Decay Width Ratios of Exotic Doubly-Heavy Hadrons in Diquark Model

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based on the paper A. Ali et al., JHEP 10 (2019) 256 [arXiv:1907.06507]

Outline

- Introduction
- Double well potential in hidden-charm tetraquarks
- Double well potential in hidden-charm pentaquarks
- Conclusions

Introduction

- At present, production, properties, and decays of heavy hadrons are intensively studied both experimentally and theoretically
- Many of them are interpreted as exotic
  - Tetraquark states:
    ($c\bar{s}d\bar{u}$): $T_{c\bar{s}}(2900)^0$, $T_{c\bar{s}}(2900)^{++}$
    ($\bar{c}c\bar{q}q$): $\chi_{c1}(3872)$, $\chi_{c1}(4140)$, $\psi(4230)$, $\psi(4360)$, etc
    ($\bar{c}c\bar{d}u$): $Z_c(3900)^\pm$, $Z_c(4020)^\pm$, $Z_c(4050)^\pm$, etc
    ($\bar{c}c\bar{s}u$): $Z_{cs}(4000)^\pm$, $Z_{cs}(4220)^\pm$
    ($cc\bar{d}\bar{u}$): $T_{cc}(3875)^+$
    ($\bar{c}c\bar{c}c$): $T_{\bar{c}c\bar{c}c}(6900)$, etc
    ($\bar{b}b\bar{d}u$): $Z_b(10610)^\pm$, $Z_b(10650)^\pm$
    ($\bar{b}b\bar{q}q$): $Y_b(10753)$
  - Pentaquark states:
    ($\bar{c}cuud$): $P_c(4312)^+$, $P_c(4335)^+$, $P_c(4440)^+$, $P_c(4457)^+$
    ($\bar{c}cuds$): $P_{cs}^{\Lambda}(4338)^0$, $P_{cs}^{\Lambda}(4459)^0$
- Waiting newcomers from LHCb, Belle-II, BES-III, Atlas, CMS

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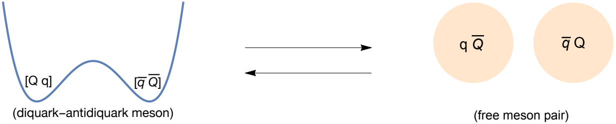
Double Well Potential in Tetraquarks

- Hypothesis: tetraquark can plausibly be represented by two diquarks in double well potential separated by a barrier [L. Maiani, A.D. Polosa & V. Riquer, Phys. Lett. B778 (2018) 247]
- There are two length scales:
  diquark radius R_{Qq} & tetraquark radius R_{4q}
- Assumed to be well separated $\lambda = R_{4q}/R_{Qq} \geq 3$
- Tunneling transitions of quarks result into strong decays
- Diquark radius R_{Qq} in tetraquark can be different from diquark radius R_{Qq}^{baryon} in baryon
- Increase of experimental resolution and statistics is crucial to support or disprove this hypothesis



Hidden-Charm Tetraquark Decays to D-Mesons

- Diquark-antidiquark system can rearrange itself into a pair of color singlets by exchanging quarks through tunneling transition
- Small overlap between constituent quarks in different wells suppresses quark-antiquark direct annihilation
- Two stage process:
  1. switch of quark and antiquark among two wells
  2. evolution of quark-antiquark pairs into mesons
- Including diquark spins (subscripts), consider the states:

$$\Psi_{\mathcal{D}}^{(1)} = [cu]_0(x)\,[\bar{c}\bar{u}]_1(y), \qquad \Psi_{\mathcal{D}}^{(2)} = \mathcal{C}\Psi_{\mathcal{D}}^{(1)} = [cu]_1(y)\,[\bar{c}\bar{u}]_0(x)$$

- After Fierz rearrangements of color and spin indices, in evident meson notations

$$\Psi_{\mathcal{D}}^{(1)} = A\,D^0\,\bar{D}^{*0} - B\,D^{*0}\,\bar{D}^0 + iC\,D^{*0}\times\bar{D}^{*0}$$
$$\Psi_{\mathcal{D}}^{(2)} = B\,D^0\,\bar{D}^{*0} - A\,D^{*0}\,\bar{D}^0 - iC\,D^{*0}\times\bar{D}^{*0}$$

