Секция ядерной физики ОФН РАН
Объединённый институт ядерных исследований

Новые результаты исследования релятивистских ядерных взаимодействий в пространстве четырехмерных скоростей

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Научная сессия секции ядерной физики Отделения физических наук Российской академии наук и Объединенного институт ядерных исследований, посвященная 300-летию Российской академии наук.
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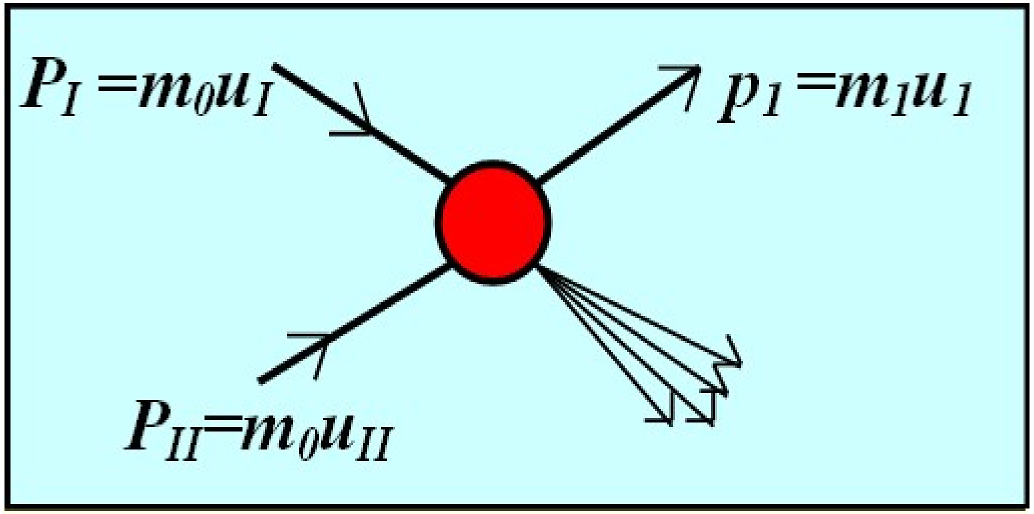
* Совместно с Г.И. Лыкасовым и А.А. Зайцевым

I + II→1 + …



$$(N_I P_I + N_{II} P_{II} - p_1)^2 = (N_I m_0 + N_{II} m_0 + M)^2$$

N_I and N_{II} are the part of the transferred four-momenta of nucleons participating in nuclei I and II.

M is the mass of the particle providing conservation of quantum numbers.

For antinuclei and **K⁻** mesons **$M = m_1$**, for **nuclear fragments $M = -m_1$**. For **K^+** mesons **$M = m_\Lambda - m_0$**. For the particles produced without accompanying antiparticles (**π mesons**) **M=0**.

A.M. Baldin, A.A. Baldin.Phys. Particles and Nuclei, 29(3), 1998, 232

$$\mathrm{I} + \mathrm{II} \rightarrow 1 + \ldots$$

$$\Pi = \min \frac{1}{2}\sqrt{(u_I N_I + u_{II} N_{II})^2}$$

u_I and u_{II} are four velocities of the nuclei I and II.

$$Ed^3\sigma/dp^3 = C_1 A_I^{\alpha(N_I)} \cdot A_{II}^{\alpha(N_{II})} \cdot \exp(-\Pi/C_2)$$

A.M. Baldin, A.I. Malakhov. Relativistic Multiparticle Processes in the Central Rapidity Region at Asimptotically High Energies. JINR Rapid Communications, 1 [87]-98 (1998) 5-12.

An analytical solution has been obtained for the similarity parameter in the central rapidity region (*y* =0):

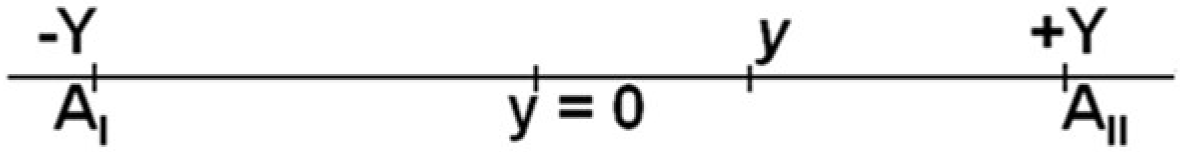
Similarity Parameter

Rapidity of interacting nuclei

$$\Pi = N \cdot ChY$$

Part of the transferred four-momentum

(The Baldin-Malakhov equation)



In the mid-rapidity region (y=0, y is the rapidity of particle 1) the analytical form for **Π** was found:

$$\Pi = N \cdot ChY$$

In this case N_I and N_{II} are equal to each other: $N_I = N_{II} = N$.

$$N = [1 + (1 + \Phi_M / \Phi^2)^{1/2}]\Phi,$$

where

$$\Phi = 2m_0(m_{1t}\, chY + M)/sh^2Y,$$

$$\Phi_M = (M^2 - m^2{}_1)/(4m^2{}_0 \cdot sh^2Y)$$

$m_{1t} = (m^2{}_1 + p_t{}^2)^{1/2}$,

Y – rapidity of interacting nuclei.

$$E\cdot(d^3\sigma/dp^3) = (1/\pi)\, d^3\sigma /(dm_{1t}^2 dy) =$$
$$= [\varphi_q(y,p_t) + \varphi_g(y,p_t)\cdot(1- \sigma_{nd}/g(s/s_0)^{\Delta})]\cdot g(s/s_0)^{\Delta}$$

σ_{nd} – cross-section of hadron production by the exchange of n-pomerons.
g – constant (~20 mbarn), S_0 ~ 1 GeV^2,
$\Delta = [\alpha_p(0)-1] \sim 0,08$

