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based on works:

[1] Phys. Rev. Lett., 129(15):151601, (2022).

[2] Phys.Lett.B 840, 137839, (2023).

[3] Phys.Rev.D 108, 12, L121701 (2023)

[4] 2401.09247 (2024)

Gravitational chiral anomaly: manifestation in hydrodynamics

Scientific session of the Russian Academy of Sciences,
JINR, Dubna, 1 to 5 April 2024

Contents

- Introduction
- Novel phenomenon: **Kinematical Vortical Effect** **(KVE)**
    - General derivation from
      the **gravitational chiral anomaly**
    - Direct verification:
      spin 1/2
      spin 3/2 **(Rarita-Schwinger-Adler model)**
- Recent development (constant curvature, **5d-Unruh** effect)
- Conclusion

Part 1

Introduction

Gravity in flat spacetime: Cheshire cat grin

“Well! I've often seen a cat without a grin,' thought Alice 'but a grin without a cat! It's the most curious thing i ever saw in my life!”

— Lewis Carroll, Alice in Wonderland

Gravity in flat spacetime: Cheshire cat grin

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— Lewis Carroll, Alice in Wonderland

CVE and CME – new anomalous transport

Consistency with **quantum anomaly**
modifies hydrodynamic equations

[V. I. Zakharov, Lect. Notes Phys.871,295(2013)]

[D.T. Son, P. Surowka, PRL, 103 (2009) 191601]

Quantum chiral anomaly (gauge part)

$$\langle \partial_\mu \hat{j}^\mu_A \rangle = -\frac{C}{8}\varepsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta}$$

Entropy current satisfies **second law** of thermodynamics $\quad \partial_\mu s^\mu \geqslant 0$

Chiral magnetic effect (CME)

CME: $\quad j^\mu = C\mu_5 B^\mu$

Current flows along the **magnetic field**

Chiral vortical effect (CVE)

CVE: $\quad j^\mu_A = C\mu^2\omega^\mu$

$$\omega^\mu = \frac{1}{2}\varepsilon^{\mu\nu\alpha\beta}u_\nu \partial_\alpha u_\beta$$

Current flows along the **vorticity**

Derivation **without entropy** current and generalization to the **second order** in gradients:

[Shi-Zheng Yang, Jian-Hua Gao, Zuo-Tang Liang, Symmetry 14 (2022) 5, 948]

[M. Buzzegoli, Lect. Notes Phys. 987, 53-93 (2021)]

Use **global equilibrium**

Modern development and the problem

What about the **gravitational chiral anomaly**?

- The gravitational chiral anomaly (unlike gauge part) grows **rapidly** with **spin**:

$$\langle \nabla_\mu \hat{j}^\mu_A \rangle_S = \frac{(S - 2S^3)}{96\pi^2 \sqrt{-g}} \varepsilon^{\mu\nu\rho\sigma} R_{\mu\nu\kappa\lambda} R_{\rho\sigma}{}^{\kappa\lambda}$$

[M. J. Duff, in First School on Supergravity (1982) arXiv:1201.0386]

[S. M. Christensen, M. J. Duff, Nucl. Phys. B 154, 301–342 (1979)]

- How does the **gravitational** chiral anomaly manifest itself in **hydrodynamics**?
- Is it possible to see the **cubic factor** $S - 2S^3$ in hydrodynamics?

Novel phenomenon: Kinematical Vortical Effect

General derivation

Hydrodynamics in curved space-time

Consider an uncharged fluid of massless particles with an **arbitrary spin** in a **gravitational field**:

fluid

4-velocity of the fluid	$u_\mu(x)$
Proper temperature	$T(x)$
Inverse temperature vector	$\beta_\mu = u_\mu / T$
Thermal vorticity tensor (analogous to the acceleration tensor)	$\varpi_{\mu\nu} = -\frac{1}{2}(\nabla_\mu \beta_\nu - \nabla_\nu \beta_\mu)$

space-time

Curved space-time metric

$g_{\mu\nu}(x)$

Riemann tensor

$R_{\mu\nu\kappa\lambda}$

We consider a medium in a state of **(global) thermodinamic equilibrium**

[F. Becattini, L. Bucciantini, E. Grossi, L. Tinti, Eur. Phys. J. C 75, 191 (2015)]

[F. Becattini, Acta Phys. Polon. B 47, 1819 (2016)]

Killing equation

$$\nabla_\mu \beta_\nu + \nabla_\nu \beta_\mu = 0$$

Very close to the **Tolman-Ehrenfest's** criterion and the **Luttinger** relation

Decomposition of the tensors

- **Components of the thermal vorticity tensor**

6 components

[M. Buzzegoli, E. Grossi, F. Becattini, JHEP 10 (2017) 091]

$$\varpi_{\mu\nu} = \epsilon_{\mu\nu\alpha\beta} w^\alpha u^\beta + \alpha_\mu u_\nu - \alpha_\nu u_\mu$$

Similar to the expansion for the electromagnetic field

- We also decompose the **Riemann tensor** into the **components:**

$$\begin{aligned} R_{\mu\nu\alpha\beta} = & \; u_\mu u_\alpha A_{\nu\beta} + u_\nu u_\beta A_{\mu\alpha} - u_\nu u_\alpha A_{\mu\beta} - u_\mu u_\beta A_{\nu\alpha} \\ & + \epsilon_{\mu\nu\lambda\rho} u^\rho (u_\alpha B^\lambda{}_\beta - u_\beta B^\lambda{}_\alpha) \\ & + \epsilon_{\alpha\beta\lambda\rho} u^\rho (u_\mu B^\lambda{}_\nu - u_\nu B^\lambda{}_\mu) \\ & + \epsilon_{\mu\nu\lambda\rho} \epsilon_{\alpha\beta\eta\sigma} u^\rho u^\sigma C^{\lambda\eta} \end{aligned}$$

20 components

Coincide with **3d** tensors in the fluid rest frame:

[L. D. Landau and E. M. Lifschits, The Classical Theory of Fields, Vol. 2, 1975]

[A. Z. Petrov, 1950]

- We consider **Ricci-flat** spaces $R_{\mu\nu} = 0$

Gradient expansion in the curved spacetime

The **gravitational chiral anomaly** has the **4th order** in gradients – it is to be related to the **3rd order** terms in gradient expansion of the axial **current**.

Hydrodynamic expansion of the axial current up to the **3rd order** in gradients:

Survive in flat spacetime

$$j_\mu^{A(3)} = \xi_1(T) w^2 w_\mu + \xi_2(T) \alpha^2 w_\mu + \xi_3(T) (\alpha w) w_\mu + \xi_4(T) A_{\mu\nu} w^\nu + \xi_5(T) B_{\mu\nu} \alpha^\nu$$

arbitrary coefficients

"gravitational" currents

See also gradient expansion for the fluid in the gravitational field, e.g.:

[P. Romatschke, Class. Quant. Grav. 27, 025006 (2010)]

[S. M. Diles, L. A. H. Mamani, A. S. Miranda, V. T. Zanchin, JHEP 2020, 1–40 (2020)]

Anomaly matching: principle

Following [D.T. Son, P. Surowka, PRL, 103 (2009) 191601]
- it would be necessary to construct the **entropy current**.

However in [Shi-Zheng Yang, Jian-Hua Gao, and Zuo-Tang Liang, Symmetry 14, 948 (2022)]

it is shown that it is possible to use the **global equilibrium** condition

$$\nabla_\mu \beta_\nu + \nabla_\nu \beta_\mu = 0$$

After that it is enough to consider ***only*** the equation for the current.

- Good for gravity, which is complicated in general case!

We use only:

$$\nabla_\mu j_A^\mu = \mathcal{N} \epsilon^{\mu\nu\alpha\beta} R_{\mu\nu\lambda\rho} R_{\alpha\beta}{}^{\lambda\rho}$$

Substitute the gradient expansion:

$$\nabla^\mu \Big(\xi_1(T) w^2 w_\mu + \xi_2(T) \alpha^2 w_\mu + \xi_3(T)(\alpha w) w_\mu + \xi_4(T) A_{\mu\nu} w^\nu + \xi_5(T) B_{\mu\nu} \alpha^\nu \Big) = 32 \mathcal{N} A_{\mu\nu} B^{\mu\nu}$$

Anomaly matching: system of equations

This **system** of linear **differential** equations has the form:

$$
\begin{aligned}
-3T\xi_1 + T^2\xi_1' + 2T\xi_3 &= 0 \\
-3T\xi_2 + T^2\xi_2' - T\xi_3 + T^2\xi_3' &= 0 \\
T^2\xi_4' + 3T\xi_5 + 2T^{-1}\xi_2 + T^{-1}\xi_3 &= 0 \\
-2T^{-1}\xi_1 - 3T\xi_4 - T\xi_5 &= 0 \\
T^2\xi_5' - T\xi_5 - T^{-1}\xi_3 &= 0 \\
-T^{-1}\xi_4 + T^{-1}\xi_5 - 32\mathscr{N} &= 0
\end{aligned}
$$

Can be solved!

Includes the factor from the **gravitational chiral anomaly**

Solution

Let's move on to the limit of **flat space-time**. Despite the absence of a gravitational field, there **remains** a contribution to the axial current induced by the **gravitational chiral anomaly:**

Zero gravitational field

Limit $R_{\mu\nu\alpha\beta} = 0$

$$j^A_\mu = \lambda_1 (\omega_\nu \omega^\nu)\omega_\mu + \lambda_2 (a_\nu a^\nu) w_\mu$$

$$\frac{\lambda_1 - \lambda_2}{32} = \mathcal{N}$$

Nonzero gravitational field

$$R_{\mu\nu\alpha\beta} \neq 0$$

$$\nabla_\mu j^\mu_A = \mathcal{N} \epsilon^{\mu\nu\alpha\beta} R_{\mu\nu\lambda\rho} R_{\alpha\beta}{}^{\lambda\rho}$$

- A new type of anomalous transport – the **Kinematical Vortical Effect (KVE)**. Does not explicitly depend on temperature and density → determined only by the **kinematics** of the flow.

