On asymptotic safety in Litim-Sannino model

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(in collaboration with **A. Mukhaeva**)

based on 2312.12128
AB,Mukaeva'24

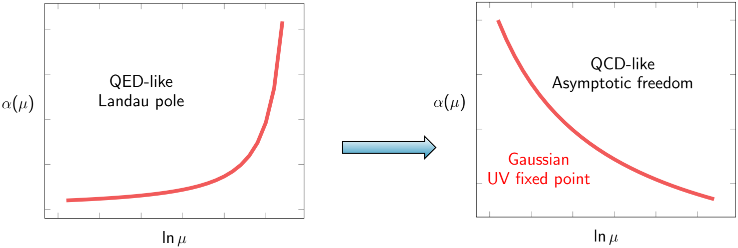
Outline

- Asymptotic freedom vs Asymptotic safety
- Litim-Sannino model
  - RG functions: state-of-the art
  - Fixed-points, critical exponents and UV critical surface
  - Veneziano limit (and beyond)
- Results and Discussion:
  - Asymptotic safety scenario at four loops
  - Conformal window updated
  - Dimension-3 operators, mixing and EOMs
- Conclusions and outlook

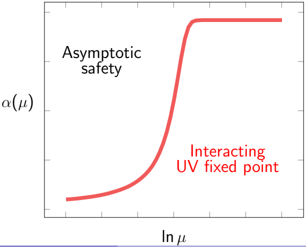
Asymptotic freedom vs Asymptotic Safety

A quest for a ultraviolet (UV) complete theory....

- Asymptotic freedom (AF) — Gross, Wilczek, Politzer'71



- Asymptotic safety (AS) — Weinberg'80
  - UV-complete quantum GR?
  - Residual UV interactions
  - Non-perturbative treatment?



Asymptotic safety in a nutshell

$$S = \int d^d x \mu^{d-\Delta_i} g_i O_i(x), \quad \alpha_i \equiv \frac{g_i^2}{16\pi^2}$$

- Renormalization-group (RG) flow in parameter space*
  $\alpha = \{\alpha_i\}$ features an interacting fixed-point (FP) α^*:

$$\partial_t \alpha_i = \beta_i(\alpha), \quad \beta_i(\alpha^*) = 0, \quad t \equiv \ln \mu - \text{RG time}$$

*can be infinite-dimensional, e.g., in effective field theories (EFT)

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$$\partial_t(\alpha_i - \alpha_i^*) = \omega_{ij}(\alpha_j - \alpha_j^*) + \ldots, \quad \omega_{ij} \equiv \partial_j \beta_i(\alpha^*)$$

eigenvalues

- UV critical surface that spans infrared (IR) relevant directions ($\theta_j^{\mathrm{IR}} < 0$) has finite dimension ($k$)

$$\alpha_i(\mu) = \alpha_i^* + \sum_{j=1}^{k} c_{ij} \left(\frac{\mu}{\mu_0}\right)^{\theta_j^{\mathrm{IR}}} + \sum_{j=k+1}^{\infty} c_{ij} \left(\frac{\mu}{\mu_0}\right)^{\theta_j^{\mathrm{UV}}} \quad (\theta^{\mathrm{IR}}, \theta^{\mathrm{UV}})$$

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- In the IR the (UV-complete) theory has k free parameters!

*can be infinite-dimensional, e.g., in effective field theories (EFT)

Asymptotic safety: Gauge-Yukawa example

Litim,Sannino'14 , see also review AB,Mukhaeva'24

$$\beta_g = \alpha_g^2(-B + C\alpha_g - D\alpha_y)$$
$$\beta_y = \alpha_y(E\alpha_y - F\alpha_g)$$

- The flow is towards IR
- Asymptotic freedom ($B > 0$)
- Gaussian (G) FP:

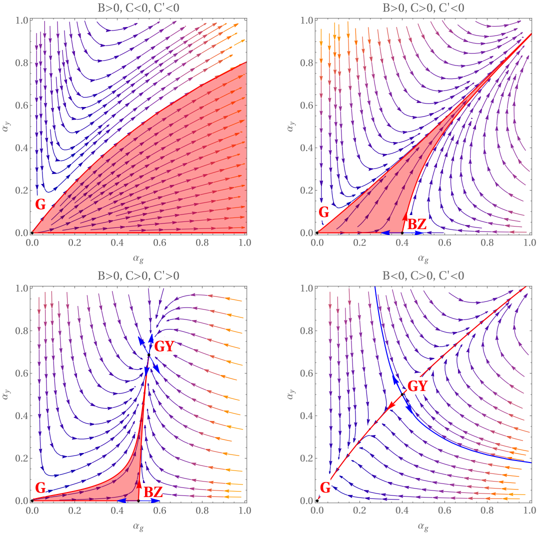
$$\alpha_g^* = \alpha_y^* = 0$$

Banks-Zaks (BZ) FP:

$$\alpha_g^* = B/C, \quad \alpha_y^* = 0$$

Gauge-Yukawa (GY) FP

$$\alpha_g^* = B/C', \quad \alpha_y^* = (F/E)\alpha_g^*$$
$$C' = C - D(F/E)$$



IR (UV) relevant eigendirections

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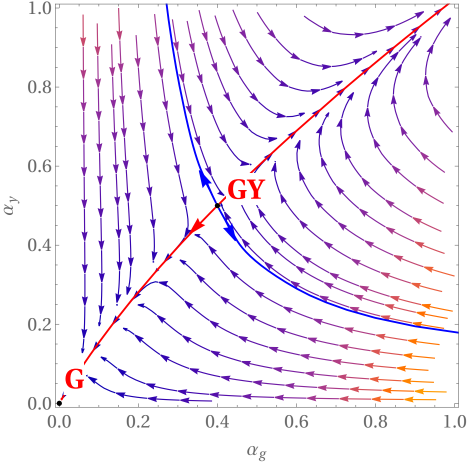
B<0, C>0, C'<0



IR (UV) relevant eigendirections

Litim-Sannino model

field	$SU(N_c)$	$U_L(N_f)$	$U_R(N_f)$
ψ_L	N_c	N_f	1
ψ_R	N_c	1	N_f
H	1	N_f	$\bar{N}_f$

- $SU(N_c)$ gauge
- $U_L(N_f) \times U_R(N_f)$ flavour
- Lagrangian (NB: $d = 4$)

$$\mathcal{L} = -\frac{1}{4} F^{a\mu\nu} F^a_{\mu\nu} + \mathcal{L}_{gf} + \mathcal{L}_{gh}, \quad \left[F^a_{\mu\nu} \equiv \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g f^{abc} G^b_\mu G^c_\nu\right]$$
$$+ \mathrm{Tr}(\bar{\psi} i \hat{D} \psi) + \mathrm{Tr}(\partial^\mu H^\dagger \partial_\mu H) - y \mathrm{Tr}[\bar{\psi}(H \mathcal{P}_R + H^\dagger \mathcal{P}_L)\psi]$$
$$- m^2 \mathrm{Tr}(H^\dagger H) - u \underbrace{\mathrm{Tr}\left[(H^\dagger H)^2\right]}_{\text{single-trace}} - v \underbrace{\left(\mathrm{Tr}\left[H^\dagger H\right]\right)^2}_{\text{double-trace}}$$

- We also study $\delta\mathcal{L}$ that breaks $U_L(N_f) \times U_R(N_f) \to U(N_f)$

$$\delta\mathcal{L} = -m_\psi \mathrm{Tr}(\bar{\psi}\psi) - \left[\frac{h_2}{2} \mathrm{Tr}(H H^\dagger H) + \frac{h_3}{2} \mathrm{Tr}(H H^\dagger) \mathrm{Tr}(H) + \mathrm{h.c.}\right]$$
$$\equiv -m_\psi O_1 - h_2 O_2 - h_3 O_3 = -\vec{\kappa} \cdot \vec{O}, \quad \vec{O} - \text{dimension-3 operators}$$

