Cornwall-Jackiw-Tomboulis effective action in (2+1)-dimensional models

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Фонд развития теоретической физики и математики

Lagrangian of Gross-Neveu model

$$L = i\overline{\psi}_k \gamma^\nu \partial_\nu \psi_k + \frac{G}{2N}\left(\overline{\psi}_k \psi_k\right)^2$$

$k = 1, ..., N$ is a number of flavours

It exhibits chiral symmetry breaking and dynamical mass generation

$$\langle \overline{\psi}\psi \rangle \neq 0$$

$$L \sim -\frac{2N}{G}\sigma(x)^2 + i\,\overline{\psi}\gamma^\mu \partial_\mu \psi - \sigma(x)\,\overline{\psi}\psi, \qquad \langle \sigma \rangle \sim \langle \overline{\psi}\psi \rangle$$

Note that the definition of chiral symmetry is slightly unusual in (2+1)-dimensions.

There exists no other 2×2 matrix anticommuting with the gamma matrices, which would allow the introduction of a γ^5-matrix in the irreducible representation.

The concept of **chiral symmetries** and their breakdown by mass terms can nevertheless be realized also in the framework of **(2+1)-dimensional quantum field theories**

by considering a **four-component reducible representation for Dirac fields**

The Dirac spinors ψ have the following form:

$$\psi(x) = \begin{pmatrix} \tilde{\psi}_1(x) \\ \tilde{\psi}_2(x) \end{pmatrix},$$

with $\tilde{\psi}_1, \tilde{\psi}_2$ being two-component spinors.
4×4 γ-matrices:

$$\gamma^\mu = diag(\tilde{\gamma}^\mu, -\tilde{\gamma}^\mu)$$

Chiral symmetry: γ^3, γ^5 and τ

There exist two matrices, γ^3 and γ^5, which anticommute with all γ^μ $(\mu = 0, 1, 2)$ and with themselves

$$\gamma^3 = \begin{pmatrix} 0\ , & I \\ I\ , & 0 \end{pmatrix}, \qquad \gamma^5 = \gamma^0\gamma^1\gamma^2\gamma^3 = i\begin{pmatrix} 0\ , & -I \\ I\ , & 0 \end{pmatrix}$$

One can also construct

$$\tau = -i\gamma^3\gamma^5 = \begin{pmatrix} I\ , & 0 \\ 0\ , & -I \end{pmatrix}$$

As a rule one introduces the source terms

$$\mathcal{S}_s = \int d^3x \Big(\overline{\psi}_k(x) J_k(x) + \overline{J}_k(x)\psi_k(y)\Big)$$

But now let us introduce bilocal source

$$\mathcal{S}_s = \int d^3x d^3y \overline{\psi}_k^{\alpha}(x) K_{\alpha}^{\beta}(x,y)\psi_{k\beta}(y)$$

$Z(K)$ is the generating functional of the Green's functions of bilocal fermion-antifermion composite operators $\overline{\psi}_k^\alpha(x)\psi_{k\beta}(y)$

$$Z(K) \equiv \exp(iNW(K)) =$$
$$= \int \mathcal{D}\overline{\psi}_k \mathcal{D}\psi_k \exp\left(i\left[I(\overline{\psi},\psi) + \int d^3x d^3y \overline{\psi}_k^\alpha(x) K_\alpha^\beta(x,y)\psi_{k\beta}(y)\right]\right)$$

where $\alpha, \beta = 1, 2, 3, 4$ are spinor indices,
$K_\alpha^\beta(x,y)$ is a bilocal source of the fermion bilinear composite field $\bar{\psi}_k^\alpha(x)\psi_{k\beta}(y)$

Generating functional can be expressed in the following form

$$Z(K) = \exp\left(iI_{int}\Big(-i\frac{\delta}{\delta K}\Big)\right)\exp\Big[N\mathrm{Tr}\ln\big(D(x,y)+K(x,y)\big)\Big]$$

$$Z(K) = \exp(iNW(K))$$

$$\exp(iNW(K)) =$$

$$= \exp\left(iI_{int}\Big(-i\frac{\delta}{\delta K}\Big)\right)\exp\left[N\,\mathrm{Tr}\ln\big(D(x,y)+K(x,y)\big)\right]$$