Direct verification:
spin 1/2

Transport coefficients and anomaly: spin 1/2

- In [GP, O.V. Teryaev, and V.I. Zakharov, JHEP, 02:146, 2019], [V. E. Ambrus, JHEP, 08:016, 2020], [A. Palermo, et al. JHEP 10 (2021) 077] and for ω^3 in [A. Vilenkin, Phys. Rev., D20:1807-1812, 1979] the following expression was obtained:

KVE

$$j_\mu^A = \left(\frac{T^2}{6} + \frac{\mu^2}{2\pi^2} - \frac{\omega^2}{24\pi^2} - \frac{a^2}{8\pi^2}\right)\omega_\mu$$

- Comparing it with the well-known anomaly [L. Alvarez-Gaume, E. Witten, Nucl. Phys., B234:269, 1984]:

$$\nabla_\mu j_A^\mu = \frac{1}{384\pi^2\sqrt{-g}}\varepsilon^{\mu\nu\alpha\beta}R_{\mu\nu\lambda\rho}R_{\alpha\beta}{}^{\lambda\rho}$$

We see that the formula is **fulfilled**:

$$\frac{\lambda_1 - \lambda_2}{32} = \mathcal{N}$$

$\Rightarrow$

$$\left(-\frac{1}{24\pi^2} + \frac{1}{8\pi^2}\right)/32 = \frac{1}{384\pi^2}$$

Correspondence between **gravity** and **hydrodynamics** is **confirmed!**

Direct verification: spin 3/2

Rarita-Schwinger-Adler model of spin 3/2

The **Rarita-Schwinger theory** – well-known theory of spin 3/2.
But this theory has a number of **pathologies.**

- For example, it **doesn't allow to construct perturbation theory!**

Solved in **[Stephen L. Adler. Phys. Rev. D, 97(4):045014, 2018]** by introducing of interaction with additional spin ½ field:

Action:

additional spin ½ field

$$S = \int d^4x \left(-\varepsilon^{\lambda\rho\mu\nu}\bar{\psi}_\lambda\gamma_5\gamma_\mu\partial_\nu\psi_\rho + i\bar{\lambda}\gamma^\mu\partial_\mu\lambda - im\bar{\lambda}\gamma^\mu\psi_\mu + im\bar{\psi}_\mu\gamma^\mu\lambda \right)$$

"coupling mass"

Anomaly was found in **[Prokhorov, Teryaev, Zakharov, Phys.Rev.D 106 (2022) 2, 025022]**

$$\nabla_\mu j^\mu_A = -\frac{19}{384\pi^2\sqrt{-g}}\varepsilon^{\mu\nu\rho\sigma}R_{\mu\nu\kappa\lambda}R_{\rho\sigma}{}^{\kappa\lambda}$$

-19 times different from the anomaly for spin ½

Zubarev density operator

Global Equilibrium Conditions

$$\nabla_\mu \beta_\nu + \nabla_\nu \beta_\mu = 0 \qquad \nabla_\mu \zeta = 0$$

⇩

Form of the density operator for a medium with rotation and acceleration

$$\widehat{\rho} = \frac{1}{Z} \exp\left[-b_\mu \widehat{P}^\mu + \frac{1}{2} \varpi_{\mu\nu} \widehat{J}^{\mu\nu} + \zeta \widehat{Q} \right]$$

Thermal vorticity tensor

Lorentz Transform Generators

$$\widehat{J}^{\mu\nu} = \int_\Sigma \mathrm{d}\Sigma_\lambda \left(x^\mu \widehat{T}^{\lambda\nu} - x^\nu \widehat{T}^{\lambda\mu} \right)$$

$$\varpi_{\mu\nu} \hat{J}^{\mu\nu} = -2\alpha^\rho \hat{K}_\rho - 2 w^\rho \hat{J}_\rho$$

$\hat{K}^\mu$ – boost (related to **acceleration**)

$\hat{J}^\mu$ – angular momentum (related to **vorticity**)

Zubarev density operator

Equillibrium perturbation theory

[M. Buzzegoli, E. Grossi, F. Becattini, JHEP 10 (2017) 091]

$$\langle \widehat{O}(x)\rangle = \frac{1}{Z}\mathrm{tr}(\widehat{\rho}\,\widehat{O}(x))_{\mathrm{ren}}$$

statistical sum:
cancellation of disconnected correlators

Perturbation theory in the third order

$$\langle \widehat{O}(x)\rangle = \langle \widehat{O}(x)\rangle_{\beta(x)} + \frac{\varpi_{\mu\nu}}{2|\beta|}\int_0^{|\beta|} d\tau \langle T_\tau J^{\mu\nu}_{-i\tau u}\widehat{O}(0)\rangle_{\beta(x),c}$$

$$+\frac{\varpi_{\mu\nu}\varpi_{\rho\sigma}}{8|\beta|^2}\int_0^{|\beta|} d\tau_x d\tau_y \langle T_\tau J^{\mu\nu}_{-i\tau_x u} J^{\rho\sigma}_{-i\tau_y u}\widehat{O}(0)\rangle_{\beta(x),c}$$

$$+\frac{\varpi_{\mu\nu}\varpi_{\rho\sigma}\varpi_{\alpha\beta}}{48|\beta|^3}\int_0^{|\beta|} d\tau_x d\tau_y d\tau_z \langle T_\tau J^{\mu\nu}_{-i\tau_x u} J^{\rho\sigma}_{-i\tau_y u} J^{\alpha\beta}_{-i\tau_z u}\widehat{O}(0)\rangle_{\beta(x),c} + \ldots$$

Imaginary time $\tau = i\,t$ *ordering*

Connected correlators

$$\langle \widehat{J}\widehat{O}\rangle_c = \langle \widehat{J}\widehat{O}\rangle - \langle \widehat{J}\rangle\langle \widehat{O}\rangle$$

KVE IN RSA THEORY: CALCULATION

- Our *goal* is to calculate the conductivities λ_1 and λ_2 in the KVE current:

$$j^{\mu}_{A,KVE} = \lambda_1(\omega_\nu\omega^\nu)\omega^\mu + \lambda_2(a_\nu a^\nu)\omega^\mu$$

- Using the described perturbation theory, we obtain:

$$\lambda_1 = -\frac{1}{6}\int_0^{|\beta|}[d\tau]\langle T_\tau \hat{J}^3_{-i\tau_x}\hat{J}^3_{-i\tau_y}\hat{J}^3_{-i\tau_z}\hat{j}^3_A(0)\rangle_{T,c}$$

$$\lambda_2 = -\frac{1}{6}\int_0^{|\beta|}[d\tau]\Big\{\langle T_\tau(\hat{K}^1_{-i\tau_x}\hat{J}^3_{-i\tau_y} + \hat{J}^3_{-i\tau_x}\hat{K}^1_{-i\tau_y})\hat{K}^1_{-i\tau_z u}\hat{j}^3_A(0)\rangle_{T,c} + \langle T_\tau\hat{K}^1_{-i\tau_x}\hat{K}^1_{-i\tau_y}\hat{J}^3_{-i\tau_z}\hat{j}^3_A(0)\rangle_{T,c}\Big\}$$

- Representing $\hat{J}_\sigma$, $\hat{K}^\mu$ through the stress-energy tensor, we obtain

$$\lambda_1 = -\frac{1}{6T^3}\Big(C^{02|02|02|3|111} + C^{02|01|01|3|122} + C^{01|02|01|3|212} + C^{01|01|02|3|221} - C^{01|01|01|3|222} - C^{01|02|02|3|211} - C^{02|01|02|3|121} - C^{02|02|01|3|112}\Big),$$

$$\lambda_2 = -\frac{1}{6T^3}\Big(C^{02|00|00|3|111} + C^{00|02|00|3|111} + C^{00|00|02|3|111} - C^{01|00|00|3|211} - C^{00|01|00|3|121} - C^{00|00|01|3|112}\Big).$$

Typical correlator to be found: *4-point one-loop function*

$$C^{\alpha_1\alpha_2|\alpha_3\alpha_4|\alpha_5\alpha_6|\lambda|ijk} = T^3\int[d\tau]d^3x\,d^3y\,d^3z\,x^i y^j z^k\langle T_\tau\hat{T}^{\alpha_1\alpha_2}(-i\tau_x,\mathbf{x})\hat{T}^{\alpha_3\alpha_4}(-i\tau_y,\mathbf{y})\hat{T}^{\alpha_5\alpha_6}(-i\tau_z,\mathbf{z})\hat{j}^\lambda_5(0)\rangle_{T,c}$$

When expanding the density operator, a shift occurs along the imaginary axis - field theory at finite temperatures.

KVE IN RSA THEORY: CALCULATION

- Apply **point splitting** to all operators (no additional contributions arise - operators satisfy free field equations):

$$\hat{T}^{\mu\nu}(X) = \lim_{X_1,X_2\to X} \mathcal{D}^{\mu\nu}_{ab(IJ)}(\tilde{\partial}_{X_1},\tilde{\partial}_{X_2})\bar{\Psi}_{aI}(X_1)\Psi_{bJ}(X_2)\,,$$

$$\hat{j}^{\mu}_A(X) = \lim_{X_1,X_2\to X} \mathcal{J}^{\mu}_{ab(IJ)}\bar{\Psi}_{aI}(X_1)\Psi_{bJ}(X_2)\,,$$

where $X_\mu = (\tau_x, -\mathbf{x})$

Fields are combined into one vector $\Psi_I = \{\tilde{\psi}_\mu, \lambda\}$ $(I = 0...4)$

The matrix element has the form of a product of **vertices** and **propagators**.

Vertices $\mathcal{J}^{\mu}_{(ij)} = i^{1-\delta_{0\mu}}\varepsilon^{ij\mu\nu}\tilde{\gamma}_\nu$

Euclidean Dirac matrices $\{\tilde{\gamma}_\mu, \tilde{\gamma}_\nu\} = 2\delta_{\mu\nu}$

$$\mathcal{D}^{\mu\nu}_{(ij)} = -\frac{1}{2}(-i)^{\delta_{0\mu}+\delta_{0\nu}}\varepsilon^{ij\nu\beta}\left(\gamma_5\tilde{\gamma}_\mu\tilde{\partial}^{X_2}_\beta - \frac{1}{4}\gamma_5\tilde{\gamma}_\beta[\tilde{\gamma}_\vartheta,\tilde{\gamma}_\mu]\left(\tilde{\partial}^{X_1}_\vartheta + \tilde{\partial}^{X_2}_\vartheta\right)\right) + (\mu\leftrightarrow\nu)$$

$0 \le (i,j) < 4$

Propagators

$$\langle T_\tau \tilde{\psi}_{a\mu}(X_1)\bar{\tilde{\psi}}_{b\nu}(X_2)\rangle_T = \sum\!\!\!\!\!\!\int_P e^{iP_\alpha(X_1-X_2)^\alpha}\frac{i}{2P^2}\left(\tilde{\gamma}_\nu \not{P}\tilde{\gamma}_\mu + 2\left[\frac{1}{m^2} - \frac{2}{P^2}\right]P_\mu P_\nu \not{P}\right)_{ab}$$

$$\langle T_\tau \tilde{\psi}_{a\mu}(X_1)\bar{\lambda}_b(X_2)\rangle_T = \sum\!\!\!\!\!\!\int_P e^{iP_\alpha(X_1-X_2)^\alpha}\frac{-P_\mu \not{P}_{ab}}{mP^2}$$

