_2^-/P_B^-$.

Main MRK properties:

- Rapidity ordering: $y(P_{A'}) \to +\infty$, $y(P_{B'}) \to -\infty$, $|y(k)| < \infty$;
- Small momentum fractions: $z_1 \sim z_2 \sim z \ll 1$, so $|\mathbf{k}_T| \ll \sqrt{s}$;
- Corresponding dominance of $\pm$ components:

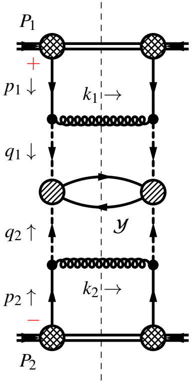
$$q_1^+ \sim O(z) \gg q_1^- \sim O(z^2),$$
$$q_2^- \sim O(z) \gg q_2^+ \sim O(z^2).$$

Two approaches to obtain this asymptotics:

- Quasi MRK approach$_{\text{[BFKL '75,76,78]}}$;
- Effective action approach$_{\text{[Lipatov '95; Lipatov and Vyazovsky '01]}}$.

[3] We use Sudakov decomposition: $p = \left(p^+ n_- + p^- n_+\right)/2 + p_T$, where $n^\pm = (1, \mathbf{0}, \mp 1)$, so that $p^\pm = (p, n^\pm)$ and $y = (1/2)\ln\left(p^+/p^-\right)$.

Parton Reggeization Approach



Auxiliary hard subprocess: $\boxed{q\,(p_1)+\bar{q}\,(p_2)\rightarrow g\,(k_1)+\mathcal{Y}\,(k)+g\,(k_2)}$, with $p_i^2=0$, $p_1^-\ll p_1^+$, $p_2^+\ll p_2^-$. Kinematic variables ($0<z_i<1$, $i=1,2$):

$$z_1=\frac{q_1^+}{p_1^+}=\frac{p_1^+-k_1^+}{p_1^+},\qquad z_2=\frac{q_2^-}{p_2^-}=\frac{p_2^--k_2^-}{p_2^-}.$$

Two main limits where $\overline{|\mathcal{M}|^2}$ can be factorized:

- **Collinear limit:** $\mathbf{k}_{T_i}^2,\mathbf{k}_T^2\ll\mu^2$ and z_i–arbitrary:

$$\overline{|\mathcal{M}|^2}_{\rm CL}\simeq\frac{4g_S^4}{\mathbf{k}_{T_1}^2\,\mathbf{k}_{T_2}^2}\,P_{qq}(z_1)\,P_{qq}(z_2)\,\frac{\overline{|\mathcal{A}|^2}}{z_1\,z_2}$$

- **Multi–Regge limit:** $z_i\ll1$ and $\mathbf{k}_{T_i}^2,\mathbf{k}_T^2$–arbitrary:

$$\overline{|\mathcal{M}|^2}_{\rm MRK}\simeq\frac{4g_S^4}{\mathbf{k}_{T_1}^2\,\mathbf{k}_{T_2}^2}\,\tilde{P}_{qq}(z_1)\,\tilde{P}_{qq}(z_2)\,\frac{\overline{|\mathcal{A}_{\rm PRA}|^2}}{z_1\,z_2}$$

Modified MRK approximation[Nefedov and Saleev '20]**:** $\mathbf{k}_{T_i}^2$, $\mathbf{k}_T^2$, and z_i–arbitrary:

$$\overline{|\mathcal{M}|^2}_{\rm MMRK}\simeq\frac{4g_S^4}{\tilde{q}_1^2\,\tilde{q}_2^2}\,P_{qq}(z_1)\,P_{qq}(z_2)\,\frac{\overline{|\mathcal{A}_{\rm PRA}|^2}}{z_1\,z_2},$$

where $\tilde{q}_i^2=\mathbf{q}_{T_i}^2/(1-z_i)$, P_{qq}–DGLAP splitting function. ***This factorization formula has both correct limits.***