G.I.Lykasov proposed to use functions depending on the similarity parameter Π as functions $\varphi(y,p_t)$:

$$\text{G. Lykasov} \longrightarrow \varphi = \varphi(\Pi)$$

Π = N·ChY

- **Ratio of antiparticle to particle yields**
- **Description of the spectra of secondary particles depending on P_T in a wide range of energies**
- **Energy dependence of the slope parameter on energy**
- **The ratio of the strange kaon yield to the pion yield**
- **Description of the particle yield depending on their rapidity**

Ratio of antiparticle to particle yields

For nuclei and nuclear fragments M = -m_1 → $$\Pi_1 = \left[\frac{m_{1T}}{m_0} chY - \frac{m_1}{m_0}\right] \frac{chY}{sh^2 Y}$$

For anti-nuclei and anti-nuclear fragments M = m_1 → $$\Pi_2 = \left[\frac{m_{1T}}{m_0} chY + \frac{m_1}{m_0}\right] \frac{chY}{sh^2 Y}$$

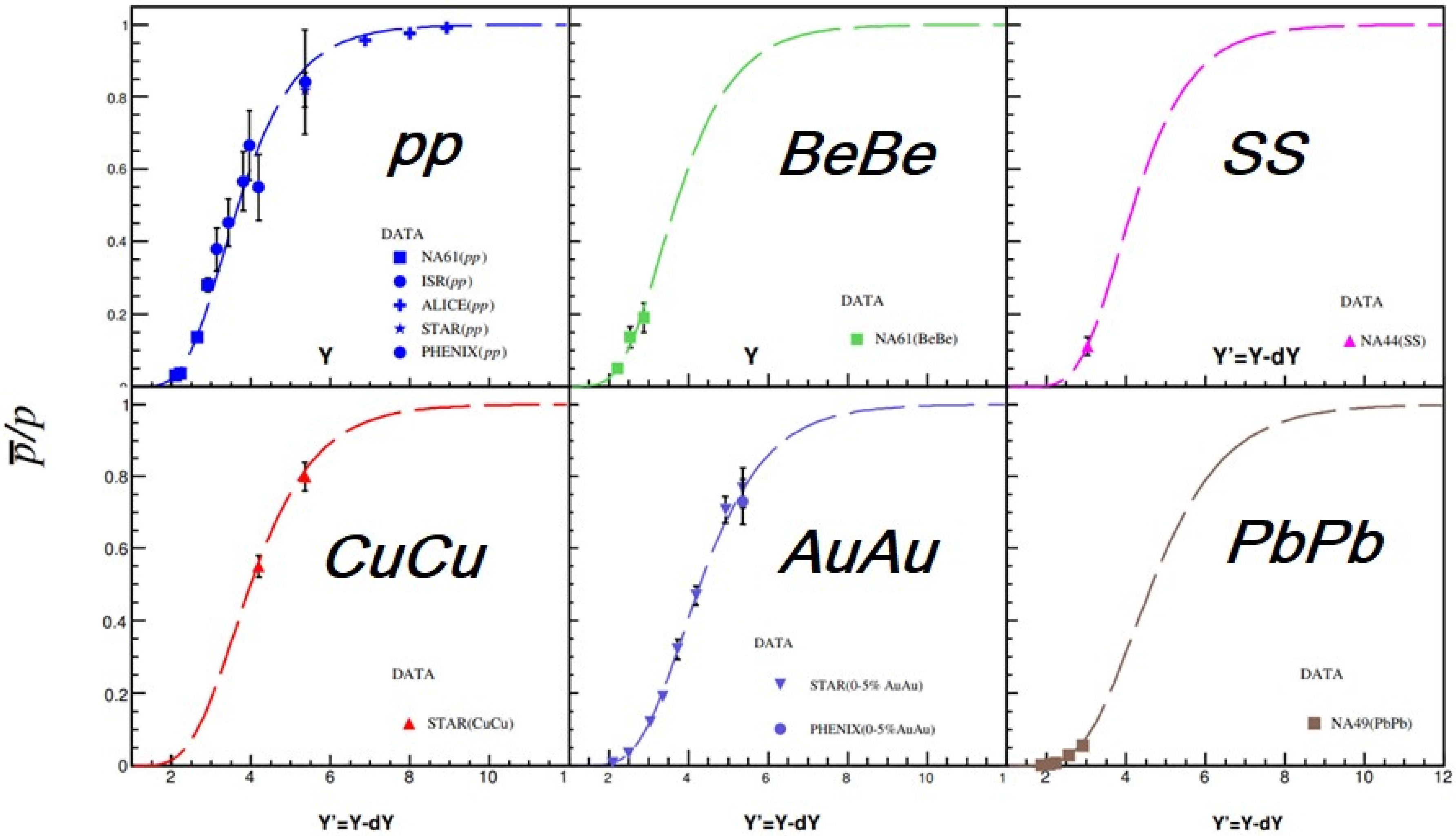
$$\text{Ratio}\left(\frac{\text{antinuclei}}{\text{nuclei}}\right) = \frac{\int_0^\infty m_{1T} \cdot C_1 \cdot A_I^{\alpha(N_I)} \cdot A_{II}^{\alpha(N_{II})} \cdot \exp\left(-\frac{\Pi_2}{C_2}\right) \cdot dm_{1T}}{\int_0^\infty m_{1T} \cdot C_1 \cdot A_I^{\alpha(N_I)} \cdot A_{II}^{\alpha(N_{II})} \cdot \exp\left(-\frac{\Pi_1}{C_2}\right) \cdot dm_{1T}}$$

In case of symmetric nuclei ($A_I = A_{II} = A$) the above relation takes the following form:

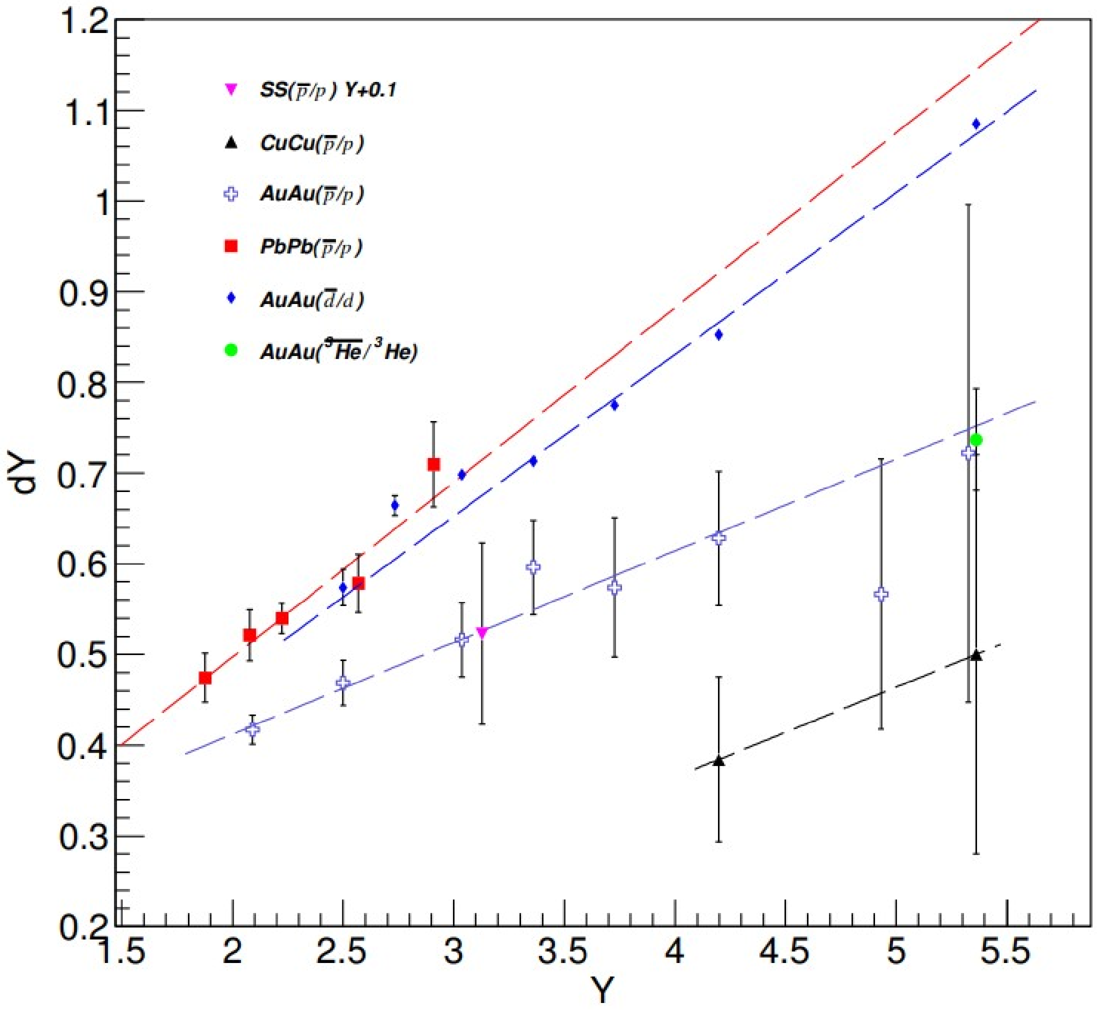
$$\mathrm{Ratio}\left(\frac{\mathrm{antibaryon}}{\mathrm{baryon}}\right) = A^{\frac{4}{3}\frac{m_1}{m_0}\frac{1}{sh^2Y}} \cdot \exp\left(-\frac{2\frac{m_1}{m_0}\cdot\frac{chY}{sh^2Y}}{C_2}\right).$$

If $A_I = A$, $A_{II} = B$, then

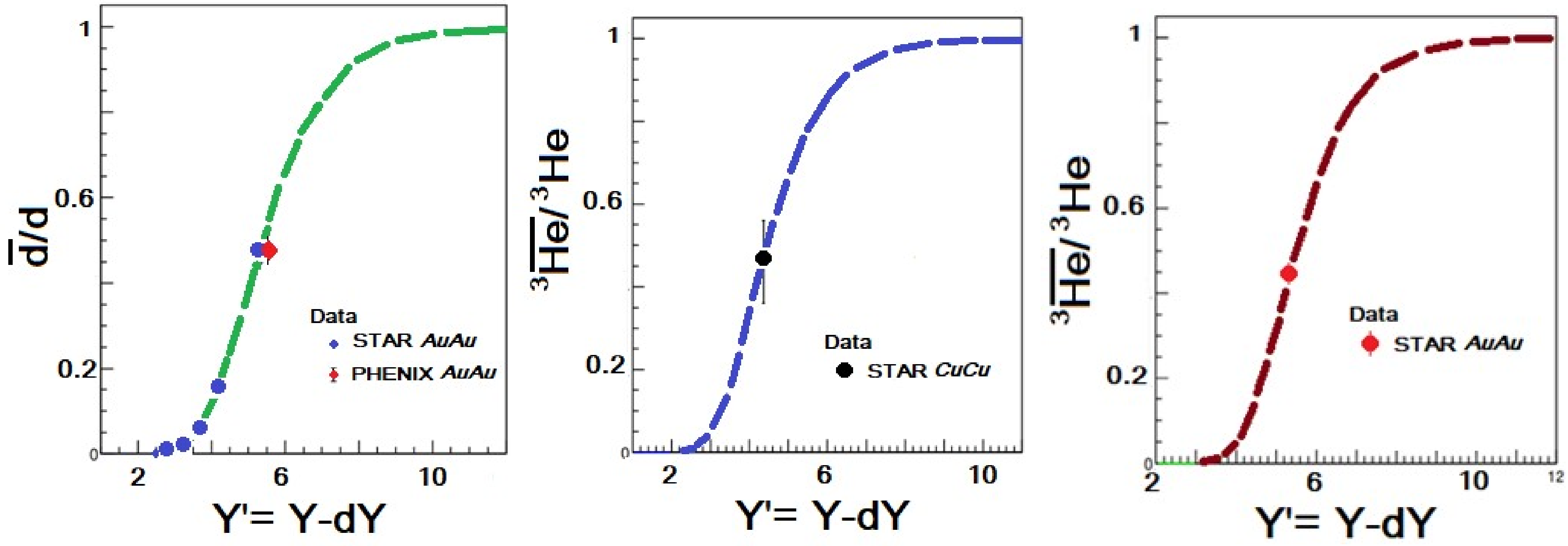
$$\mathrm{Ratio}\left(\frac{\mathrm{antibaryon}}{\mathrm{baryon}}\right) = (A\cdot B)^{\frac{2}{3}\frac{m_1}{m_0}\frac{1}{sh^2Y}} \cdot \exp\left(-\frac{2}{C_2}\frac{m_1}{m_0}\cdot\frac{chY}{sh^2Y}\right).$$