Mixed terms are non-zero

$$\langle T_\tau \lambda_a(X_1)\bar{\lambda}_b(X_2)\rangle_T = 0$$

here $P_\mu = (p_n, -\mathbf{p})\,,\ p_n = (2n+1)\pi T$

Field λ is **non-propagating**

KVE IN RSA THEORY: CALCULATION

- The remaining actions are done explicitly: integration over imaginary time τ_x, τ_y and τ_z , over the angles in $d^3p = \sin(\vartheta) p^2 dp\, d\vartheta\, d\phi$, differentiation over the momentum variables. Finally, we obtain, in particular, for $C^{02|02|02|3|111}$

$$C^{02|02|02|3|111} = \frac{T}{480\pi^2} \int \frac{dp\, p\, e^{p/T}}{(1+e^{p/T})^5} \left\{ 126 - 291\frac{p}{T} - 472\frac{p^2}{T^2} + \left[126 + 873\frac{p}{T} + 5192\frac{p^2}{T^2}\right] e^{p/T} \right.$$
$$\left. + \left[-126 + 873\frac{p}{T} - 5192\frac{p^2}{T^2}\right] e^{2p/T} + \left[-126 - 291\frac{p}{T} + 472\frac{p^2}{T^2}\right] e^{3p/T} \right\} = \frac{177 T^3}{80\pi^2}$$

- Calculating other diagrams we obtain

$$\lambda_1 = -\frac{1}{6}\left(2\cdot\frac{177}{80\pi^2} + 6\cdot\frac{353}{240\pi^2}\right) = -\frac{53}{24\pi^2},$$

$$\lambda_2 = -\frac{1}{6}\left(\frac{33}{40\pi^2} + \frac{53}{80\pi^2} + \frac{1}{2\pi^2} + \frac{3}{4\pi^2} + \frac{47}{80\pi^2} + \frac{17}{40\pi^2}\right) = -\frac{5}{8\pi^2}$$

Thus, the **KVE** in the **RSA theory** has the form:

$$j^{\mu}_{A,KVE} = \left(-\frac{53}{24\pi^2}\omega^2 - \frac{5}{8\pi^2}a^2\right)\omega^{\mu}$$

KVE vs Gravitational Anomaly

The obtained formula for **cubic gradients** (KVE):

$$j_\mu^{A(3)} = \left(-\frac{53}{24\pi^2}\omega^2 - \frac{5}{8\pi^2}a^2\right)\omega_\mu$$

Gravitational chiral **anomaly**:

$$\nabla_\mu j_A^\mu = \frac{-19}{384\pi^2\sqrt{-g}}\varepsilon^{\mu\nu\alpha\beta}R_{\mu\nu\lambda\rho}R_{\alpha\beta}{}^{\lambda\rho}$$

$$\frac{\lambda_1 - \lambda_2}{32} = \mathcal{N}$$

Direct **verification**:

$$\left(-\frac{53}{24\pi^2} + \frac{5}{8\pi^2}\right)/32 = -\frac{19}{384\pi^2}$$

Coincidence of hydrodynamics and gravitational anomaly!

- The relationship between the transport coefficients in a **vortical accelerated fluid** and the **gravitational** chiral **anomaly** is shown!
- Verification in a **nontrivial** case with higher spins and interaction.

Recent Development

Generalization to (anti)de Sitter space

- Going **beyond approximation** $R_{\mu\nu} = 0$ **[Khakimov, Prokhorov, Teryaev, Zakharov,2401.09247 (2024)]**

$$\frac{\lambda_1 - \lambda_2}{32} = \mathcal{N}$$

-- **anomaly-hydro** relation remains valid

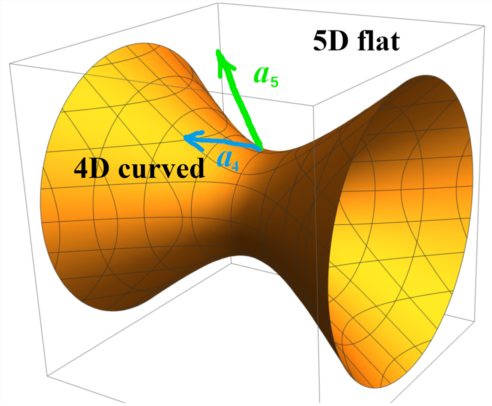
$$j^A_\mu \sim a^2 \omega_\mu \quad \Longleftrightarrow \quad j^A_\mu \sim R\omega_\mu$$

-- equivalence principle in higher orders

- **5-dimensional Unruh effect**: **[Khakimov, Prokhorov, Teryaev, Zakharov, Phys.Rev.D 108, 12, L121701 (2023)]**

The temperature measured by an accelerated observer in **(A)dS space** is determined by the 5-dimensional acceleration!

[S. Deser and O. Levin, Phys. Rev. D, 59:064004, 1999]



Hydrodynamic expansion for the stress-energy tensor:

$$\langle \hat{T}^{\mu\nu} \rangle_{s=1/2} = \left[\frac{7\pi^2}{180} T^4 + \frac{1}{72} \left(|a|^2 + \frac{R}{12} \right) T^2 - \frac{17}{2880\pi^2} \left(|a|^2 + \frac{R}{12} \right)^2 \right] \left(4u^\mu u^\nu - g^{\mu\nu} \right) + \frac{11}{960\pi^2} \left(\frac{R}{12} \right)^2 g^{\mu\nu}$$

$$\langle \hat{T}^{\mu\nu} \rangle (T = T_{UR}) = \frac{k}{4} R^2 g^{\mu\nu}$$

has a vacuum form ⇒

$$T_{UR} = \frac{a_5}{2\pi} = \sqrt{T_U^2 + T_R^2} = \frac{\sqrt{|a|^2 + R/12}}{2\pi}$$

Conclusion

Conclusion

- The relationship between the hydrodynamic current in the third order of gradient expansion $\lambda_1(\omega_\nu\omega^\nu)\omega_\mu$ and $\lambda_2(a_\nu a^\nu)w_\mu$, the **Kinematical Vortical Effect (KVE)**, and the **gravitational chiral anomaly** has been established:
    - The axial current in a flat space-time in a vortical and accelerated fluid turns out to be associated with a quantum violation of current conservation in a curved space-time.
- The obtained formula has been **verified** directly for **spin 1/2** and **3/2**.
- This demonstrates the interplay of **infrared** and **ultraviolet** phenomena. And can be interpreted as a demonstration of **equivalence principle** (for **higher orders** in gradients of metrics and **quantum loop** effects).
- It is shown that the effects survive when there is also constant curvature. The role of five-dimensional acceleration is demonstrated for the case of an accelerated observer in (A)dS space.

Additional slides

Outlook

- Signatures of **gravitational** chiral **anomaly without gravity?** Contribution to the vortical **polarization**?

  - To observe cubic terms, it is necessary that the gradients (acceleration, vorticity, magnetic field...) give a contribution at least of the order of temperature

$$\omega, a \sim (0.1 - 0.6)T$$

[A. Zinchenko, A. Sorin, O. Teryaev, M. Baznat, J.Phys.Conf.Ser. 1435 (2020)]

[F. Becattini, M. Buzzegoli, G. Inghirami, I. Karpenko, A. Palermo, Phys.Rev.Lett. 127 (2021) 27, 272302]

However, the cubic terms are also suppressed by the **numerical factor**

KVE: $$j^{\mu}_{A,S=1/2} = \left(-\frac{1}{24\pi^2}\omega^2 - \frac{1}{8\pi^2}a^2\right)\omega^{\mu}$$

- The **good** news: for higher spins (e.g. 3/2) it is enhanced by **cubic growth** with spin (related to anomaly growth):

$$\lambda_1^S = \frac{S - 8S^3}{12\pi^2}$$

- **But**: should be generalized to massive particles (omega baryon is heavy).

- **Idea:** consider massless **quasiparticles** with spin 3/2 in semymetals?

[I. Boettcher, Phys. Rev. Lett. 124, 127602 (2020)]

Outlook

- Other anomalies?

Weyl anomaly

$$\langle T^\mu_\mu \rangle = c_e F^{\mu\nu} F_{\mu\nu} + c_g R^{\mu\nu\alpha\beta} R_{\mu\nu\alpha\beta} + \ldots$$

Transport from the Weyl anomaly in **boundary QFT**:

[Chong-Sun Chu, Rong-Xin Miao, Phys.Rev.Lett. 121 (2018) 25, 251602]

Torsion anomaly $\mathscr{A} = \langle \nabla_\mu J_5^\mu \rangle$

$$\mathscr{A} = \frac{2}{(4\pi)^2} \left[\nabla_\mu \mathscr{K}^\mu + \frac{1}{48} \varepsilon_{\rho\sigma\alpha\beta} R^{\alpha\beta}_{\cdot\,\cdot\,\mu\nu} R^{\mu\nu\rho\sigma} + \frac{1}{6} \eta^2 \varepsilon^{\mu\nu\alpha\beta} S_{\mu\nu} S_{\alpha\beta} \right]$$

[Physical aspects of the space–time torsion, I.L. Shapiro, 2001]

- Exist in condensed matter

Polarisation as a sign of the vorticity

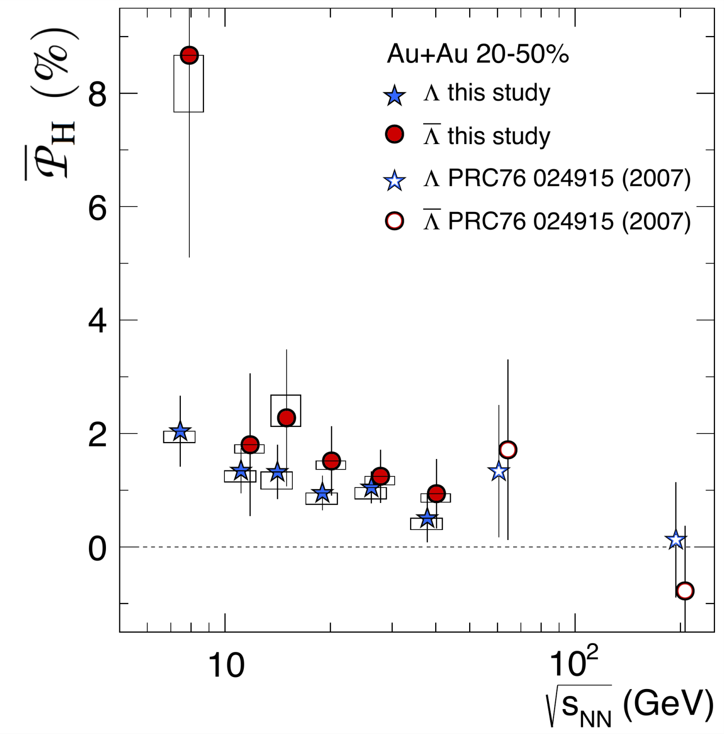
The **vorticity** leads to **polarization** of hadrons:

1. Through the chiral vortical effect (CVE) and chiral anomaly (**Dubna group**)
[M. Baznat, K. Gudima, A. Sorin, O. Teryaev, Phys.Rev.C 97 (2018) 4, 041902]

CVE: $Q_5^s \sim < \Pi_0^{\Lambda,lab} >$

2. Acceleration also leads to polarization
[F. Becattini, V. Chandra, L. Del Zanna, E. Grossi Annals Phys., 338:32–49, 2013]

$$\langle \Pi_\mu(p) \rangle \simeq \frac{1}{8}\epsilon_{\mu\rho\sigma\tau}\frac{p^\tau}{m}\frac{\int d\Sigma_\lambda\, p^\lambda\, n_F(1-n_F)\partial^\rho\beta^\sigma}{\int d\Sigma_\lambda\, p^\lambda n_F}$$



[Nature, 548:62–65, 2017]

Chiral vortical effect for spin 3/2: result

After summation of all the contributions, the next formula for CVE for **spin 3/2** is obtained from *Zubarev operator* **[2109.06048]**:

$$\langle \hat{j}^{\nu}_{A} \rangle^{(1)} = \left(\frac{5T^2}{6} + \frac{5\mu^2}{2\pi^2} \right) \omega^{\nu}$$

Chiral anomaly for **spin 3/2** was *[Stephen L. Adler. Phys. Rev. D, 97(4):045014, 2018]*:

$$\langle \partial_{\mu} \hat{j}^{\mu}_{A} \rangle = -\frac{5}{16\pi^2} \varepsilon^{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta}$$

The relationship with anomaly is checked for higher spins!!!

