Lepton angular coefficients in the $Z-$boson production in the high-energy factorization approach

Omelyanchuk S.[1], Saleev V.[1,2]

[1]Samara University
[2]Joint Institute for Nuclear Research

April 3, 2024
JINR, Dubna

Introduction

Inclusive $Z-$boson production at the LHC

$$p + p \to Z + X$$

Drell-Yan processes as tools to search PDFs (in CPM, TMD PM and k_T-fatorization (HEF))

- $q + \bar{q} \to \gamma^* \to \mu^+ + \mu^-$
- $q + \bar{q} \to Z \to \mu^+ + \mu^-$
- $g + g \to H \to \gamma + \gamma$

$$Q = \sqrt{(q_{\mu^+} + q_{\mu^-})^2}, \quad m_Z, \quad m_H >> \Lambda_{QCD}$$

Final states in the DY processes are colorless $\Rightarrow$ hard-soft factorization

Hadronization in the final state is absent $\Rightarrow$ more simple for experimental analysis

Introduction

CMS, ATLAS and LHCb at 7, 8 and 13 TeV

- Total cross sections
- Transverse momentum spectra of $Z-$boson
- Lepton angular coefficients $A_0 - A_7$ in $Z \to \mu^+\mu^-(e^+e^-)$

NLO, NLO+jet, NNLO calculations in the CPM, $p_T > 5$ GeV

- LO: $q + \bar{q} \to Z$
- NLO: $q + g \to q + Z$ + 1-loop QCD and EW corrections
- NNLO: $g + g \to Z + q + \bar{q}$ + 2-loop QCD and EW corrections
- At small-p_T one has the resummation of large $\log(m_Z/p_T)$
- Monte-Carlo "kitchen", which includes Parton Shower (PS), Underline Events, DPS effects ...

NLO+NNLL calculations in the TMD PM, $0 < p_T < 15$ GeV

LO calculations in the HEF, all $p_T << \sqrt{s}$

Factorization approaches: CPM, TMD PM and HEF

Collinear parton model

- $p_T >> q_{1,2T} \sim 1$ GeV and $\mu_F \sim \sqrt{m_Z^2 + p_T^2}$

$$d\sigma^{LO}(pp \to ZX) = \int dx_1 \int dx_2 f_q(x_1, \mu_F) f_{\bar{q}}(x_2, \mu_F) d\hat{\sigma}(q + \bar{q} \to Z + g) + \mathcal{O}(\Lambda_{QCD}^2/\mu_F^2)$$

- DGLAP evolutions for $f_{q,g}(x, \mu_F)$ with k_T-ordering during QCD evolution of partons equations

Factorization approaches: CPM, TMD PM and HEF

TMD PM originally by Collins, Soper, Sterman

- $p_T \sim q_{1,2T}$ and $\Lambda_{QCD} << p_T << \mu_F$

$$d\sigma^{TMD}(pp \to ZX) = \int dx_1 d^2q_{1T} \int dx_2 d^2q_{2T} F_q(x_1, q_{1T}, \mu_F, \mu_Y) \times \times F_{\bar{q}}(x_2, q_{2T}, \mu_F, \mu_Y) d\hat{\sigma}(q+\bar{q} \to Z) + \mathscr{O}(< q_T^2 > / \mu_F^2)$$

- $$\frac{d\sigma^{TMD}(p+p \to ZX)}{dp_T dy} = \sigma_0(s, M_Z, \mu_F) \int \frac{d^2\mathbf{b}}{(2\pi)^2} e^{i\mathbf{b}\cdot\mathbf{q}_T} \times \tilde{F}_{q/P_1}(x_1, \mathbf{b}, \mu, \zeta_1) \tilde{F}_{\bar{q}/P_2}(x_2, \mathbf{b}, \mu, \zeta_2) + Y(M_Z, y, \mathbf{p}_T) + \text{ suppressed corrections}$$

where $\tilde{F}_{q/P}(x, \mathbf{b}, \mu, \zeta)$ is universal TMD PDFs with evolution.

$$\tilde{F}_{q/P}(x, \mathbf{b}, \mu, \zeta) \leftrightarrows F_{q/P}(x, \mathbf{k}_T, \mu, \zeta)$$

Factorization approaches: CPM, TMD PM and HEF

HEF for hard processes in Multi-Regge-Kinematics, $\Lambda_{QCD} << \mu_F << \sqrt{s}$ or $x << 1$

HEF $\rightarrow$ Parton Reggeization Approach (PRA)

Parton Reggeization Approach smoothly interpolate between regions $p_T << \mu_F$ and $p_T \sim \mu_F$

$$d\sigma^{PRA}(pp \to ZX) = \int \frac{dx_1}{x_1} \frac{d^2 q_{1T}}{\pi} \int \frac{dx_2}{x_2} \frac{d^2 q_{2T}}{\pi} \Phi_Q(x_1, \vec{q}_{1T}, \mu_F^2) \Phi_{\bar{Q}}(x_2, \vec{q}_{2T}, \mu_F^2) d\hat{\sigma}^{PRA}(Q\bar{Q} \to Z)$$

$$q_{1,2}^{\mu} = x_{1,2} P_{1,2}^{\mu} + q_{1,2T}^{\mu}, \quad q_{1,2T} \neq 0, \quad q_{1,2}^2 = q_{1,2T}^2 = -\vec{q}_{1,2T}^{\,2}$$