Parton Reggeization Approach

Substituting $\overline{|\mathcal{M}|^2}_{\text{MMRK}}$ into the CPM factorization formula and changing variables, one can obtain:

$$d\sigma = \int_0^1 \frac{dx_1}{x_1} \int \frac{d^2\mathbf{q}_{T_1}}{\pi} \Phi_q(x_1,t_1,\mu^2) \int_0^1 \frac{dx_2}{x_2} \int \frac{d^2\mathbf{q}_{T_2}}{\pi} \Phi_{\bar{q}}(x_2,t_2,\mu^2) \times d\hat{\sigma}_{\text{PRA}} + \mathcal{O}\left(\frac{\Lambda^2}{\mu^2}, \frac{\mu^2}{s}\right),$$

here $x_1 = q_1^+/P_1^+$ and $x_2 = q_2^-/P_2^-$, $t_i = \mathbf{q}_{T_i}^2$, and

$$\Phi_i(x,t,\mu^2) = \frac{\alpha_S(\mu)}{2\pi} \frac{T_i(t,\mu^2,x)}{t} \times \sum_j \int_x^1 dz\, P_{ij}(z)\, F_j\left(\frac{x}{z},t\right) \times \theta\left(\Delta(t,\mu) - z\right),$$

is a ***unintegrated PDF (uPDF)*** calculated in ***(Modified) Kimber–Martin–Ryskin–Watt***[KMR '01; MRW '03] model based on *last–step evolution* mechanism.

- $T_i(t,\mu^2,x)$ IS A *SUDAKOV FORMFACTOR*, WHICH REGULARIZES COLLINEAR DIVERGENCES AND $t \to 0$ DIVERGENCE AND SATISFIES BOUNDARY CONDITIONS: $T_i(t=0,\mu^2,x) = 0$ AND $T_i(t=\mu^2,\mu^2,x) = 1$. THE EXACT SOLUTION FOR *SUDAKOV FORMFACTOR* WAS FOUND IN[Nefedov and Saleev '20].
- $\Delta(t,\mu) = \mu/\left(\mu+\sqrt{t}\right)$ IS A *KMR–CUTOFF FUNCTION* WHICH ENSURES *RAPIDITY ORDERING OF THE LAST EMITTED PARTON AND PARTICLES PRODUCED IN THE HARD PROCESS AND REGULARIZES IR–DIVERGENCES.*

The partonic cross section $d\hat{\sigma}_{\text{PRA}}$ is expressed in terms of squared amplitude $\overline{|\mathcal{A}_{\text{PRA}}|^2}$ calculated in the ***Lipatov's EFT for Multi–Regge processes***[Lipatov '95] in a standard way.

Quarkonium production

Due to high mass $M_{\psi[1S]} \simeq 3.10$ GeV and $M_{\Upsilon[1S]} \simeq 9.46$ GeV, these states can be considered as *non–relativistic* in some potential models, f.e., with *Cornell potential*:

$$\mathcal{V}_{\text{Cornell}} = -C_F \frac{\alpha_S(1/r)}{r} + \sigma r, \qquad \text{one can obtain:} \quad \boxed{\alpha_S^2(m_q v) \simeq v^2.}$$

The velocity expansion for quarkonium eigenstate:

$$|\mathcal{Q}[q\bar{q}[1S]]\rangle = \mathcal{O}(v^0)\,|q\bar{q}[^3S_1^1]\rangle + \mathcal{O}(v^1)\,|q\bar{q}[^3P_J^8]+g\rangle + \mathcal{O}(v^{3/2})\,|q\bar{q}[^1S_0^8]+g\rangle + \mathcal{O}(v^2)\,|q\bar{q}[^3S_1^8]+gg\rangle + \ldots,$$

NRQCD (*non–relativistic EFT*) factorization formula[Bodwin, Braaten, Lepage '95]**:**

$$\boxed{d\sigma_{\mathcal{Q}} = \sum_n d\sigma_{q\bar{q}[n]} \times \left\langle \mathcal{O}_n^{\mathcal{Q}} \right\rangle,}$$

here $\langle \mathcal{O}_n^{\mathcal{Q}} \rangle$ is LDME (squared NRQCD amplitude of quarkonia production with quantum numbers n).

▶ NOTE THAT COLOUR–SINGLET LDME ARE LO IN POWERS OF v FOR S WAVE STATE.