- A, B, and C are non-perturbative coefficients associated to barrier penetration amplitudes for different total spins of u and $\bar{u}$

Hidden-Charm Tetraquark Decays to Charmonia

- Tunneling transition of light quarks

$$X_u \sim \frac{1}{\sqrt{2}}\left[\Psi_{\mathcal{D}}^{(1)} + \Psi_{\mathcal{D}}^{(2)}\right] = \frac{A+B}{\sqrt{2}}\left[D^0\bar{D}^{*0} - D^{*0}\bar{D}^0\right]$$

- Tunneling transition of heavy quarks

$$X_u \sim a\, iJ/\psi \times (\omega + \rho^0)$$

- Tunneling amplitude in leading semiclassical approximation, $\mathcal{A}_M \sim e^{-\sqrt{2ME}\ell}$, where E and ℓ are barrier height and extension
- For constituent quark masses, m_q and m_c, $E = 100$ MeV and $\ell = 2$ fm, the ratio of amplitides squared

$$R = [a/(A+B)]^2 \sim (\mathcal{A}_{m_c}/\mathcal{A}_{m_q})^2 \sim 10^{-3}$$

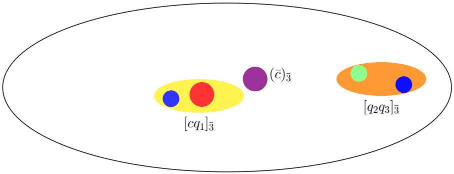
- With decay momenta $p_\rho \simeq 124$ MeV and $p_{DD^*} \simeq 2$ MeV

$$\frac{\Gamma(X(3872) \to J/\psi\, \rho)}{\Gamma(X(3872) \to D\bar{D}^*)} = \frac{p_\rho}{p_{DD^*}} R \sim 0.1$$

- Experiment [PDG]: $B_{\rm exp}(X(3872) \to J/\psi\, \rho) = (3.8 \pm 1.2)\%$
  $B_{\rm exp}(X(3872) \to D\bar{D}^*) = (37 \pm 9)\%$

Existing theoretical models of pentaquarks

- Several dynamical models of pentaquarks are suggested:
  1. baryon-meson model (molecular pentaquark);
  2. triquark-diquark model;
  3. diquark-diquark-antiquark model;
  4. ...
- For example, in the diquark-diquark-antiquark model, dynamics is determined by interaction of light diquark $[q_2 q_3]$, heavy diquark $[c q_1]$ and *c*-antiquark, where q_i is one of the light *u*-, *d*- or *s*-quarks [A. Ali et al., JHEP 10 (2019) 256]



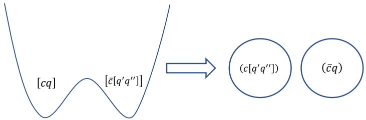
Double Well Potential in Pentaquarks

- Hypothesis: pentaquark can be represented by heavy diquark and heavy triquark in double well potential separated by barrier [A. Ali et. al., JHEP 10 (2019) 256]
- There are two triquark-diquark representations

$$\Psi_1^D = \frac{1}{\sqrt{3}} \left[\frac{1}{\sqrt{2}} \epsilon_{ijk} \bar{c}^i \left[\frac{1}{\sqrt{2}} \epsilon^{jlm} c_l q_m \right] \right] \left[\frac{1}{\sqrt{2}} \epsilon^{knp} q'_n q''_p \right] \equiv [\bar{c}\,[cq]]\,[q' q'']$$

$$\Psi_2^D = \frac{1}{\sqrt{3}} \left[\frac{1}{\sqrt{2}} \epsilon_{ikj} \bar{c}^i \left[\frac{1}{\sqrt{2}} \epsilon^{knp} q'_n q''_p \right] \right] \left[\frac{1}{\sqrt{2}} \epsilon^{jlm} c_l q_m \right] \equiv [\bar{c}\,[q' q'']]\,[cq]$$