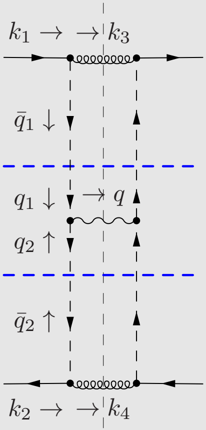
Instead of TMD PM, initial partons in HEF are off-mass-shell (Reggeized) and $q_T \sim p_T \sim \mu_F$

For details of the PRA, see following publications:

- M.A. Nefedov, N.N. Nikolaev and V.A. Saleev, Drell-Yan lepton pair production at high energies in the Parton Reggeization Approach, Phys. Rev. D **87** (2013) no.1, 014022
- M. Nefedov and V. Saleev, Off-shell initial state effects, gauge invariance and angular distributions in the Drell-Yan process// Phys. Lett. B **790**, 551-556 (2019)
- M. A. Nefedov and V. A. Saleev, High-Energy Factorization for Drell-Yan process in pp and $p\bar{p}$ collisions with new Unintegrated PDFs // Phys. Rev. D **102** (2020), 114018

Parton Reggeization Approach

MRK factorization, $z_{1,2} << 1$ and $\tilde{q}_2^+ << \tilde{q}_2^-$, $\tilde{q}_1^- << \tilde{q}_1^+$, $\mathbf{q}_T^2/Q^2$ can be arbitrary, $Q^2, Q_T^2 << S$



MMRK factorization, from $2 \rightarrow 3$ to $2 \rightarrow 1$ with Reggeized off-shell quarks

We consider auxiliary hard CPM subprocesses:

$$q + \bar{q} \rightarrow \gamma^* + g + g \Rrightarrow Q + \bar{Q} \rightarrow \gamma^*$$

$$g + g \rightarrow \gamma^* + q + \bar{q} \Rrightarrow Q + \bar{Q} \rightarrow \gamma^*$$

$$q + g \rightarrow \gamma^* + q + g \Rrightarrow Q + \bar{Q} \rightarrow \gamma^*$$

$$\Gamma_\mu^{(+-)}(q_1, q_2) = \gamma_\mu - \hat{q}_1 \frac{n_\mu^-}{q_2^-} - \hat{q}_2 \frac{n_\mu^+}{q_1^+} \quad (1)$$

The Fadin-Sherman vertex [V. S. Fadin and V. E. Sherman, JETP Lett. 23, 599 (1976)] which we have applied in the case of Drell-Yan process for the first time in [M. Nefedov, N. Nikolaev, and V. Saleev, Phys. Rev. D 87, 876 014022 (2013)]

LO mMRK factorization formula

Auxiliary hard CPM subprocess:

$$q(k_1)+\bar{q}(k_2)\to g(k_3)+\gamma^*(q)+g(k_4),$$

where $k_1^2=0$, $k_1^-=0$, $k_2^2=0$, $k_2^+=0$.
Kinematic variables ($0<z_{1,2}<1$):

$$z_1=\frac{k_1^+-k_3^+}{k_1^+},\quad z_2=\frac{k_2^--k_4^-}{k_2^-},$$

There are two limits where $\overline{|\mathcal{M}|^2}$ factorizes:

1. **Collinear limit:** $\mathbf{k}_{T3,4}^2,\mathbf{q}_T^2\ll Q^2$, $z_{1,2}$ – arbitrary,
2. **Multi-Regge limit:** $z_{1,2}\ll 1$, $\mathbf{k}_{T3,4}^2$, $\mathbf{q}_T^2$ – arbitrary.

LO mMRK factorization formula

Multi-Regge limit: $z_{1,2} \ll 1$, $\mathbf{k}_{T3,4}^2$ – arbitrary:

$$\overline{|\mathscr{M}|^2}_{\rm MRK} \simeq \frac{4g_s^4}{\mathbf{k}_{T3}^2\mathbf{k}_{T4}^2}\tilde{P}_{qq}(z_1)\tilde{P}_{qq}(z_2)\frac{\overline{|\mathscr{A}_{PRA}|^2}}{z_1 z_2},$$

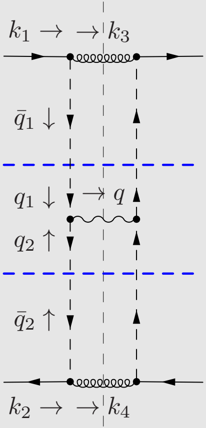
where $\overline{|\mathscr{A}_{PRA}|^2}$ is the **gauge-invariant** amplitude $Q_+(q_1)+\bar{Q}_-(q_2)\to\gamma^*(q)$ with **Reggeized (off-shell)** partons in the initial state.

In MRK regime we can use Lipatov's Effective Field Theory of Reggeized gluons and Reggeized quarks to obtain $\mathscr{A}_{PRA}$

- E. A. Kuraev, L. N. Lipatov and V. S. Fadin, Sov. Phys. JETP **44**, 443-450 (1976)
- L. N. Lipatov, Nucl. Phys. B452, 369 (1995).
- L. N. Lipatov and M. I. Vyazovsky, Nucl. Phys. B597, 399 (2001).
- V. S. Fadin and V. E. Sherman, JETP Lett. 23, 599 (1976)
- E. N. Antonov, L.N. Lipatov, E.A. Kuraev, and I. O. Cherednikov, Nucl. Phys. B721, 111 (2005).
- M.A. Nefedov, ReggeQCD model-file for FeynArts (2017-2019).