CJT effective action of the composite bilocal and bispinor operator $\bar{\psi}_k^{\alpha}(x)\psi_{k\beta}(y)$ is defined as a functional $\Gamma(S)$ of the **full fermion propagator** $S_{\beta}^{\alpha}(x,y)$ by **Legendre transformation** of the functional $W(K)$

$$\Gamma(S) = W(K) - \int d^3x d^3y S_{\beta}^{\alpha}(x,y) K_{\alpha}^{\beta}(y,x),$$

$$S_{\beta}^{\alpha}(x,y) = \frac{\delta W(K)}{\delta K_{\alpha}^{\beta}(y,x)}.$$

$S(x, y)$ is the full fermion propagator at $K(x, y) = 0$

One can show for CJT effective action $\Gamma(S)$

$$\frac{\delta \Gamma(S)}{\delta S^{\alpha}_{\beta}(x, y)} = -K^{\beta}_{\alpha}(y, x)$$

If bilocal sources $K^{\beta}_{\alpha}(y, x)$ are zero, the full fermion propagator is a solution of

$$\frac{\delta \Gamma(S)}{\delta S^{\alpha}_{\beta}(x, y)} = 0.$$

we calculate the effective action pertubatively

$$\Gamma(S) = -i\mathrm{Tr}\ln\big(-iS^{-1}\big) + \int d^3x d^3y S^{\alpha}_{\beta}(x,y) D^{\beta}_{\alpha}(y,x)$$

$$+\frac{G}{2}\int d^3x \Big[\mathrm{tr} S(x,x)\Big]^2 - \frac{G}{2N}\int d^3x \ \mathrm{tr}\Big[S(x,x)S(x,x)\Big].$$

The stationary equation for the CJT effective action

$$0 = i\left[S^{-1}\right]_{\alpha}^{\beta}(x,y)+D_{\alpha}^{\beta}(x,y)+G\delta_{\alpha}^{\beta}\delta(x-y)\,\mathrm{tr}S(x,y)-\frac{G}{N}S_{\alpha}^{\beta}(x,y)\delta(x-y).$$

$S(x,y)$ is a translationary invariant operator

$$\overline{(S^{-1})_{\alpha}^{\beta}}(p)-ip_{\nu}(\gamma^{\nu})_{\alpha}^{\beta} = iG\delta_{\alpha}^{\beta}\int\frac{d^3q}{(2\pi)^3}\,\mathrm{tr}\overline{S}(q)-i\frac{G}{N}\int\frac{d^3q}{(2\pi)^3}\overline{S_{\alpha}^{\beta}}(q)$$

Let us explore, using the CJT approach, the possibility of mass term

$$\overline{S^{-1}} = i(\hat{p} + m_D), \quad \text{i.e.} \quad \overline{S} = -i\frac{\hat{p} + m_D}{p^2 - m_D^2}$$

$\mathcal{P}$ - symmetric $\mathcal{T}$ - symmetric

Break chiral symmetries Γ^5 and Γ^3

UV divergence can be removed from the gap equations if we require the following behavior of the bare coupling constant $G \equiv G(\Lambda)$ vs Λ

$$\frac{1}{G(\Lambda)} = \frac{4N-1}{2N\pi^2}\left(\Lambda + g_D\frac{\pi}{2} + g_D\mathcal{O}\left(\frac{g_D}{\Lambda}\right)\right)$$

where g_D is a finite Λ-independent and renormalization group invariant quantity, and it can also be considered as a new free parameter of the model.

