Color transparency in hard pd collisions

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Outline:

- Introduction: phenomenon of color transparency (CT)

- The process d(p,2p)n at large momentum transfer: generalized eikonal approximation (GEA), quantum diffusion model of CT, separation of the hard (quark counting) and soft (Landshoff) amplitudes

- Nuclear transparency, tensor analyzing power

- Summary and outlook

Based on PRC 107, 014605 (2023) [arXiv:2208.08832]

Related talks by ***Nikolay Pivnyuk***
and ***Stepan Shimansky***
on Tuesday afternoon
nuclear physics session

Scientific session of the nuclear physics section
of the Department of Physical Sciences
of the Russian Academy of Sciences,
BLTP, JINR, Dubna, April 1-5, 2024

Introduction

Hard process : $Q^2 \gg 1\ \mathrm{GeV}^2$

- Quark-gluon d.o.f.
- Point-like $q\bar{q}$ and qqq configurations (PLCs): $r_\perp \sim 1/Q$

Color dipole – proton cross section in the pQCD limit $(r_\perp \to 0)$ $\sigma_{q\bar{q}} \propto r_\perp^2 \sim 1/Q^2$

L. Frankfurt, G.A. Miller, M. Strikman, PLB 304, 1 (1993)

Color transparency (CT): a quark configuration in the final or initial state of a high momentum transfer exclusive process interacts with nucleons with reduced cross section.

For review of CT see

L. Frankfurt, G.A. Miller, M. Strikman, Annu. Rev. Nucl. Part. Sci. 44, 501 (1994);
P. Jain, B. Pire, J.P. Ralston, Phys. Rept. 271, 67 (1996);
D. Dutta, K. Hafidi, M. Strikman, Prog. Part. Nucl. Phys. 69, 1 (2013)

CT has been predicted for the binary semi-exclusive processes with large momentum transfer

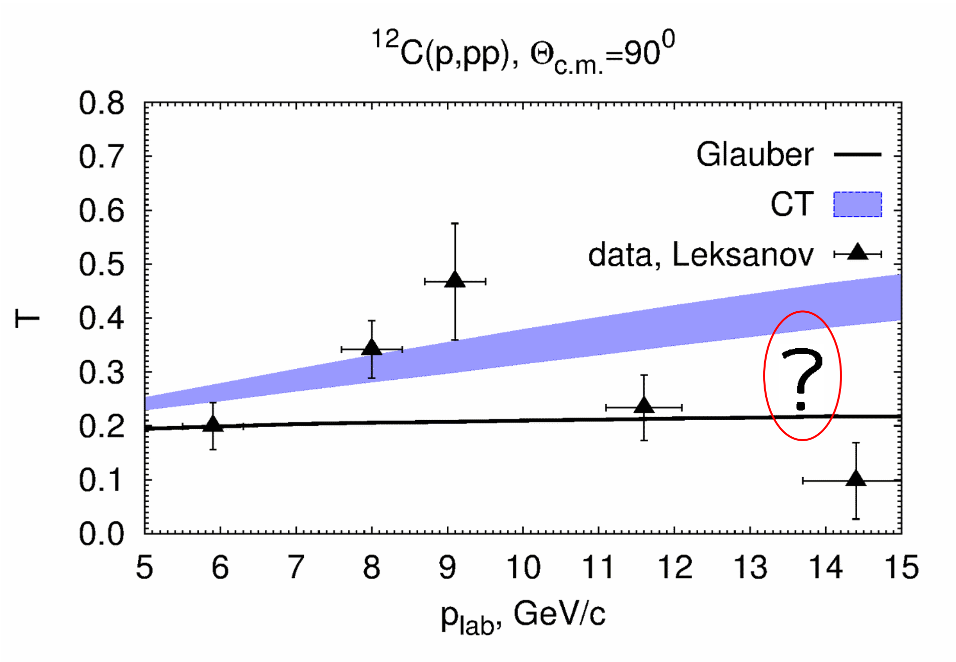
$$h + A \to h + p + (A-1)^*$$

S.J. Brodsky, 1982; A.H. Mueller, 1982

Nuclear transparency:

$$T = \frac{\sigma}{\sigma_{\rm IA}}\ , \quad \sigma_{\rm IA} \simeq Z\sigma_p$$

Neglecting Fermi motion



Data: EVA@AGS,
A. Leksanov et al.,
PRL 87, 212301 (2001).

Decrease of T at high p_{lab} is not understood:

- could be due to stronger absorption of the large-size quark configurations produced by Landshoff mechanism, J.P. Ralston, B. Pire, PRL 61, 1823 (1988);
- or due to intermediate (very broad, Γ~ 1 GeV) 6q$c\bar{c}$ resonance formation with mass ~ 5 GeV, S.J. Brodsky, G.F. de Teramond, PRL 60, 1924 (1988).

Deuteron target:

- ISI and FSI are small, however, the PLCs will likely not expand too much on the length scale < 1.5 fm (internucleon distances in the deuteron contributing to the rescattering amplitudes) for momenta above several GeV/c, i.e. they are likely to be frozen.

- Possibilities to study CT in several large-angle processes:

d(e,e'p)n – **V.V. Anisovich, L.G. Dakhno, M.M. Giannini, PRC 49, 3275 (1994);**
L.L. Frankfurt, W.R. Greenberg, G.A. Miller, M.M. Sargsian, M.I. Strikman, Z. Phys. A352, 97 (1995)

Proposal for JLab: **S. Li et al, MDPI Physics 4, 1426 (2022) [arXiv:2209.14400]**

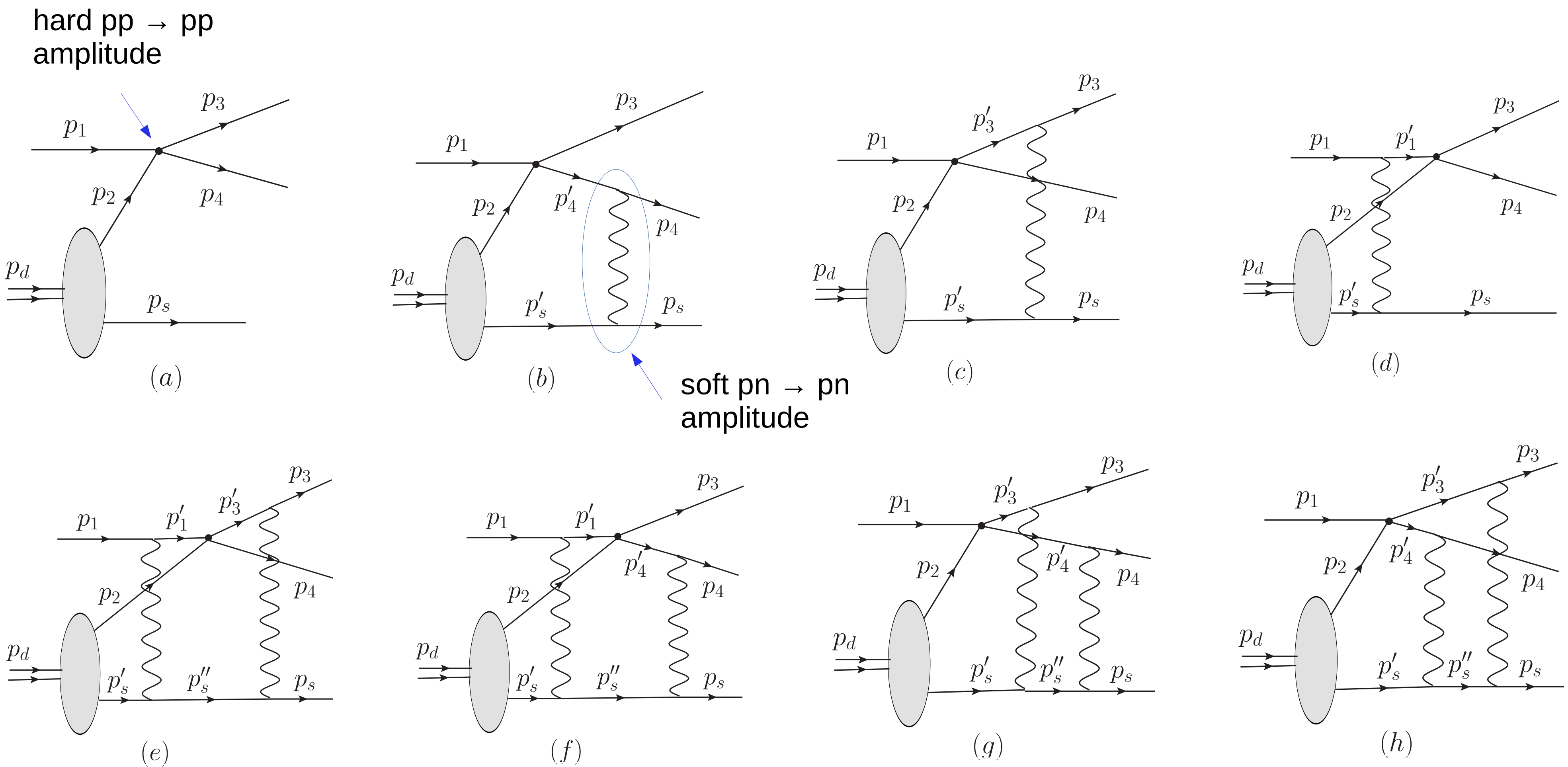
d(p,2p)n - **L.L. Frankfurt, E. Piasetzky, M.M. Sargsian, M.I. Strikman, PRC 56, 2752 (1997);**
AL PRC 107, 014605 (2023)