Some open «puzzles»:

- J/ψ polarization[Butenschoen, Kniehl '12];
- Pair J/ψ production[He, Kniehl, Nefedov, Saleev '21];
- Octet contribution in η_c hadroproduction[Butenschoen, Kniehl '12].
- Associated $J/\psi + \psi[2S]$ production[Aaij et.al '23] **(?)**;

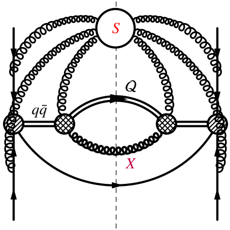
Improved Color Evaporation Model (ICEM)[Ma and Vogt '16]

ICEM assumes «democracy» between different $|q\bar{q}[n]\rangle$ states:

$$\mathcal{O}(v^0) \text{ for } {}^1S_0^{1,8}, {}^3S_1^{1,8} \text{ states.}$$

Picture of heavy quarkonia production in the ICEM:

$$\boxed{\frac{d\sigma_Q}{d^3p} = \mathcal{F}^Q \times \int_{M_Q}^{2M_H} dM\, d^3p'\, \delta^{(3)}\left(\mathbf{p} - \frac{M_Q}{M}\mathbf{p}'\right) \frac{d\sigma_{q\bar{q}}}{dM\, d^3p'} + \mathcal{O}\left(\lambda^2/m_q^2\right)}$$



S–exchanged and *X–emitted gluons*

- Production of $q\bar{q}$ pair with momentum p' in hard process;
- Introduction of soft scale λ ($m_q \gg \lambda \gg \Lambda$):

$$p' = p + p_S + p_X, \qquad \text{soft part}: p_{S,X}^2 > \lambda^2,$$

  Assume isotropic emission of the exchanged gluons $p_S^0 \simeq 0$ and due to $p_X^0 > 0$: $M > M_Q$;
- Based on analysis of quarkonia spectroscopy, ICEM assumes that all $q\bar{q}$ ***pairs below*** $H\bar{H}$ ***threshold hadronize to quarkonium with probability*** $\mathcal{F}^Q$;
  - THRESHOLD EFFECTS ARE TAKEN INTO ACCOUNT BY THE CONDITION $p_S^0 \simeq 0$.
- By counting powers of λ: $p_{S,X}^\mu, p^i \sim \mathcal{O}(\lambda)$, $p^0 \sim \mathcal{O}(m_q)$: $\boxed{p = (M_Q/M)\,p' + \mathcal{O}(\lambda^2/m_q^2)}$.
  - THIS MATCHING CONDITION IMPROVES p_T–SCALING.

Inclusive $\psi[1S]$ and $\psi[2S]$ production in the ICEM

We obtain the following values of $\mathcal{F}^{\psi}$ and $\mathcal{F}^{\psi'}$ for $\psi[1S]$ and $\psi[2S]$ states respectively:

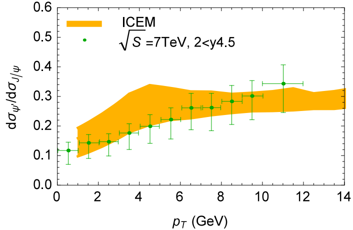
$$\mathcal{F}^{\psi} \simeq 0.02 \quad \text{and} \quad \mathcal{F}^{\psi'} \simeq 0.06$$

As it was obtained in[Cheung and Vogt '18]:

$$\mathcal{F}^{\Upsilon[1S]} < \mathcal{F}^{\Upsilon[2S]} < \mathcal{F}^{\Upsilon[3S]}$$

The absence of hierarchy, f.e., as in CSM, is associated with a lower limit of integration M_Q, since $Q[n'S] > Q[nS]$ for $n' > n$.

But the ratio $\psi[2S]$ to $\psi[1S]$ is «ok»[Ma and Vogt '16]:



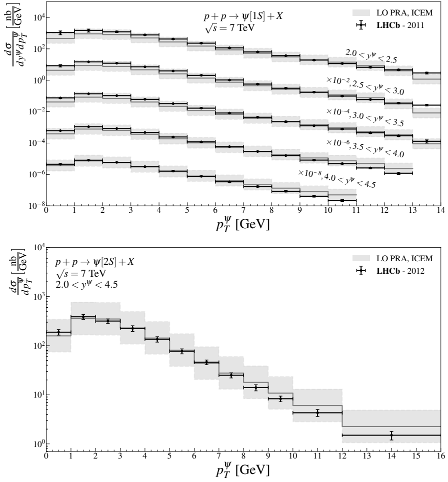
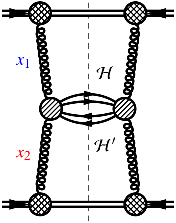
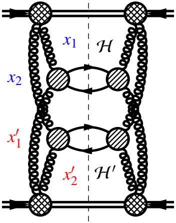


Figure 1: The data are from LHCb Collaboration[Aaij et.al. '11,12].