Generalization to (anti)de Sitter space

Previously, we considered **Ricci-flat** background $R_{\mu\nu} = 0$

Let us generalize to the case with constant scalar curvature **(A)dS** $R_{\mu\nu} = \Lambda g_{\mu\nu}$

Using the same method (gradient expansion and conservation relations), we obtain

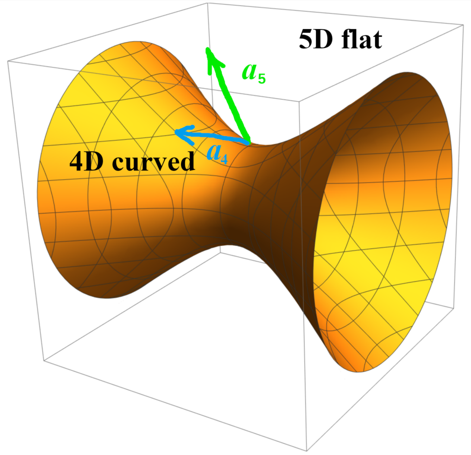
$$\langle \hat{T}^{\mu\nu} \rangle_{s=1/2} = \left[\frac{7\pi^2}{180} T^4 + \frac{1}{72} \left(|a|^2 + \frac{R}{12} \right) T^2 - \frac{17}{2880\pi^2} \left(|a|^2 + \frac{R}{12} \right)^2 \right] \left(4u^\mu u^\nu - g^{\mu\nu} \right) + \frac{11}{960\pi^2} \left(\frac{R}{12} \right)^2 g^{\mu\nu}$$

for spin 1/2. The general case: [R.V. Khakimov, G.Yu. Prokhorov, O.V. Teryaev, V.I. Zakharov, 2023, 2308.08647].

At temperature T_{UR}, the stress-energy tensor has a vacuum form:

$$\langle \hat{T}^{\mu\nu} \rangle (T = T_{UR}) = \frac{k}{4} R^2 g^{\mu\nu}$$

$$T_{UR} = \frac{a_5}{2\pi} = \sqrt{T_U^2 + T_R^2} = \frac{\sqrt{|a|^2 + R/12}}{2\pi}$$



The temperature measured by an accelerated observer in (A)dS space is determined by the 5-dimensional acceleration! [S. Deser and O. Levin, Phys. Rev. D, 59:064004, 1999]

Duality relations are obtained that between the accelerated fluid to the effects of constant curvature.

KVE: ARBITRARY SPIN

Is it possible to obtain a general formula for KVE for an arbitrary spin?

General formulas

$$\nabla_\mu j^\mu_{A,S} = \frac{(S-2S^3)}{96\pi^2\sqrt{-g}}\varepsilon^{\mu\nu\rho\sigma}R_{\mu\nu\kappa\lambda}R_{\rho\sigma}{}^{\kappa\lambda}$$

$$\lambda_1 - \lambda_2 = 32\mathcal{N}$$

Special cases

$$j^\mu_{A,S=1/2} = \left(-\frac{1}{24\pi^2}\omega^2 - \frac{1}{8\pi^2}a^2\right)\omega^\mu$$

$$j^\mu_{A,RSA} = \left(-\frac{53}{24\pi^2}\omega^2 - \frac{5}{8\pi^2}a^2\right)\omega^\mu$$

But it should be taken into account that the RSA theory includes **two degrees** of freedom **with spin ½**, then:

$$j^\mu_{A,S=3/2} = \left(-\frac{51}{24\pi^2}\omega^2 - \frac{3}{8\pi^2}a^2\right)\omega^\mu$$

Hypothesis:

$$\lambda_2 \sim S$$

$$\lambda_2^S = -\frac{S}{4\pi^2}$$

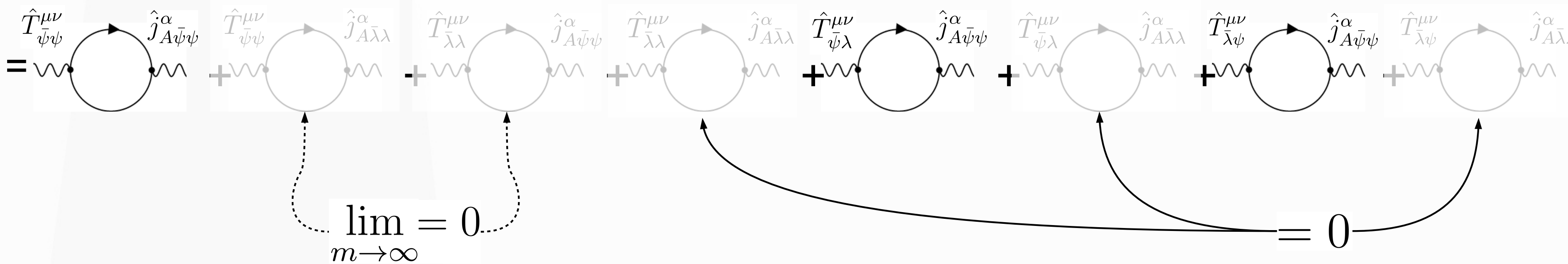
$$\lambda_1^S = \frac{S-8S^3}{12\pi^2}$$

Differs from a cubic dependence on spin, which could be *naively* expected from the term

$$\Delta H = -\mathbf{\Omega}\cdot\mathbf{S}$$

Chiral vortical effect for spin 3/2: quantum correlators

$$W = W_{\bar\psi\psi\bar\psi\psi} + W_{\bar\psi\psi\bar\lambda\lambda} + W_{\bar\lambda\lambda\bar\psi\psi} + W_{\bar\lambda\lambda\bar\lambda\lambda} + W_{\bar\psi\lambda\bar\psi\psi} + W_{\bar\psi\lambda\bar\lambda\lambda} + W_{\bar\lambda\psi\bar\psi\psi} + W_{\bar\lambda\psi\bar\lambda\lambda}$$



SET and axial current

$\hat j^{\alpha}_{A\bar\psi\psi}$ $\hat j^{\alpha}_{A\bar\lambda\lambda}$

$$j^{\mu}_{A} = -i\varepsilon^{\lambda\rho\nu\mu}\bar\psi_\lambda\gamma_\nu\psi_\rho + \bar\lambda\gamma_\mu\gamma_5\lambda$$

$$\begin{aligned}
T^{\mu\nu} = {} & \frac{1}{2}\varepsilon^{\lambda\alpha\beta\rho}\bar\psi_\lambda\gamma_5(\gamma^\mu\delta^\nu_\alpha + \gamma^\nu\delta^\mu_\alpha)\partial_\beta\psi_\rho \\
& + \frac{1}{8}\partial_\eta\Big(\varepsilon^{\lambda\alpha\beta\rho}\bar\psi_\lambda\gamma_5\gamma_\alpha([\gamma^\eta,\gamma^\mu]\delta^\nu_\beta + [\gamma^\eta,\gamma^\nu]\delta^\mu_\beta)\psi_\rho\Big) \quad \Big\} \hat T^{\mu\nu}_{\bar\psi\psi} \\
& + \frac{i}{4}\Big(\bar\lambda\gamma^\nu\partial^\mu\lambda - \partial^\mu\bar\lambda\gamma^\nu\lambda + \bar\lambda\gamma^\mu\partial^\nu\lambda - \partial^\nu\bar\lambda\gamma^\mu\lambda\Big) \quad \Big\} \hat T^{\mu\nu}_{\bar\lambda\lambda} \\
& + \frac{i}{2}m\Big(\bar\psi^\mu\gamma^\nu\lambda - \bar\lambda\gamma^\mu\psi^\nu + \bar\psi^\nu\gamma^\mu\lambda - \bar\lambda\gamma^\nu\psi^\mu\Big).
\end{aligned}$$

$\hat T^{\mu\nu}_{\bar\psi\lambda}$ $\hat T^{\mu\nu}_{\bar\lambda\psi}$

Discussion

It also turns out that in the current

$$j_{\mu}^{A(3)} = \lambda_1 \omega^2 \omega_{\mu} + \lambda_2 a^2 \omega_{\mu} + \lambda_3 (a\omega)\omega_{\mu} + \lambda_4 A_{\mu\nu}\omega^{\nu} + \lambda_5 B_{\mu\nu} a^{\nu}$$

- **Difference** of **flat-space terms** is to be equal to **difference** of **curved-space terms:**

$$\lambda_1 - \lambda_2 = \lambda_5 - \lambda_4$$

- At the finite mass, λ begin to depend on mass and temperature:

[G. Y. Prokhorov, O. V. Teryaev, V. I. Zakharov, JHEP 02 (2019)]

Method of conformal correlation functions: gravitational anomaly

We will consider the case of points on one 4-axis:

$$x_\mu = x\, e_\mu\,,\; y_\mu = y\, e_\mu\,,\; z_\mu = z\, e_\mu$$

Then the correlator should look like:

$$\langle T\, \hat{T}^{\mu\nu}(x)\hat{T}^{\sigma\rho}(y)\hat{j}^{\omega}_{A}(z)\rangle = \mathscr{A}\Big(4(x-y)^5 \times (x-z)^3(y-z)^3\Big)^{-1} e_\vartheta \Big(\eta^{\nu\rho}\varepsilon^{\sigma\vartheta\mu\omega} + \eta^{\mu\rho}\varepsilon^{\sigma\vartheta\nu\omega} + \eta^{\nu\sigma}\varepsilon^{\vartheta\mu\rho\omega} + \eta^{\mu\sigma}\varepsilon^{\vartheta\nu\rho\omega} - 6e^2(e^\nu e^\rho \varepsilon^{\sigma\vartheta\mu\omega} + e^\mu e^\rho \varepsilon^{\sigma\vartheta\nu\omega} + e^\sigma e^\nu \varepsilon^{\vartheta\mu\rho\omega} + e^\sigma e^\mu \varepsilon^{\vartheta\nu\rho\omega})\Big)$$