- Can be measured at NICA SPD and BM@N at JINR, FAIR, J-PARC, HIAF

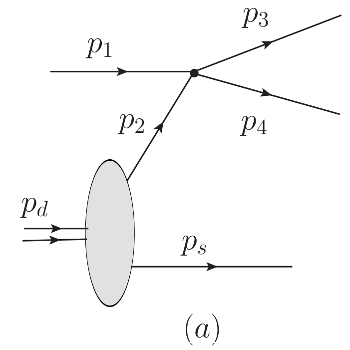
d($\bar{p}$,$\pi^-\pi^0$)p – **AL, M.I. Strikman, EPJA 56, 21 (2020)**

- Can be measured at PANDA@FAIR

Partial amplitudes:



Impulse approximation (IA) amplitude:



$$M^{(a)} = M_{\rm hard}(s_{\rm hard}, t_{\rm hard}) \frac{i\Gamma_{d\to pn}(p_d, p_s)}{(p_2)^2 - m^2 + i\epsilon} \ ,$$

$$s_{\rm hard} = (p_3 + p_4)^2 \ , \ t_{\rm hard} = (p_1 - p_3)^2 \ , \ u_{\rm hard} = (p_1 - p_4)^2$$

$$t_{\rm hard} \simeq u_{\rm hard} \simeq -s_{\rm hard}/2 \qquad \Theta_{c.m.} \simeq 90^\circ$$

Non-relativistic treatment of the deuteron wave function (DWF) in the deuteron rest frame for the on-shell spectator neutron:

$$\frac{i\Gamma_{d\to pn}(p_d, p_s)}{(p_2)^2 - m^2 + i\epsilon} = \left(\frac{2E_s m_d}{p_2^0}\right)^{1/2} (2\pi)^{3/2} \phi^{\lambda_d}(\boldsymbol{p}_2, \lambda_2, \lambda_s) \ , \quad \boldsymbol{p}_2 = -\boldsymbol{p}_s \ , \quad E_s \equiv (m^2 + \boldsymbol{p}_s^2)^{1/2} \ , \quad \lambda_d = 0, \pm 1$$

DWF:

$$\phi^{\lambda_d}(\boldsymbol{p}_2, \lambda_2, \lambda_s) = \frac{1}{\sqrt{4\pi}} \left[u(p_2) + \frac{w(p_2)}{\sqrt{8}} S(\boldsymbol{p_2}) \right] \chi^{\lambda_d} \ ,$$

$$\chi^{\pm 1} = \delta_{\pm 1/2, \lambda_p} \delta_{\pm 1/2, \lambda_n}$$

$$\chi^0 = \frac{1}{\sqrt{2}} (\delta_{1/2,\lambda_p} \delta_{-1/2,\lambda_n} + \delta_{-1/2,\lambda_p} \delta_{1/2,\lambda_n})$$

$$S(\boldsymbol{p}) = \frac{3(\boldsymbol{\sigma}_{\lambda_2\lambda_p} \boldsymbol{p})(\boldsymbol{\sigma}_{\lambda_s\lambda_n} \boldsymbol{p})}{p^2} - \boldsymbol{\sigma}_{\lambda_2\lambda_p} \boldsymbol{\sigma}_{\lambda_s\lambda_n}$$

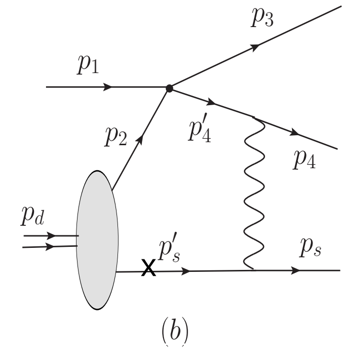
- spin tensor operator

$$\int d^3p \sum_{\lambda_2, \lambda_s} |\phi^{\lambda_d}(\boldsymbol{p}, \lambda_2, \lambda_s)|^2 = 1 \ .$$

$$M^{(a)} \simeq 2m^{1/2} M_{\rm hard}(s_{\rm hard}, t_{\rm hard}) (2\pi)^{3/2} \phi(-\boldsymbol{p}_s) = 2m^{1/2} M_{\rm hard}(s_{\rm hard}, t_{\rm hard}) \int d^3r \, {\rm e}^{i\boldsymbol{p}_s \boldsymbol{r}} \phi(\boldsymbol{r}) \ , \quad \boldsymbol{r} = \boldsymbol{r}_2 - \boldsymbol{r}_s$$

nucleon mass

Amplitude with rescattering of an outgoing proton:



- momentum transfer in soft rescattering is small, $M_{\rm hard}$ can be factorized out of the four momentum integral

$$M^{(b)} = M_{\rm hard}(s_{\rm hard}, t_{\rm hard}) \int \frac{d^4 p'_s}{(2\pi)^4} \frac{\Gamma_{d\to pn}(p_d, p'_s) M_{\rm el}(p_4, p_s, p'_4)}{((p_2)^2 - m^2 + i\epsilon)((p'_4)^2 - m^2 + i\epsilon)((p'_s)^2 - m^2 + i\epsilon)}$$

- static neutron approximation: neglect the dependence of the soft rescattering amplitude $M_{\rm el}$ on the energy $p_s'^0$ of neutron

- perform integration over $p_s'^0$ by closing the contour in the lower part of $p_s'^0$ complex plane (pole approximation, $(p'_s)^2 = m^2$)

$$M^{(b)} = -\frac{M_{\rm hard}(s_{\rm hard}, t_{\rm hard})}{m^{1/2}} \int \frac{d^3 k}{(2\pi)^3} \frac{(2\pi)^{3/2} \phi(-\boldsymbol{p}'_s) M_{\rm el}(|\boldsymbol{p}_4|, t)}{(p'_4)^2 - m^2 + i\epsilon} \ ,$$

$$\boldsymbol{k} = \boldsymbol{p}_s - \boldsymbol{p}'_s, \quad \boldsymbol{k}_t = \boldsymbol{k} - (\boldsymbol{k}\boldsymbol{p}_4)\boldsymbol{p}_4/|\boldsymbol{p}_4|^2, \quad t = -k_t^2$$