SPS and DPS





Standard approach to study multi–particle production is to use so–called ***single parton scattering (SPS)***:

$$d\sigma^{\rm SPS}_{\mathcal{H}\mathcal{H}'} = \left[f_g(x_1,\mu^2)\times f_g(x_2,\mu^2)\right]\otimes d\hat{\sigma}_{gg\to\mathcal{H}\mathcal{H}'}(x_1,x_2)$$

There is another approach, this is so–called ***double parton scattering (DPS)***:

$$d\sigma^{\rm DPS}_{\mathcal{H}\mathcal{H}'} = \frac{1}{(1+\delta_{\mathcal{H}\mathcal{H}'})\,\sigma_{\rm eff}}\left[\underbrace{D_{gg}(x_1,x_2,\mu^2)}_{f_g(x_1,\mu^2)\times f_g(x_2,\mu^2)}\times\underbrace{D_{gg}(x_1',x_2',\mu^2)}_{f_g(x_1',\mu^2)\times f_g(x_2',\mu^2)}\right]$$

$$\otimes\left[d\hat{\sigma}_{gg\to\mathcal{H}}(x_1,x_1')\times d\hat{\sigma}_{gg\to\mathcal{H}'}(x_2,x_2')\right]$$

$$\simeq \frac{d\sigma^{\rm SPS}_{\mathcal{H}}\times d\sigma^{\rm SPS}_{\mathcal{H}'}}{(1+\delta_{\mathcal{H}\mathcal{H}'})\,\sigma_{\rm eff}}$$

- This formula is based on the following assumptions:
  - hard part factorization: $d\hat{\sigma}_{ijkl\to\mathcal{H}\mathcal{H}'} = d\hat{\sigma}_{ik\to\mathcal{H}}\times d\hat{\sigma}_{jl\to\mathcal{H}'}$
  - decomposition of gDPDF's: $\Gamma_{ij}(x_1,x_2,\mu_1^2,\mu_2^2;\mathbf{b}_1,\mathbf{b}_2) = D_{ij}(x_1,x_2,\mu_1^2,\mu_2^2)\times f(\mathbf{b}_1)f(\mathbf{b}_2)$
    $\sigma_{\rm eff} = \left[\int d^2\mathbf{b}\; T^2(\mathbf{b})\right]^{-1}$ is a free parameter $[2,25]$ mb
- For details see **backup slides**.

KaTie

There are two ways to perform calculations in the PRA:

- Semi–analytic with ReggeQCD$_{\text{[Nefedov '16]}}$ + numerical integration (f.e., with CUBA$_{\text{[Hahn '15]}}$ library):
  - The Feynman rules of Lipatov's EFT up to the order

$$\mathcal{O}(e^n\, g_S^m), \qquad n+m \leq 4$$

    are implemented in ReggeQCD model–file for the FeynArts package;

  THE COMPLETE SET OF $c\bar{c}c\bar{c}$ PRODUCTION DIAGRAMS IN LIPATOV'S EFT IS ABOUT 72–TOO MUCH FOR ANALYTICAL CALCULATIONS.

- Fully numerical with KaTie$_{\text{[Hameren '18]}}$:
  - Fully numerical method for calculating gauge invariant amplitudes with off–shell initial states based on spinor amplitudes formalism and recurrence relations of the Britto–Cachazo–Feng–Witten (BCFW) type;
  - Order of diagrams up to:

$$\mathcal{O}(e^n\, g_S^m), \qquad n+m \leq 4$$

  - Collinear PDF sets from LHAPDF;
  - uPDF sets from TMDlib and from own grid files;
  - Opportunities to study multipartonic interactions.

These two methods are equivalent at the stage of numerical calculations$_{\text{[Nefedov, Shipilova, and Saleev]}}$.