Dynamical mass generation

- at $g_D > 0$ its global minimum lies at the point $m_D = 0$, and no dynamical mass generation

- at $g_D < 0$ the global minimum is achieved at $m_D = |g_D|$

$$m_D = |g_D|$$

One could define dimensionless bare coupling constant

$$\lambda = \Lambda G(\Lambda)$$

The β-function is

$$\beta(\Lambda) = \Lambda \frac{\partial \lambda(\Lambda)}{\partial \Lambda}, \qquad \beta(\Lambda) = \frac{\lambda}{\lambda_D}(\lambda_D - \lambda)$$

where $\lambda_D = \frac{2N\pi^2}{4N-1}$

there exists a nonzero UV-stable fixed point λ_D in the model

Renormalization group equation

At rather large values of Λ

$$\lambda(\Lambda) - \lambda_D \sim -\frac{g_D}{\Lambda}$$

- at $\lambda > \lambda_D$ — **chiral symmetry is broken**
- at $\lambda < \lambda_D$ — **symmetry of the model remains intact**

Let us explore, using the CJT approach, the possibility of mass term

$$\overline{S^{-1}} = i(\hat{p} + \tau m_H), \quad \text{i.e.} \quad \overline{S} = -i\frac{\hat{p} + \tau m_H}{p^2 - m_H^2}$$

$\mathcal{P}$ - breaking **$\mathcal{T}$ - symmetric**

Keep chiral symmetries Γ^5 and Γ^3 intact

the UV divergence can be removed from the gap equations if we require the following behavior of the bare coupling constant $G \equiv G(\Lambda)$ vs Λ

$$\frac{1}{G(\Lambda)} = -\frac{1}{2N\pi^2}\left(\Lambda + g_H\frac{\pi}{2} + g_H\mathcal{O}\left(\frac{g_H}{\Lambda}\right)\right)$$

where g_H is a finite Λ-independent and renormalization group invariant quantity, and it can also be considered as a new free parameter of the model.

- at $g_H > 0$ its global minimum lies at the point $m_H = 0$, and no dynamical generation of Haldane mass

- at $g_H < 0$ the global minimum is achieved at $m_H = |g_H|$

$$m_H = |g_H|$$

At rather large values of Λ

$$\lambda(\Lambda) - \lambda_H \sim \frac{2\pi^2 N g_H}{\Lambda}$$

where $\lambda_H = -2N\pi^2$

- at $\lambda > \lambda_H$ — **parity remains intact**
- at $\lambda < \lambda_H$ — **parity is broken**

Since $\lambda_H \to -\infty$ at $N \to \infty$

we may conclude that **in the limit of large** N the (2+1)-D GN model cannot have a **P-odd phase** and **Haldane mass cannot arise dynamically**

Let us explore the possibility that the solution of the gap equation has the form

$$\overline{S^{-1}} = i(\hat{p} + i\gamma^5 m_5 + i\gamma^3 m_3), \quad \text{i.e.} \quad \overline{S} = -i\frac{\hat{p} + i\gamma^5 m_5 + i\gamma^3 m_3}{p^2 - (m_3^2 + m_5^2)}$$

It corresponds to a dynamically generated mass term of the form $\mathcal{M}_H = \left(m_5\overline{\psi}i\gamma^5\psi + m_3\overline{\psi}i\gamma^3\psi\right)$ in the Lagrangian

Since m_5 and m_3 are some real numbers, this mass term is a Hermitian one.

Dynamical generation of masses m_3 and m_5

- at $g > 0$ only a trivial solution of the gap equations exists, $m_3 = m_5 = 0$, and all discrete symmetries of the model remain intact

- at $g < 0$

$$m_3 = |g| \cos\alpha, \qquad m_5 = |g| \sin\alpha$$

(where $0 \leq \alpha \leq \pi/2$ is some arbitrary fixed angle)

Physical fermion mass

At $g < 0$ in all above mentioned cases (at arbitrary values of the angle parameter α)
the genuine physical fermion mass, which is indeed a pole of the fermion propagator, is equal to

$$M_F = \sqrt{m_3^2 + m_5^2} \equiv |g|$$

At rather large values of Λ

$$\lambda(\Lambda) - \lambda_{35} \sim \frac{2\pi^2 N g}{\Lambda}$$

where $\lambda_{35} = 2N\pi^2$

- at $\lambda > \lambda_{35}$ – $m_5\overline{\psi}i\gamma^5\psi + m_3\overline{\psi}i\gamma^3\psi$ **mass term is dynamically generated**
- at $\lambda < \lambda_{35}$ – **symmetric phase**

