Spatially inhomogeneous phase transition in rotating gluon plasma

A. A. Roenko[1],

in collaboration with

V. V. Braguta, M. N. Chernodub,

[1] Joint Institute for Nuclear Research, Bogoliubov Laboratory of Theoretical Physics
roenko@theor.jinr.ru

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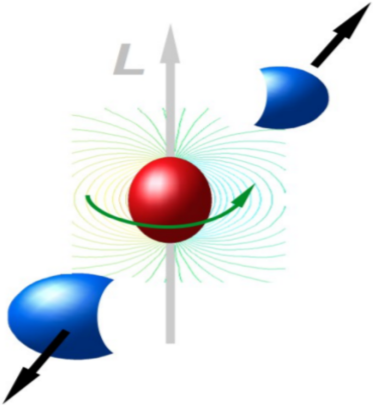
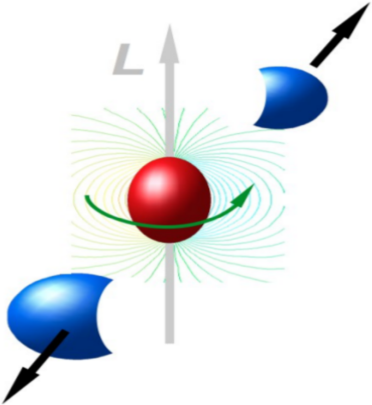
Introduction

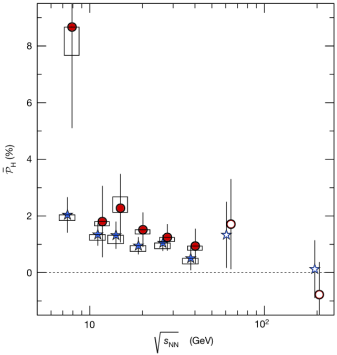
- In non-central heavy ion collisions creation of QGP with angular momentum is expected.



Introduction

- In non-central heavy ion collisions creation of QGP with angular momentum is expected.
- The rotation occurs with relativistic velocities.





[L. Adamczyk et al. (STAR), Nature **548**, 62–65 (2017), arXiv:1701.06657 [nucl-ex]]

$\langle \omega \rangle \sim 6$ MeV ($\sqrt{s_{NN}}$-averaged)

Critical temperature in rotating QCD

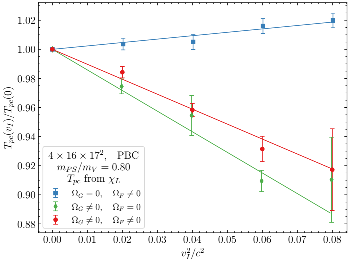
All* theoretical models assume rigid rotation, $\Omega \neq 0$. Mostly the global T_c is measured.

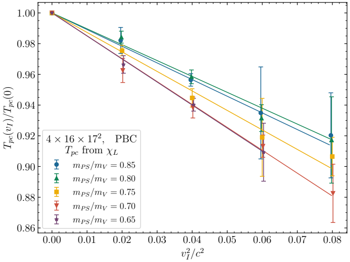
Our lattice results for gluodynamics show that the confinement critical temperature increases with rotation

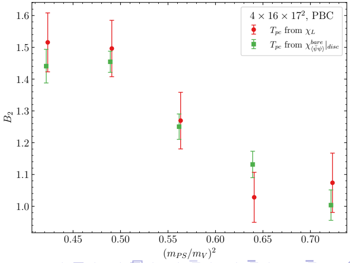
- V. V. Braguta, A. Y. Kotov, D. D. Kuznedelev, and A. A. Roenko, JETP Lett. **112**, 6–12 (2020)
- V. V. Braguta, A. Y. Kotov, D. D. Kuznedelev, and A. A. Roenko, Phys. Rev. D **103**, 094515 (2021), arXiv:2102.05084 [hep-lat]
- V. Braguta, A. Y. Kotov, D. Kuznedelev, and A. Roenko, PoS **LATTICE2021**, 125 (2022), arXiv:2110.12302 [hep-lat]

Lattice results for QCD: the chiral and deconfinement critical temperatures both increase with rotation (decrease with imaginary rotation); fermions and gluons have opposite influence on T_c.