Litim-Sannino model: Veneziano limit

RG flow:

$$\partial_t \alpha_i = \beta_i(\alpha, N_c, N_f) = \beta_i(\alpha, \epsilon, N_c), \qquad \epsilon \equiv \frac{N_f}{N_c} - \frac{11}{2}$$

- 't Hooft-like coupling:

$$\alpha_g = \frac{g^2 N_c}{(4\pi^2)}, \quad \alpha_y = \frac{y^2 N_c}{(4\pi^2)}, \quad \alpha_u = \frac{u N_f}{(4\pi^2)}, \quad \alpha_v = \frac{v N_f^2}{(4\pi^2)}.$$

- Veneziano limit: $N_{f,c} \to \infty$, constant N_f/N_c
- Small expansion parameter $|\epsilon| \ll 1$
- Given $\beta_i(N_c \to \infty)$ compute FP as series in ϵ:

$$\alpha_x^* = \sum_{i=1}^{\infty} c_x^{(i)} \epsilon^i, \text{ perturbative control!}$$

LS model: Veneziano limit (and beyond)

- 1 loop gauge coupling running: $\partial_t \alpha_g = \alpha_g{}^2 \left[\frac{4}{3}\epsilon + \ldots\right]$
- To find $c_x^{(n)}$ one needs to consider

$$\left.\begin{array}{cccc} (n+1) & \text{- loop} & \beta_g & \text{gauge} \\ n & \text{- loop} & \beta_y & \text{Yukawa} \\ n & \text{- loop} & \beta_{u,v} & \text{scalar} \end{array}\right\} \quad (n+1, n, n) \text{ - scheme}$$

- State-of-the-art result: 433 - scheme

$$\alpha_x^* = \underset{211}{c_x^{(1)}\epsilon} + \underset{322}{c_x^{(2)}\epsilon^2} + \underset{433}{c_x^{(3)}\epsilon^3}$$

Litim,Sannino'14	Bond,Litim, Medina,Steudtner'17	Litim,Riyaz, Stamou,Steudtner'23, AB,Mukhaeva'23

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Bond,Litim, Medina'21 — Bond,Litim, Medina'21 — AB,Mukhaeva'24

- Finite-N_c corrections

$$\mathbf{f_x^{(n)}}(\mathbf{N_c}) : \qquad \lim_{N_c \to \infty} \mathbf{f_x^{(n)}}(\mathbf{N_c}) = 1$$

LS model: Veneziano limit

- Given $\alpha^* = \alpha^*(\epsilon, N_c)$, one computes:

$$\theta_1(\alpha^*) < 0 < \theta_{2,3,4}(\alpha^*) \text{ - one IR-relevant eigendirection}$$
$$\Delta_O = d_O + \gamma^*_O, \gamma^*_O \equiv \gamma_O(\alpha^*) \text{ - critical dimensions of operators}$$

- In Veneziano limit (coefficients are known analytically):

$$\alpha_g{}^* = 0.456\epsilon + 0.781\epsilon^2 + 6.610\epsilon^3,$$
$$\alpha_y{}^* = 0.211\epsilon + 0.508\epsilon^2 + 3.322\epsilon^3,$$
$$\alpha_u{}^* = 0.200\epsilon + 0.440\epsilon^2 + 2.693\epsilon^3,$$
$$-\alpha_v{}^* = 0.137\epsilon + 0.632\epsilon^2 + 4.313\epsilon^3,$$

Conformal window : $\alpha^*(\epsilon_{\text{ perturbativity }}) = 1?$

Bound(s) on $\epsilon_{\max}$ $[\alpha_u{}^* + \alpha_v{}^*](\epsilon_{\text{vacuum stability}}) = 0?$