- From color algebra, these states are related, $\Psi_2^D = -\Psi_1^D$, but other internal dynamical properties can be different
- Color connection of quarks in Ψ_1^D is used for mass spectrum
- Ψ_2^D color structure is suitable for study strong decays

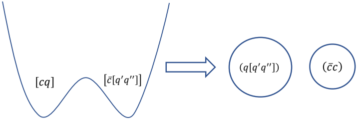


Double Well Potential in Pentaquarks

- Color-singlet combinations are meson-baryon alternatives

$$\Psi_1^H = \left(\frac{1}{\sqrt{3}}\,\bar{c}^i c_i\right)\left[\frac{1}{\sqrt{6}}\,\epsilon^{jkl} q_j q'_k q''_l\right] \equiv (\bar{c}c)\,[qq'q'']$$
$$\Psi_2^H = \left(\frac{1}{\sqrt{3}}\,\bar{c}^i q_i\right)\left[\frac{1}{\sqrt{6}}\,\epsilon^{jkl} c_j q'_k q''_l\right] \equiv (\bar{c}q)\,[cq'q'']$$
$$\Psi_3^H = \left(\frac{1}{\sqrt{3}}\,\bar{c}^i q'_i\right)\left[\frac{1}{\sqrt{6}}\,\epsilon^{jkl} c_j q_k q''_l\right] \equiv (\bar{c}q')\,[cqq'']$$
$$\Psi_4^H = \left(\frac{1}{\sqrt{3}}\,\bar{c}^i q''_i\right)\left[\frac{1}{\sqrt{6}}\,\epsilon^{jkl} c_j q_k q'_l\right] \equiv (\bar{c}q'')\,[cqq']$$

- Ψ_1^H and Ψ_2^H only satisfy HQS condition
- Light $[q'q'']$-diquark is transmitted intact, retaining its spin quantum number, from b-baryon to pentaquark



Double Well Potential in Pentaquarks

- Keeping the color of the light diquark unchanged, convolution of two Levi-Civita tensors entering the triquark gives

$$\Psi_1^D = -\frac{\sqrt{3}}{2}\left[\Psi_1^H + \Psi_2^H\right],$$

- Color reconnection is not enough to reexpress pentaquark operator as direct product of the meson and baryon operators
- Spins of quarks and diquarks should be projected onto definite hadronic spin states
- One needs to know Dirac structure of pentaquark operators to undertake the Fierz transformations in Dirac space
- Exemplify this by considering $P_c(4312)$ pentaquark

Mass Predictions for Unflavored Pentaquarks

J^P	AAAPR	AAAR	J^P	AAAPR	AAAR
	$S_{ld} = 0, L = 0$			$S_{ld} = 1, L = 1$	
$1/2^-$	3830 ± 34	4086 ± 42	$1/2^+$	4144 ± 37	3970 ± 50
	4150 ± 29	4162 ± 38		4209 ± 37	4174 ± 44
$3/2^-$	4240 ± 29	4133 ± 55		4465 ± 32	4198 ± 50
	$S_{ld} = 1, L = 0$			4530 ± 32	4221 ± 40
$1/2^-$	4026 ± 31	4119 ± 42		4564 ± 33	4240 ± 50
	4346 ± 25	4166 ± 38		4663 ± 32	4319 ± 43
	4436 ± 25	4264 ± 41	$3/2^+$	4187 ± 37	
$3/2^-$	4026 ± 31	4072 ± 40		4250 ± 37	
	4346 ± 25	4300 ± 40		4508 ± 32	
	4436 ± 25	4342 ± 40		4570 ± 32	
$5/2^-$	4436 ± 25	4409 ± 40		4511 ± 33	
	$S_{ld} = 0, L = 1$			4566 ± 32	
$1/2^+$	4030 ± 39	4030 ± 62		4656 ± 32	
	4351 ± 35	4141 ± 44	$5/2^+$	4260 ± 37	4450 ± 44
	4430 ± 35	4217 ± 40		4581 ± 32	4524 ± 41
$3/2^+$	4040 ± 39			4601 ± 32	4678 ± 44
	4361 ± 35			4656 ± 32	4720 ± 44
	4440 ± 35		$7/2^+$	4672 ± 32	
$5/2^+$	4457 ± 35	4510 ± 57			