- V. V. Braguta, A. Kotov, A. Roenko, and D. Sychev, PoS **LATTICE2022**, 190 (2023), arXiv:2212.03224 [hep-lat]







Lattice formulation of rotating QCD

We study gluodynamics in the co-rotating reference frame (it rotates with angular velocity Ω around z-axis) $\to$ external gravitational field [A. Yamamoto and Y. Hirono, Phys. Rev. Lett. **111**, 081601 (2013), arXiv:1303.6292 [hep-lat]]

$$g^E_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & y\Omega_I \\ 0 & 1 & 0 & -x\Omega_I \\ 0 & 0 & 1 & 0 \\ y\Omega_I & -x\Omega_I & 0 & 1 + r^2\Omega_I^2 \end{pmatrix}, \tag{1}$$

where $r^2 = x^2 + y^2$, and the angular velocity is put in the purely imaginary form $\Omega_I = -i\Omega$ to avoid the sign problem. Observables are calculated from first principles as follows

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int DA\, \mathcal{O}[A] \exp\left(- S_G[A] \right), \qquad Z = \int DA \, \exp\left(- S_G[A] \right). \tag{2}$$

We use tree-level improved (Symanzik) lattice gauge action; the lattice size is $N_t \times N_z \times N_s^2$;

The notations $R \equiv a(N_s - 1)/2$ and $v_I \equiv \Omega_I R$ are used.

The gluon action has the following form:

$$S = \frac{1}{4g_0^2}\int d^4x\,\sqrt{g_E}\,g_E^{\mu\nu}g_E^{\alpha\beta}F^a_{\mu\alpha}F^a_{\nu\beta} \equiv S_0 + S_1\Omega_I + S_2\frac{\Omega_I^2}{2}\,, \tag{3}$$

where

$$S_0 = \frac{1}{4g_0^2}\int d^4x F^a_{\mu\nu}F^a_{\mu\nu}\,, \tag{4}$$

$$S_1 = \frac{1}{g_0^2}\int d^4x\left[yF^a_{xy}F^a_{y\tau} + yF^a_{xz}F^a_{z\tau} - xF^a_{yx}F^a_{x\tau} - xF^a_{yz}F^a_{z\tau}\right], \tag{5}$$

$$S_2 = \frac{1}{g_0^2}\int d^4x\left[r^2(F^a_{xy})^2 + y^2(F^a_{xz})^2 + x^2(F^a_{yz})^2 + 2xyF^a_{xz}F^a_{zy}\right], \tag{6}$$

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Sign problem

- The Euclidean action is complex-valued function with real rotation ($S_1 \neq 0$)!
- The Monte–Carlo simulations are conducted with imaginary angular velocity $\Omega_I = -i\Omega$.
- The results are analytically continued to the region of the real angular velocity ($\Omega_I^2 = -\Omega^2$, $v_I^2 = -v_R^2$).

Rotating reference frame: temperature

- Tolman-Ehrenfest effect: In gravitational field the temperature isn't a constant in space at thermal equilibrium:

$$T(r)\sqrt{g_{00}} = T_0 = const\,,$$

- For the rotating system one gets

$$T(r) = \frac{T_0}{\sqrt{1-\Omega^2 r^2}} = \frac{T_0}{\sqrt{1+\Omega_I^2 r^2}}\,. \tag{7}$$

where T_0 is the temperature at the rotation axis $(r=0)$. Below $T_0 \equiv T = 1/N_t a$.

- One could expect, that the rotation effectively warm up the periphery of the modeling volume

$$T(r) > T(r=0),$$

and as a result, from relativity, the critical temperature should decreases: $T_c^{TE}(r) = T_{c0}\sqrt{1-\Omega^2 r^2}$

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- Inhomogeneous phases are predicted by some phenomenological models with $T_c(r) = T_c^{TE}(r)$:

  2+1 cQED: M. N. Chernodub, Phys. Rev. D **103**, 054027 (2021), arXiv:2012.04924 [hep-ph]
  Holography: N. R. F. Braga and O. C. Junqueira, (2023), arXiv:2306.08653 [hep-th]

The Polyakov loop is an order parameter in gluodynamics.

$$L(x,y) = \frac{1}{N_z} \sum_z \mathrm{Tr} \left[\prod_{\tau=0}^{N_t-1} U_4(\vec{r},\tau) \right] , \qquad L = \frac{1}{N_s^2} \sum_{x,y} L(x,y) \,. \tag{8}$$

In confinement $\langle L \rangle = 0$; in deconfinement $\langle L \rangle \neq 0$ ($\mathbb{Z}_3$ center symmetry is broken).