Associated $\psi[1S]+\psi[1S]$

To extract $\mathcal{F}^{\psi\psi}$ and $\sigma_{\rm eff}$ we perform fit with two free paramaters:

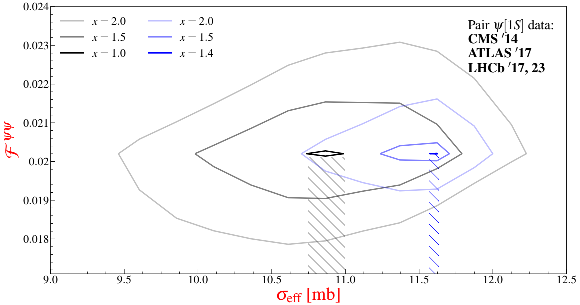
$$\sigma(\mathcal{F}^{\psi\psi},\sigma_{\rm eff}) = \sigma^{\rm SPS}(\mathcal{F}^{\psi\psi}) + \sigma^{\rm DPS}(\mathcal{F}^{\psi},\sigma_{\rm eff})$$

while $\mathcal{F}^{\psi}$ *is fixed in inclusive production.*

We introduce the function x:

$$x = \sum_k \frac{|\,\sigma_k^{\rm exp} - \sigma_k(\mathcal{F}^{\psi\psi},\sigma_{\rm eff})\,|}{\Delta\sigma_k^{\rm exp}},$$

where is a sum over the experimental data at high energies $\sqrt{s} = 7-13$ TeV.



Black isolines corresponds to the previous extracted value and **blue** to the new one:

$$\boxed{\sigma_{\rm eff} \simeq \mathbf{11.0}\ \text{mb} \quad \vee \quad \sigma_{\rm eff} \simeq \mathbf{11.5}\ \text{mb},}$$

within the hard scale variatian, this two values are equivalent, and at high energies we obtain:

$$\boxed{\mathcal{F}^{\psi\psi} \simeq \mathcal{F}^{\psi}}$$

Results

LO PRA + NRQCD $\vee$ NLO* CPM + CSM $\vee$ LO PRA + ICEM

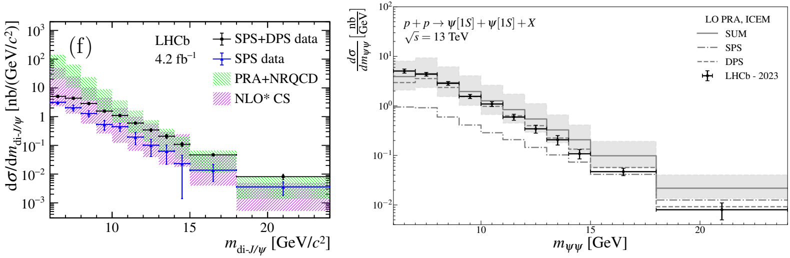


Figure 2: The left plot is from[Aaij et.al. '23].

Predictions in LO PRA+NRQCD and CSM+NLO* CPM are performed only taking into account the SPS contribution!

Results

NLO* CPM + CSM $\vee$ LO PRA + ICEM

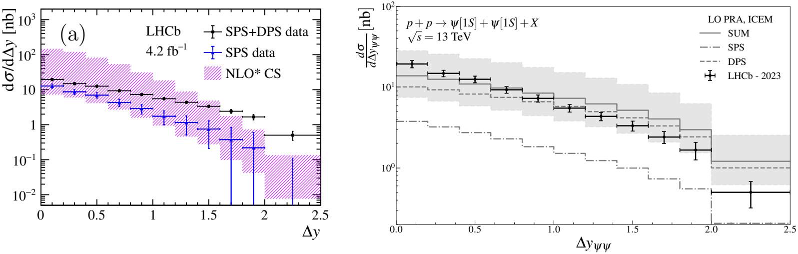


Figure 3: The left plot is from[Aaij et.al. '23].

Predictions in CSM+NLO* CPM are performed only taking into account the SPS contribution!

Results

NLO* CPM + CSM $\quad\vee\quad$ LO PRA + ICEM

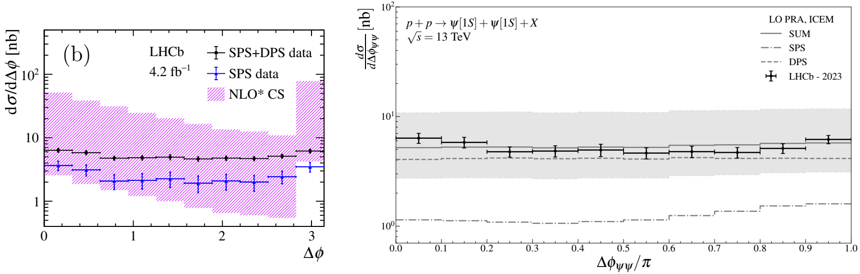


Figure 4: The left plot is from[Aaij et.al. '23].