Method of conformal correlation functions: gravitational anomaly

Let's again decompose all the operators depending on the set of the fields:

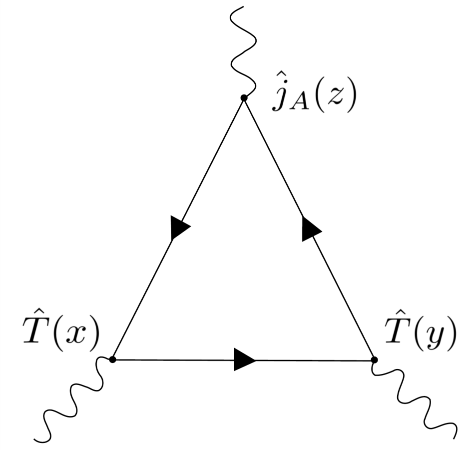
$$\hat{T}^{\mu\nu} = \hat{T}^{\mu\nu}_{\bar{\psi}\psi} + \hat{T}^{\mu\nu}_{\bar{\lambda}\lambda} + \hat{T}^{\mu\nu}_{\bar{\psi}\lambda} + \hat{T}^{\mu\nu}_{\bar{\lambda}\psi}$$

$$\hat{j}^{\mu}_{A} = \hat{j}^{\mu}_{A\bar{\psi}\psi} + \hat{j}^{\mu}_{A\bar{\lambda}\lambda}$$

Then the three-point function decomposes into **32 terms**:

$$\langle T\hat{T}^{\mu\nu}(x)\hat{T}^{\sigma\rho}(y)\hat{j}^{\omega}_{A}(z)\rangle_c = \langle \hat{T}_{\bar{\psi}\psi}\hat{T}_{\bar{\psi}\psi}\hat{j}^{A}_{\bar{\psi}\psi}\rangle + (31\,\text{terms})$$

A typical diagram (different SETs in the vertices):



Method of conformal correlation functions: gravitational anomaly

As a result, we have for the independent correlators:

$$\begin{aligned}&\langle T\,\hat{T}^{\mu\nu}_{\bar\psi\psi}(x)\hat{T}^{\sigma\rho}_{\bar\psi\psi}(y)\hat{j}^{\omega}_{A\bar\psi\psi}(z)\rangle_c = \frac{1}{4\pi^6(x-y)^5(x-z)^4(y-z)^4}e_\vartheta(28e^2x^2e^\mu e^\sigma\varepsilon^{\vartheta\nu\rho\omega} - \varepsilon^{\sigma\vartheta\nu\omega}(2e^2e^\mu e^\rho(-14x^2\\
&+9z(x+y)+19xy-14y^2-9z^2)+(26x^2-3z(x+y)-49xy+26y^2+3z^2)\eta^{\mu\rho})+2e^2e^\nu(14x^2-19xy\\
&-9xz+14y^2-9yz+9z^2)(e^\rho\varepsilon^{\sigma\vartheta\mu\omega}+e^\sigma\varepsilon^{\vartheta\mu\rho\omega})-38e^2xye^\mu e^\sigma\varepsilon^{\vartheta\nu\rho\omega}-18e^2xze^\mu e^\sigma\varepsilon^{\vartheta\nu\rho\omega}+28e^2y^2e^\mu e^\sigma\varepsilon^{\vartheta\nu\rho\omega}\\
&-18e^2yze^\mu e^\sigma\varepsilon^{\vartheta\nu\rho\omega}+18e^2z^2e^\mu e^\sigma\varepsilon^{\vartheta\nu\rho\omega}-26x^2\eta^{\nu\rho}\varepsilon^{\sigma\vartheta\mu\omega}-26x^2\eta^{\mu\sigma}\varepsilon^{\vartheta\nu\rho\omega}-(26x^2-3z(x+y)-49xy\\
&+26y^2+3z^2)\eta^{\nu\sigma}\varepsilon^{\vartheta\mu\rho\omega}+49xy\eta^{\nu\rho}\varepsilon^{\sigma\vartheta\mu\omega}+49xy\eta^{\mu\sigma}\varepsilon^{\vartheta\nu\rho\omega}+3xz\eta^{\nu\rho}\varepsilon^{\sigma\vartheta\mu\omega}+3xz\eta^{\mu\sigma}\varepsilon^{\vartheta\nu\rho\omega}-26y^2\eta^{\nu\rho}\varepsilon^{\sigma\vartheta\mu\omega}\\
&-26y^2\eta^{\mu\sigma}\varepsilon^{\vartheta\nu\rho\omega}+3yz\eta^{\nu\rho}\varepsilon^{\sigma\vartheta\mu\omega}+3yz\eta^{\mu\sigma}\varepsilon^{\vartheta\nu\rho\omega}-3z^2\eta^{\nu\rho}\varepsilon^{\sigma\vartheta\mu\omega}-3z^2\eta^{\mu\sigma}\varepsilon^{\vartheta\nu\rho\omega})\,,\end{aligned}$$

$$\begin{aligned}&\langle T\,\hat{T}^{\mu\nu}_{\bar\psi\psi}(x)\hat{T}^{\sigma\rho}_{\bar\psi\lambda}(y)\hat{j}^{\omega}_{A\bar\psi\psi}(z)\rangle_c = \frac{1}{4\pi^6(x-y)^5(x-z)^4(y-z)^4}e_\vartheta(4e^2x^2e^\mu e^\sigma\varepsilon^{\vartheta\nu\rho\omega} - \varepsilon^{\sigma\vartheta\nu\omega}(2e^2e^\mu e^\rho(-2x^2\\
&+3z(7x+y)-17xy+7y^2-12z^2)+(10x^2+7xy-27xz-13y^2+19yz+4z^2)\eta^{\mu\rho})+2e^2e^\nu(2x^2\\
&+17xy-21xz-7y^2-3yz+12z^2)(e^\rho\varepsilon^{\sigma\vartheta\mu\omega}+e^\sigma\varepsilon^{\vartheta\mu\rho\omega})+34e^2xye^\mu e^\sigma\varepsilon^{\vartheta\nu\rho\omega}-42e^2xze^\mu e^\sigma\varepsilon^{\vartheta\nu\rho\omega}\\
&-14e^2y^2e^\mu e^\sigma\varepsilon^{\vartheta\nu\rho\omega}-6e^2yze^\mu e^\sigma\varepsilon^{\vartheta\nu\rho\omega}+24e^2z^2e^\mu e^\sigma\varepsilon^{\vartheta\nu\rho\omega}-10x^2\eta^{\nu\rho}\varepsilon^{\sigma\vartheta\mu\omega}-10x^2\eta^{\mu\sigma}\varepsilon^{\vartheta\nu\rho\omega}-(10x^2+7xy\\
&-27xz-13y^2+19yz+4z^2)\eta^{\nu\sigma}\varepsilon^{\vartheta\mu\rho\omega}-7xy\eta^{\nu\rho}\varepsilon^{\sigma\vartheta\mu\omega}-7xy\eta^{\mu\sigma}\varepsilon^{\vartheta\nu\rho\omega}+27xz\eta^{\nu\rho}\varepsilon^{\sigma\vartheta\mu\omega}+27xz\eta^{\mu\sigma}\varepsilon^{\vartheta\nu\rho\omega}\\
&+13y^2\eta^{\nu\rho}\varepsilon^{\sigma\vartheta\mu\omega}+13y^2\eta^{\mu\sigma}\varepsilon^{\vartheta\nu\rho\omega}-19yz\eta^{\nu\rho}\varepsilon^{\sigma\vartheta\mu\omega}-19yz\eta^{\mu\sigma}\varepsilon^{\vartheta\nu\rho\omega}-4z^2\eta^{\nu\rho}\varepsilon^{\sigma\vartheta\mu\omega}-4z^2\eta^{\mu\sigma}\varepsilon^{\vartheta\nu\rho\omega})\,,\end{aligned}$$

$$\langle T\hat{T}^{\mu\nu}_{\bar\psi\lambda}(x)\hat{T}^{\sigma\rho}_{\bar\psi\lambda}(y)\hat{j}^{\omega}_{A\bar\psi\psi}(z)\rangle_c = \frac{4e^2e_\vartheta(e^\nu(e^\rho\varepsilon^{\sigma\vartheta\mu\omega}+e^\sigma\varepsilon^{\vartheta\mu\rho\omega})+e^\mu(e^\rho\varepsilon^{\sigma\vartheta\nu\omega}+e^\sigma\varepsilon^{\vartheta\nu\rho\omega}))}{\pi^6(x-y)^3(x-z)^4(y-z)^4}$$

$$\begin{aligned}&\langle T\,\hat{T}^{\mu\nu}_{\bar\psi\lambda}(x)\hat{T}^{\sigma\rho}_{\bar\lambda\psi}(y)\hat{j}^{\omega}_{A\bar\psi\psi}(z)\rangle_c = \frac{5}{2\pi^6(x-y)^3(x-z)^4(y-z)^4}e_\vartheta(-2e^2e^\mu e^\rho\varepsilon^{\sigma\vartheta\nu\omega}-2e^2e^\nu(e^\rho\varepsilon^{\sigma\vartheta\mu\omega}+e^\sigma\varepsilon^{\vartheta\mu\rho\omega})\\
&-2e^2e^\mu e^\sigma\varepsilon^{\vartheta\nu\rho\omega}+\eta^{\nu\rho}\varepsilon^{\sigma\vartheta\mu\omega}+\eta^{\mu\rho}\varepsilon^{\sigma\vartheta\nu\omega}+\eta^{\sigma\nu}\varepsilon^{\vartheta\mu\rho\omega}+\eta^{\sigma\mu}\varepsilon^{\vartheta\nu\rho\omega})\,.\end{aligned}$$

Each term **differs** from what we need.