Description of the yield ratios of anti-p/p with one value of constant C_2=0.146 taking into account dY (Y) dependences.



The dependence of the rapidity loss of dY on the rapidity of Y. The dotted lines are linear approximations of dY (Y)=p0+p1·Y .



Description of the yield ratios of anti-d/d, anti-He^3/He^3 with one value of constant C_2=0.146.

A.I. Malakhov and A.A. Zaitsev. Journal of Experimental and Theoretical Physics, 2022, Vol. 135, No. 2, pp. 209–214.

Description of the spectra of secondary particles depending on P_T in a wide range of energies

Using relativistic invariant variables s, p_t and ch(Y) = $\sqrt{s}/2m_0$ dependence we have obtained the following form for Π:

$$\Pi = \left\{ \frac{m_{1t}}{2m_0\delta} + \frac{M}{\sqrt{s}\delta} \right\} \left\{ 1 + \sqrt{1 + \frac{M^2 - m_1^2}{m_{1t}^2}\delta} \right\}$$

$$\delta = 1 - 4m_0^2/s, \quad m_t^2 = p_t^2 + m_1^2$$

Baldin-Malakhov-Lykasov equation

At large $\sqrt{s}$ >>1 GeV:

$$\Pi = \frac{m_{1t}}{2m_0(1 - 4m_0^2/s)} \left\{ 1 + \sqrt{1 + \frac{M^2 - m_1^2}{m_{1t}^2}(1 - 4m_0^2/s)} \right\}$$

G.I. Lykasov, A.I. Malakhov. Self-consistent analysis of hadron production in pp and AA collisions at mid-rapidity. Eur.Phys. J. A54, 187 (2018).

For π-mesons at $p_t^2 \gg m_1^2$:

$$\Pi \simeq \frac{m_{1t}}{m_0(1 - 4m_0^2/s)}$$

$$E\cdot(d^3\sigma/dp^3) = (1/\pi)\, d^3\sigma\,/(dm_{1t}{}^2 dy) =$$

$$= [\varphi_q(y,p_t) + \varphi_g(y,p_t)\cdot(1- \sigma_{nd}/g(s/s_0)^{\Delta})]\cdot g(s/s_0)^{\Delta}$$

σ_{nd} – cross-section of hadron production by the exchange of n-pomerons.
g – constant (~20 mbarn), S_0 ~ 1 GeV^2,
$\Delta = [\alpha_p(0)-1]$ ~ 0,08

G.I.Lykasov proposed to use functions depending on the similarity parameter Π as functions $\varphi(y,p_t)$:

G. Lykasov $\longrightarrow \varphi = \varphi(\Pi)$

The first part of inclusive spectrum (Soft QCD (quarks)) is related to the function ϕ_q(y=0, Π), which is fitted by the following form [*]:

$$\phi_q(y=0,\Pi) \;=\; A_q exp(-\Pi/C_q)\ ,$$

where $A_q = 3.68\ (GeV/c)^{-2}, C_q = 0.147$

The function $\phi_g(y=0, \Pi)$ related to the second part (Soft QCD (gluons)) of the spectrum is fitted in the following form [*]:

$$\phi_g(y=0,\Pi) \;=\; A_g\sqrt{m_{1t}} exp(-\Pi/C_g)\ ,$$

where $A_g = 1.7249\ (GeV/c)^{-2}, C_g = 0.289$

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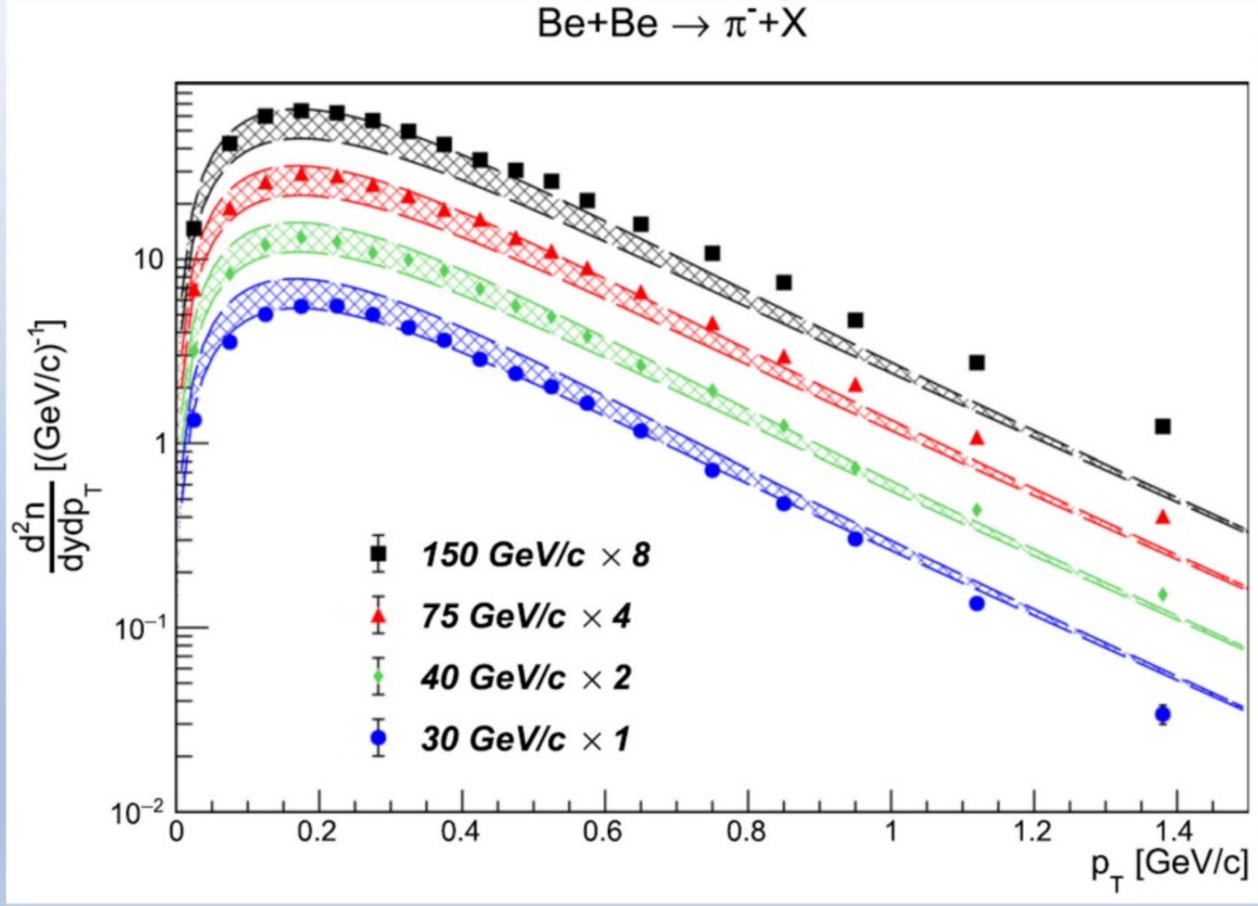
[*] **V. A. Bednyakov, A. A. Grinyuk, G. I. Lykasov, M. Pogosyan. Int.J.Mod.Phys., A27 (2012) 1250042.**

DESCRIPTION OF p_t SPECTRA OF PIONS AND KAONS IN *BeBe* COLLISIONS

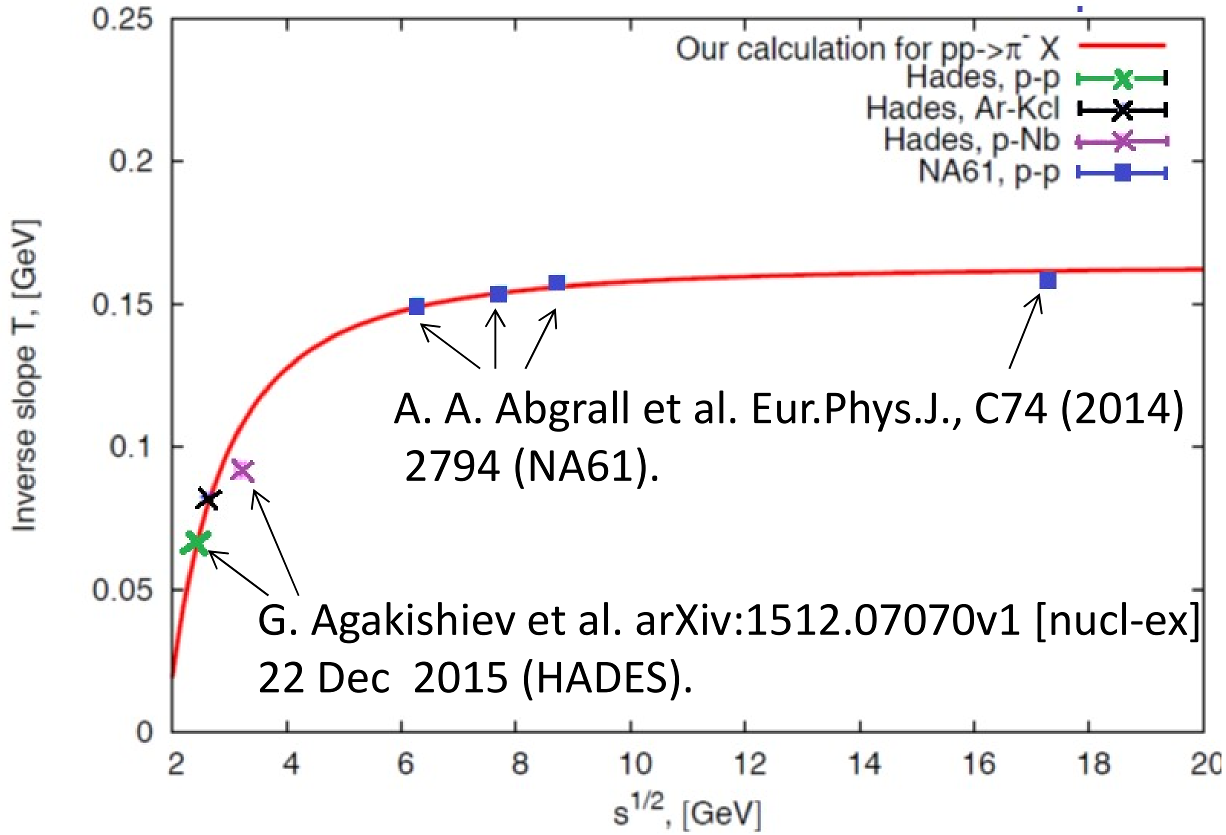


G. I. Lykasov, A. I. Malakhov, A. A. Zaitsev. Ratio of kaon-to-pion production cross-sections in BeBe collisions as a function of √s. Eur. Phys. J. A (2022) 58:112