Since $\lambda_{35} \to \infty$ at $N \to \infty$

we may conclude that **in the limit of large** N there is no **dynamical** $m_5\overline{\psi}i\gamma^5\psi + m_3\overline{\psi}i\gamma^3\psi$ **mass term generation**

Spontaneous non-Hermiticity in Gross-Neveu model

$$\mathcal{M}_H = im_5\overline{\psi}(x)\gamma^5\psi(x) + im_3\overline{\psi}(x)\gamma^3\psi(x)$$

$$\mathcal{M}_{NH1} = im_5\overline{\psi}(x)\gamma^5\psi(x) + m_3\overline{\psi}(x)\gamma^3\psi(x)$$

$\mathcal{PT}$ **- symmetric**

$$\mathcal{M}_{NH2} = m_5\overline{\psi}(x)\gamma^5\psi(x) + im_3\overline{\psi}(x)\gamma^3\psi(x)$$

$\mathcal{PT}$ **- breaking**

Let us explore, using the CJT approach, the possibility of the dynamic appearance of a non-Hermitian and $\mathcal{PT}$ symmetric mass term $\mathcal{M}_{NH1}$

$$\overline{S^{-1}} = i(\hat{p} + i\gamma^5 m_5 + \gamma^3 m_3), \quad \text{i.e.} \quad \overline{S} = -i\frac{\hat{p} + i\gamma^5 m_5 + \gamma^3 m_3}{p^2 - (m_5^2 - m_3^2)}$$

where m_3 and m_5 are real quantities.

Suppose that $m_5^2 \geq m_3^2$

- at $g > 0$ its global minimum lies at the point $m_5 = m_3 = 0$, and dynamical mass generation is absent
- at $g < 0$ the global minimum is achieved at arbitrary (m_3, m_5) point such that $m_5^2 - m_3^2 = g^2$

$$m_3 = |g| \sinh\beta, \quad m_5 = |g| \cosh\beta$$

Note that such a structure of the global minimum point of the model appears due to the emergent symmetry of the CJT effective potential with respect to non-Unitary transformations

$$\begin{pmatrix} m_5 \\ m_3 \end{pmatrix} \to \begin{pmatrix} \cosh\beta & \sinh\beta \\ \sinh\beta & \cosh\beta \end{pmatrix} \begin{pmatrix} m_5 \\ m_3 \end{pmatrix}.$$

the non-Hermitian but $\mathcal{PT}$-odd mass term $\mathcal{M}_{NH2}$

$$\overline{S^{-1}} = i(\hat{p} + \gamma^5 m_5 + i\gamma^3 m_3), \quad \text{i.e.} \quad \overline{S} = -i\frac{\hat{p} + \gamma^5 m_5 + i\gamma^3 m_3}{p^2 - (m_3^2 - m_5^2)}$$

where m_3 and m_5 are real quantities.

Suppose that $m_5^2 \leq m_3^2$

It can be shown in exactly the same way that for the same dependence of the bare coupling constant G vs Λ, there exists a nontrivial solution of the renormalized stationary (Dyson-Schwinger) equation

- at $g < 0$ of the non-Hermitian but $\mathcal{PT}$-odd mass term $\mathcal{M}_{NH2}$ in the model.

$$m_3 = |g| \cosh \omega, \quad m_5 = |g| \sinh \omega$$

$\sqrt{m_3^2 - m_5^2} \equiv |g|$

fermion pole mass $M_F = \sqrt{m_3^2 - m_5^2} \equiv |g|$.

at $\lambda > \lambda_{35}$ – **non-Hermitian mass terms could be dynamically generated**

where $\lambda_{35} = 2N\pi^2$

Since $\lambda_{35} \to \infty$ at $N \to \infty$

we may conclude that **in the limit of large** N there is no **dynamical generation of non-Hermitian mass terms**