Gravitational chiral anomaly in RSA theory

- **How to explain the factor -19?**
- How does it **relate** to **previous** calculations?

$$\langle \nabla_\mu \hat{j}^\mu_A \rangle_{RS} = \frac{-21}{384\pi^2\sqrt{-g}} \varepsilon^{\mu\nu\rho\sigma} R_{\mu\nu\kappa\lambda} R_{\rho\sigma}{}^{\kappa\lambda}$$

[M. J. Duff, 1201.0386] [J.J.M. Carrasco, et al. JHEP, 07:029, 2013]

"ghostless" contribution [J.J.M. Carrasco, et al. JHEP, 07:029, 2013]

$$-19 = -20 + 1$$

Contribution of spin 1/2

$$-19 = -21 + 2$$

KVE IN RSA THEORY: CALCULATION

Propagator at finite temperature

$$\langle T_\tau \Psi_{aI_1}(X)\overline{\Psi}_{bI_2}(Y)\rangle = \sum\!\!\!\!\!\!\!\int_P e^{iP(X-Y)} G_{ab(I_1 I_2)}(P)$$

contains either **2nd or 4th order poles:**

$$G(P) = G_1(P) + G_2(P)\,,\ G_1 \sim 1/P^2\,,\ G_2 \sim 1/P^4$$

Formulas for summation over Matsubara frequencies

[M. Buzzegoli, thesis 2020. arXiv: 2004.08186]

$$\sum_{\omega_n=(2n+1)\pi T} \frac{f(\omega_n)e^{i\omega_n \tau}}{\omega_n^2 + E^2} = \frac{1}{2ET}\sum_{s=\pm 1} f(-isE)e^{\tau s E}\Big[\theta(-s\tau) - n_F(E)\Big]$$

$$\sum_{\omega_n=(2n+1)\pi T} \frac{f(\omega_n)e^{i\omega_n \tau}}{(\omega_n^2 + E^2)^2} = \frac{1}{T}\sum_{s=\pm 1} e^{\tau s E}\left\{\frac{f(-isE)}{4E^2}n_F'(E) + \frac{(1-s\tau E)f(-isE) + isEf'(-isE)}{4E^3}\Big[\theta(-s\tau) - n_F(E)\Big]\right\}$$

Fermi-Dirac distribution

KVE IN RSA THEORY: CALCULATION

Let us consider (for illustration) the contribution of the first of the term from the Wick's theorem and only from $G_1 \sim 1/P^2$. After **summation over the Matsubara** frequencies, we obtain:

$$C^{\alpha_1\alpha_2|\alpha_3\alpha_4|\alpha_5\alpha_6|\lambda|ijk}_{\text{Wick1,G1}} = -T^3 \int [d\tau] d^3x d^3y d^3z \, x^i y^j z^k \times \int \frac{d^3p_1 d^3p_2 d^3p_3 d^3p_4}{16 E_1 E_2 E_3 E_4} e^{-i\mathbf{p}_1(\mathbf{y}-\mathbf{x})-i\mathbf{p}_2\mathbf{x}-i\mathbf{p}_3(\mathbf{z}-\mathbf{y})+i\mathbf{p}_4\mathbf{z}}$$
$$\times \sum_{s_n=\pm1} e^{s_1E_1(\tau_y-\tau_x)+s_2E_2\tau_x+s_3E_3(\tau_z-\tau_y)-s_4E_4\tau_z} \Big\{\theta(-s_1[\tau_y-\tau_x]) - n_F(E_1)\Big\}$$
$$\times \Big\{\theta(-s_2\tau_x) - n_F(E_2)\Big\} \Big\{\theta(-s_3[\tau_z-\tau_y]) - n_F(E_3)\Big\} \Big\{\theta(-s_4\tau_z) - n_F(E_4)\Big\}$$
$$\times \operatorname*{tr}_{I,a} \Big\{\mathcal{D}^{\alpha_1\alpha_2}(-i\widetilde{P}_1, i\widetilde{P}_2) G_1(\widetilde{P}_2) \mathcal{J}^\lambda G_1(\widetilde{P}_4) \mathcal{D}^{\alpha_5\alpha_6}(-i\widetilde{P}_4, i\widetilde{P}_3) G_1(\widetilde{P}_3) \mathcal{D}^{\alpha_3\alpha_4}(-i\widetilde{P}_3, i\widetilde{P}_1) G_1(\widetilde{P}_1)\Big\}$$

where

$$\widetilde{P}^n_\mu = (-is_nE_1, -\mathbf{p}_1), \; E_n = |\mathbf{p}_n|$$

Contains an **explicit dependence on coordinates**.

- Can be absorbed into **derivatives** by integration by parts

$$\int d^3p_1 d^3p_2 d^3p_3 d^3p_4 d^3x d^3y d^3z \, x^i y^j z^k e^{-i\mathbf{p}_1(\mathbf{y}-\mathbf{x})-i\mathbf{p}_2\mathbf{x}-i\mathbf{p}_3(\mathbf{z}-\mathbf{y})+i\mathbf{p}_4\mathbf{z}} f(\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3,\mathbf{p}_4) =$$
$$= i(2\pi)^9 \int d^3p \left(\frac{\partial^3}{\partial p_4^k \partial p_2^i \partial p_3^j} + \frac{\partial^3}{\partial p_4^k \partial p_2^i \partial p_4^j}\right) f(\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3,\mathbf{p}_4)\Big|_{\substack{\mathbf{p}_4=\mathbf{p}_1\\ \mathbf{p}_3=\mathbf{p}_1\\ \mathbf{p}_2=\mathbf{p}_1}} .$$

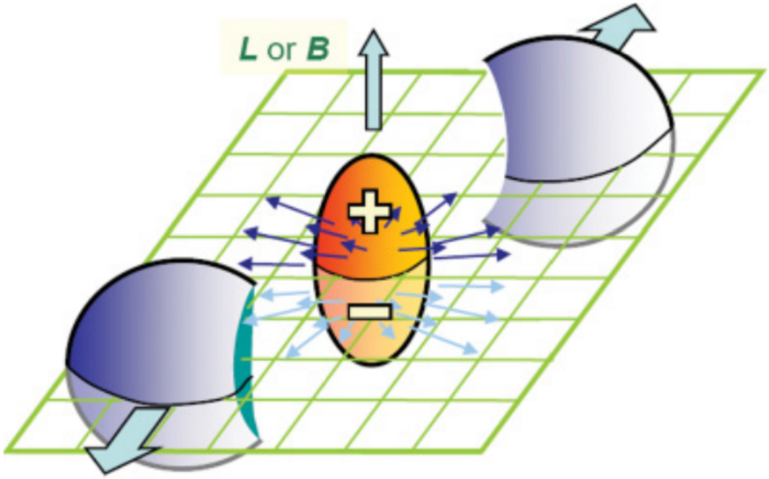
There remains only **one** integral over the momentum.

Introduction

In noncentral collisions of heavy ions, **huge magnetic fields** and a **huge angular momentum** arise. *Differential* rotation - different at different points: **vorticity** and vortices.

– Rotation 25 orders of magnitude *faster* than the rotation of the Earth:

the vorticity is about 10^{22} sec^{-1}



- **Acceleration** is of the **same order** of magnitude as vorticity (as the components of the same tensor).

[I. Karpenko and F. Becattini, Nucl. Phys., A982:519–522, 2019]

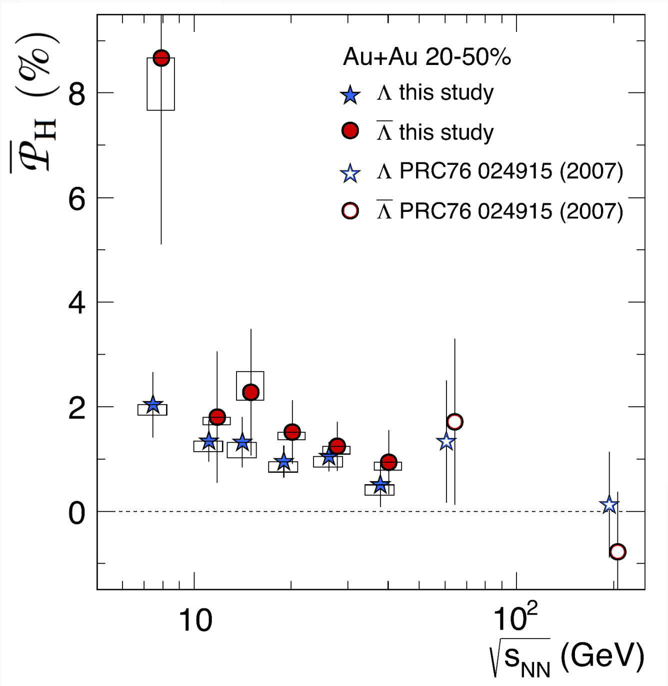
- *Another mechanism*: accelerations due to the tension of the hadron string (acceleration is much higher – of the order of Λ_{QCD})

$$e^+e^- \to \gamma^* \to q\bar{q} \to \text{ hadrons}$$

[P. Castorina, D. Kharzeev, H. Satz, Eur. Phys. J., C52:187–201, 2007]

Introduction

The orbital angular momentum transforms into polarization - an analogue of the Barnett effect.



[Nature 548 (2017) 62-65 arXiv:1701.06657 [nucl-ex]]

STAR Collaboration

- Generation of **hyperon polarization**.
- Both **vorticity** and **acceleration** are essential for polarization.
- Pioneering theoretical prediction: **[Z. T. Liang and X. N. Wang, Phys. Rev. Lett. 94, 102301 (2005)]**
- For recent development see: **[Jian-Hua Gao, Shi-Zheng Yang, 2308.16616]**
- Also described based on **Chiral Vortical Effect** (**CVE**) – Dubna group **[Rogachevsky, Sorin, Teryaev, Phys.Rev.C82(2010) 054910],**

CVE: $$\langle j_\mu^5 \rangle = \left(\frac{T^2}{6} + \frac{\mu^2}{2\pi^2}\right)\omega_\mu$$

– **Qualitative and quantitative correspondence!**
– Polarization from quantum anomaly ~ *spin crisis* and *gluon anomaly*: **[Efremov, Soffer, Teryaev, Nucl.Phys.B 346 (1990) 97-114]**

proton spin → hyperon polarization,
gluon field → chemical potential*4-velocity

- Also described on the basis of a thermodynamic approach: **[I. Karpenko, F. Becattini, Nucl.Phys.A 982 (2019) 519-522]**

Gravitational anomaly in thermal CVE

The (possible) answer was obtained in different approaches:

- From the holography. **[K. Landsteiner, E. Megias, F. Pena-Benitez, "Gravitational Anomaly and Transport," Phys. Rev. Lett. 107, 021601 (2011)]**

- In **[S.P. Robinson, F. Wilczek. Phys. Rev. Lett., 95:011303, 2005]** Hawking radiation is associated with a gravitational anomaly: it is necessary to integrate the anomaly from the horizon to infinity + the condition for the consellation of the currents on the horizon.

  In **[M. Stone, J. Kim. Phys. Rev., D98(2):025012, 2018]** the derivation was generalized to 3+1 dimensional gravitational chiral anomaly and (analogue) of rotating black hole.

- From the condition of the translational invariance of the Euclidean vacuum.

