Equation of state of rotating QCD and Moment of inertia of quark-gluon plasma

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2 April, 2024

In collaboration with

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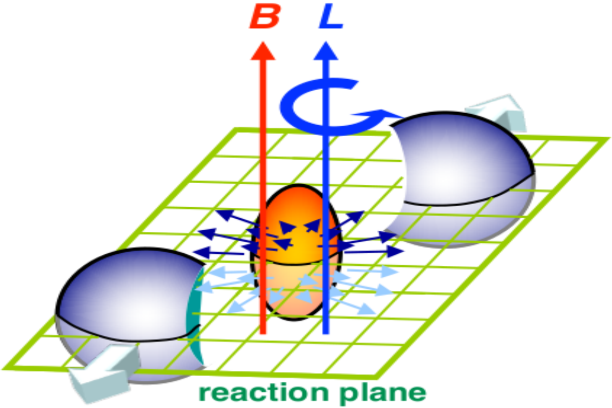
JETP Lett. 117 (2023) 9, 639-644, , e-Print: 2303.03147 [hep-lat] accepted to Phys.Lett.B, e-Print: 2310.16036 [hep-ph]

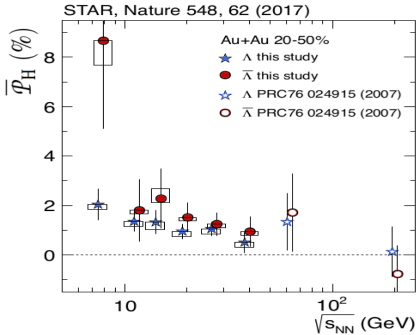
* next talk by Artem Roenko

Rotation of QGP in heavy ion collisions



- QGP is created with non-zero angular momentum in non-central collisions

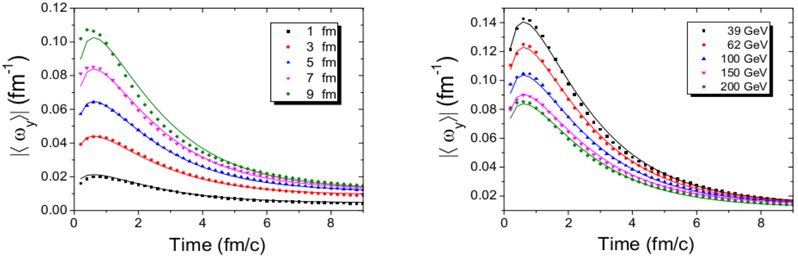
Rotation of QGP in heavy ion collisions



Angular velocity from STAR (Nature 548, 62 (2017))

- $\Omega = (P_\Lambda + P_{\bar\Lambda})\frac{k_B T}{\hbar}$ (Phys. Rev. C 95, 054902 (2017))
- $\Omega \sim 10$ MeV ($v \sim c$ at distances 10-20 fm, $\sim 10^{22} s^{-1}$)
- Relativistic rotation of QGP

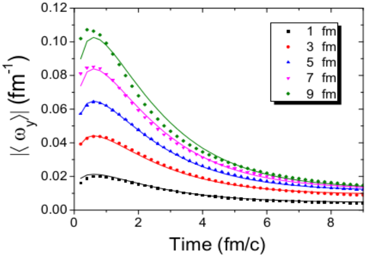
Rotation of QGP in heavy ion collisions

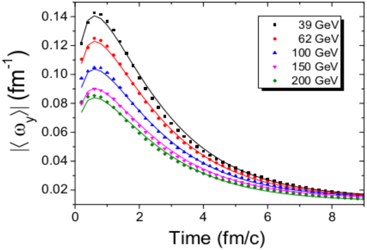


Hydrodynamic simulations (Phys.Rev.C 94, 044910 (2016))

- Au-Au: *left* $\sqrt{s} = 200$ GeV, *right* $b = 7$ fm,
- $\Omega \sim (4 - 28)$ MeV
- Relativistic rotation of QGP

Rotation of QGP in heavy ion collisions





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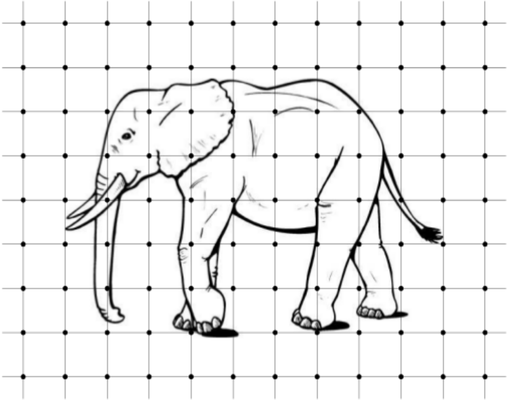
How relativistic rotation influences QCD?