Energy dependence of the slope parameter on energy

$$E(d^3\sigma/dp^3) \sim \exp(-m_t/T), \quad T = Const$$

$$E(d^3\sigma/dp^3) \sim \exp(-\Pi/C_2) = \exp(-m_{1t}/[C_2 m_0(1-4m_0^2/s)]$$

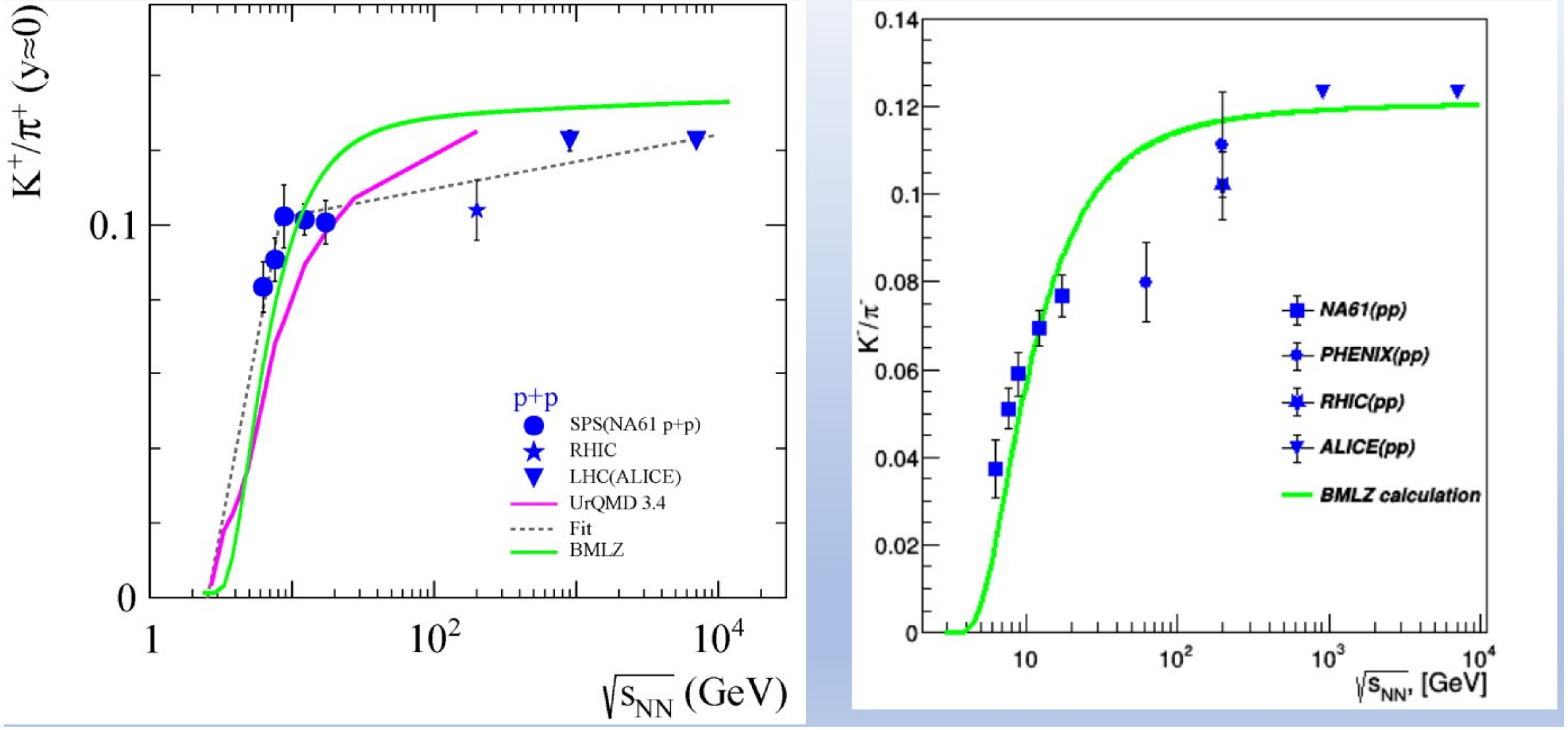


$$T = C_2 m_0(1-4m_0^2/s)$$

G.I. Lykasov, A.I. Malakhov. Self-consistent analysis of hadron production in pp and AA collisions at mid-rapidity. Eur.Phys.J.A 54 (2018) 11, 187.

The ratio of the strange kaon yield to the pion yield

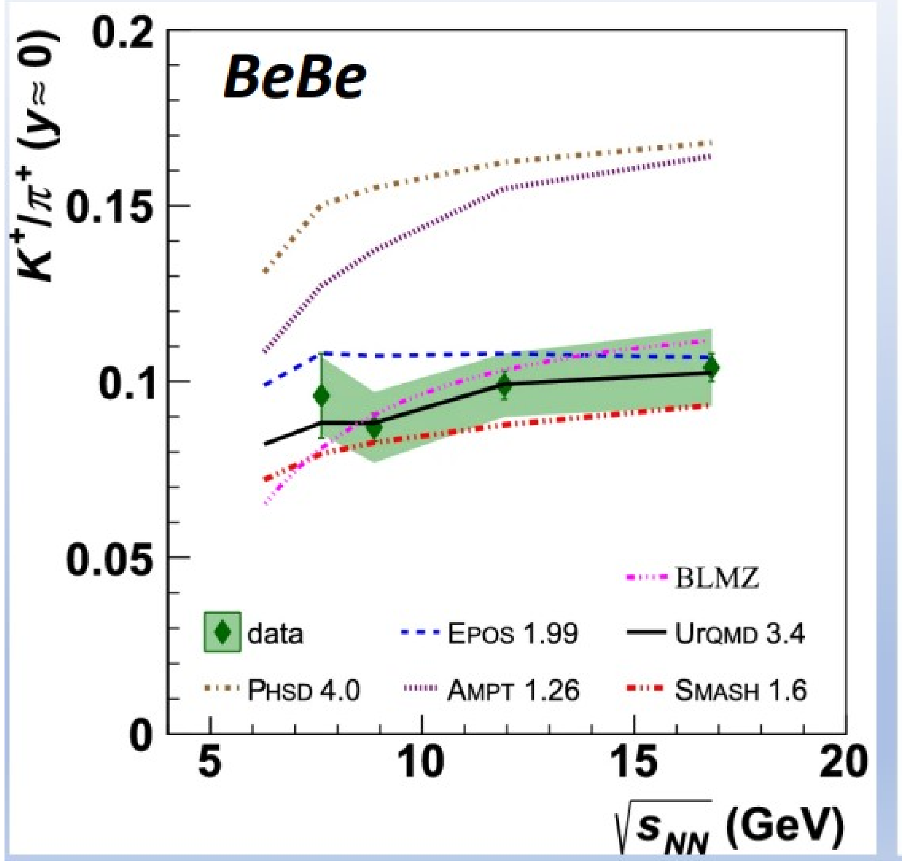
Ratios of kaons to pions in *pp* collisions as functions of $\sqrt{s}$