The critical temperature is associated with the peak of the Polyakov loop susceptibility

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Causality restriction

- Minkowski metric: $g_{00} = 1 - \Omega^2 r^2 \quad \Rightarrow \quad \Omega r < 1, \quad$ or $\quad v_R = \Omega R < 1/\sqrt{2}$
- Euclidean metric: $g_{44} = 1 + \Omega_I^2 r^2 \quad \Rightarrow \quad$ no restriction (analytic continuation is allowed for $v_I < 1/\sqrt{2}$)
- Boundary conditions are important! (we check two different types of b.c.: open/periodic)

To investigate inhomogeneous phases the lattices of huge size is needed ($N_s/N_t \gtrsim 18$) !

Inhomogeneous phases for imaginary rotation

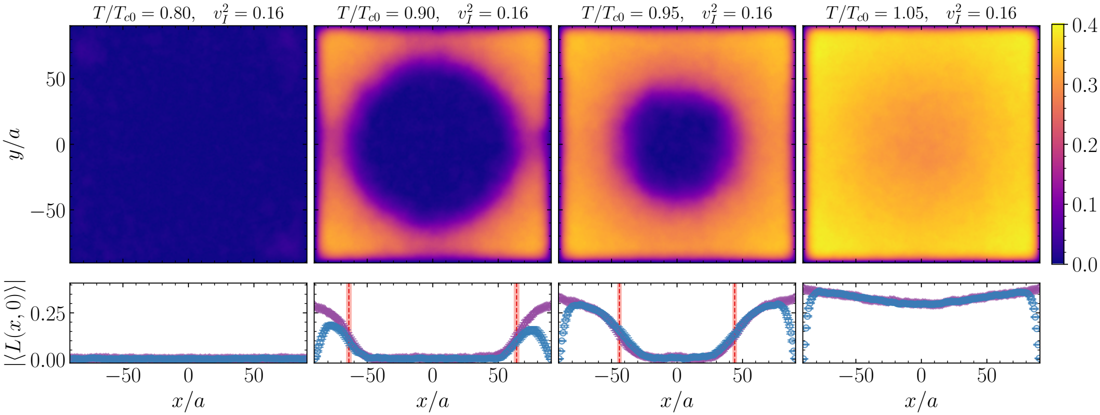


Figure: The distribution of the local Polyakov loop in x, y-plane for lattice of size $5 \times 30 \times 181^2$ with open boundary conditions at the fixed imaginary velocity at the boundary $v_I^2 \equiv (\Omega_I R)^2 = 0.16$ and different on-axis temperatures.

- As the (on-axis) temperature increases, the radius of the inner confining region shrinks.
- Boundary is screened.
- Local thermalization takes place; Phase transition occurs as a vortex evolution.

Inhomogeneous phases for imaginary rotation

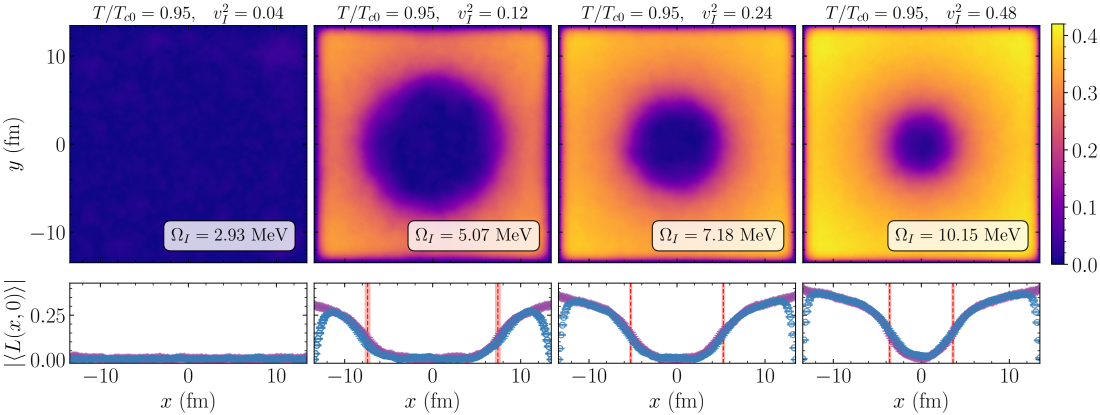
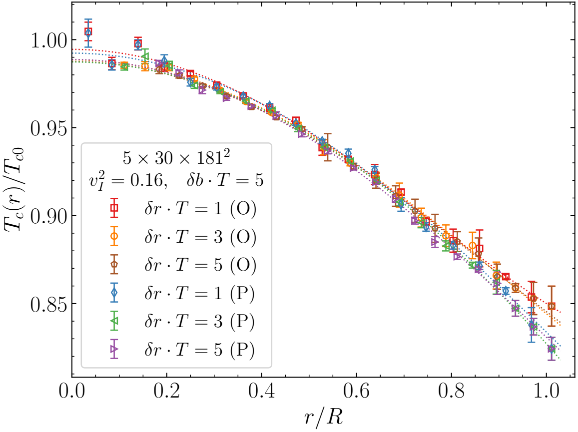