Lattice QCD



Lattice simulation

- Allows to study strongly interacting systems
- Based on the first principles of quantum field theory
- Powerful due to modern supercomputers and algorithms

Lattice simulation of QCD

- We study QCD in thermodynamical equilibrium
- The system is in the finite volume
- Calculation of the partition function

$$Z \sim \int DU e^{-S_G(U)} \prod_{i=u,d,s\ldots} \det\left(\hat{D}_i(U) + m_i\right)$$

- Monte Carlo calculation of the integral
- Carry out continuum extrapolation $a \to 0$
- Uncertainties (discretization and finite volume effects) can be systematically reduced
- The first principles based approach. No assumptions!
- Parameters: g^2 and masses of quarks

Modern lattice simulation of QCD

$$Z_l = \int DU e^{-S_G(U)} \times \det\left(\hat{\mathbf{D}}(\mathbf{U}) + \mathbf{m}\right) = \int DU e^{-E_{eff}(U)}$$

- Lattices
  - 96×48^3
  - Variables: $96 \cdot 48^3 \cdot 4 \cdot 8 \sim 300 \cdot 10^6$
  - Matrices: $100 \cdot 10^6 \times 100 \cdot 10^6$
- Dynamical u, d, s, c–quarks
- Physical masses of u, d, s, c–quarks
- Lattice spacing $a \sim 0.05\,\mathrm{fm}$

Applications

- Spectroscopy
- Matrix elements and correlations functions
- Thermodynamic properties of QCD
- Transport properties of QCD
- Phase transitions
- Nuclear physics
- Properties of QCD under extreme conditions (magnetic field, baryon density, relativistic rotation,...)
- Topological properties
- ...

Study of rotating QGP

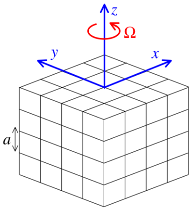
- Our aim: study rotating QCD within lattice simulations
- Rotating QCD at thermodynamic equilibrium
  - At the equilibrium the system rotates with some Ω
  - The study is conducted in **the reference frame which rotates with QCD matter**
  - QCD in external gravitational field
- **Boundary conditions are very important!**

Details of the simulations

- Gluodynamics is studied at thermodynamic equilibrium in external gravitational field
- The metric tensor

$$g_{\mu\nu} = \begin{pmatrix} 1 - r^2\Omega^2 & \Omega y & -\Omega x & 0 \\ \Omega y & -1 & 0 & 0 \\ -\Omega x & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

- Geometry of the system: $N_t \times N_z \times N_x \times N_y = N_t \times N_z \times N_s^2$



Details of the simulations

- Partition function ($\hat{H}$ is conserved)

$$Z = \mathrm{Tr}\ \exp\left[-\beta \hat{H}\right] = \int DA\ \exp\left[-S_G\right]$$

- Euclidean action

$$S_G = -\frac{1}{2g_{YM}^2} \int d^4x \sqrt{g_E}\, g_E^{\mu\nu} g_E^{\alpha\beta} F_{\mu\alpha}^{(a)} F_{\nu\beta(a)}$$

$$S_G = \frac{1}{2g_{YM}^2} \int d^4x \mathrm{Tr}\Big[(1 - r^2\Omega^2) F_{xy}^a F_{xy}^a + (1 - y^2\Omega^2) F_{xz}^a F_{xz}^a +$$

$$+(1 - x^2\Omega^2) F_{yz}^a F_{yz}^a + F_{x\tau}^a F_{x\tau}^a + F_{y\tau}^a F_{y\tau}^a + F_{z\tau}^a F_{z\tau}^a -$$

$$-2iy\Omega(F_{xy}^a F_{y\tau}^a + F_{xz}^a F_{z\tau}^a) + 2ix\Omega(F_{yx}^a F_{x\tau}^a + F_{yz}^a F_{z\tau}^a) - 2xy\Omega^2 F_{xz} F_{zy}\Big]$$