Spontaneous symmetry breaking in Thirring model

Lagrangian of Thirring model

$$L = \overline{\Psi}_k \gamma^\nu i \partial_\nu \Psi_k - \frac{G}{2N} \left(\overline{\Psi}_k \gamma^\mu \Psi_k \right) \left(\overline{\Psi}_l \gamma_\mu \Psi_l \right)$$

$k = 1, ..., N$ is a number of flavours

It is invariant under the transformations

$$U(N),\ U(2N), \quad \Gamma_3,\ \Gamma_5,\ \mathcal{P}$$

The stationary equation for the CJT effective action

$$-i\Big[S^{-1}\Big]^{\beta}_{\alpha}(x,y) - D^{\beta}_{\alpha}(x,y) = -G\big(\gamma^{\rho}\big)^{\beta}_{\alpha}\mathrm{tr}\big[\gamma_{\rho}S(x,y)\big]\delta^{3}(x-y)$$

$$+\frac{G}{N}\big[\gamma^{\rho}S(x,y)\gamma_{\rho}\big]^{\beta}_{\alpha}\delta^{3}(x-y)$$

$S(x,y)$ is a translationary invariant operator

$$-i\overline{(S^{-1})^{\beta}_{\alpha}}(p) - (\hat{p})^{\beta}_{\alpha} \quad = \quad -G\big(\gamma^{\rho}\big)^{\beta}_{\alpha}\int\frac{d^{3}q}{(2\pi)^{3}}\ \mathrm{tr}\big[\gamma_{\rho}\overline{S}(q)\big]$$

$$+\frac{G}{N}\int\frac{d^{3}q}{(2\pi)^{3}}\ \big[\gamma^{\rho}\overline{S}(q)\gamma_{\rho}\big]^{\beta}_{\alpha}$$

Let us explore, using the CJT approach, the possibility of mass term

$$i\overline{S^{-1}} = (\hat{p} + m_D + m_H\tau) \;\; = \begin{pmatrix} \tilde{p} + m_D + m_H, & 0 \\ 0 \;\;, & -\tilde{p} + m_D - m_H \end{pmatrix}$$

i. e.

$$\overline{S}(p) = -i \begin{pmatrix} \frac{\tilde{p} - m_D - m_H}{p^2 - (m_D + m_H)^2}, & 0 \\ 0 \;\;, & \frac{-\tilde{p} - m_D + m_H}{p^2 - (m_D - m_H)^2} \end{pmatrix}$$

CJT effective potential

$$V(S)\int d^3x \equiv -\Gamma(S)\Big|_{\text{transl.}-\text{inv. S(x,y)}}$$

for Thirring model is

CJT effective potential has the following symmetries

$$m_D \to -m_D,\;\; m_H \to -m_H \qquad m_H \leftrightarrow m_D$$

- at $g > 0$ its global minimum lies at the point $m_D = 0$ and $m_H = 0$, and no dynamical mass generation

- at $g < 0$ the global minimum is achieved at

$$(m_D = -g/2, m_H = 0) \quad \text{and} \quad (m_D = 0, m_H = -g/2)$$

- at $\lambda > \lambda_0$ — **symmetries of the model are broken**
- at $\lambda < \lambda_0$ — **symmetry of the model remains intact**

$\lambda < \lambda_0$ - symmetry of the model remains intact

Since $\lambda_0 \to \infty$ at $N \to \infty$

we may conclude that **in the limit of large** N there is no **dynamical mass generation** in Thirring model

The global minimum is achieved at

$$(m_D = -g/2, m_H = 0) \quad \text{and} \quad (m_D = 0, m_H = -g/2)$$

and is degenerate

Spontaneous symmetry breaking in generalized Thirring model

Lagrangian of Thirring model

$$L = \overline{\Psi}_k \gamma^\nu i \partial_\nu \Psi_k - \frac{G_v}{2N} \left(\overline{\Psi}_k \gamma^\mu \Psi_k \right) \left(\overline{\Psi}_k \gamma_\mu \Psi_k \right) +$$