Predictions in CSM+NLO* CPM are performed only taking into account the SPS contribution!

Results

Based on previous results, we can choose upper value as $\boxed{\mathcal{F}^{\psi\psi'} \simeq \mathcal{F}^{\psi'}}$, which corresponds to the *maximum contribution of the SPS mechanism*, while *the DPS contribution is fixed* due to fixed parameters.

$$\text{LO PRA} + \text{NRQCD} \quad \vee \quad \text{NLO}^\star \text{ CPM} + \text{CSM} \quad \vee \quad \text{LO PRA} + \text{ICEM}$$

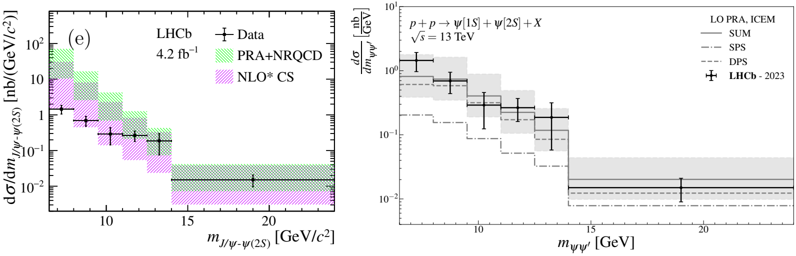


Figure 5: The left plot is from [Aaij et.al. '23].

Predictions in LO PRA+NRQCD and CSM+NLO* CPM are performed only taking into account the SPS contribution!

Results

$\text{NLO}^\star$ CPM + CSM $\vee$ LO PRA + ICEM

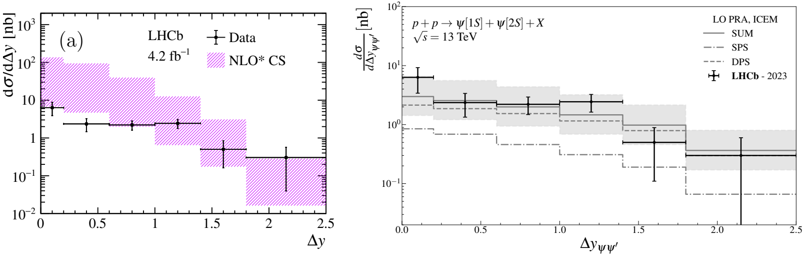


Figure 6: The left plot is from[Aaij et.al. ′23].

Predictions in CSM+NLO* CPM are performed only taking into account the SPS contribution!

Results

$$\text{NLO}^{\star}\ \text{CPM} + \text{CSM} \quad \vee \quad \text{LO PRA} + \text{ICEM}$$

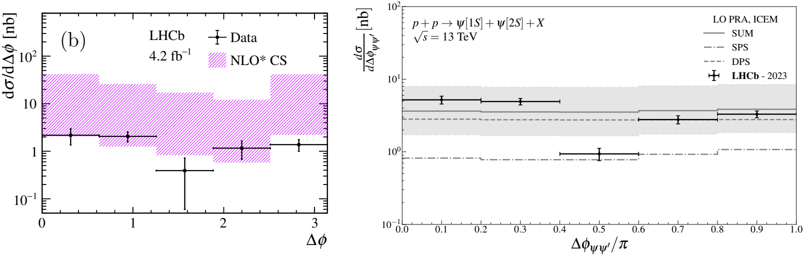


Figure 7: The left plot is from[Aaij et.al. ′23].

Predictions in CSM+NLO⋆ CPM are performed only taking into account the SPS contribution!