LO mMRK factorization formula

mMRK



Modified MRK approximation: $z_{1,2}$ and $\mathbf{k}_{T3,4}^2$ – arbitrary

:

$$\overline{|\mathscr{M}|^2}_{\rm mMRK} \simeq \frac{4g_s^4}{\tilde{q}_1^2\tilde{q}_2^2} P_{qq}(z_1)P_{qq}(z_2)\frac{\overline{|\mathscr{A}_{PRA}|^2}}{z_1 z_2},$$

where $\tilde{q}_{1,2}^2 = \mathbf{q}_{T1,2}^2/(1-z_{1,2})$, $P_{qq}(z)$ – DGLAP $q \rightarrow q$ splitting function. This factorization formula has correct **Collinear** and **MRK** limits!

Factorization formula

Substituting the $\overline{|\mathscr{M}|^2}_{\rm MMRK}$ to the factorization formula of CPM and changing the variables we get:

$$d\sigma = \int\limits_0^1 \frac{dx_1}{x_1} \int \frac{d^2\mathbf{q}_{T1}}{\pi} \Phi_q^{(\rm tree)}(x_1, t_1, \mu_F^2, \mu_Y^2) \times \int\limits_0^1 \frac{dx_2}{x_2} \int \frac{d^2\mathbf{q}_{T2}}{\pi} \Phi_{\bar{q}}^{(\rm tree)}(x_2, t_2, \mu_F^2, \mu_Y^2) \cdot d\hat{\sigma}_{\rm PRA},$$

where $x_1 = q_1^+/P_1+$, $x_2 = q_2^-/P_2^-$, $\Phi^{(\rm tree)}(x, t, \mu_Y^2)$ – "tree-level" **unintegrated PDFs**. The partonic cross-section in PRA is written as:

$$d\hat{\sigma}_{\rm PRA} = \frac{\overline{|\mathscr{A}_{PRA}|^2}}{2Sx_1x_2} \cdot (2\pi)^4 \delta\left(\frac{1}{2}\left(q_1^+ n_- + q_2^- n_+\right) + q_{T1} + q_{T2} - q(\gamma^*)\right) d\Phi(\gamma^*).$$

Note that **flux-factor** $2x_1x_2S$ for **off-shell** initial-state partons should be used, $\mu_F = \mu_Y = \mu$.

Tree-level unintegrated PDFs:

$$\Phi_i^{(\rm tree)}(x, t, \mu) = \frac{\alpha_s(\mu)}{2\pi} \frac{1}{t} \sum_{j=q,\bar{q},g} \int\limits_x^1 dz\, P_{ij}(z) F_j\left(\frac{x}{z}, \mu_F^2\right) \theta\left(\Delta(t,\mu) - z\right),$$

where $F_i(x,\mu) = xf_j(x,\mu)$. The θ-functions enforce the rapidity-ordering between particles in the final-state $y_3 > y_{\gamma^*} > y_4$, for our MMRK approximation for the squared amplitude and kinematics to be applicable. The KMR cutoff function defines the region of applicability of mMRK:

$$\Delta(t, \mu^2) = \frac{\mu}{\mu + \sqrt{t}}.$$

Unintegrated PDFs with exact normalization

To resolve a divergence problem of $\Phi_i^{(\mathrm{tree})}(x,t,\mu_Y^2)$

we follow the standard definition of the uPDF in BFKL formalism and require that:

$$\int_0^{\mu^2} dt\ \Phi_i(x,t,\mu^2) = F_i(x,\mu^2),$$

which is equivalent to:

$$\Phi_i(x,t,\mu^2) = \frac{d}{dt}\left[T_i(t,\mu^2,x)F_i(x,t)\right],$$

where $T_i(t,\mu^2,x)$ is usually referred to as *Sudakov form-factor*, satisfying the boundary conditions $T_i(t=0,\mu^2,x)=0$ and $T_i(t=\mu^2,\mu^2,x)=1$.

We ask exact equivalence between last ones and following (*KMRW*) prescription:

$$\Phi_i(x,t,\mu) = \frac{\alpha_s(t)}{2\pi}\frac{T_i(t,\mu^2,x)}{t}\sum_{j=q,\bar{q},g}\int_x^1 dz\ P_{ij}(z)F_j\left(\frac{x}{z},t\right)\theta\left(\Delta(t,\mu^2)-z\right),$$

Unintegrated PDFs with exact normalization

The solution for Sudakov form-factor

$$T_i(t,\mu^2,x) = \exp\left[-\int_t^{\mu^2} \frac{dt'}{t'} \frac{\alpha_s(t')}{2\pi} \left(\tau_i(t',\mu^2) + \Delta\tau_i(t',\mu^2,x)\right)\right]$$

with

$$\tau_i(t,\mu^2) = \sum_j \int_0^1 dz\; zP_{ji}(z)\theta(\Delta(t,\mu^2)-z), \qquad \Delta\tau_i(t,\mu^2,x) = \sum_j \int_0^1 dz\; \theta(z-\Delta(t,\mu^2))\left[zP_{ji}(z) - \frac{F_j\left(\frac{x}{z},t\right)}{F_i(x,t)} P_{ij}(z)\theta(z-x)\right].$$

mMRK PDFs versus KMRW PDFs

- The Sudakov form-factor in mMRK uPDFs depends on x
- The rapidity-ordering condition is imposed both on quarks and gluons, while in KMRW-approach it is imposed only on gluons
- The condition $x << 1$ becomes not so strong as for KMRW uPDFs