$$+ \frac{G_s}{2N} \left(\overline{\Psi}_k \tau \Psi_k \right)^2$$

$k = 1, ..., N$ is a number of flavours, and $\tau = -i\gamma^3\gamma^5$

It is invariant under the transformations

$$U(N), \; U(2N), \quad \Gamma_3, \; \Gamma_5, \; \mathcal{P}$$

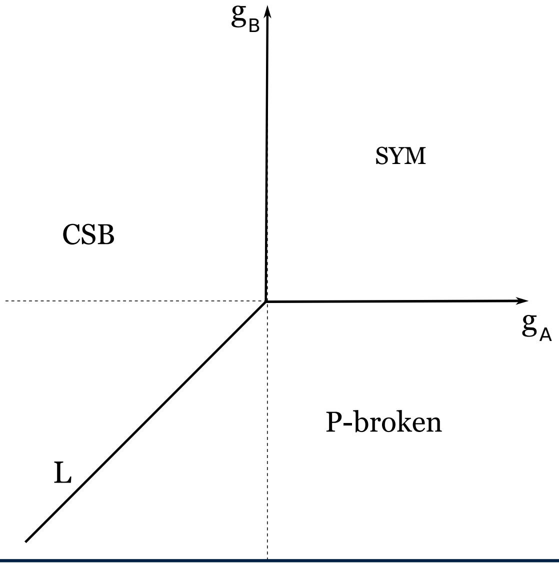
Let us explore, using the CJT approach, the possibility of mass term

$$i\overline{S^{-1}} = (\hat{p} + m_D + m_H\tau) \;\; = \begin{pmatrix} \tilde{p} + m_D + m_H, & 0 \\ 0 \;\;, & -\tilde{p} + m_D - m_H \end{pmatrix}$$

i. e.

$$\overline{S}(p) = -i \begin{pmatrix} \frac{\tilde{p} - m_D - m_H}{p^2 - (m_D + m_H)^2}, & 0 \\ 0 \;\;, & \frac{-\tilde{p} - m_D + m_H}{p^2 - (m_D - m_H)^2} \end{pmatrix}$$

Phase diagram of generalized Thirring model

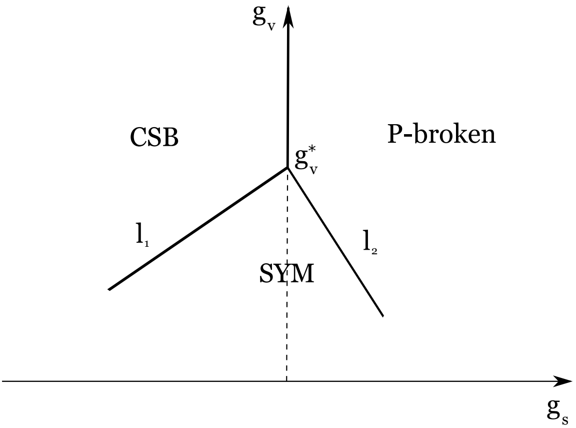


Phase diagram of generalized Thirring model

In terms of other more natural and physically acceptable dimensionless coupling constants

$g_s \equiv \Lambda G_s$

$g_v \equiv \Lambda G_v$



Spontaneous non-hermiticity in Thirring model

Let us explore, using the CJT approach, the possibility of mass term

$$\mathcal{M}_H = \overline{\Psi}_k (m_H \tau + m_D + i m_5 \gamma^5 + i m_3 \gamma^3) \Psi_k$$

i. e.

$$\overline{S^{-1}}(p) = i \left(\hat{p} + m_H \tau + m_D + i m_5 \gamma^5 + i m_3 \gamma^3 \right)$$

- at $g > 0$ its global minimum lies at the point

  $m_D = 0$, $m_H = 0$, $m_3 = 0$ and $m_5 = 0$

  and no dynamical mass generation

- at $g < 0$ the global minimum is achieved at

  $(m_H = -g/2, \Sigma = 0)$ and $(m_H = 0, \Sigma = -g/2)$

$$\Sigma^2 \equiv m_D^2 + m_5^2 + m_3^2 = g^2/4$$

the non-Hermitian mass term $\mathcal{M}_{NH}$

$$\mathcal{M}_{NH} = \overline{\Psi}_k(m_H\tau + \eta \cdot m_D + \vartheta \cdot im_5\gamma^5 + \kappa \cdot im_3\gamma^3)\Psi_k$$

where each of the multipliers η, ϑ, κ is either 1 or i and all mass parameters m_H, m_D, m_5, m_3 are real quantities