Conclusions

- Working in the LO PRA and ICEM, taking into account the SPS and the DPS mechanisms, we have obtained self agreement description of the pair $\psi[1S]+\psi[1S]$ and $\psi[1S]+\psi[2S]$, associated $\mathcal{Q}[1S]+D$ and $\psi[1S]+Z/W$ productions;
- In this study, we confirm early obtained numerical values for parameters of the ICEM, $\mathcal{F}^{\psi}\simeq 0.02$ and the DPS pocket formula, $\sigma_{\text{eff}}\simeq 11$ mb, and extracted one new $\mathcal{F}^{\psi'}\simeq 0.06$;
- We found the contribution of the DPS to be dominant over the SPS one in the LHCb experiment kinematics in the pair charmonia production processes, such that:

$$R_{\psi[1S]\psi[1S]}\simeq 0.3, \qquad R_{\psi[1S]\psi[2S]}\simeq 0.2$$

- The absence of hierarchy between ICEM hadronization factors $\mathcal{F}^{\psi[1S]}$ and $\mathcal{F}^{\psi[2S]}$ does not affect the processes of associated $\psi[1S]+\psi[2S]$ production;

For details see Phys.Rev.D **106** (2022) 11, 114006.

Thank you for your attention!

Modified Kimber–Martin–Ryskin–Watt uPDF

We follow the standard definition of uPDF in BFKL formalism:

$$\boxed{\int_0^{\mu^2} dt\ \Phi_i(x,t,\mu^2) = F_i(x,\mu^2)} \quad \Longleftrightarrow \quad \boxed{\Phi_i(x,t,\mu^2) = \frac{d}{dt}\left[T_i(t,\mu^2,x)\times F_i(x,t)\right],}$$

where $F_i(x,\mu^2) = xf_i(x,\mu^2)$ and $T_i(t,\mu^2,x)$ is a usually reffered as a ***Sudakov formfactor***, satysfying boundary conditions:

$$T_i(t=0,\mu^2,x)=0 \quad \text{and} \quad T_i(t=\mu^2,\mu^2,x)=1.$$

- SUDAKOV FORMFACTOR REGULARIZES COLLINEAR DIVERGENCES.

(Modified) Kimber–Martin–Ryskin–Watt (MKMRW)[KMR '01; MRW '03] prescription to obtain uPDF from collinear one is based on the mechanism of *last step evolution* (final k_T–dependent parton radiation) and strong angular ordering:

$$\boxed{\Phi_i(x,t,\mu^2) = \frac{\alpha_S(\mu)}{2\pi}\,\frac{T_i(t,\mu^2,x)}{t}\times\sum_j\int_x^1 dz\ P_{ij}(z)\ F_j\left(\frac{x}{z},t\right)\times\theta\left(\Delta(t,\mu)-z\right),}$$

where $\Delta(t,\mu)=\mu/\left(\mu+\sqrt{t}\right)$ is a *KMR–cutoff function* which ensures *rapidity ordering of the last emitted parton and particles produced in the hard process.*

- KMR–CUTOFF $\Delta(t,\mu)$ REGULARIZES IR–DIVERGENCES.

Modified Kimber–Martin–Ryskin–Watt uPDF

The solution for Sudakov formfactor[Nefedov and Saleev '20]:

$$T_i(t,\mu^2,x) = \exp\left[-\int_t^{\mu^2} \frac{dt'}{t'} \frac{\alpha_S(t')}{2\pi} \left(\tau_i(t',\mu^2) + \Delta\tau_i(t',\mu^2,x)\right)\right],$$

with

$$\tau_i(t,\mu^2) = \sum_j \int_0^1 dz\, zP_{ji}(z)\theta(\Delta(t,\mu^2)-z),$$

$$\Delta\tau_i(t,\mu^2,x) = \sum_j \int_0^1 dz\, \theta(z-\Delta(t,\mu^2))\left[zP_{ji}(z) - \frac{F_j\left(\frac{x}{z},t\right)}{F_i(x,t)} P_{ij}(z)\theta(z-x)\right].$$

Conclusion: PRA smoothly interpolates QCD predictions between high–energy and low–energy regions as well as between small–p_T and large–p_T of final particles and allows us to study processes described non–Abelian QCD structures.

Previously, PRA was used to describe production of: dijets, diphotons, open beauty / charm $B\bar{B}/D\bar{D}$, Drell–Yan pairs, pair J/ψ, and many more… (see [Nefedov, Saleev, et.al.])