- express the propagator of the fast proton in the eikonal form:

$$(p'_4)^2 - m^2 + i\epsilon = (p_4 + p_s - p'_s)^2 - m^2 + i\epsilon = 2p_4(p_s - p'_s) + (p_s - p'_s)^2 + i\epsilon = 2|\boldsymbol{p}_4|(p_s'^{\tilde z} - p_s^{\tilde z} + \Delta_4 + i\epsilon) \ , \ \boldsymbol{e}_{\tilde z} \upuparrows \boldsymbol{p}_4 \ ,$$

$$\Delta_4 \equiv \frac{E_4(E_s - E'_s)}{|\boldsymbol{p}_4|} + \frac{(p_s - p'_s)^2}{2|\boldsymbol{p}_4|} \simeq \frac{(E_4 - m)(E_s - m)}{|\boldsymbol{p}_4|} \ , \quad E_i \equiv \sqrt{m^2 + \boldsymbol{p}_i^2} \ .$$

neglect Fermi motion in the deuteron (GEA), Glauber limit: $\Delta_4 = 0$

$$\Rightarrow \quad M^{(b)} = \frac{M_{\rm hard}(s_{\rm hard}, t_{\rm hard})}{2|\boldsymbol{p}_4| m^{1/2}} \int d^3 r \Theta(-\tilde z) \phi(\boldsymbol{r}) \mathrm{e}^{i\boldsymbol{p}_s \boldsymbol{r} - i\Delta_4 \tilde z} \int \frac{d^2 k_t}{(2\pi)^2} \mathrm{e}^{-i\boldsymbol{k}_t \tilde{\boldsymbol{b}}} i M_{\rm el}(|\boldsymbol{p}_4|, t) \ ,$$

$$\boldsymbol{r} = \boldsymbol{r}_2 - \boldsymbol{r}_s \qquad \tilde z = \boldsymbol{r}\boldsymbol{p}_4/|\boldsymbol{p}_4| \qquad \tilde{\boldsymbol{b}} = \boldsymbol{r} - (\boldsymbol{r}\boldsymbol{p}_4)\boldsymbol{p}_4/|\boldsymbol{p}_4|^2$$

Hard pp → pp scattering amplitude   *J.P. Ralston, B. Pire, 1988*

$$M_{\rm hard} = M_{\rm QC} + M_{\rm L} = M_{\rm QC}(1+R(s))$$

quark counting component ~ s^{-4}
minimally connected graphs,
small-size configurations (PLCs)

Landshoff component – independent qq scattering,
disconnected graphs, Sudakov effects, **large-size configurations**

V.A. Matveev, R.M. Muradyan, A.N. Tavkhelidze, 1973;
S.J. Brodsky, G.R. Farrar, 1973

Only a part of rescattering amplitudes $\propto \mathrm{M_{QC}}$ ***is influenced by CT !***

$$R(s) = M_{\rm L}/M_{\rm QC} = \frac{\rho_1\sqrt{s}}{2} \mathrm{e}^{\pm i \underbrace{(\phi(s)+\delta_1)}_{\text{chromo-Coulomb phase shift}}}, \quad \rho_1 = 0.08\ \mathrm{GeV}^{-1}, \quad \delta_1 = -2$$

$$\phi(s) = \frac{\pi}{0.06}\log\left[\log\left(\frac{s}{\Lambda^2_{\rm QCD}}\right)\right], \quad \Lambda_{\rm QCD} = 0.1\ \mathrm{GeV}$$

Cross section parameterization
L. Frankfurt, E. Piasetsky,
M. Sargsian, M. Strikman,
1995

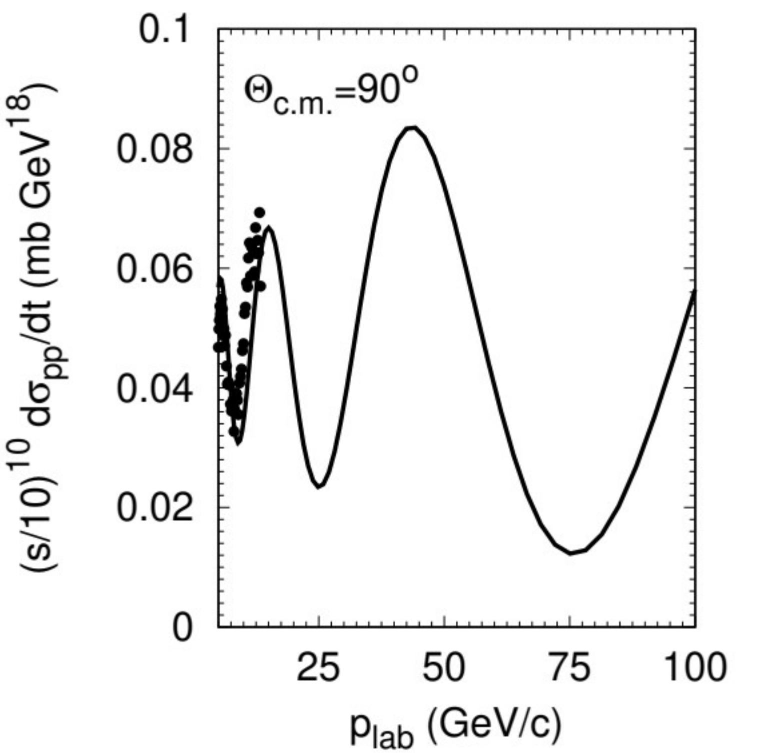
$$\frac{d\sigma_{pp}}{dt} = \frac{d\sigma^{\rm QC}_{pp}}{dt}|1+R(s)|^2 \underbrace{\cancel{F(s,\Theta_{c.m.})}}_{\approx 1 \text{ for } s > 15\ \mathrm{GeV}^2\ (p_{\rm lab} > 7\ \mathrm{GeV/c})},$$

$$\frac{d\sigma^{\rm QC}_{pp}}{dt} = 45\frac{\mu\mathrm{b}}{\mathrm{GeV}^2}\left(\frac{10\,\mathrm{GeV}^2}{s}\right)^{10}\left(\frac{4m^2-s}{2t}\right)^{4\gamma}$$

$$\gamma = 1.6$$

Assume spin-independent hard amplitude,
non-polarized proton beam:

$$M_{\rm hard} = \left(16\pi(s-4m^2)s\frac{d\sigma^{\rm QC}_{pp}}{dt}\right)^{1/2}[1+R(s)]\,\delta_{\lambda_1\lambda_3}\delta_{\lambda_2\lambda_4}$$



Data: *C.W. Akerlof et al., Phys. Rev. 159, 1138 (1967)*

Color transparency in the pn elastic scattering amplitude:

Quantum diffusion model of CT: ***G.R. Farrar, H. Liu, L.L. Frankfurt, M.I. Strikman, PRL 61, 686 (1988); L.L. Frankfurt, W.R. Greenberg, G.A. Miller, M.M. Sargsian, M.I. Strikman, ZPA 352, 97 (1995)***

Without CT (GEA): $$M_{\rm el}(|\boldsymbol{p}|,t) = 2|\boldsymbol{p}|m\sigma_{pn}^{\rm tot}(i+\rho_{\rm pn})\mathrm{e}^{B_{\rm pn}t/2}$$

$$\boldsymbol{r} = \boldsymbol{r}_2 - \boldsymbol{r}_s$$

With CT:

$$M_{\rm el}(|\boldsymbol{p}|,t,l) = 2|\boldsymbol{p}|m\,\sigma_{pn}^{\rm eff}(l)\,(i+\rho_{\rm pn})\mathrm{e}^{B_{\rm pn}t/2}\,\frac{G(t\cdot\frac{\sigma_{pn}^{\rm eff}(l)}{\sigma_{pn}^{\rm tot}})}{G(t)}\ ,\quad l = |\boldsymbol{r}\boldsymbol{p}|/|\boldsymbol{p}|$$

$$\sigma_{pn}^{\rm eff}(l) = \sigma_{pn}^{\rm tot}\left(\left[\frac{l}{l_c} + \frac{Q_0^2}{Q^2}\left(1-\frac{l}{l_c}\right)\right]\Theta(l_c-l) + \Theta(l-l_c)\right)\ ,\quad Q_0 \simeq 1\ \mathrm{GeV}$$

$$Q^2 = \min(-t_{\rm hard}, -u_{\rm hard})$$ - hard scale

$$l_c = \frac{1}{\sqrt{m_{res}^2+|\boldsymbol{p}|^2} - \sqrt{m^2+|\boldsymbol{p}|^2}} \simeq \frac{2|\boldsymbol{p}|}{m_{res}^2 - m^2} \equiv \frac{2|\boldsymbol{p}|}{\Delta M^2}$$ - coherence length