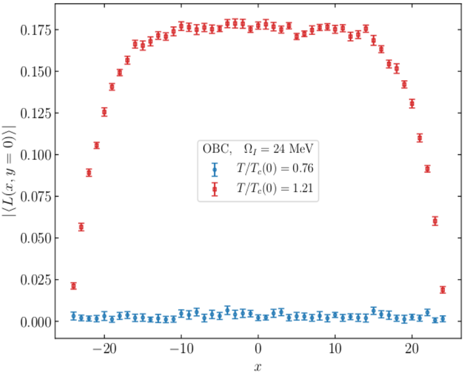
Details of the simulations

Boundary conditions

- **Periodic b.c.:**
  - $U_{x,\mu} = U_{x+N_i,\mu}$
  - Not appropriate for the field of velocities of rotating body
- **Dirichlet b.c.:**
  - $U_{x,\mu}\big|_{x\in\Gamma} = 1, \quad A_\mu\big|_{x\in\Gamma} = 0$
  - Violate Z_3 symmetry
- **Neumann b.c.:**
  - Outside the volume $U_P = 1, \quad F_{\mu\nu} = 0$

- *The dependence on boundary conditions is the property of all approaches*
- *One can expect that boundary conditions influence our results considerably, but their influence is restricted due to the screening*

Screening of boundary conditions



Details of the simulations

Sign problem

$$S_G = \frac{1}{2g_{YM}^2}\int d^4x \mathrm{Tr}\big[(1-r^2\Omega^2)F^a_{xy}F^a_{xy} + (1-y^2\Omega^2)F^a_{xz}F^a_{xz}+$$

$$+(1-x^2\Omega^2)F^a_{yz}F^a_{yz} + F^a_{x\tau}F^a_{x\tau} + F^a_{y\tau}F^a_{y\tau} + F^a_{z\tau}F^a_{z\tau}-$$

$$-2iy\Omega(F^a_{xy}F^a_{y\tau} + F^a_{xz}F^a_{z\tau}) + 2ix\Omega(F^a_{yx}F^a_{x\tau} + F^a_{yz}F^a_{z\tau}) - 2xy\Omega^2 F_{xz}F_{zy}\big]$$

- The Euclidean action has imaginary part (**sign problem**)
- Simulations are carried out at imaginary angular velocities $\Omega \to i\Omega_I$
- The results are analytically continued to real angular velocities
- This approach works up to sufficiently large Ω

EoS of rotating gluodynamics

- Free energy of rotating QGP

$$F(T,R,\Omega) = F_0(T,R) + C_2\Omega^2 + ...$$

- The moment of inertia

$$C_2 = -\frac{1}{2}I_0(T,R), \quad I_0(T,\Omega) = -\frac{1}{\Omega}\left(\frac{\partial F}{\partial \Omega}\right)_{T,\Omega\to 0}$$

- Instead of $I_0(T,R)$ we calculate $K_2 = -\frac{I_0(T,R)}{F_0(T,R)R^2}$
- Sign of K_2 concides with the sign of $I_0(T,R)$

EoS of rotating gluodynamics

- Classical moment of inertia

$$I_0(R) = \int_V d^3x x_\perp^2 \rho_0(x_\perp)$$

- Related to the trace of EMT $T^\mu_\mu = \rho_0(x_\perp)c^2$
- Generation of mass scale in QCD and scale anomaly

$$T^\mu_\mu \sim \langle G^2 \rangle \sim \langle H^2 + E^2 \rangle$$

- In QCD the gluon condensate $\langle G^2 \rangle \neq 0$
- *One could anticipate:* $\rho_0 \sim \langle H^2 + E^2 \rangle$?