The PRA works as TMD PM at the $p_T << \mu << \sqrt{s}$ and as CPM at the $p_T \sim \mu$ and $x < 1$

$$d\sigma^{PRA}(pp \to ZX) = \int \frac{dx_1}{x_1} \frac{d^2q_{1T}}{\pi} \int \frac{dx_2}{x_2} \frac{d^2q_{2T}}{\pi} \Phi_Q(x_1,\vec{q}_{1T},\mu_F^2)\Phi_{\bar{Q}}(x_2,\vec{q}_{2T},\mu_F^2) d\hat{\sigma}^{PRA}(Q\bar{Q} \to Z)$$

Z-boson production in the LO PRA

M. A. Nefedov and V. A. Saleev, High-Energy Factorization for Drell-Yan process in pp and $p\bar{p}$ collisions with new Unintegrated PDFs // Phys. Rev. D 102 (2020), 114018

$$Q + \bar{Q} \to Z(\gamma^*) \to \mu^+ + \mu^-$$

$$\overline{|\mathcal{M}_{q\bar{q}}|^2} = \frac{(4\pi\alpha)^2}{4N_c}\left\{\frac{e_q^2}{\hat{s}^2}M^2_{VV,VV} + \frac{1}{(\hat{s}-M_Z^2)^2+M_Z^2\Gamma_Z^2}\left[2\left(C^{(q)}_{AA}C^{(q)}_{VV}+C^{(q)}_{AV}C^{(q)}_{VA}\right)M^2_{VV,AA}\right.\right.$$
$$\left. + \left((C^{(q)}_{AA})^2+(C^{(q)}_{AV})^2+(C^{(q)}_{VA})^2+(C^{(q)}_{VV})^2\right)M^2_{VV,VV} + \left((C^{(q)}_{AA})^2+(C^{(q)}_{VA})^2\right)\Delta M^2\right]$$
$$\left. + \frac{4(\hat{s}-M_Z^2)e_q}{\hat{s}((\hat{s}-M_Z^2)^2+M_Z^2\Gamma_Z^2)}\left[C^{(q)}_{VV}M^2_{VV,VV}+C^{(q)}_{AA}M^2_{VV,AA}\right]\right\},$$

where $C^{(q)}_{ab} = C^{(a)}_q C^{(b)}_{Zl^-}$ is the product of quark and lepton coupling-factors (17), while

$$M^2_{VV,VV} = \frac{8}{q_1^+q_2^-}[2(p_1^-q_1^+)^2(q_1^+q_2^- - t_1) - 2p_1^-q_1^+q_2^-(q_1^+(m_l^2-\hat{t}-t_1) + p_1^+(2q_1^+q_2^- + \hat{t}+\hat{u}-2m_l^2))$$
$$+ q_2^-(2(p_1^+)^2q_2^-(q_1^+q_2^- - t_2) + 2p_1^+q_1^+q_2^-(t_2+\hat{u}-m_l^2) + q_1^+(2m_l^2(q_1^+q_2^-+\hat{s}) - \hat{s}(\hat{t}+\hat{u})))],$$
$$M^2_{VV,AA} = 8\hat{s}(\hat{u}-\hat{t}+2(p_1^+q_2^- - p_1^-q_1^+)),$$
$$\Delta M^2 = 32m_l^2(\hat{t}+\hat{u}-2m_l^2),$$

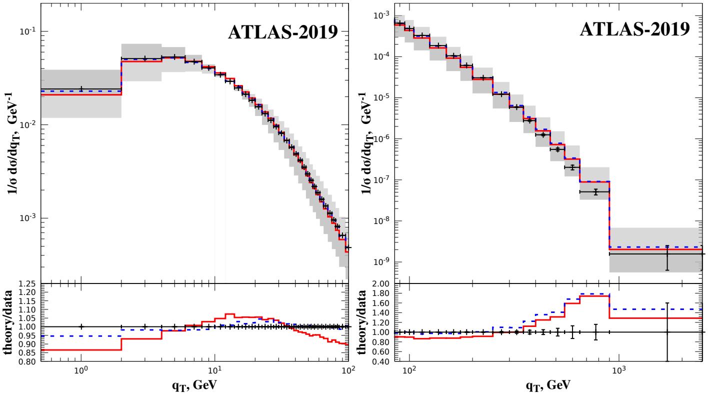
$$M^2_{VV,VV} = \frac{8}{q_1^+q_2^-}[2(p_1^-q_1^+)^2(q_1^+q_2^- - t_1) - 2p_1^-q_1^+q_2^-(q_1^+(m_l^2-\hat{t}-t_1) + p_1^+(2q_1^+q_2^- + \hat{t}+\hat{u}-2m_l^2))$$
$$+ q_2^-(2(p_1^+)^2q_2^-(q_1^+q_2^- - t_2) + 2p_1^+q_1^+q_2^-(t_2+\hat{u}-m_l^2) + q_1^+(2m_l^2(q_1^+q_2^-+\hat{s}) - \hat{s}(\hat{t}+\hat{u})))],$$
$$M^2_{VV,AA} = 8\hat{s}(\hat{u}-\hat{t}+2(p_1^+q_2^- - p_1^-q_1^+)),$$
$$\Delta M^2 = 32m_l^2(\hat{t}+\hat{u}-2m_l^2),$$

where $\hat{s} = (q_1+q_2)^2$, $\hat{t} = (q_1-p_1)^2$, $\hat{u} = (q_2-p_1)^2$ and $t_{1,2} = \mathbf{q}^2_{T1,2}$. If instead of the process (36) one consi process $\bar{Q}(q_1) + Q(q_2) \to l^+ + l^-$, the overall sign of the Lorentz-structure $M^2_{VV,AA}$ should be flipped.