$|\boldsymbol{p}| \gg m_{res}, m$

$\Delta M^2 \simeq 1\ \mathrm{GeV}^2$ - from pion transparency studies at JLab

$\Delta M^2 \simeq 2-3\ \mathrm{GeV}^2$ - from recent JLab ^{12}C(e,e'p) data analysis, S. Li et al., MDPI Physics 4, 1426 (2022) [arXiv:2209.14400]

$$G(t) = \frac{1}{(1-t/0.71\ \mathrm{GeV}^2)^2}$$ - electric formfactor of the proton

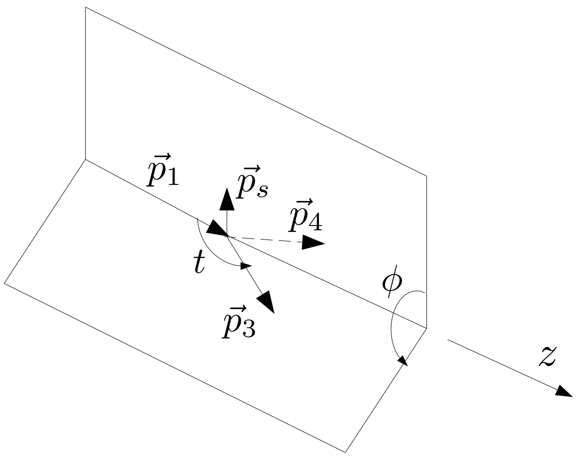
Kinematic variables

$\alpha_s = \frac{2(E_s - p_s^z)}{m_d}$ - the light cone variable ($\alpha_s/2$ = momentum fraction of the deuteron carried by the spectator neutron in the infinite momentum frame where the deuteron moves fast backward)

p_{st} - the transverse momentum of the spectator neutron

$\phi = \phi_3 - \phi_s$ - the relative azimuthal angle between the scattered proton and spectator neutron

$t = (p_1 - p_3)^2 \equiv t_{\rm hard}$ - Mandelstam variable



The deuteron rest frame

Default choice: $\alpha_s = 1$ - transverse kinematics which minimizes relativistic corrections to the DWF and maximizes ISI/FSI, see **L.L. Frankfurt et al, PRC 56, 2752 (1997)**

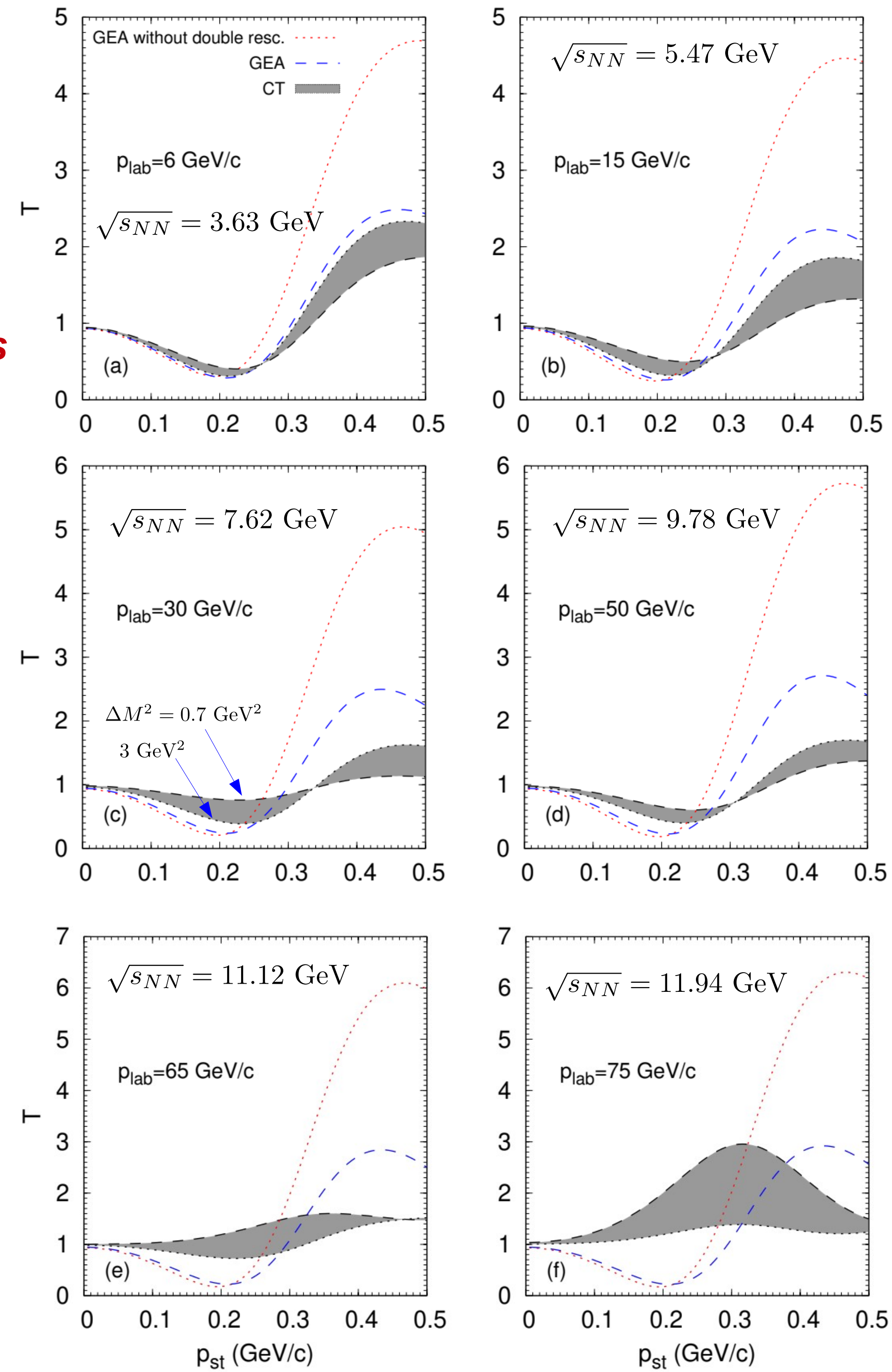
$t = (4m^2 - s)/2, \; s = (p_3 + p_4)^2 \equiv s_{\rm hard}$
- corresponds to $\Theta_{c.m.} = 90^\circ$

$\phi = 180^\circ$ - in-plane kinematics

Nuclear transparency vs transverse momentum of spectator neutron

$$T = \frac{\sigma}{\sigma_{\rm IA}} = \frac{\overline{|M^{(a)} + M^{(b)} + M^{(c)} + M^{(d)} + M^{(g)} + M^{(h)}|^2}}{\overline{|M^{(a)}|^2}}$$

- *absorptive ISI/FSI at small p_{st} due the interference between the IA and single-rescattering amplitudes*
- *enhancement at large p_{st} due to the single-rescattering amplitudes squared*
- *destructive interference of the single- and double-rescattering amplitudes, important at large p_{st}*
- *GEA-transparencies do not much depend on p_{lab} (parameters of soft NN scattering amplitude are rather weakly p_{lab}-dependent)*
- *CT-transparencies tend to unity (IA-limit) with increasing p_{lab} up to p_{lab}≈30 GeV/c and then start to deviate from unity again*
- *this “anomaly” is due to the fact that CT influences only the QC part of the amplitude and not the Landshoff part*