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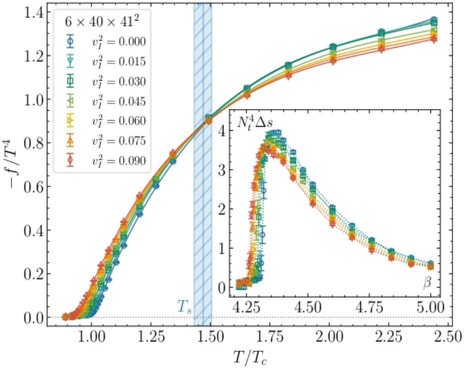
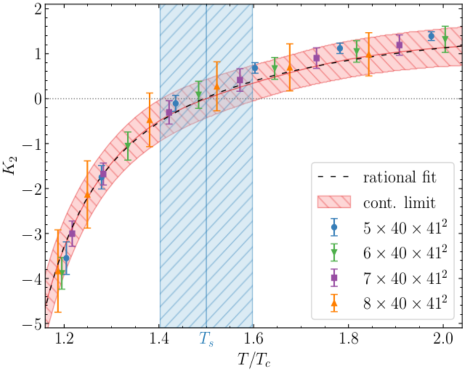
- In QCD the gluon condensate $\langle G^2 \rangle \neq 0$
- *One could anticipate:* $\rho_0 \sim \langle H^2 + E^2 \rangle$?
- $I_2 = I_{mech} + I_{magn}$ *valid for QCD!*
  $I_{mech} = \langle J_z^2 \rangle - (\langle J_z \rangle)^2$
  $I_{magn} = \frac{1}{3} \int d^3x r^2 \langle H^2 \rangle$

Calculation of free energy on the lattice

- $F = -T \log Z$ impossible to calculate on the lattice
- $\frac{\partial F}{\partial \beta} \sim \langle \Delta s(\beta) \rangle = s(\beta)_T - s(\beta)_{T=0}, \quad \beta = \frac{6}{g^2}$
- $\frac{F(T)}{T^4} \sim \int_{\beta_0}^{\beta_1} d\beta' \langle \Delta s(\beta') \rangle$

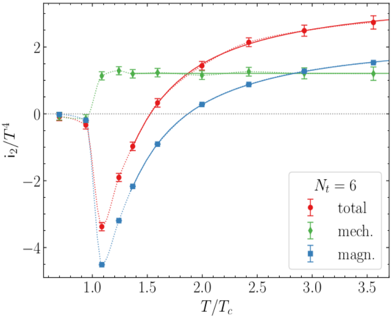


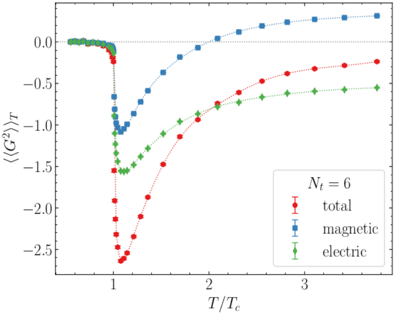
Moment of inertia of gluon plasma



- $I(T, R) = -F_0(T, R) K_2 R^2$
- $I < 0$ for $T < 1.5T_c$ and $I > 0$ for $T > 1.5T_c$
- The region of $I < 0$ is related to magnetic condensate and the scale anomaly
- We believe that the same is true for QCD

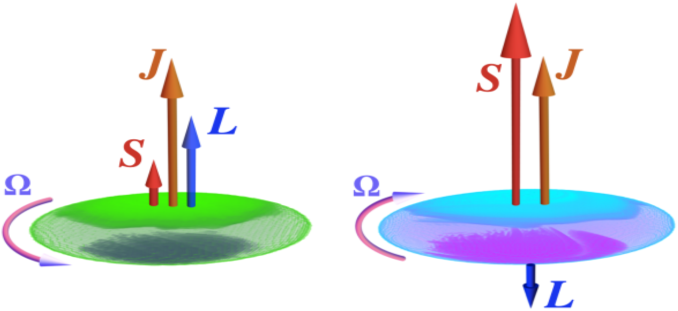
Moment of inertia of gluon plasma





- $i_2 = \frac{I_2}{VR_\perp^2}, \quad I_2 = I_{mech} + I_{magn}$
  $I_{mech} = \langle J_z^2\rangle - (\langle J_z\rangle)^2$
  $I_{magn} = \frac{1}{3}\int d^3x r^2 \langle H^2\rangle$
- $\langle G^2\rangle_T = \langle E^2\rangle_T + \langle H^2\rangle_T$

Negative Barnett effect(?)



- $J = I_2 \Omega = -\left(\frac{\partial F}{\partial \Omega}\right)_T$
- $\mathbf{J} = \mathbf{L} + \mathbf{S}, \quad \mathbf{L} \parallel \boldsymbol{\Omega}$

Conclusion

- Lattice study of rotating gluodynamics has been carried out
- We calculated the moment of inertia of GP. It is negative at temperatures $T < 1.5T_c$ and positive at larger temperatures
- We believe that all observed effects remain in QCD

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THANK YOU!