Large N_f and $\{\beta\}$-decomposed representations for the Adler function in QCD

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Plan

- Large N_f expansion and leading renormalon chains naive nonabelianization $O(1/N_f^k)$ effects ; generalization of BLM and study of high order PT QCD effects
- $\{\beta\}$ decomposed represntations of coefficients generalization of BLM (PMC); diagrammatic and non-diagrammatic realizations
- relation of large N_f and β-expansion and ambiguities (model dependence)
- Comments on the PMC disfavouring by the phenomenological e^+e^- D-function "data" and (not yet checked) Bjorken poalized sum rule preliminary study
- Comment on analogy with Adler (1972) clarification on status of Finite QED Program Johnson, Baker , Willey et al (63 up to 70s)

The basic definitions

$$D(L, a_s) = -\frac{d\Pi(L, a_s)}{d\ln Q^2} = Q^2 \int\limits_0^\infty ds \frac{R^{th}_{e^+e^-}(l, a_s)}{(s+Q^2)^2},$$

$$R^{th}_{e^+e^-}(l, a_s) = \sigma_{tot}(l, a_s)/\sigma_0(e^+e^- \to \mu^+\mu^-)$$

$$\left(\frac{\partial}{\partial \ln \mu^2} + \beta(a_s)\frac{\partial}{\partial a_s}\right) D(L, a_s) = 0,$$

$$\frac{\partial a_s}{\partial \ln \mu^2} = \beta(a_s) = -\sum_{n\geq 0} \beta_n a_s^{n+2}.$$

$$D\left(a_s(Q^2)\right) = \left(\sum_i q_i^2\right) D^{ns}\left(a_s(Q^2)\right) + \left(\sum_i q_i\right)^2 D^{si}\left(a_s(Q^2)\right)$$

The a_s^4 -result Baikov,Chetyrkin, Kuhn (2010)=BChK group ;

The $\overline{MS}$-scheme large N_f

In the $\overline{MS}$-scheme the expansions read:

$$D^{ns}(a_s) = 1 + d_{10}a_s + (d_{20} + d_{21}N_f)a_s^2 + (d_{30} + d_{31}N_f + d_{32}N_f^2)a_s^3$$
$$+(d_{40} + d_{41}N_f + d_{42}N_f^2 + d_{43}N_f^3)a_s^4$$

Grunberg,Kataev (91); Grunberg (92); Kataev (92); Beneke, Braun (95) ; Brodsky,Wu (2012) d_{n0}- scale-invariant contribution $d_{10} = +1$ Grunberg, K generalized BLM machinery $d_{20} = \frac{1}{12} \approx 0.085$; $d_{30} \approx = -23.227$; $d_{40} = +82.344$ (sign ! ; order of magnitude !)l; In agreement with β-expanded model (see next page) and R_δ Brodsky, Wu et al (12) $a(\mu^2) = a(\mu_\delta) + \sum_{n\geq 1} \frac{1}{n} \frac{d^n a_(\mu_\delta^2)}{dln(\mu_\delta^2)}(-\delta)^n$ (Goriachuk, K, Molokoedov (22)) Renormalon chain $\overline{MS}$ QED result Broadhurst (92) ; QCD Broadhurst,K(93) d_{nk} at $k = n - 1$ assymptotic QCD series study by e.g. Laenen et al (23); Ayala, Cvetic (23, 24); Caprini (24)

The $\{\beta\}$-expansion PT approach for the RG-invariant quantities Mikhailov (04-07) up to now

Consider the PT expansion

$$D^{ns}(a_s) = 1 + d_1 a_s + d_2 a_s^2 + d_3 a_s^3 + d_4 a_s^4 + O(a_s^5)$$

In the MS-like schemes β-expansion prescription is:

$$d_1 = d_1[0]$$
$$d_2 = \beta_0 d_2[1] + \mathbf{d_2[0]} - \textbf{ the Basis of BLM procedure}$$
$$d_3 = \beta_0^2 d_3[2] + \beta_1 d_3[0,1] + \beta_0 d_3[1] + \mathbf{d_3[0]},$$
$$d_4 = \beta_0^3 d_4[3] + \beta_2 d_4[0,0,1] + \beta_1\beta_0 d_4[1,1] + \beta_0^2 d_4[2] + \beta_1 d_4[0,1]$$
$$+\beta_0 d_4[1] + \mathbf{d_4[0]}$$

$\{\beta\}$-expansions suggested by Mikhailov (Quarks2004, JHEP(07)) Further on Bakulev,Mikhailov, Stefanis(10) ; Kataev, Mikhalov M(12-16); Kataev,Molokoedov (23) ; Cvetic,Kataev(16); Brodsky,Wu, Mojaza et al(12-23)