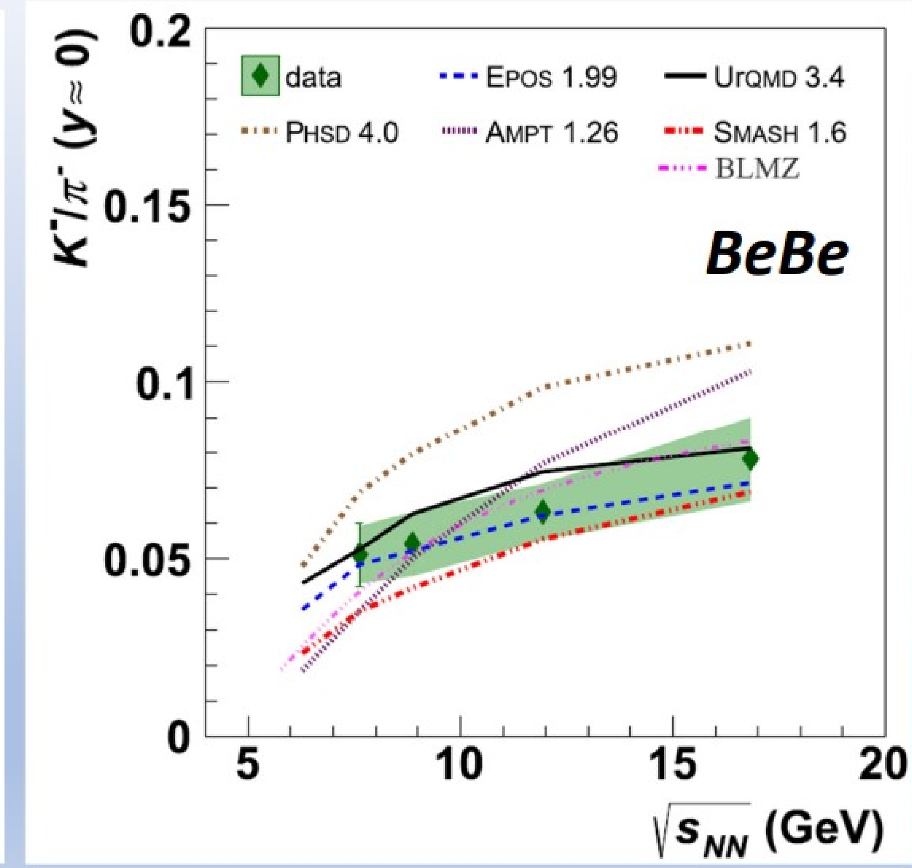


BMLZ – Baldin-Malakhov-Lykasov-Zaitsev model.

G.I. Lykasov, A.I. Malakhov, A.A. Zaitsev. Ratio of cross-sections of kaons to pions produced in pp collisions as a function of √s. Eur.Phys.J.A 57 (2021) 3, 91.

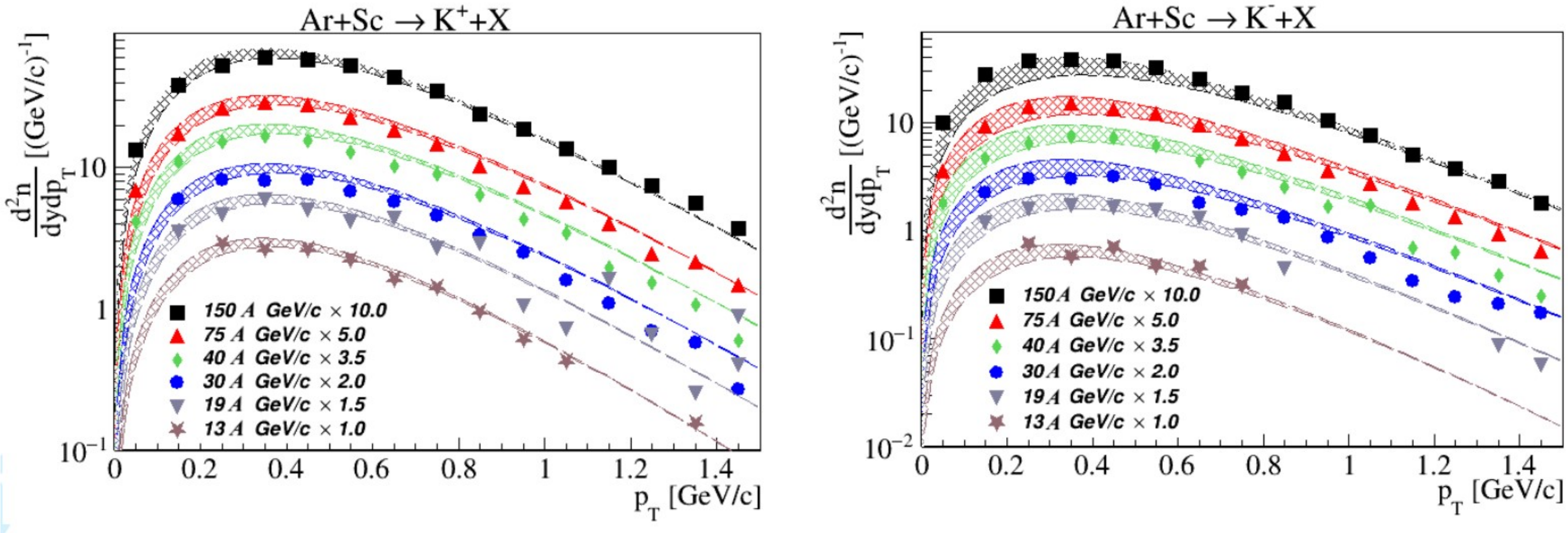
Ratios of kaons to pions as functions of $\sqrt{s}$