Pair charmonia production in the ICEM

Cross section for pair heavy quarkonia $\mathcal{Q}[(q\bar{q})]+\mathcal{Q}'[(q\bar{q})']$ production in the ICEM:

$$d\sigma_{QQ'} = \mathcal{F}^{QQ'} \times \int_{M_Q}^{2M_H} dM \int_{M_{Q'}}^{2M_{H'}} dM' \frac{d\sigma_{(q\bar{q})(q\bar{q})'}}{dM\, dM'}$$

But how $\mathcal{F}^{Q}$, $\mathcal{F}^{Q'}$, and $\mathcal{F}^{QQ'}$ parameters are related?

- Since that $r_{Q[1S]} \sim 0.3$ fm, they cannot hadronize independent at characteristic distance 1 fm.

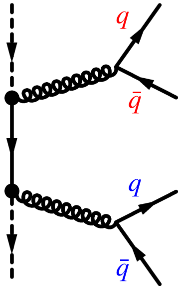
According to the principle of quantum identity, $(q\bar{q})_{\mathrm{f}}$ and $(q\bar{q})'_{\mathrm{f}'}$ pairs hadronize in two different ways:

- The flavours are different $\mathrm{f} \neq \mathrm{f}'$:

$$\mathcal{F}^{QQ'} = \mathcal{F}^{Q} \times \mathcal{F}^{Q'}$$

- The flavours are the same $\mathrm{f} = \mathrm{f}'$:

$$\mathcal{F}^{QQ} \neq \mathcal{F}^{Q} \times \mathcal{F}^{Q}$$



These relations were numerically checked in[A.C. and Saleev '22,23].

Hard multipartonic interactions

Standard approach to study multi–particle production is to use so called *single parton scattering (SPS)*:

$$d\sigma^{\rm SPS}_{\mathcal{H}\mathcal{H}'} = \sum_{i,j} \int dx_1 dx_2 \left[f_i(x_1,\mu^2) \times f_j(x_2,\mu^2) \right] \times d\hat{\sigma}_{ij\to\mathcal{H}\mathcal{H}'}(x_1,x_2)$$

But the proton is a composite system, so *double parton scattering (DPS)* is possible:

$$d\sigma^{\rm DPS}_{\mathcal{H}\mathcal{H}'} = \frac{1}{1+\delta_{\mathcal{H}\mathcal{H}'}} \sum_{i,j,k,l} \int dx_1 dx_2 dx_1' dx_2' d^2\mathbf{b}_1 d^2\mathbf{b}_2 d^2\mathbf{b} \left[\Gamma_{ij}(x_1,x_2,\mu_1^2,\mu_2^2;\mathbf{b}_1,\mathbf{b}_2) \times \Gamma_{kl}(x_1',x_2',\mu_1^2,\mu_2^2;\mathbf{b}_1',\mathbf{b}_2') \right]$$

$$\times \left[d\hat{\sigma}_{ik\to\mathcal{H}}(x_1,x_1',\mu_1^2) \times d\hat{\sigma}_{jl\to\mathcal{H}'}(x_2,x_2',\mu_2^2) \right],$$

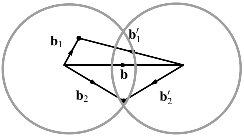
here Γ_{ij} are *generalized double parton distribution functions (dPDF's)*, $\mathbf{b}_i$–distances in transverse plane.

Standard assumptions for phenomenological study:

- Generalized dPDF's decomposition:

$$\boxed{\Gamma_{ij}(x_1,x_2,\mu_1^2,\mu_2^2;\mathbf{b}_1,\mathbf{b}_2) = D_{ij}(x_1,x_2,\mu_1^2,\mu_2^2) \times f(\mathbf{b}_1) f(\mathbf{b}_2),}$$

$$\int d^2\mathbf{b}_i\, d^2\mathbf{b}\; f(\mathbf{b}_i) f(\mathbf{b}_i - \mathbf{b}) = \int d^2\mathbf{b}\; T(\mathbf{b}) = 1$$



- dPDF's factorization in terms of PDF's:

$$\boxed{D_{ij}(x_1,x_2,\mu_1^2,\mu_2^2) = f_i(x_1,\mu_1^2) \times f_j(x_2,\mu_2^2),} \qquad \sigma_{\rm eff} = \left[\int d^2\mathbf{b}\; T^2(\mathbf{b}) \right]^{-1}$$ – «parton interaction area»

▶ $\sigma_{\rm eff}$ IS A FREE PARAMETER, EXTRACTED VALUES ARE VERY DIFFERENT $\sigma_{\rm eff} \in [2,25]$ mb.