Figure: The distribution of the local Polyakov loop in x, y-plane for lattice of size $5 \times 30 \times 181^2$ with open boundary conditions at the fixed on-axis temperature $T = 0.95\,T_{c0}$ and different imaginary angular frequencies; $R = 13.5$ fm.

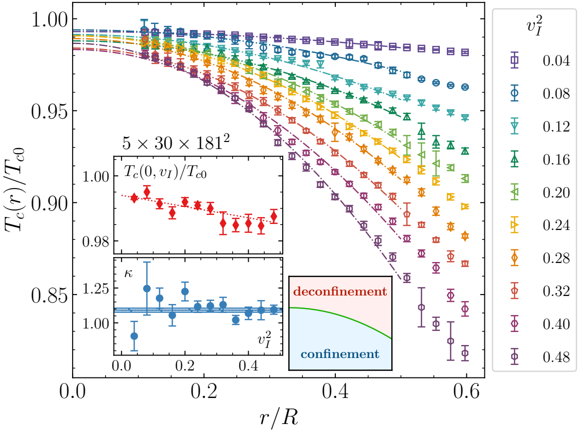
- Imaginary rotation produces the deconfinement at the periphery while the central region stays in confinement phase (for $T < T_{c0}$).
- The deconfinement region approaches the rotation axis with the increase of Ω_I;

We split the system into thin cylinders of width δr and measure local critical temperature



- Results for different width $\delta r \cdot T = 1, 2, 3, 4, 5$ are in agreement.
- Thin layer of width δb is skipped from boundary.
- There is minor difference on b.c. at $r/R \sim 1$

In the central region of lattice the local critical temperature does not depends on b.c.



For a moderate radius $r/R \lesssim 0.5$, the fitting function is

$$\frac{T_c(r,\Omega_I)}{T_{c0}} = \frac{T_c(\Omega_I)}{T_{c0}} - \kappa(\Omega_I)\,(\Omega_I r)^2\,, \qquad (10)$$

- $T_c(0) \approx T_{c0}$ with few percent accuracy:
  - Finite correlation length and finite R.
  - Averaging over layer with δr.

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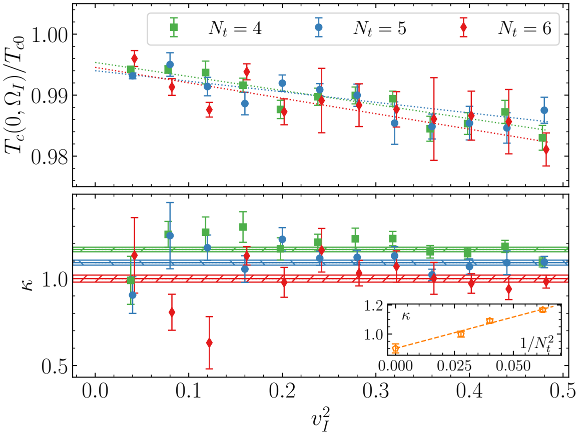

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- $T_c(0) \approx T_{c0}$ with few percent accuracy:
  - Finite correlation length and finite R.
  - Averaging over layer with δr.
- Vortical curvature in continuum limit

$$\kappa = 0.901(32)\,.$$

It is not TE

Decomposition of rotating action

The action of rotating gluons has the following structure

$$S_G = S_0 + \lambda_1 S_1 \Omega_I + \lambda_2 S_2 \Omega_I^2 \,, \tag{11}$$

where we introduce switching factors λ_1, λ_2.

- The first operator S_1 is an angular momentum of gluons, constructed using the stress-energy tensor.
- The second operator S_2 is related to an average square of the chromomagnetic fields F_{ij}^2.