BLMZ model:

Lykasov, G.I., Malakhov, A.I. and Zaitsev, A.A. Ratio of kaon-to-pion production cross-sections in BeBe collisions as a function of √s. Eur. Phys. J. A 58, 112 (2022).



G.I. Lykasov, A.I. Malakhov and A.A. Zaitsev. Production of charged kaons in ArSc collisions. http://arxiv.org/abs/2402.03260

Description of the particle yield depending on their rapidity

A.I. Malakhov, G.I. Lykasov. Mid-rapidity dependence of hadron production in p-p and A−A collisions. Eur.Phys.J.A56 (2020) 4, 114.

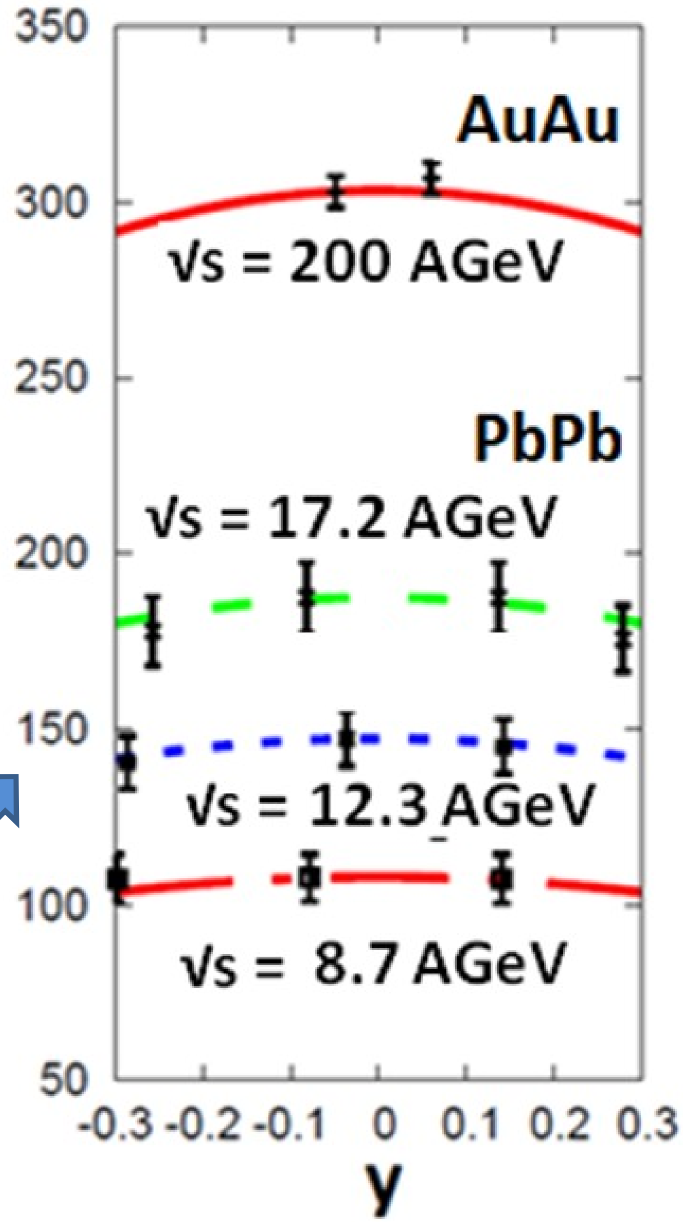
$$\Pi = N \cdot \mathrm{ch}Y$$

$$N = \{1 + [1 + (\Phi_M / \Phi^2)]^{1/2}\}\Phi$$

$$\Phi \approx \{(1/m_0)[m_{1t}\,\mathrm{ch}y \cdot \mathrm{ch}Y + M]\} \cdot [1/(2\mathrm{sh}^2 Y)]$$

$$\Phi_M = (M^2 - m_1^2)/(4 m_0^2 \mathrm{sh}^2 Y)$$

Pion y-spectra in AuAu (RHIC) and PbPb (SPS) collision.
J. Cleymans, et al., Phys. Rev. C78, 017901 (2008).



Conclusion

Inclusive spectra of the pions and kaons produced in pp, BeBe and kaons in ArSc collisions as functions of their transverse momentum p_T , have been calculated within the approach based on the assumption of the similarity of inclusive spectra of the hadrons produced in nucleus-nucleus collisions in the mid-rapidity region taking into account the quark-gluon dynamics in nucleon-nucleon interactions.

As a result, we have obtained a good description:

- **depending on p_T of secondary particles spectra in a wide range of energies;**
- **energy dependence of the inverse slope parameter;**
- **ratio of the strange kaon yield to the pion yield.**

This approach has also allowed us to describe:

- **particle yield depending on their rapidity;**
- **ratio of antiparticle to particle yields.**

Thank you for the attention!