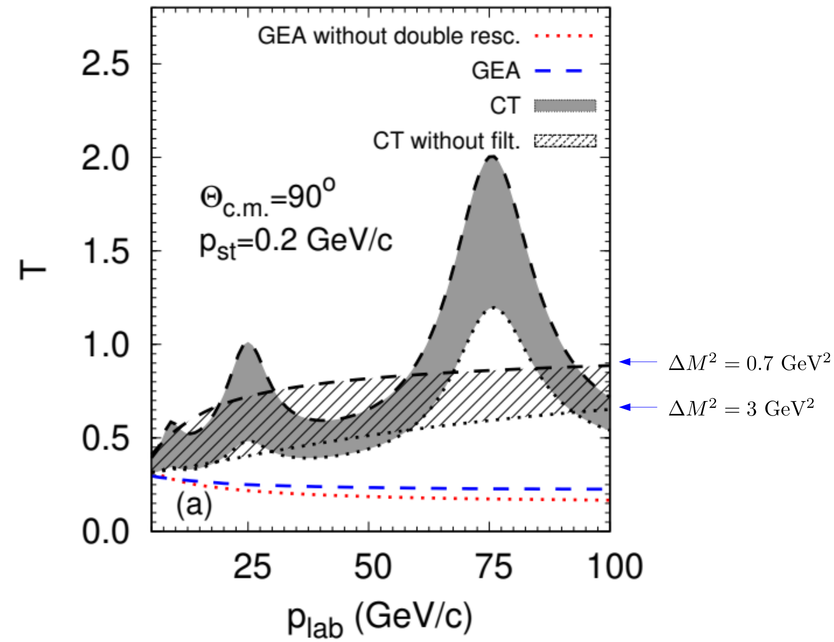


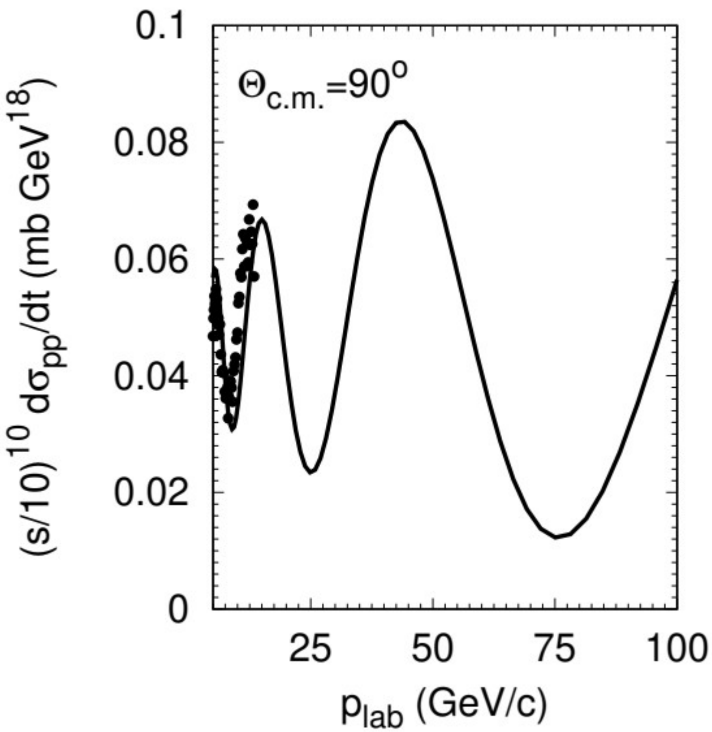
- out-of-phase oscillations relative to the elementary cross section due to σ_{IA} in the denominator

- very similar to the nuclear filtering of the Landshoff component for heavy nuclei
J.P. Ralston, B. Pire, PRL 61, 1823 (1988)

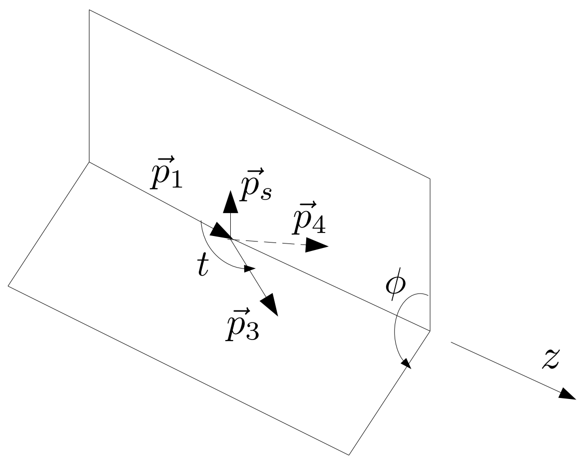
- "antiabsorptive" behavior (i.e. T > 1) at p_{lab}≈75 GeV/c due to the constructive interference of the IA amplitude and the Landshoff part of the single-rescattering amplitudes

- monotonic increase w/o nuclear filtering (i.e. when CT affects both the Landshoff and QC parts of hard scattering amplitude)

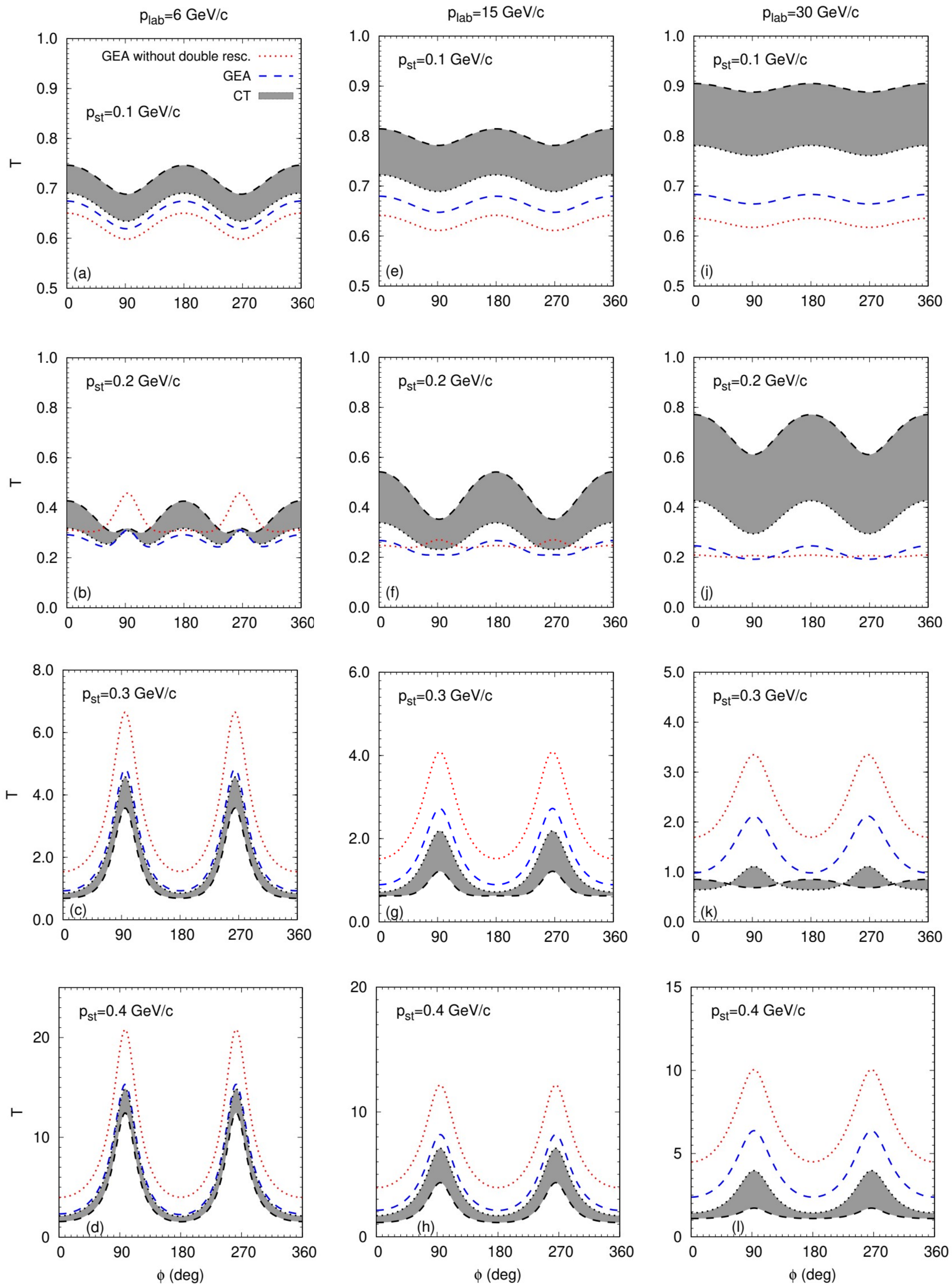




Dependence of the **transparency** on the azimuthal angle between the scattered proton and spectator neutron



- *enhanced single-rescattering amplitudes for outgoing protons (3 and 4) for φ=90° and 270° when $\vec{p}_s \simeq \vec{k}_t$*
- *at small p_{st} this leads to the increased absorption while at large p_{st} – to the increased yield at φ=90° and 270°*
- *CT effects grow with p_{lab} and become strongest at p_{lab}≈30 GeV/c*
- *reasonable agreement with* **L.L. Frankfurt et al, PRC 56, 2752 (1997)** *at p_{lab}=6 and 15 GeV/c*