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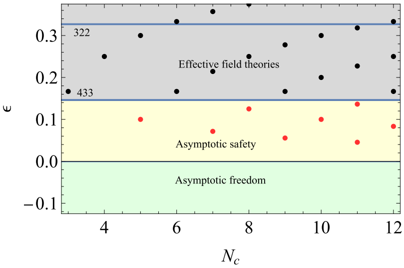
- In Veneziano limit (coefficients are known analytically):

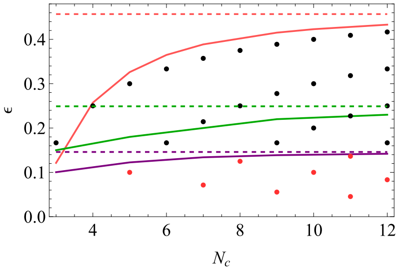
$$\begin{aligned} -\theta_1 &= 0.608\epsilon^2 - 0.707\epsilon^3 - 6.947\epsilon^4, \quad \theta_1(\epsilon_{\text{merge}}) = 0? \\ \theta_2 &= 2.737\epsilon + 6.676\epsilon^2 + 22.120\epsilon^3, \\ \theta_3 &= 2.941\epsilon + 1.041\epsilon^2 + 5.137\epsilon^3, \\ \theta_4 &= 4.039\epsilon + 9.107\epsilon^2 + 38.646\epsilon^3 \end{aligned}$$

LS model: Conformal window updated

- perturbativity of couplings $0 < |\alpha^*| \lesssim 1$
- no FP merger (collision of UV and IR FP $\Leftrightarrow \theta = 0$)
- vacuum stability $\alpha_u{}^* > 0$ and $\alpha_u{}^* + \alpha_v{}^* > 0$

$\alpha_g{}^*$
θ_1
$\alpha_u{}^*+\alpha_v{}^*$





Pairs of (N_c, N_f) compatible with AS: [N_c dependence of $\epsilon_{\max}$]

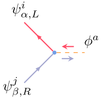
$(N_c, N_f) = (5, 28), (7, 39), (8, 45), (9, 50), (10, 56), (11, 61), (11, 62), (12, 67)$

Litim-Sannino model: Feynman rules

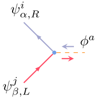
Ways to find β-functions in LS model within perturbation theory[†]:

- Use (at least partially) general (template) results
  432 - arbitrary QFT: AB,Pikelner'21 Davies,Herren,Thomsen'21
- 3-loop scalar $\beta_{u,v}$: direct computation of diagrams, utilizing, e.g.

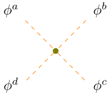
$$H = \phi^a T^a, \qquad H^\dagger = \phi^a \bar{T}^a, \qquad \bar{T}^a \equiv T^{a\dagger}$$



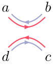
$$= -iyT^a_{ij}\delta_{\alpha\beta},$$

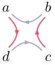


$$= -iy \cdot \bar{T}^a_{ij} \cdot \delta_{\alpha\beta}$$



$$= -i2\{u \cdot [\underbrace{\mathrm{Tr}(T^a\bar{T}^b)\,\mathrm{Tr}(T^c\bar{T}^d)} + 11 \text{ perms.}] + v \cdot [\underbrace{\mathrm{Tr}(T^a\bar{T}^b T^c\bar{T}^d)} + 11 \text{ perms.}]\}$$





+ DIANA (diagram generation) + MATAD (vacuum integrals)

[†]In dimensional regularization and the $\overline{\mathrm{MS}}$ scheme.