[See Talk by V. Braguta]

The following regimes of the rotation are possible:

i1) $\lambda_1 = 1, \quad \lambda_2 = 0; \quad v^2 < 0$

i2) $\lambda_1 = 0, \quad \lambda_2 = 1; \quad v^2 < 0$

i3) $\lambda_1 = 1, \quad \lambda_2 = 1; \quad v^2 < 0$

Note that in case i2) there is, actually, no sign problem:

r2) $\lambda_1 = 0, \quad \lambda_2 = 1; \quad v^2 > 0$

Imaginary vs real rotation for decomposed action

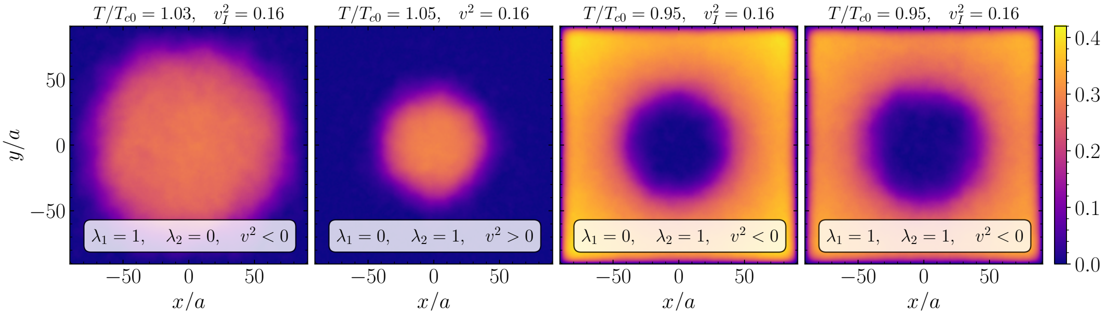
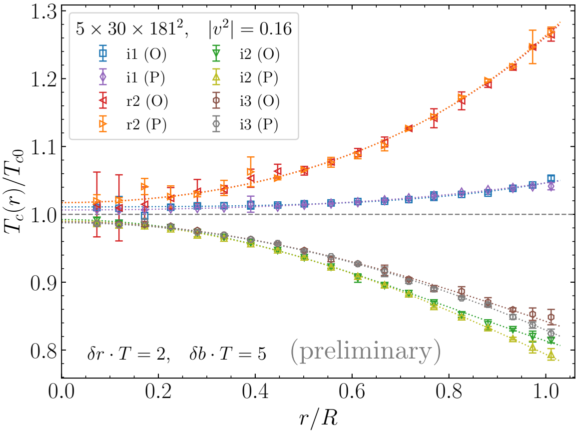


Figure: The distribution of the local Polyakov loop in x, y-plane for lattice size $5 \times 30 \times 181^2$, open boundary conditions OBC at fixed velocity $|v^2| = 0.16$ and different values of switching factors λ_1, λ_2.

- The radius of the inner region in regime i2) is slightly smaller, than in i3). Phase arrangement is the same.
- In regimes i1) and r2) rotation produces confinement phase in the outer region at $T > T_{c0}$.

Imaginary vs real rotation for decomposed action

The behaviour of the local critical temperature in these regimes is different.



- κ_{i1} takes small negative value
- κ_{i2} and κ_{i3} are positive, with $\kappa_{i2} > \kappa_{i3}$.
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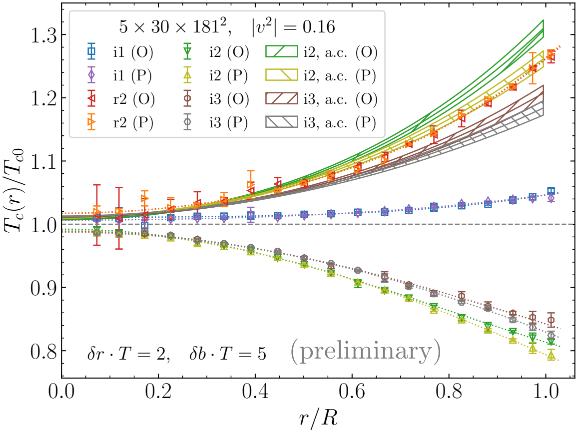