  [K. Jensen, R. Loganayagam, A. Yarom, JHEP 02, 088 (2013)]

$$j_A^\nu = (\sigma_T T^2 + \sigma_\mu \mu^2)\omega^\nu$$

$$\epsilon^{\mu\nu\alpha\beta} = \frac{1}{\sqrt{-g}}\varepsilon^{\mu\nu\alpha\beta}$$

$$\boxed{\sigma_T = 64\pi^2 \mathcal{N}} \Longleftrightarrow \nabla_\mu j_A^\mu = \mathcal{N}\,\epsilon^{\mu\nu\alpha\beta} R_{\mu\nu\lambda\rho} R_{\alpha\beta}{}^{\lambda\rho}$$

Gravitational anomaly in thermal CVE

Works for the Dirac field!

$$j_A^\mu = \left(\frac{T^2}{6} + \frac{\mu^2}{2\pi^2}\right)\omega^\mu \qquad \sigma_T = 64\pi^2 N$$

[L. Alvarez-Gaume, E. Witten, Nucl. Phys. B234 (1984) 269]

$$\nabla_\mu j_A^\mu = \frac{1}{384\pi^2\sqrt{-g}}\varepsilon^{\mu\nu\rho\sigma}R_{\mu\nu\kappa\lambda}R_{\rho\sigma}{}^{\kappa\lambda}$$

But there is **problem** for higher spins: *Rarita-Schwinger-Adler* theory for **spin 3/2**

[S. L. Adler, Phys. Rev. D 97 (4) (2018) 045014]

[G. Y. Prokhorov, O. V. Teryaev, V. I. Zakharov, Phys. Rev. D 105 (4) (2022) L041701]

$$\Rightarrow \quad j_A^\nu = \left(\frac{5}{6}T^2 + \frac{5}{2\pi^2}\mu^2\right)\omega^\nu$$

[G. Y. Prokhorov, O. V. Teryaev, V. I. Zakharov, Phys. Rev. D 106 (2) (2022) 025022]

$$\Rightarrow \quad \nabla_\mu j_A^\mu = -\frac{19}{384\pi^2\sqrt{-g}}\varepsilon^{\mu\nu\rho\sigma}R_{\mu\nu\kappa\lambda}R_{\rho\sigma}{}^{\kappa\lambda}$$

$$\sigma_T \neq 64\pi^2 N$$

Question:

In hydrodynamics, the **cubic** dependence on the **spin** from the **gravitational** anomaly **is not visible?**

Decomposition of the tensors: gravity

We also decompose the **Riemann tensor** into the **components:**

$$
\begin{aligned}
R_{\mu\nu\alpha\beta} = & \; u_\mu u_\alpha A_{\nu\beta} + u_\nu u_\beta A_{\mu\alpha} - u_\nu u_\alpha A_{\mu\beta} - u_\mu u_\beta A_{\nu\alpha} \\
& + \epsilon_{\mu\nu\lambda\rho} u^\rho (u_\alpha B^\lambda{}_\beta - u_\beta B^\lambda{}_\alpha) \\
& + \epsilon_{\alpha\beta\lambda\rho} u^\rho (u_\mu B^\lambda{}_\nu - u_\nu B^\lambda{}_\mu) \\
& + \epsilon_{\mu\nu\lambda\rho} \epsilon_{\alpha\beta\eta\sigma} u^\rho u^\sigma C^{\lambda\eta}
\end{aligned}
$$

20 components

Coincide with 3d tensors in the fluid rest frame ("**Petrov expansion**"):

[A. Z. Petrov, 1950] **[L. D. Landau and E. M. Lifschits, The Classical Theory of Fields, Vol. 2, 1975]**

Inverse formulas:

$$A_{\mu\nu} = u^\alpha u^\beta R_{\alpha\mu\beta\nu}$$

Symmetric tensor

6 components

$$B_{\mu\nu} = \frac{1}{2} \epsilon_{\alpha\mu\eta\rho} u^\alpha u^\beta R_{\beta\nu}{}^{\eta\rho}$$

Nonsymmetric traceless pseudotensor dual to the Riemann tensor

8 components

$$C_{\mu\nu} = \frac{1}{4} \epsilon_{\alpha\mu\eta\rho} \epsilon_{\beta\nu\lambda\gamma} u^\alpha u^\beta R^{\eta\rho\lambda\gamma}$$

Double dual symmetric Riemann tensor

6 components

Decomposition of the tensors: gravity

- **Properties**

$$A_{\mu\nu} = A_{\nu\mu}\,, \quad C_{\mu\nu} = C_{\nu\mu}\,, \quad B^{\mu}{}_{\mu} = 0\,,$$
$$A_{\mu\nu}u^{\nu} = C_{\mu\nu}u^{\nu} = B_{\mu\nu}u^{\nu} = B_{\nu\mu}u^{\nu} = 0\,.$$

The gravitational field is external. For simplicity, we impose an **additional condition**:

(for example, the field around a black hole) $R_{\mu\nu} = 0$

- **Additional properties** appear, similar to

[L. D. Landau and E. M. Lifschits, The Classical Theory of Fields, Vol. 2, 1975]

$$A_{\mu\nu} = -C_{\mu\nu}\,, \quad A^{\mu}_{\mu} = 0\,, \quad B_{\mu\nu} = B_{\nu\mu}$$

There are **10 independent** components left

Derivatives

Using the condition of global equilibrium and relations for the gravitational field (the Bianchi identity, etc.), we obtain for the derivatives:

Luttinger relation

[J. M. Luttinger, Phys. Rev. 135, A1505–A1514 (1964)]

$$\nabla_\mu T = T^2 \alpha_\mu ,$$

$$\nabla_\mu u_\nu = T(\epsilon_{\mu\nu\alpha\beta} u^\alpha w^\beta + u_\mu \alpha_\nu) ,$$

$$\nabla_\mu w_\nu = T(-g_{\mu\nu}(w\alpha) + \alpha_\mu w_\nu) - T^{-1} B_{\nu\mu} ,$$

$$\nabla_\mu \alpha_\nu = T\Big(w^2(g_{\mu\nu} - u_\mu u_\nu) - \alpha^2 u_\mu u_\nu - w_\mu w_\nu - u_\mu \eta_\nu - u_\nu \eta_\mu\Big) + T^{-1} A_{\mu\nu} ,$$

$$\nabla^\mu (A_{\mu\nu} w^\nu) = -3T B_{\mu\nu} w^\mu w^\nu - T^{-1} A_{\mu\nu} B^{\mu\nu} ,$$

$$\nabla^\mu (B_{\mu\nu} \alpha^\nu) = 3T A_{\mu\nu} w^\mu \alpha^\nu + T^{-1} A_{\mu\nu} B^{\mu\nu} - T B_{\mu\nu} w^\mu w^\nu - T B_{\mu\nu} \alpha^\mu \alpha^\nu$$

In **blue** – known terms:

[Shi-Zheng Yang, Jian-Hua Gao, and Zuo-Tang Liang, Symmetry 14, 948 (2022)]

In **red** – **new** terms.

Anomaly matching: system of equations

The divergence of the axial current transforms into the sum of **independent** terms:

$$
\begin{aligned}
\nabla_\mu j^\mu_{A(3)} = &\ (\alpha w) w^2 \left(-3T\xi_1 + T^2 \xi_1' + 2T\xi_3\right) \\
&+ (\alpha w)\alpha^2 \left(-3T\xi_2 + T^2\xi_2' - T\xi_3 + T^2\xi_3'\right) \\
&+ A_{\mu\nu}\alpha^\mu w^\nu \left(T^2\xi_4' + 3T\xi_5 + 2T^{-1}\xi_2 + T^{-1}\xi_3\right) \\
&+ B_{\mu\nu} w^\mu w^\nu \left(-2T^{-1}\xi_1 - 3T\xi_4 - T\xi_5\right) \\
&+ B_{\mu\nu}\alpha^\mu \alpha^\nu \left(T^2\xi_5' - T\xi_5 - T^{-1}\xi_3\right) \\
&+ A_{\mu\nu} B^{\mu\nu} \left(-T^{-1}\xi_4 + T^{-1}\xi_5\right) \\
= &\ 32\mathcal{N} A_{\mu\nu} B^{\mu\nu} \,.
\end{aligned}
$$

Principle:

In this case, the divergence is equal to the anomaly: additional terms - *macroscopic* - cannot violate the equation from the fundamental *microscopic* theory.

The coefficient in front of each pseudocalar must be equal to zero - a **system of equations** for the unknown coefficients $\xi_n(T)$.

Anomaly matching: solution

Since the theory does not include **dimensional** parameters other than temperature:

$$\xi_1 = T^3 \lambda_1 \quad \xi_2 = T^3 \lambda_2 \quad \xi_3 = T^3 \lambda_3 \quad \xi_4 = T \lambda_4 \quad \xi_5 = T \lambda_5$$

The **current** will be:

$$j_\mu^{A(3)} = \lambda_1 \omega^2 \omega_\mu + \lambda_2 a^2 \omega_\mu + \lambda_3 (a\omega) \omega_\mu + \lambda_4 A_{\mu\nu} \omega^\nu + \lambda_5 B_{\mu\nu} a^\nu$$

The solution looks like:

$$\frac{\lambda_1 - \lambda_2}{32} = \mathcal{N}$$

$$\lambda_4 = 8\mathcal{N} + \frac{\lambda_1}{2}$$

$$\lambda_5 = -24\mathcal{N} + \frac{\lambda_1}{2}$$

$$\lambda_3 = 0$$

was also shown in

[G. Y. Prokhorov, O. V. Teryaev, V. I. Zakharov, JHEP 02 (2019)]

Discussion

- **Arbitrary** massless fields with **arbitrary** spin were considered:
- Only **conservation relation** for the current was used.

General exact result

- Although the effect is associated with gravitational anomaly, it exists in **flat space-time** (the **Cheshire cat grin**).

- In contrast to **CVE** and the **gauge** anomaly case, for **KVE** the factor from the **gravitational** anomaly is split into **two** conductivities:

$$\lambda_1 - \lambda_2 = 32\mathscr{N}$$

- Conservation laws lead to the interplay of **infrared** (e.g. vortical current) and **ultraviolet** (quantum anomaly) effects.