Deuteron tensor analyzing powers: $A_{\alpha\beta} = \frac{\mathrm{Sp}(MS_{\alpha\beta}M^\dagger)}{\mathrm{Sp}(MM^\dagger)}$, $\alpha, \beta = x, y, z$

$S_{\alpha\beta} = \frac{3}{2}(S_\alpha S_\beta + S_\beta S_\alpha) - 2\delta_{\alpha\beta}$ - spin-quadrupole operator,

S_α - deuteron spin matrices

$$A_{zz} = \frac{\sigma(+1) + \sigma(-1) - 2\sigma(0)}{\sigma(+1) + \sigma(-1) + \sigma(0)}$$ (spin asymmetry)

$\sigma(\lambda_d)$ - differential cross section for the fixed projection λ_d of deuteron spin on z-axis (along the proton beam)

In the IA for a spin-independent hard amplitude, the tensor analyzing power is fully determined by the DWF:

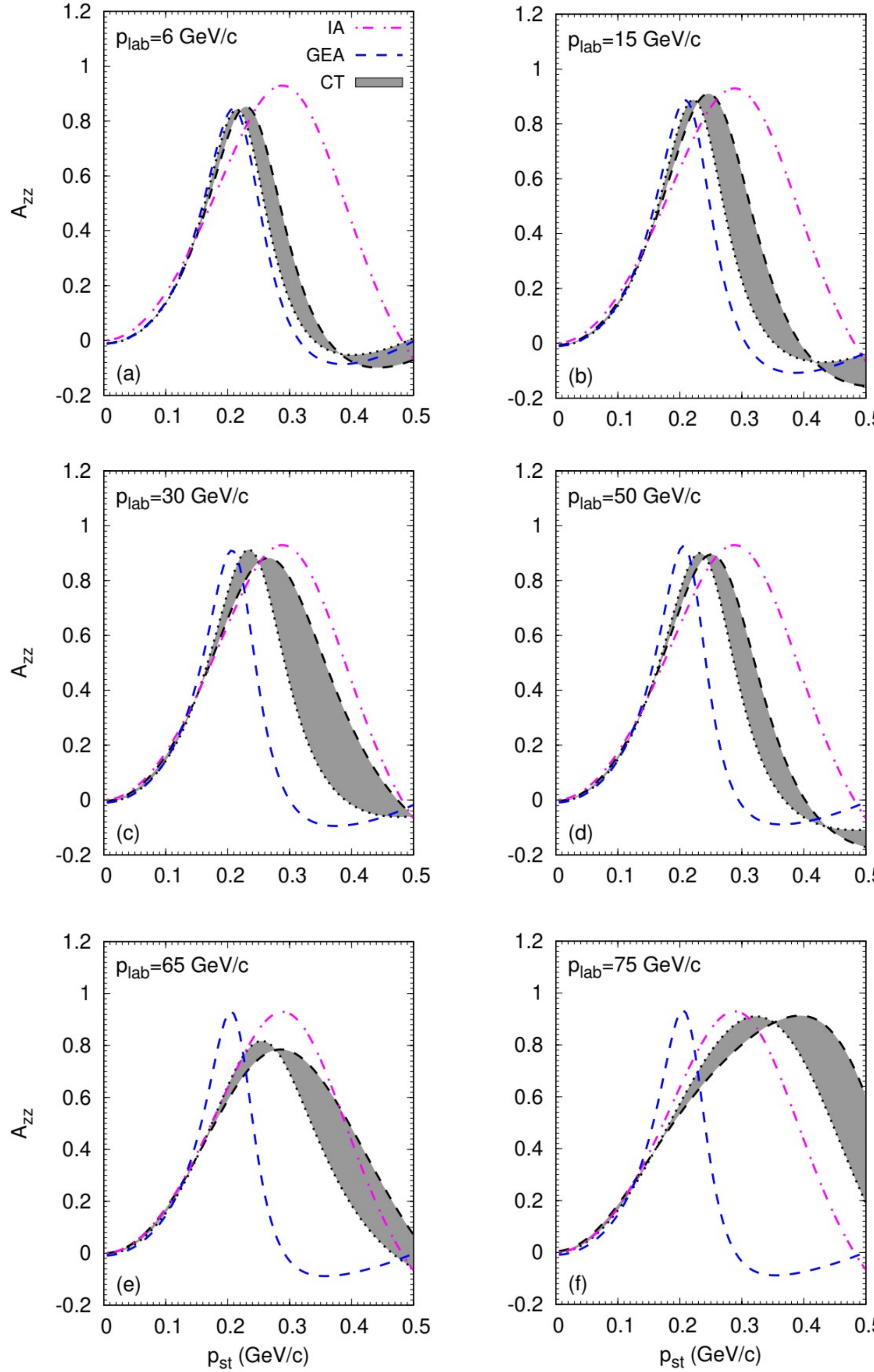
$$A_{zz}^{IA} = \frac{|\phi^{+1}(-\boldsymbol{p}_s))|^2 + |\phi^{-1}(-\boldsymbol{p}_s))|^2 - 2|\phi^0(-\boldsymbol{p}_s))|^2}{|\phi^{+1}(-\boldsymbol{p}_s))|^2 + |\phi^{-1}(-\boldsymbol{p}_s))|^2 + |\phi^0(-\boldsymbol{p}_s))|^2}$$
$$= \frac{(3(p_s^z/p_s)^2 - 1)(\sqrt{2}u(p_s)w(p_s) - w^2(p_s)/2)}{u^2(p_s) + w^2(p_s)} .$$

Thus, it probes the D-state component of the DWF.

Dependence of the **tensor analyzing power** on the transverse momentum of the spectator neutron

- shift of the peak from p_{st}=0.3 GeV/c to p_{st}=0.2 GeV/c and reduced width due to ISI/FSI in the GEA calculations

- pronounced CT effects due to the D-state dominance in A_{zz} (favors shorter distances in the deuteron)