Z-boson production in the LO PRA

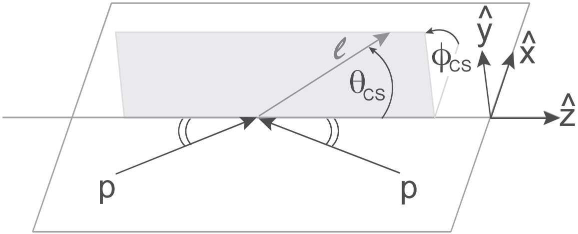
The total cross section, $80 < M_{l\bar{l}} < 100$ GeV

$$K_{LO} = \frac{\sigma^{exp}}{\sigma^{theory}_{LO}} \simeq 1.7$$



Z-boson production in the LO PRA

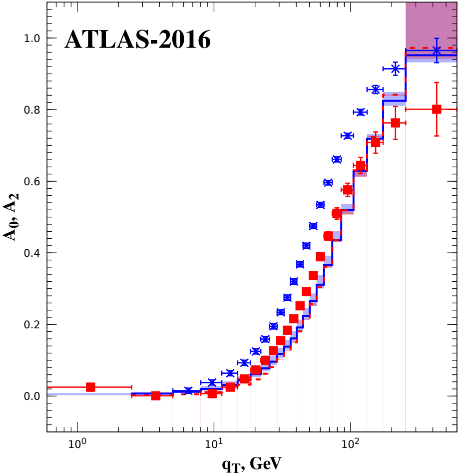
To obtain angular coefficients A_n we use the analytical harmonic-projectors method



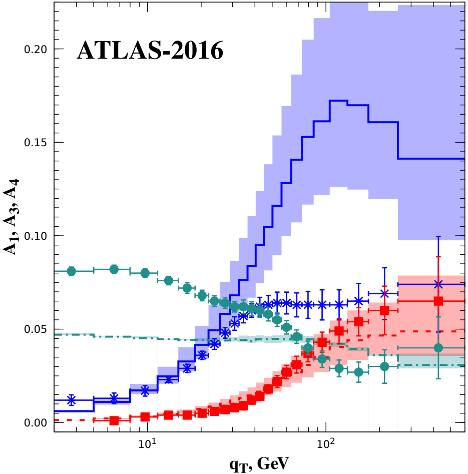
$$
\begin{aligned}
\frac{d\sigma}{dQ d\mathbf{q}_T^2 dy d\Omega_l} &= \frac{3}{16\pi}\frac{d\sigma}{dQ d\mathbf{q}_T^2 dy}\Big\{(1+\cos^2\theta_l)+\frac{A_0}{2}(1-3\cos^2\theta_l)\\
&+ A_1\sin 2\theta_l\cos\phi_l+\frac{A_2}{2}\sin^2\theta_l\cos 2\phi_l + A_3\sin\theta_l\cos\phi_l + A_4\cos\theta_l\\
&+ A_5\sin^2\theta_l\sin 2\phi_l + A_6\sin 2\theta_l\sin\phi_l + A_7\sin\theta_l\sin\phi_l\Big\},
\end{aligned}
$$

Z-boson production in the LO PRA

Angular coefficients A_0 (blue) and A_2 (red)

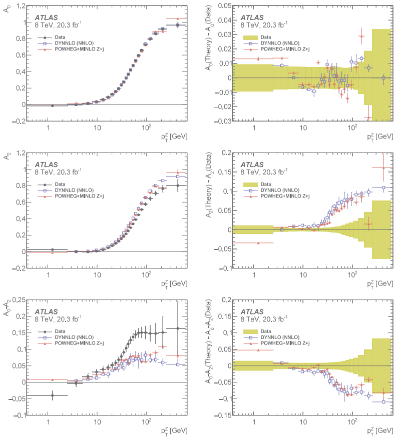


Angular coefficients A_1 (blue), A_3 (red) and A_4 (green)

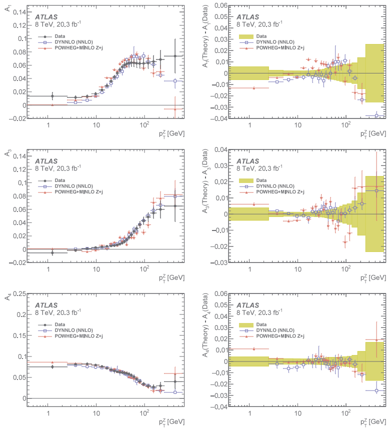


Z-boson production in the NLO CPM

Angular coefficients $A_{0,2}$: DYNNLO and POWHEG NLO+j



Angular coefficients $A_{1,3,4}$: DYNNLO and POWHEG NLO+j



Z-boson production in the NLO* PRA

Quark-gluon (Compton-like) scattering is studied

$$R + Q \to Z + q \to q + \mu^+ + \mu^-$$

$$R + \bar{Q} \to Z + \bar{q} \to \bar{q} + \mu^+ + \mu^-$$

Quark-antiquark annihilation is neglected due to smallness (+ necessity of virtual corrections)

$$\bar{Q} + Q \to Z + g \to g + \mu^+ + \mu^-$$

KaTie

MC parton-level event generator KaTie

- A. van Hameren, "KaTie : For parton-level event generation with k_T-dependent initial states," Comput. Phys. Commun. **224** (2018), 371-380
- uPDFs from TMDlib: https://tmdlib.hepforge.org/
- Output files are in LHEF format (Les Houches Event File)

Abstract

KATIE is a parton-level event generator for hadron scattering processes that can deal with partonic initial-state momenta with an explicit transverse momentum dependence causing them to be space-like. Provided with the necessary transverse momentum dependent parton density functions, it calculates the tree-level off-shell matrix elements and performs the phase space importance sampling to produce weighted events, for example in the Les Houches Event File format. It can deal with arbitrary processes within the Standard Model, for up to at least four final-state particles. Furthermore, it can produce events for single-parton scattering as well as for multi-parton scattering.