The procedures of finding terms of the $\{\beta\}$-expansion; diagrammatic; Mikhailov (04-07; up to now) but may have "theory" ambiguity

The problem appears at the N^2LO QCD:

$$d_3 = d_{32}n_f^2 + d_{31}n_f + d_{30} \to \beta_0^2 \; d_3[2] + \beta_1 d_3[0,1] + \beta_0 d_3[1,0] + d_3[0],$$

where $\beta_0 = \beta_{00} + \beta_{10}n_f$, $\beta_1 = \beta_{10} + \beta_{11}n_f$. How to get from single n_f- term two terms $\beta_1 d_3[0,1] + \beta_0 d_3[1]$. Mikhailov(07): Apply **additional degree of freedom**, i.e. $n_{\tilde{g}}$ flavour number of multiplet of *MSSM gluino*.
In this case $\beta_0 = \beta_0(n_f, n_{\tilde{g}})$, $\beta_1 = \beta_0(n_f, n_{\tilde{g}})$ are known analytically (Clavelli,Surguladze(97) and $d_3(n_f, n_{\tilde{g}})$; eQCD D-function evaluated analytically by Chetyrkin(97); Chetyrkin (2023) ; Zoller (2016)- β-function. The procedure has unique solutions (Mikhailov(07)): Model dependence may exist Kataev, Molokoedov (23-24), Miklhailov (24-in progress) Bednyakov (2015 and 2024 now in progress)) $d_{20} = \frac{1}{12} \approx 0.085$; $d_{30} \approx= -35.87(\ notmodel - independent)$; $d_{40} \approx -98$ (sign ! ; order of magnitude ! DIFFER BY SIGN FROM model built)l;

Non-diagrammatic representations not only for the D^{ns} in not only QCD

Whether expansion in powers of conformal anomaly $\beta(a_s)/a_s$, where $\beta(a_s) = -\sum_{j\geq 0}\beta_j a_s^{j+2}$ is valid for the D^{ns}? Cvetic, Kataev (16); K,Mikhailov (09-12) motivated

$$D^{ns}(a_s) = 1 + \sum_{n\geq 0}\left(\frac{\beta(a_s)}{a_s}\right)^n D_n(a_s)$$

$$D_n(a_s) = \sum_{r=1}^{r-n} a_s^r \sum_{k=1}^{r} D_n^{(r)}[k, r-k] C_F^k C_A^{r-k} + a_s^4 \delta_{n0} \times$$
$$\left(D_0^{(4)}[F,A]\frac{d_F^{abcd} d_A^{abcd}}{d_R} + D_0^{(4)}[F,F]\frac{d_F^{abcd} d_F^{abcd}}{d_R} n_f \right)$$

Why not to subdivide this $a_s^4 n_f$-dependent term as

$$\delta_{n0} D_0^{(4)}[F,F] n_f = \left(\delta_{n0}\frac{11C_A}{4T_F} D_0^{(4)}[F,F] + \delta_{n1}\frac{3}{T_F} D_1^{(4)}[F,F] \right)$$

with $D_0^{(4)}[F,F] = D_1^{(4)}[F,F]$? This contradicts QED limit- there is no such δ_{n1} contribution from light-by-light-type subgraph.

The $\{\beta\}$ expanded QCD terms for D^{ns} in $SU(N_c)$ non-diagrammatic and diagarammatic (!) differences

Using the $\overline{MS}$-scheme factorized representation, Cvetic,Kataev(16). The results differ in part from obtained in QCD+gluino theory (Mikhailov (07))

$$d_1[0] = \frac{3}{4}C_F \; d_2[0] = \left(-\frac{3}{32}C_F^2 + \frac{1}{16}C_F C_A\right) \; d_2[1] = \left(\frac{33}{8} - 3\zeta_3\right)C_F$$

$$d_3[0] = -\frac{69}{128}C_F^3 - \left(\frac{101}{256} - \frac{33}{16}\zeta_3\right)C_F^2 C_A \neq +\mathbf{\frac{71}{64}C_F^2 C_A}$$

$$-\left(\frac{53}{192} + \frac{33}{16}\zeta_3\right)C_F C_A^2 \neq +\left(\mathbf{\frac{523}{768}} - \mathbf{\frac{27}{8}}\boldsymbol{\zeta_3}\right)\mathbf{C_F C_A^2}$$

$$d_3[1] = \left(-\frac{111}{64} - 12\zeta_3 + 15\zeta_5\right)C_F^2 \neq \left(-\mathbf{\frac{27}{8}} - \mathbf{\frac{39}{4}}\boldsymbol{\zeta_3} + \underline{\mathbf{15}\boldsymbol{\zeta_5}}\right)\mathbf{C_F^2}$$

$$+\left(\frac{83}{32} + \frac{5}{4}\zeta_3 - \frac{5}{2}\zeta_5\right)C_F C_A \neq \left(-\mathbf{\frac{9}{64}} + \mathbf{5}\boldsymbol{\zeta_5} - \underline{\mathbf{\frac{5}{2}}\boldsymbol{\zeta_5}}\right)\mathbf{C_F C_A}$$

$$d_3[0,1] = \left(\frac{33}{8} - 3\zeta_3\right)C_F \neq \left(\mathbf{\frac{101}{16}} - \mathbf{6}\boldsymbol{\zeta_3}\right)\mathbf{C_F} d_3[2] = \left(\frac{151}{6} - 19\zeta_3\right)C_F$$