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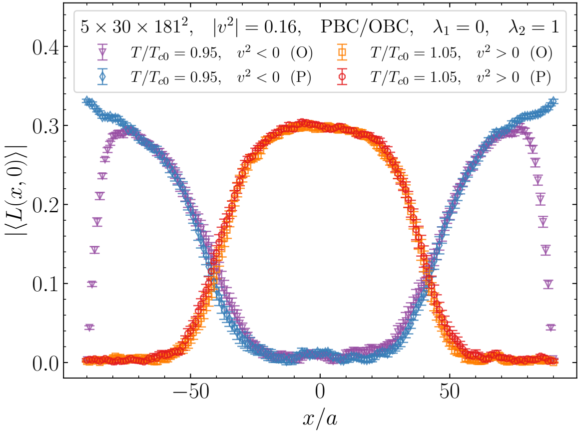
- Contribution from S_2 is predominant. The results resemble decomposition of I. [See Talk by V. Braguta]
- Real Ω results (r2) are in agreement with a.c. of imaginary Ω_I results (i2) for $r/R \lesssim 0.5$ (where quadratic fit works).

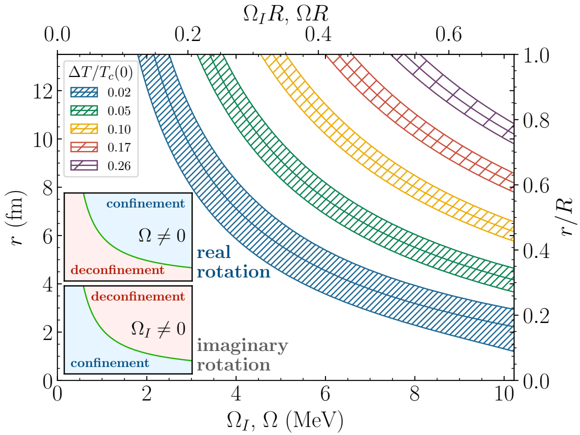
The Polyakov loop distribution for real and imaginary rotation (S_1 term is omitted)



- (r2): $T = T_{c0} + \Delta T$ for real rotation $v^2 = 0.16$
- (i2): $T = T_{c0} - \Delta T$ for imaginary rotation $v_I^2 = 0.16$

Confinement $\leftrightarrow$ deconfinement with approximately the same boundary.

Using continuum limit results one can plot a phase diagram in (Ω, r) plane (example for $R = 13.5$ fm).



The diagram has the same form for imaginary angular velocity (at $T = T_{c0} - \Delta T$)

$$\frac{T_c(r, \Omega_I)}{T_{c0}} = \frac{T_c(\Omega_I)}{T_{c0}} - \kappa(\Omega_I)\,(\Omega_I r)^2\,,$$

and real angular velocity (at $T = T_{c0} + \Delta T$):

$$\frac{T_c(r, \Omega)}{T_{c0}} = \frac{T_c(\Omega)}{T_{c0}} + \kappa(\Omega)\,(\Omega r)^2\,.$$

The upper right corner of diagram will be slightly affected by $\mathcal{O}(\Omega^4)$ corrections.

Conclusions

- Using lattice simulation, we found the mixed confinement-deconfinement phase in rotating SU(3) gluodynamics at thermal equilibrium.
- The local critical temperature is determined mainly by the local linear velocity of rotational motion:

$$\frac{T_c(r,\Omega_I)}{T_{c0}} = \frac{T_c(\Omega_I)}{T_{c0}} - \kappa(\Omega_I)\,(\Omega_I r)^2 + \mathcal{O}(\Omega^4)\,, \tag{12}$$

  where $\kappa = 0.901 \pm 0.032$ in continuum limit (universal for any b.c.). The results $\mathcal{O}(\Omega^4)$ are on the way.
- The local critical temperature on the axis $T_c(0)$ is T_{c0} with few percent accuracy. It is not TE.
- This behaviour originates from chromomagnetic term S_2 (same as for $I < 0$ – magnetic gluon condensate?? negative spin-vortical coupling??). [See Talk by V. Braguta]
- Local thermalization takes place; Phase transition occurs as a evolution of vortex of new phase.
- Inhomogeneous phases may appear even for a slow rotation (if R is large enough), and we expect similar picture for QCD.
- Previous results for T_c should be understood as bulk-averaged values.

V. V. Braguta, M. N. Chernodub, and A. A. Roenko, (2023), arXiv:2312.13994 [hep-lat]

Another details coming soon: 2404.XXXXX

Thank you for your attention!