KaTie

MC parton-level event generator KaTie

1. The approach to obtaining gauge invariant amplitudes with off-shell initial state partons in scattering at high energies was proposed in the Ref. [A. van Hameren, P. Kotko, and K. Kutak, JHEP 01, 078 (2013), 1211.0961]. The method is based on the use of spinor amplitudes formalism and recurrence relations of the Britto-Cachazo-Feng-Witten (BCFW) type.
2. This formalism for numerical amplitude generation is equivalent to amplitudes built according to Feynman rules of the Lipatov EFT at the level of tree diagrams. It has been tested numerically at least for $2 \to 2$ and $2 \to 3$ [Nefedov, Saleev, Shipilova, Hameren, Kutak, Maciula, Szczurek, 2019].
3. The accuracy of our numerical calculations using KaTie for total proton-proton cross sections is equal to 0.1%.

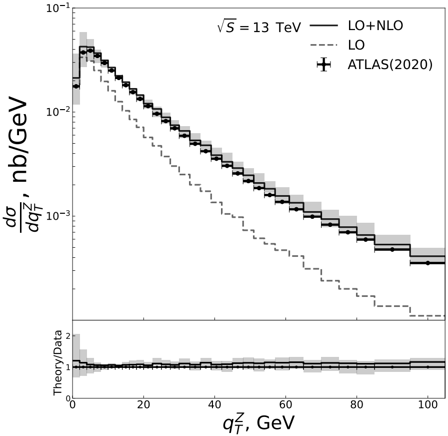
Results

Total cross sections

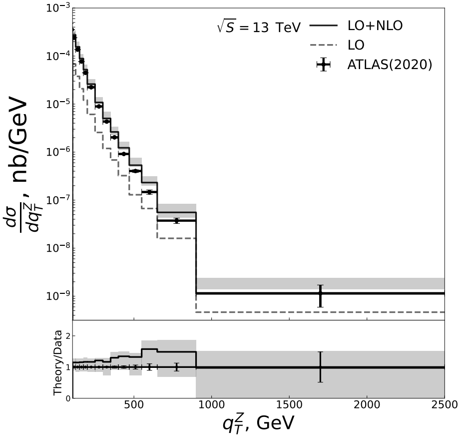
Collaboration	$\sqrt{S}$ TeV	σ^{exp} [nb]	σ^{LO} [nb]	σ^{NLO} [nb]	σ^{theor} [nb]
ATLAS(2017)	7	$0.477^{+0.011}_{-0.011}$	$0.304^{+0.024}_{-0.024}$	$0.190^{+0.055}_{-0.049}$	$0.494^{+0.030}_{-0.042}$
cuts:	$p_T^\mu > 20$ GeV	$\|\eta^\mu\| < 2.4$	$\|\eta^{q/\bar{q}}\| < 2.4$	$46 < m_{\mu^+\mu^-} < 150$ GeV	
CMS(2014)	8	$0.410^{+0.030}_{-0.030}$	$0.301^{+0.007}_{-0.024}$	$0.150^{+0.041}_{-0.038}$	$0.451^{+0.016}_{-0.030}$
cuts:	$p_T^\mu > 25$ GeV	$\|\eta^\mu\| < 2.1$	$\|\eta^{q/\bar{q}}\| < 2.1$	$60 < m_{\mu^+\mu^-} < 120$ GeV	
ATLAS(2020)	13	$0.731^{+0.027}_{-0.027}$	$0.454^{+0.021}_{-0.047}$	$0.342^{+0.087}_{-0.083}$	$0.796^{+0.040}_{-0.062}$
cuts:	$p_T^\mu > 27$ GeV	$\|\eta^\mu\| < 2.5$	$\|\eta^{q/\bar{q}}\| < 2.5$	$66 < m_{\mu^+\mu^-} < 116$ GeV	
LHCb(2022)	13	$0.196^{+0.06}_{-0.06}$	$0.150^{+0.003}_{-0.012}$	$0.053^{+0.017}_{-0.015}$	$0.203^{+0.005}_{-0.011}$
cuts:	$p_T^\mu > 25$ GeV	$2.0 < \|\eta^\mu\| < 4.5$	$2.0 < \|\eta^{q/\bar{q}}\| < 2.5$	$60 < m_{\mu^+\mu^-} < 120$ GeV	
CMS(2017)	8	$0.440^{+0.030}_{-0.030}$	$0.327^{+0.012}_{-0.030}$	$0.169^{+0.044}_{-0.042}$	$0.496^{+0.014}_{-0.030}$
cuts:	$p_T^\mu > 20$ GeV	$\|\eta^\mu\| < 2.1$	$\|\eta^{q/\bar{q}}\| < 2.1$	$60 < m_{\mu^+\mu^-} < 120$ GeV	
$\mu = m_T^Z/2$					