Double Well Potential in Pentaquarks

- Diquark-diquark-antiquark operators with spinless heavy and light diquarks

$$\Psi_1^{H(1)}(x,y) = \frac{1}{3}\left(\tilde{c}^i(x)\,\sigma_2\right)\left(c_i(y)\,\sigma_2\,q_k(y)\right)\,d_0^k(x)$$
$$\Psi_2^{H(1)}(x,y) = \frac{1}{3}\left(\tilde{c}^i(x)\,\sigma_2\right)\left(c_k(y)\,\sigma_2\,q_i(y)\right)\,d_0^k(x)$$

- For the lowest lying pentaquark, $q = u$ and $d_0 = [u\,C\,\gamma_5\,d]$, being scalar diquark
- Quarks are considered in the non-relativistic limit
- After Fierz transformation of Pauli matrices and suppressing position dependence, they can be rewritten in terms of hadrons

$$\Psi_1^{H(1)} = -\frac{i}{\sqrt{2}}\left[a\,\eta_c + b\,(\boldsymbol{\sigma}\,\boldsymbol{J}/\boldsymbol{\psi})\right]p, \quad \Psi_2^{H(1)} = -\frac{i}{\sqrt{2}}\left[A\,\bar{D}^0 + B\left(\boldsymbol{\sigma}\,\bar{\boldsymbol{D}}^{*0}\right)\right]\Lambda_c^+$$

- A and B (a and b) are non-perturbative coefficients associated with barrier penetration amplitudes for light (heavy) quark
- They are equal in the limit of naive Fierz coupling

Double Well Potential in Pentaquarks

- Similarly, diquark-diquark-antiquark operators containing heavy diquark with $S_{hd} = 1$ and light diquark $S_{ld} = 0$

$$\boldsymbol{\Psi}_1^{H(2)}(x,y) = \frac{1}{3}\left(\tilde{c}^i(x)\,\sigma_2\right)\left(c_i(y)\,\sigma_2\,\boldsymbol{\sigma}\,q_k(y)\right)d_0^k(x)$$

$$\boldsymbol{\Psi}_2^{H(2)}(x,y) = \frac{1}{3}\left(\tilde{c}^i(x)\,\sigma_2\right)\left(c_k(y)\,\sigma_2\,\boldsymbol{\sigma}\,q_i(y)\right)d_0^k(x)$$

- Being direct product of spinor and vector, they need to be devided into two states with spins $J = 1/2$ and $J = 3/2$
- For $P_c(4312)$ interpreted as $J^P = 3/2^-$ pentaquark, decompositions in term of hadrons are as follows

$$\boldsymbol{\Psi}_1^{H(3/2)} = \frac{i\sqrt{2}}{3}\left\{b'\,\boldsymbol{J}/\psi - 2ic'\left[\boldsymbol{\sigma}\times\boldsymbol{J}/\psi\right]\right\}p$$

$$\boldsymbol{\Psi}_2^{H(3/2)} = -\frac{i\sqrt{2}}{3}\left\{B'\,\bar{\boldsymbol{D}}^{*0} - 2iC'\left[\boldsymbol{\sigma}\times\bar{\boldsymbol{D}}^{*0}\right]\right\}\Lambda_c^+$$

- $P_c(4312)$ is mainly decaying either to $J/\psi\, p$ final state, in which it was observed, or to $\Lambda_c^+\,\bar{D}^{*0}$

Hidden-Charm Pentaquark Decays

- Tunneling amplitude in leading semiclassical approximation, $\mathcal{A}_M \sim e^{-\sqrt{2ME}\ell}$, where E and ℓ are barrier height and extension
- For constituent quark masses, m_u and m_c, $E = 100$ MeV and $\ell = 2$ fm, the ratio of amplitides squared

$$R_{\rm penta} = \frac{|b'|^2 + 4|c'|^2}{|B'|^2 + 4|C'|^2} \sim \left(\frac{\mathcal{A}_{m_c}}{\mathcal{A}_{m_u}}\right)^2 \sim 10^{-3} \sim R$$