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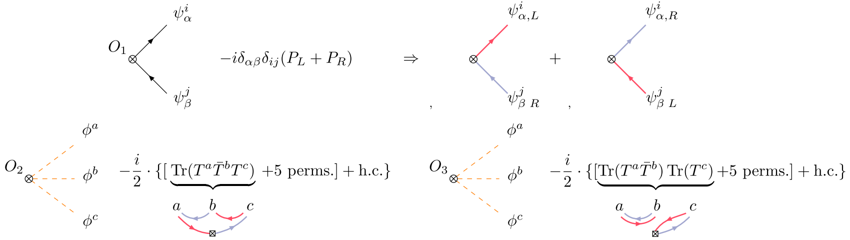

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Dimension-3 operators: peculiarities

- Unitarity bound on Δ_O for O with canonical dimension d_O:

$$\Delta_O = d_O + \gamma_O(\alpha^*) > 1$$

Mack'77

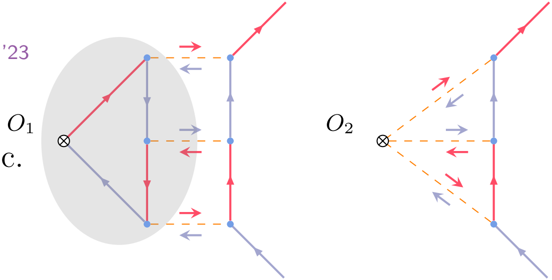
- We consider dimension-3 operators and account for mixing

$O_1 = \mathrm{Tr}(\bar{\psi}\psi)$ Litim et al'23

$O_2 = \frac{1}{2}\mathrm{Tr}(HH^\dagger H) + \mathrm{h.c.}$

$O_3 = \frac{1}{2}\mathrm{Tr}(HH^\dagger)\mathrm{Tr}(H) + \mathrm{h.c.}$

$\vec{O} \equiv \{O_1, O_2, O_3\}$



$$\gamma^*_{O_1} = \underbrace{-0.2105\epsilon}_{\text{unitarity bound?}} + \underbrace{0.4628\epsilon^2 + 0.3669\epsilon^2}_{\text{can not be trusted}}$$

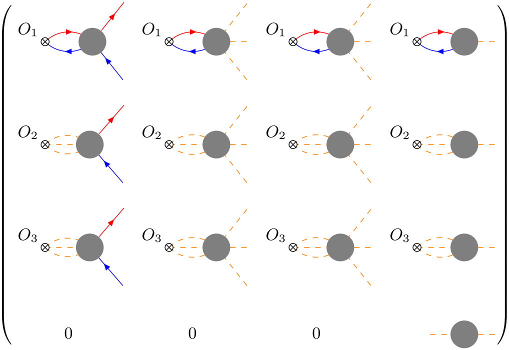
Litim et al'23

Dimension-3 operators: mixing and EOMs

- Mixing of 4 operators: $O_4 = 1/2\partial^2 \mathrm{Tr}(H) + \mathrm{h.c.}$

$$[O_i]_R = (\bar{Z}_O)_{ij}(O_j)_{\mathrm{bare}}, \quad \text{matrix: } \bar{\gamma}_O \equiv -(\partial_t \bar{Z}_O) \cdot \bar{Z}_O^{-1}$$

$$\bar{Z}_O = \begin{pmatrix} Z_O & \vec{Z} \\ \vec{0} & Z_H^{-1/2} \end{pmatrix} \equiv \mathcal{K}\mathcal{R}'$$



- We demostrate that $\bar{\gamma}_O^*$ besides two non-trivial eigenvalues $\gamma_{3,4}$ have two eigenvalues $\gamma_{1,2} = \pm\gamma_H^*$ $[\gamma_H \equiv \frac{1}{2}\partial_t \log Z_H^{1/2}]$

AB,Mukhaeva'24

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$$\begin{aligned} \gamma_H^* = \gamma_1 = -\gamma_2 &= 0.21053\epsilon + 0.46247\epsilon^2 + 2.47105\epsilon^3, \\ \gamma_3 &= 2.22982\epsilon + 3.88519\epsilon^2 + 20.5012\epsilon^3, \\ \gamma_4 &= 1.68082\epsilon + 0.98321\epsilon^2 + 5.03949\epsilon^3. \end{aligned}$$