Z-boson spectra in LO and NLO* of PRA

ATLAS, $\sqrt{s} = 13$ TeV, $p_T^Z < 100$ GeV

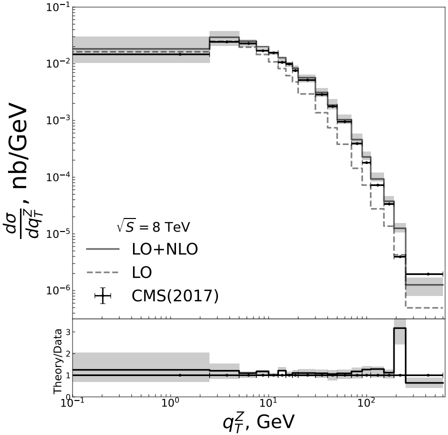


ATLAS, $\sqrt{s} = 13$ TeV, $p_T^Z > 100$ GeV

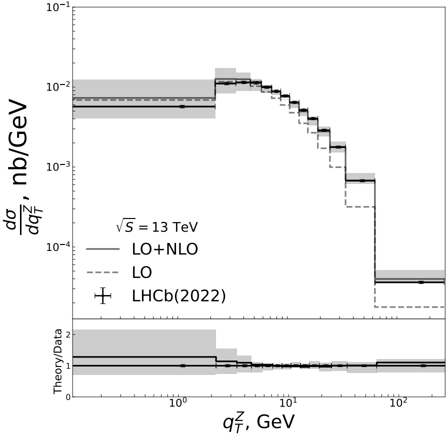


Z-boson spectra in LO and NLO* of PRA

CMS, $\sqrt{s} = 8$ TeV



LHCb, $\sqrt{s} = 13$ TeV



Z-boson spectra in LO and NLO* of PRA

Event File is transformed to the Collins-Soper frame of $Z-$boson

$$
\begin{aligned}
E^{\ell}(CS) &= \frac{M}{2} \\
p_x^{\ell}(CS) &= \frac{M}{2}\,\sin\theta_{CS}^{\ell}\cos\phi_{CS}^{\ell} = \frac{(p_T^{\ell})^2-(p_T^{\overline{\ell'}})^2}{2p_T\sqrt{1+r^2}} \equiv \frac{(p_x^{\ell}-p_x^{\overline{\ell'}})}{2\sqrt{1+r^2}} \\
p_y^{\ell}(CS) &= \frac{M}{2}\,\sin\theta_{CS}^{\ell}\sin\phi_{CS}^{\ell} = p_y^{\ell} \equiv -p_y^{\overline{\ell'}} \\
p_z^{\ell}(CS) &= \frac{M}{2}\,\cos\theta_{CS}^{\ell} = \frac{(p_z^{\ell}E^{\overline{\ell'}}-p_z^{\overline{\ell'}}E^{\ell})}{M\sqrt{1+r^2}}\,.
\end{aligned}
$$

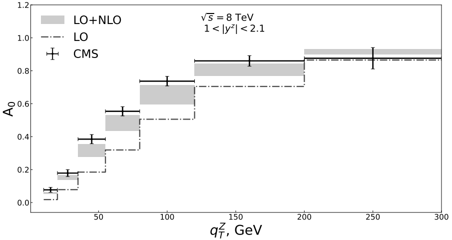
Master formulae for A_n

$$
\langle P(\theta_l,\phi_l)\rangle = \frac{1}{N_{ev}}\sum_{k}^{N_{ev}} P(\theta_l^{(k)},\phi_l^{(k)})
$$

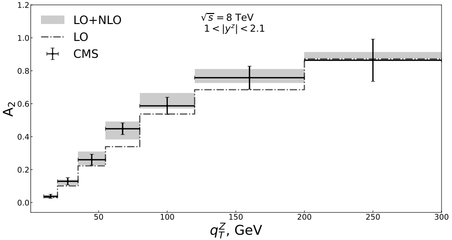
$$
\begin{aligned}
&\langle\frac{1}{2}(1-3\cos^2\theta)\rangle = \frac{3}{20}(A_0-\frac{2}{3});\quad \langle\sin 2\theta\cos\phi\rangle = \frac{1}{5}A_1;\quad \langle\sin^2\theta\cos 2\phi\rangle = \frac{1}{10}A_2;\\
&\langle\sin\theta\cos\phi\rangle = \frac{1}{4}A_3;\quad \langle\cos\theta\rangle = \frac{1}{4}A_4;\quad \langle\sin^2\theta\sin 2\phi\rangle = \frac{1}{5}A_5;\\
&\langle\sin 2\theta\sin\phi\rangle = \frac{1}{5}A_6;\quad \langle\sin\theta\sin\phi\rangle = \frac{1}{4}A_7.
\end{aligned}
$$