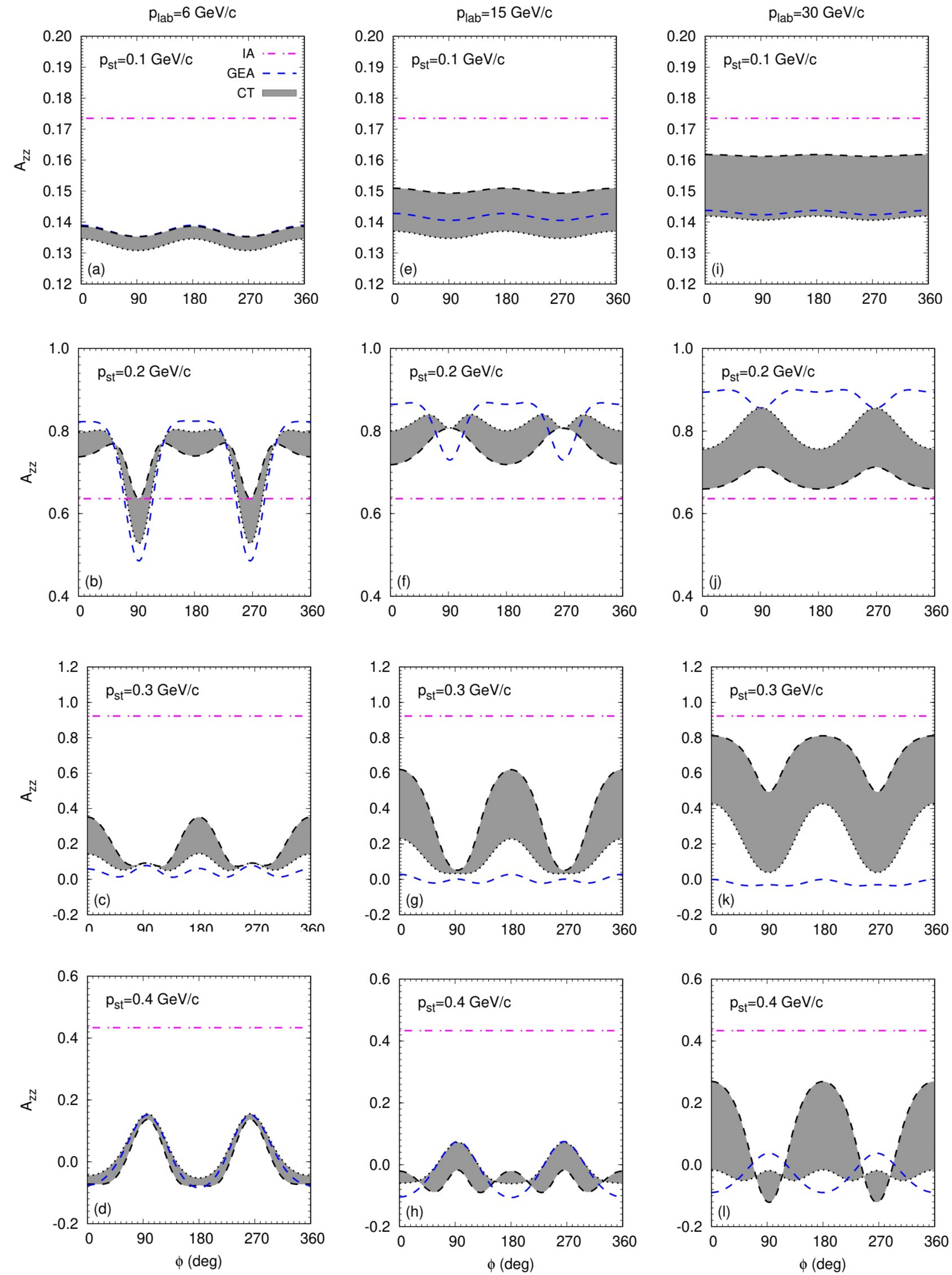


Dependence of the **tensor analyzing power** on the azimuthal angle between the scattered proton and spectator neutron

- *in the GEA, A_{zz} behaves similar to T as a function of φ both at small and large p_{st}*

- *the influence of CT is strongest at p_{st} ≈0.3 GeV/c*

- *p_{lab} = 15-30 GeV/c seems to be optimal for the studies of CT effects*



Summary

- Calculations for the d(p,2p)n large-angle process at p_{lab}=6-75 GeV/c ($\sqrt{s_{NN}}$=3.6-12 GeV) are performed on the basis of the generalized eikonal approximation. The effects of CT are included within the quantum diffusion model, taking into account the interference of small- and large-size qqq configurations.

- Similar to the case of heavier nuclear targets, the Landshoff component of the hard $pp \to pp$ amplitude is effectively filtered-out that leads to the oscillation pattern of the nuclear transparency as a function of p_{lab} at small transverse momentum of the spectator neutron.

- The azimuthal dependence of the nuclear transparency and of the tensor analyzing power are especially sensitive to the CT effects.

Outlook

- d+d collisions (better at first stage of NICA SPD)

- pA collisions for $A \geq 3$ (stronger ISI/FSI, CT should be more pronounced)

Thank you for your attention !

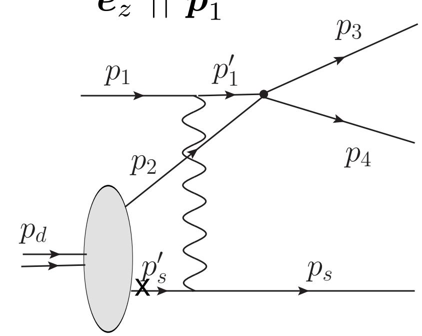
Backup

Amplitude with rescattering of the incoming proton:

$$M^{(d)} = \frac{M_{\rm hard}(s_{hard},t_{hard})}{2p_1^z m^{1/2}} \int d^3r \Theta(z) \phi(\boldsymbol{r}) \mathrm{e}^{i\boldsymbol{p}_s\boldsymbol{r} - i\Delta_1 z} \int \frac{d^2k_t}{(2\pi)^2} \mathrm{e}^{-i\boldsymbol{k}_t\boldsymbol{b}} iM_{\rm el}(|\boldsymbol{p}_1|,k_t) \ ,$$

$$\Delta_1 = \frac{E_1(E_s - E_s')}{p_1^z} - \frac{(p_s' - p_s)^2}{2p_1^z} \simeq \frac{(E_1+m)(E_s-m)}{p_1^z} \ .$$

$\boldsymbol{e}_z \upuparrows \boldsymbol{p}_1$



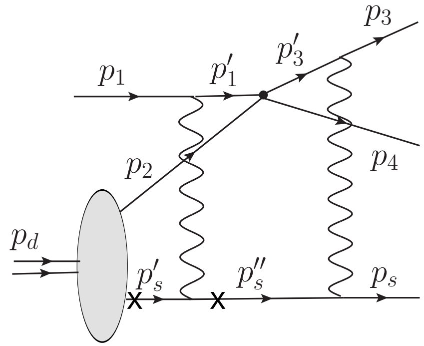
(d)

$\boldsymbol{k}_t = \boldsymbol{p}_{st} - \boldsymbol{p}_{st}'$

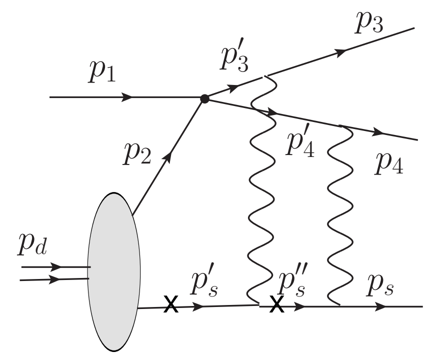
Double rescattering amplitudes:

$$M^{(e)} = 0$$

in collinear geometry
see **L.L. Frankfurt et al, PRC 56, 2752 (1997)**



(e)



(g)

$\boldsymbol{k}_t' = \boldsymbol{p}_{st}'' - \boldsymbol{p}_{st}', \quad \boldsymbol{k}_t'' = \boldsymbol{p}_{st} - \boldsymbol{p}_{st}''$

$$M^{(g)} = \frac{M_{\rm hard}(s_{\rm hard},t_{\rm hard})}{8p_4^z p_3^z m^{3/2}} \int d^3r \Theta(-z) \phi(\boldsymbol{r}) \mathrm{e}^{i\boldsymbol{p}_s\boldsymbol{r}}$$
$$\times \int \frac{d^2k_t'}{(2\pi)^2} \mathrm{e}^{-i\boldsymbol{k}_t'\boldsymbol{b}} iM_{\rm el}(p_3^z,k_t') \int \frac{d^2k_t''}{(2\pi)^2} \mathrm{e}^{-i\boldsymbol{k}_t''\boldsymbol{b}} iM_{\rm el}(p_4^z,k_t'') \mathrm{e}^{-i(\Delta_4+\Delta_3)z} \ ,$$

$$\Delta_3 \simeq \frac{(E_3-m)(E_s''-m)}{p_3^z} - \frac{\boldsymbol{p}_{3t}\boldsymbol{k}_t'}{p_3^z} \ , \quad \Delta_4 \simeq \frac{(E_4+E_s)(E_s-E_s'')}{p_4^z} - \frac{(\boldsymbol{p}_{4t}+\boldsymbol{p}_{st})\boldsymbol{k}_t''}{p_4^z} \ .$$

Note that at high energies the sum $\Delta_3 + \Delta_4$ becomes independent on the energy E_s'' of intermediate spectator.
Set $E_s'' = (E_s+m)/2$ in numerical calculations (transparency is quite weakly sensitive to E_s'').