- at $g > 0$

$$
\begin{array}{lll}
\text{(i)} & \overline{\Psi}_k(m_D - m_5\gamma^5 + im_3\gamma^3)\Psi_k & \text{where} \quad m_D^2 + m_3^2 = m_5^2, \\
\text{(ii)} & \overline{\Psi}_k(im_D + im_5\gamma^5 + im_3\gamma^3)\Psi_k & \text{where} \quad m_5^2 + m_3^2 = m_D^2, \\
\text{(iii)} & \overline{\Psi}_k(im_D - m_5\gamma^5 + im_3\gamma^3)\Psi_k & \text{where} \quad m_D^2 + m_5^2 = m_3^2, \\
\text{(iv)} & \overline{\Psi}_k(im_D + im_5\gamma^5 - m_3\gamma^3)\Psi_k & \text{where} \quad m_D^2 + m_3^2 = m_5^2, \\
\text{(v)} & \overline{\Psi}_k(m_D - m_5\gamma^5 - m_3\gamma^3)\Psi_k & \text{where} \quad m_5^2 + m_3^2 = m_D^2
\end{array}
$$

the corresponding to non-Hermitian mass term $\overline{\Psi}_k(m_D + im_5\gamma^5 - m_3\gamma^3)\Psi_k$ fermion propagator $\overline{S}(p)$ looks like

$$
\overline{S}(p) = -i(\hat{p} + m_D + i\gamma^5 m_5 - \gamma^3 m_3)/p^2
$$

- at $g < 0$

In the case $m_H = -g/2$ and $\widetilde{\Sigma} = 0$

$$
\begin{array}{lll}
\text{(i)} & \overline{\Psi}_k(m_H\tau + m_D + im_5\gamma^5 - m_3\gamma^3)\Psi_k & \text{with} \quad m_D^2 + m_5^2 = m_3^2, \\
\text{(ii)} & \overline{\Psi}_k(m_H\tau + m_D - m_5\gamma^5 + im_3\gamma^3)\Psi_k & \text{with} \quad m_D^2 + m_3^2 = m_5^2, \\
\text{(iii)} & \overline{\Psi}_k(m_H\tau + im_D - m_5\gamma^5 + im_3\gamma^3)\Psi_k & \text{with} \quad m_D^2 + m_5^2 = m_3^2, \\
\text{(iv)} & \overline{\Psi}_k(m_H\tau + im_D + im_5\gamma^5 - m_3\gamma^3)\Psi_k & \text{with} \quad m_D^2 + m_3^2 = m_5^2, \\
\text{(v)} & \overline{\Psi}_k(m_H\tau + im_D + im_5\gamma^5 + im_3\gamma^3)\Psi_k & \text{with} \quad m_5^2 + m_3^2 = m_D^2, \\
\text{(vi)} & \overline{\Psi}_k(m_H\tau + m_D - m_5\gamma^5 - m_3\gamma^3)\Psi_k & \text{with} \quad m_5^2 + m_3^2 = m_D^2
\end{array}
$$

the corresponding fermion propagator $\overline{S}(p)$ looks like

$$
\overline{S}(p) = -i\,\frac{\hat{p} + im_D + i\gamma^5 m_5 - \gamma^3 m_3}{p^2 + M_F^2}
$$

where $M_F = |g|/2$

Conclusions and Thanks for the Attention

- There has been studied the possibility of the dynamical appearance of both Hermitian and non-Hermitian mass terms in the originally Hermitian massless (2+1)-dimensional GN model
- the effect of spontaneous non-Hermiticity can be detected only outside the large-N expansion technique
- There has been shown that parity breaking Haldane mass can be generated dynamically in the model