Angular coefficients in LO and NLO* of PRA, CMS

CMS, $\sqrt{s} = 8$ TeV, $1 < |y^Z| < 2.1$

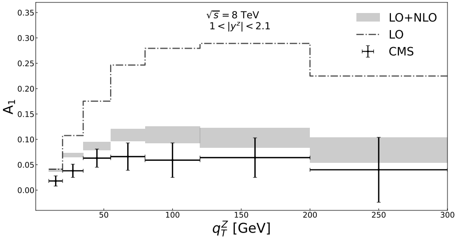


CMS, $\sqrt{s} = 8$ TeV, $1 < |y^Z| < 2.1$

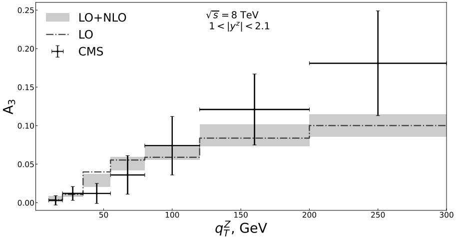


Angular coefficients in LO and NLO* of PRA, CMS

CMS, $\sqrt{s} = 8$ TeV, $1 < |y^Z| < 2.1$

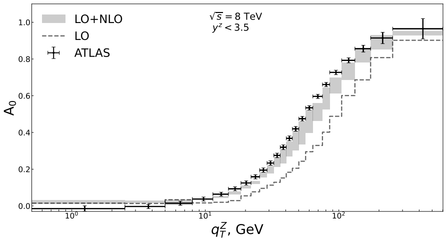


CMS, $\sqrt{s} = 8$ TeV, $1 < |y^Z| < 2.1$

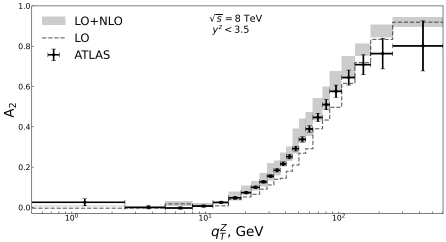


Angular coefficients in LO and NLO* of PRA, ATLAS

ATLAS, $\sqrt{s} = 8$ TeV, $|y^Z| < 3.5$

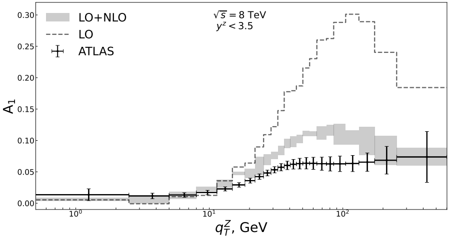


ATLAS, $\sqrt{s} = 8$ TeV, $|y^Z| < 3.5$

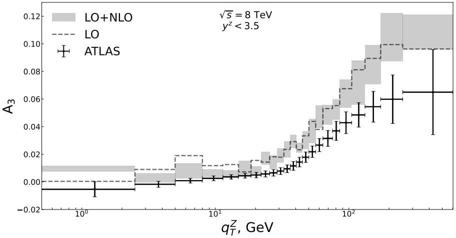


Angular coefficients in LO and NLO* of PRA, ATLAS

ATLAS, $\sqrt{s} = 8$ TeV, $|y^Z| < 3.5$

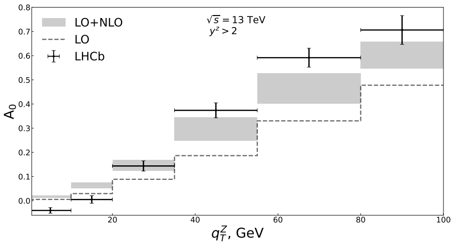


ATLAS, $\sqrt{s} = 8$ TeV, $|y^Z| < 3.5$

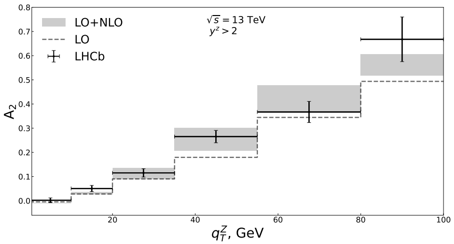


Angular coefficients in LO and NLO* of PRA, LHCb

LHCb, $\sqrt{s} = 13$ TeV, $2 < y^Z < 4.5$

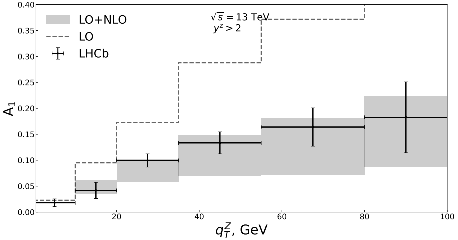


LHCb, $\sqrt{s} = 13$ TeV, $2 < y^Z < 4.5$

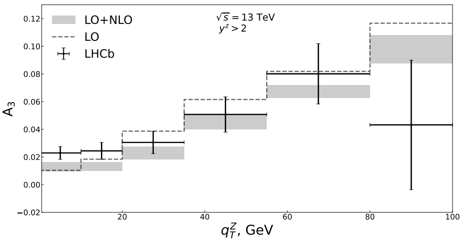


Angular coefficients in LO and NLO* of PRA, LHCb

LHCb, $\sqrt{s} = 13$ TeV, $2 < y^Z < 4.5$



LHCb, $\sqrt{s} = 13$ TeV, $2 < y^Z < 4.5$



Conclusions

- The lepton angular coefficients A_n are calculated in NLO* k_T-factorization for the first time.
- The NLO* correction improves the description of angular coefficients measured by CMS, ATLAS and LHCb collaborations at the LHC
- The best choice for hard scale is $\mu = m_T^Z/2$.
- PRA predictions coincide well with data up to $p_T \sim 200$ GeV
- We demonstrate (*once more*) the equivalence of semi-analytical calculations based on Lipatov Effective Action in high energy QCD and numerical modeling in the MC KaTie.
- Processes $pp \to Z + j,\ Z + j + j, ..$ are in study
- The results for $pp \to Z + J/\psi(\Upsilon)$ are published: A. Chernyshev and V. Saleev, Int. J. Mod. Phys. A **38**, no.35n36, 2350193 (2023)

Thank you for your attention!