An estimate of event rate at SPD-NICA

$p_{\rm lab} = 30$ GeV/c ($\sqrt{s_{NN}} = 7.6$ GeV)

For $\Theta_{c.m.} = 90^\circ$ and $p_{st} = 0.2$ GeV/c

$$\alpha_s \frac{d^4\sigma}{d\alpha_s\, dt\, d\phi\, p_{st} dp_{st}} \simeq 10^{-6} \mu\mathrm{b/GeV}^4$$

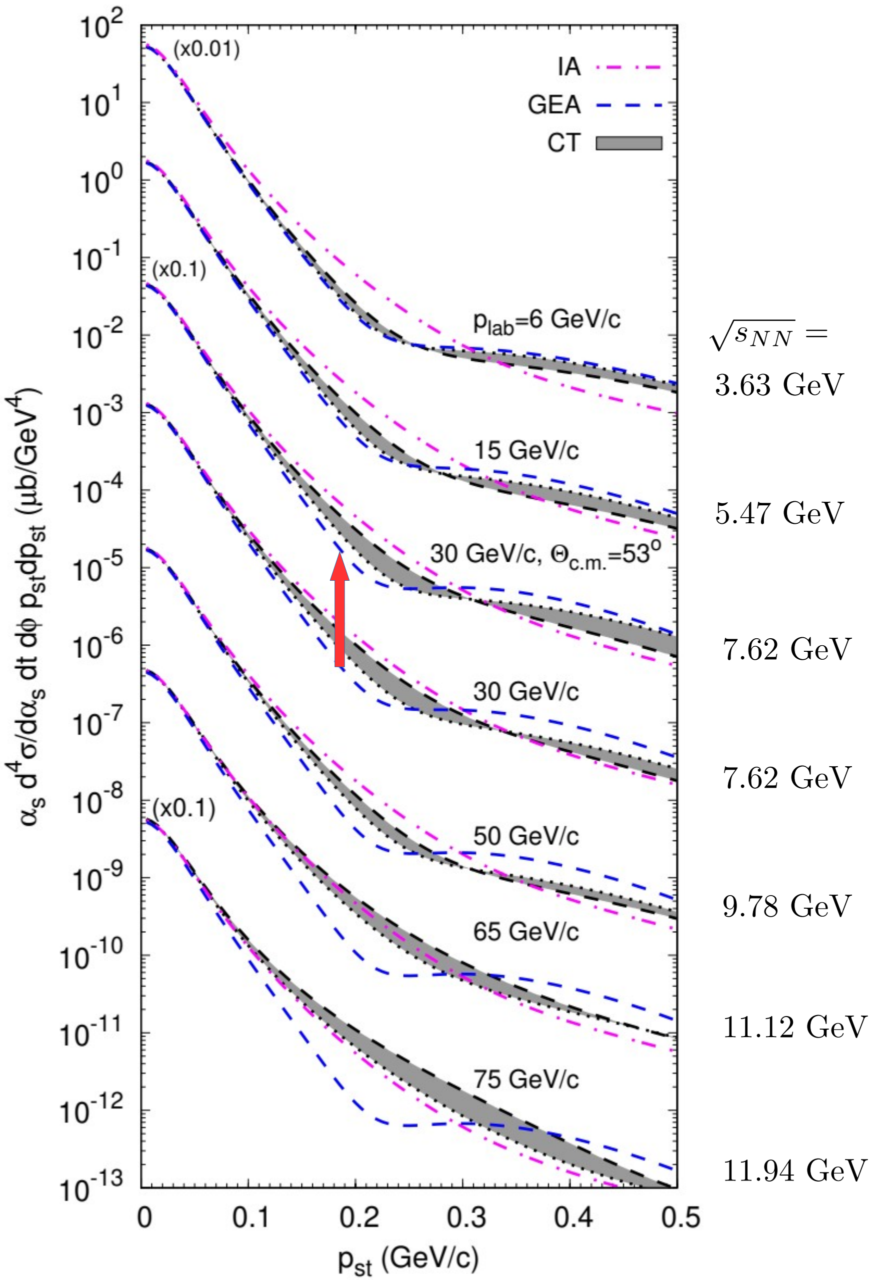
$\sigma \simeq 5$ fb in the ranges $\Delta\alpha_s = 0.2$, $\Delta t = 3$ GeV^2, $\Delta\phi = \pi/3$, $\Delta p_{st} = 0.04$ GeV/c.

3 events/year for $L = 2 \cdot 10^{31}$ cm^{-2} s^{-1}

Too low because $d\sigma_{pp}^{\rm QC}/dt$ quickly drops with |t|.
Smaller |t| needed.

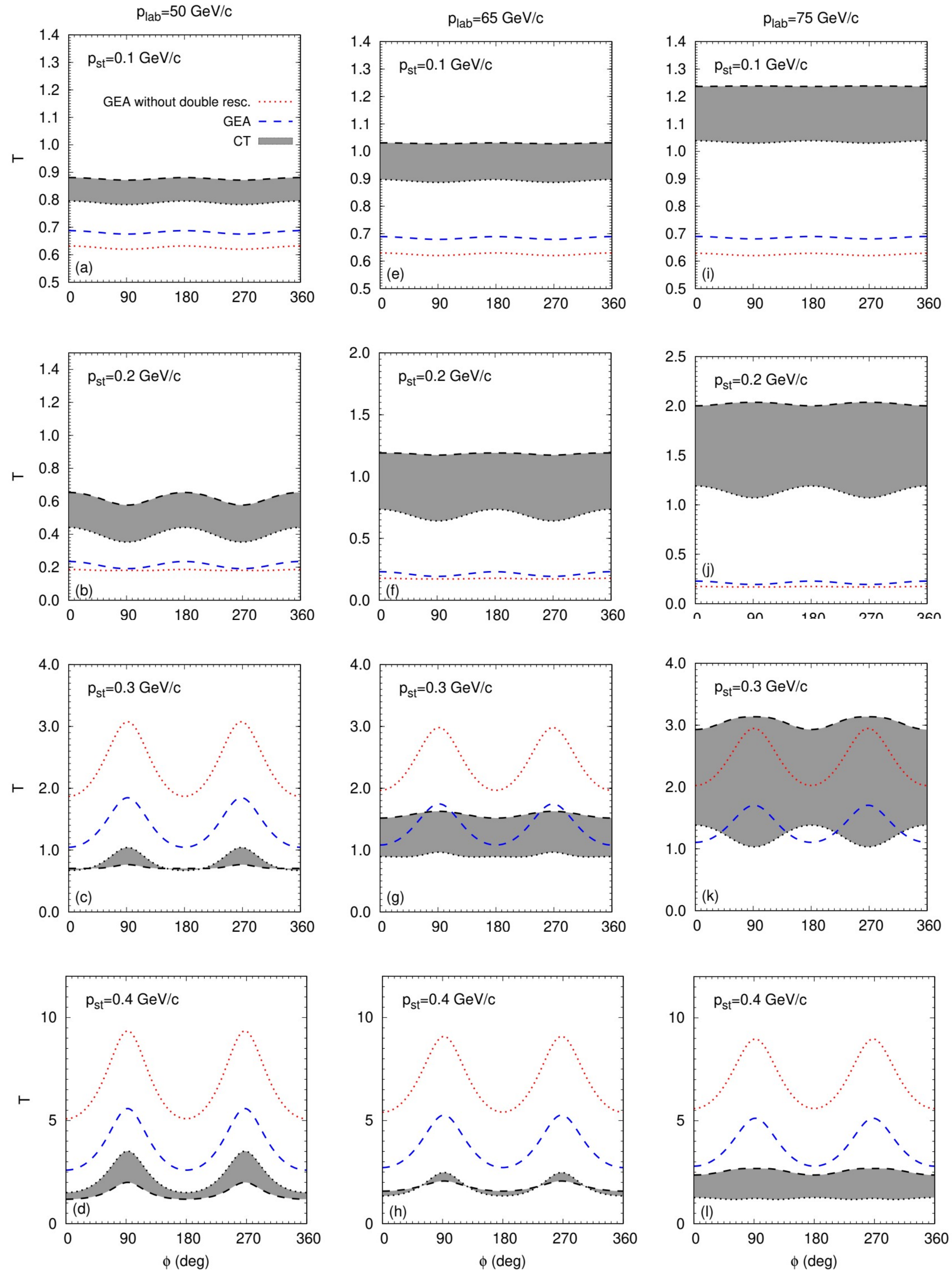
Several events/day

for $\Theta_{c.m.} = 53^\circ$, i.e. for $t = 0.4(4m^2 - s)/2$



- between p_{lab}=30 and 50 GeV/c
the transparency changes quite weakly

- a tendency to isotropy at higher p_{lab}
in the calculations with CT



- *at higher beam momenta, the GEA gives the saturation of φ-dependence of A_{zz}*

- *in calculations with CT A_{zz} tends to isotropy in φ*