The underlined contributions are the same- they are

The $\{\beta\}$ expansion QCD expression for d_4 was also obtained

We present model dependent one from Cvetic, Kataev (2016)

$$d_4[0] = \Bigg[\left(\frac{4157}{2048} + \frac{3}{8}\zeta_3\right)C_F^4 - \left(\frac{3509}{1536} + \frac{73}{128}\zeta_3 + \frac{165}{32}\zeta_5\right)C_F^3 C_A$$
$$+\left(\frac{9181}{4608} + \frac{299}{128}\zeta_3 + \frac{165}{64}\zeta_5\right)C_F^2 C_A^2 - \left(\frac{30863}{36864} + \frac{147}{128}\zeta_3 - \frac{165}{64}\zeta_5\right)C_F C_A^3$$
$$+\left(\frac{3}{16} - \frac{1}{4}\zeta_3 - \frac{5}{4}\zeta_5\right)\frac{d_F^{abcd}d_A^{abcd}}{d_R} + \left(-\frac{13}{16} - \zeta_3 + \frac{5}{2}\zeta_5\right)\frac{d_F^{abcd}d_F^{abcd}}{d_R}n_f$$

Where the difference from diagrammatic related expression is ?
In **three** structures $C_F^3 C_A$, $C_F^2 C_A^2$ and $C_F C_A^3$(*main by sign*!)
(based on discussions by AK with Molokoedov (@23) and Mikhailov (@23-24)

PMC vs massless $\overline{MS}$: Adler function at n_f=3 Kataev-Molokoedov PRD (23)

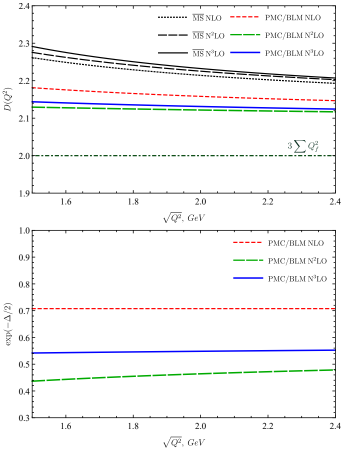
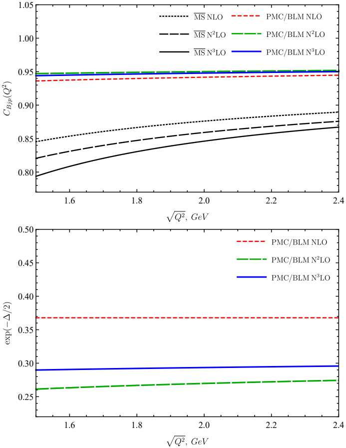


Figure: (1a) The dependence of the PT Adler function $D(Q^2)$ on $\sqrt{Q^2}$ at $n_f = 3$ in the massless limit. (1b) The dependence of the factor $\exp(-\Delta/2)$ on $\sqrt{Q^2}$ at $n_f = 3$.

PMC vs massless $\overline{MS}$: Bjorken polarized SR at n_f=3 (preliminary) $S_{Bjp}(Q^2) = \frac{1}{6}(g_A/g_V)C_{Bjp}(Q^2)$ by Kataev-Molokoedov or vise versa (demonstrated @ 2024)



Effects of conformal symmetry violation by both PT and non-PT effects ARE NOT SEEN in PMC but ARE SEEN in NATURE (!) PMC essentail problem (not explained by HO PT masless effects)

Conclusions

- Analogy with Finite QED Program treatment by Adler. Analogy with trouble of PMC (to be checked and considered with care)
- Is it possible to understand better the existing essential model difference in coefficients of β-expanded terms of PT series ? May give the hint to clarifying effects of subleading renormalon chains
- Leading renormalon chains desribe nicely effects ofv growth of PT coefficvients of Eucledian PT series
- Claim of α_s CERN Working group gided with participation of Michelangelo Mangano (2024). We should take into account in α_s extraction "scale systematics" or "missing higher order systematics" or ... (up to possible todays discussions)
- Questions, Comments are Wellcomed