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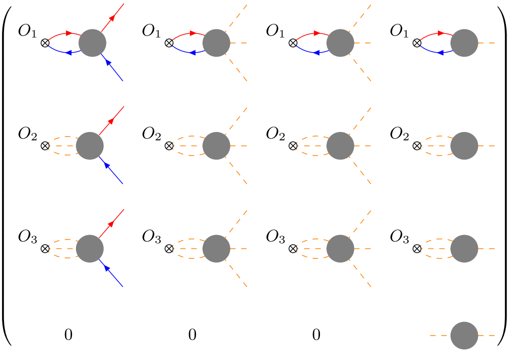


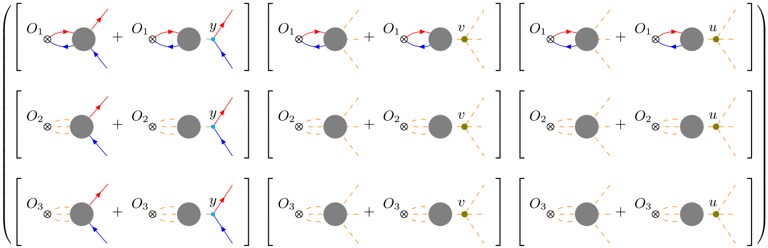
- Equation of motion:

$$(O_4)_{\rm bare} + \vec{\Lambda}_{bare} \cdot (\vec{O})_{bare} = 0, \quad \vec{\Lambda}_{\rm bare} = (y/2, 2u, 2v)_{\rm bare}$$

Dimension-3 operators: mixing and EOMs

- Mixing of 3 operators $\vec{O}$ with account of EOM

$$[\vec{O}]_R = Z_O \cdot (\vec{O})_{\mathrm{bare}} + \vec{Z} \cdot (O_4)_{\mathrm{bare}}, \quad (O_4)_{\mathrm{bare}} = -\vec{\Lambda}_{bare} \cdot (\vec{O})_{bare}$$



$$[\vec{O}]_R = \tilde{Z}_O (\vec{O})_{\mathrm{bare}}, \quad \tilde{Z}_O = Z_O - \vec{Z} \otimes \vec{\Lambda}_{\mathrm{bare}}, \quad \tilde{\gamma}_O = -\partial_t \tilde{Z}_O \cdot \tilde{Z}_O{}^{-1}$$

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$\tilde{\gamma}^*_O\ (3 \times 3)$ $\qquad$ $\bar{\gamma}^*_O\ (4 \times 4)$ $\qquad$ $\gamma^*_O\ (3 \times 3)$

$\gamma_1 \xleftrightarrow{+\gamma^*_H} \gamma_1, \gamma_2 \xleftrightarrow{-\gamma^*_H} \gamma_2$

$\gamma_3, \gamma_4 \qquad \gamma_3, \gamma_4 \qquad \gamma_3, \gamma_4$

descendant of $\mathrm{Tr}(H) + \mathrm{h.c.}$ $\qquad$ $\mathrm{Tr}(\delta S/\delta H) + \mathrm{h.c.}$ zero onshell

$$[\vec{O}]_R = \tilde{Z}_O (\vec{O})_{\mathrm{bare}}, \quad \tilde{Z}_O = Z_O - \vec{Z} \otimes \vec{\Lambda}_{\mathrm{bare}}, \quad \tilde{\gamma}_O = -\partial_t \tilde{Z}_O \cdot \tilde{Z}_O{}^{-1}$$

Conclusions and outlook

- Computed all β_x in 433 - scheme
- Computed α^* and θ_i up to $\mathcal{O}(\epsilon^3)$ [confirmed Litim et al'23]
  finite-N_c corrections (for the first time)
- Updated Conformal Window [vacuum stability]
- Anomalous dimensions of dimension-3 operators
  correct treatment with mixing
  no threat to unitarity
- Tree-level vacuum stability $\rightarrow$ effective potential?
  Steudtner'24
- General result for 3-loop self-coupling β-function?
- What about 544?

Thank you for attention!