- With decay momenta $p_p \simeq 660$ MeV and $p_{\Lambda_c} \simeq 200$ MeV

$$\frac{\Gamma(P_c(4312) \to J/\psi\, p)}{\Gamma(P_c(4312) \to \Lambda_c^+ \bar{D}^{*0})} = \frac{p_p}{p_{\Lambda_c}} R_{\rm penta} \sim 10^{-3}$$

- If this approach is correct, $P_c(4312)$ should be searched in $\Lambda_b^0 \to \Lambda_c^+ \bar{D}^{*0} K^-$ decay
- This can also be applied to decays of $P_{cs}(4459)$ pentaquark

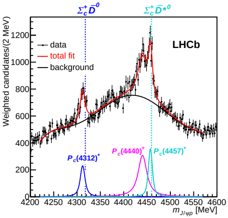
Summary

- Quark-Diquark approach is working quite successful in getting mass predictions of heavy baryons and doubly-heavy exotic hadrons
- Decay width of tetraquarks with hidden charm or bottom can be explained within the diquark model due to a barrier between heavy diquark and heavy antidiquark
- Decay width of pentaquarks with hidden charm or bottom can be also explained within the quark-diquark model due to a barrier between heavy diquark and heavy triquark
- If this approach is correct, $P_c(4312)$-state, considered as a ground-state pentaquark, should be also searched in $\Lambda_b^0 \to \Lambda_c^+ \bar{D}^{*0} K^-$ decay with good chances to be found

Backup Slides

- Λ_b-baryon decay $\Lambda_b \to p + J/\psi + K^-$ was studied on 9 times more data based on Run 1 and 2 than on Run 1
- Three narrow peaks were observed in $m_{J/\psi\, p}$ distribution

State	Mass [MeV]	Width [MeV]	(95% CL)	$\mathcal{R}$ [%]
$P_c(4312)^+$	$4311.9 \pm 0.7^{+6.8}_{-0.6}$	$9.8 \pm 2.7^{+3.7}_{-4.5}$	(< 27)	$0.30 \pm 0.07^{+0.34}_{-0.09}$
$P_c(4440)^+$	$4440.3 \pm 1.3^{+4.1}_{-4.7}$	$20.6 \pm 4.9^{+8.7}_{-10.1}$	(< 49)	$1.11 \pm 0.33^{+0.22}_{-0.10}$
$P_c(4457)^+$	$4457.3 \pm 0.6^{+4.1}_{-1.7}$	$6.4 \pm 2.0^{+5.7}_{-1.9}$	(< 20)	$0.53 \pm 0.16^{+0.15}_{-0.13}$

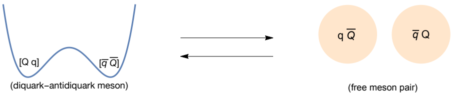


- $P_c(4312)$ is a new resonance
- $P_c(4450)$ splits into $P_c(4440)$ and $P_c(4457)$
- $P_c(4380)$ under question
- Spin-parities are unknown yet

- Theoretical Interpretations of 3 Narrow Pentaquarks: Molecular, Hadrocharmonium & Compact Multiquark Pictures

Double Well Potential in Tetraquarks

- Hypothesis: tetraquark can plausibly be represented by two diquarks in double well potential separated by a barrier
[L. Maiani, A.D. Polosa & V. Riquer, Phys. Lett. B778 (2018) 247]
- Arguments in favor:
  1. At large distances, diquarks interact like QCD point charges
  2. Confining forces are the same as for quark and antiquark
  3. At shorter distances, forces among constituents in diquarks (e. g. attraction between quarks and antiquarks) reduce the diquark binding energies
  4. These effects increase at decreasing distance and produce repulsion among diquark and antidiquark, i. e. increasing component in potential at decreasing distance
  5. If this effect wins against the decrease due to the color attraction, the barrier is produced



List of hadrons observed at the LHCb

P. Koppenburg [LHCb Collab.], LHCb-FIGURE-2021-001, 2021 (2023 update)