Rarita-Schwinger-Adler model of spin 3/2

- The interaction **shifts the pole** in the Dirac bracket!

$$[\Psi_i(\vec{x}), \Psi_j^\dagger(\vec{y})]_D = -\,i\left[(\delta_{ij} - \frac{1}{2}\sigma_i\sigma_j)\delta^3(\vec{x}-\vec{y}) - \overrightarrow{D}_{\vec{x}\,i}\frac{\delta^3(\vec{x}-\vec{y})}{m^2 + g\vec{\sigma}\cdot\vec{B}(\vec{x})}\overleftarrow{D}_{\vec{y}\,j}\right]$$

Contribution of interaction with an additional field

- The **stress-energy tensor** can be obtained by varying with respect to the metric

$$T^{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta S}{\delta g_{\mu\nu}}$$

$$T^{\mu\nu} = \frac{1}{2}\varepsilon^{\lambda\nu\beta\rho}\bar{\psi}_\lambda\gamma_5\gamma^\mu\partial_\beta\psi_\rho + \frac{1}{8}\partial_\eta\left(\varepsilon^{\lambda\alpha\nu\rho}\bar{\psi}_\lambda\gamma_5\gamma_\alpha[\gamma^\eta,\gamma^\mu]\psi_\rho\right) + \frac{i}{4}\left(\bar{\lambda}\gamma^\nu\partial^\mu\lambda - \partial^\mu\bar{\lambda}\gamma^\nu\lambda\right) + \frac{i}{2}m\left(\bar{\psi}^\mu\gamma^\nu\lambda - \bar{\lambda}\gamma^\nu\psi^\mu\right) + (\mu\leftrightarrow\nu)\,.$$

Traceless unlike the usual Rarita-Schwinger field $T^\mu_\mu = 0$

- The currents can be obtained from Noether's theorem. The **axial current** can be constructed for the $U(1)_A$ transformation:

$$j^\mu_A = -i\varepsilon^{\lambda\rho\nu\mu}\bar{\psi}_\lambda\gamma_\nu\psi_\rho + \bar{\lambda}\gamma_\mu\gamma_5\lambda$$

Gravitational chiral anomaly in RSA theory

[G. Y. Prokhorov, O. V. Teryaev, V. I. Zakharov, Phys. Rev. D 106 (2) (2022) 025022]

Summing 9 correlates (contributions of different), we will obtain:

$$\langle T\hat{T}^{\mu\nu}(x)\hat{T}^{\sigma\rho}(y)\hat{j}^{\omega}_{A}(z)\rangle_c = -19\Big(4\pi^6(x-y)^5 \times (x-z)^3(y-z)^3\Big)^{-1} e_{\vartheta}\Big(\eta^{\nu\rho}\varepsilon^{\sigma\vartheta\mu\omega}+\eta^{\mu\rho}\varepsilon^{\sigma\vartheta\nu\omega} +\eta^{\nu\sigma}\varepsilon^{\vartheta\mu\rho\omega}+\eta^{\mu\sigma}\varepsilon^{\vartheta\nu\rho\omega}-6e^2(e^{\nu}e^{\rho}\varepsilon^{\sigma\vartheta\mu\omega} +e^{\mu}e^{\rho}\varepsilon^{\sigma\vartheta\nu\omega}+e^{\sigma}e^{\nu}\varepsilon^{\vartheta\mu\rho\omega}+e^{\sigma}e^{\mu}\varepsilon^{\vartheta\nu\rho\omega})\Big)$$

$\Leftrightarrow$

Matches the form we want!

$$\langle T\hat{T}^{\mu\nu}(x)\hat{T}^{\sigma\rho}(y)\hat{j}^{\omega}_{A}(z)\rangle = \mathscr{A}\Big(4(x-y)^5 \times (x-z)^3(y-z)^3\Big)^{-1} e_{\vartheta}\Big(\eta^{\nu\rho}\varepsilon^{\sigma\vartheta\mu\omega}+\eta^{\mu\rho}\varepsilon^{\sigma\vartheta\nu\omega} +\eta^{\nu\sigma}\varepsilon^{\vartheta\mu\rho\omega}+\eta^{\mu\sigma}\varepsilon^{\vartheta\nu\rho\omega}-6e^2(e^{\nu}e^{\rho}\varepsilon^{\sigma\vartheta\mu\omega} +e^{\mu}e^{\rho}\varepsilon^{\sigma\vartheta\nu\omega}+e^{\sigma}e^{\nu}\varepsilon^{\vartheta\mu\rho\omega}+e^{\sigma}e^{\mu}\varepsilon^{\vartheta\nu\rho\omega})\Big)$$

We can determine the factor in the anomaly:

$$\mathscr{A}_{RSA} = -19\mathscr{A}_{s=1/2} = -\frac{19}{\pi^6}$$

$$\langle\nabla_{\mu}\hat{j}^{\mu}_{A}\rangle_{RSA} = \frac{-19}{384\pi^2\sqrt{-g}}\varepsilon^{\mu\nu\rho\sigma}R_{\mu\nu\kappa\lambda}R_{\rho\sigma}{}^{\kappa\lambda}$$

–19 times different from the anomaly for spin ½

KVE IN RSA THEORY: CALCULATION

Substituting the **split** form of the operators into the typical correlator C, we obtain:

$$C^{\alpha_1\alpha_2|\alpha_3\alpha_4|\alpha_5\alpha_6|\lambda|ijk} = -T^3\int[d\tau]d^3x d^3y d^3z\, x^i y^j z^k \lim_{X,Y,Z,F} \mathcal{D}^{\alpha_1\alpha_2}_{a_1a_2(I_1I_2)}(\tilde{\partial}_{X_1},\tilde{\partial}_{X_2})\mathcal{D}^{\alpha_3\alpha_4}_{a_3a_4(I_3I_4)}(\tilde{\partial}_{Y_1},\tilde{\partial}_{Y_2})\mathcal{D}^{\alpha_5\alpha_6}_{a_5a_6(I_5I_6)}(\tilde{\partial}_{Z_1},\tilde{\partial}_{Z_2})$$
$$\times\mathcal{J}^{\lambda}_{a_7a_8(I_7I_8)}\langle T_\tau\overline{\Psi}_{a_1I_1}(X_1)\Psi_{a_2I_2}(X_2)\overline{\Psi}_{a_3I_3}(Y_1)\Psi_{a_4I_4}(Y_2)\overline{\Psi}_{a_5I_5}(Z_1)\Psi_{a_6I_6}(Z_2)\overline{\Psi}_{a_7I_7}(F_1)\Psi_{a_8I_8}(F_2)\rangle_{T,c}$$

We transform the average of 8 fields using the **Wick theorem**

$$\langle\overline{\Psi}(X)\Psi(X)\overline{\Psi}(Y)\Psi(Y)\overline{\Psi}(Z)\Psi(Z)\overline{\Psi}(F)\Psi(F)\rangle_c = -\langle\Psi(Y)\overline{\Psi}(X)\rangle\langle\Psi(X)\overline{\Psi}(F)\rangle\langle\Psi(Z)\overline{\Psi}(Y)\rangle\langle\Psi(F)\overline{\Psi}(Z)\rangle + (5\text{ terms})$$

- Initially there are 6×5^8 terms, but we should take into account, that:

1) some terms are zero because they include propagator $\langle\lambda\bar{\lambda}\rangle = 0$
2) some vertices are zero, e.g. $\mathcal{D}^{\mu\nu}_{(ij)} = 0$ if $i = j \neq 4$
3) some terms are zero in the limit $m\to\infty$ if the total number of propagators $\langle\lambda\bar{\psi}\rangle$ and $\langle\psi\bar{\lambda}\rangle$ is greater than the total number of vertices $\mathcal{D}_{(4i)}$ and $\mathcal{D}_{(i4)}$

Then there are:

94752 terms for each C-correlator in λ_1

31152 terms for each C-correlator in λ_2

Generalization to (anti)de Sitter space

Previously, we considered Ricci-flat background $R_{\mu\nu} = 0$

Let us generalize to the case with constant scalar curvature (A)dS $R_{\mu\nu} = \Lambda g_{\mu\nu}$

Using the same method (gradient expansion and conservation relations), we obtain

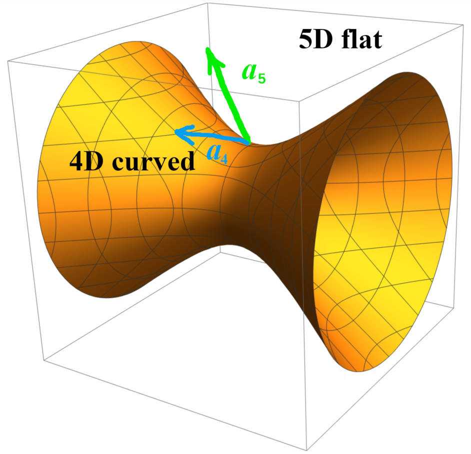
$$\langle \hat{T}^{\mu\nu} \rangle_{s=1/2} = \left[\frac{7\pi^2}{180} T^4 + \frac{1}{72}\left(|a|^2 + \frac{R}{12}\right) T^2 - \frac{17}{2880\pi^2}\left(|a|^2 + \frac{R}{12}\right)^2 \right] \left(4u^\mu u^\nu - g^{\mu\nu}\right) + \frac{11}{960\pi^2}\left(\frac{R}{12}\right)^2 g^{\mu\nu}$$

for spin 1/2. The general case: **[R.V. Khakimov, G.Yu. Prokhorov, O.V. Teryaev, V.I. Zakharov, 2023, 2308.08647]**.

At temperature T_{UR}, the stress-energy tensor has a vacuum form:

$$\langle \hat{T}^{\mu\nu} \rangle (T = T_{UR}) = \frac{k}{4} R^2 g^{\mu\nu}$$

$$T_{UR} = \frac{a_5}{2\pi} = \sqrt{T_U^2 + T_R^2} = \frac{\sqrt{|a|^2 + R/12}}{2\pi}$$



The temperature measured by an accelerated observer in (A)dS space is determined by the 5-dimensional acceleration! **[S. Deser and O. Levin, Phys. Rev. D, 59:064004, 1999]**

Duality relations are obtained that between the accelerated fluid to the effects of constant curvature.

Rarita-Schwinger-Adler model of spin 3/2

The **Rarita-Schwinger theory** – well-known theory of spin 3/2.
But this theory has a number of **pathologies.**

Generalized Hamiltonian dynamics: **Dirac bracket** instead of **Poisson bracket**

$$[F(\vec{x}), G(\vec{y})]_D = [F(\vec{x}), G(\vec{y})] - \int d^3w d^3z [F(\vec{x}), \chi^\dagger(\vec{w})] M^{-1}(\vec{w}, \vec{z}) [\chi(\vec{z}), G(\vec{y})]$$

$$M(\vec{x}, \vec{y}) = [\chi(\vec{x}), \chi^\dagger(\vec{y})]$$

- There is **singularity** in a Dirac bracket in weak gauge field limit for RS-theory!

Doesn't allow to construct perturbation theory!

Solved in **[Stephen L. Adler. Phys. Rev. D, 97(4):045014, 2018]** by introducing of interaction with additional spin ½ field:

$$S = \int d^4x \left(-\varepsilon^{\lambda\rho\mu\nu} \bar{\psi}_\lambda \gamma_5 \gamma_\mu \partial_\nu \psi_\rho + i\bar{\lambda}\gamma^\mu \partial_\mu \lambda - im\bar{\lambda}\gamma^\mu \psi_\mu + im\bar{\psi}_\mu \gamma^\mu \lambda \right)$$