

ОФН РАН

Исследование общего потенциала Хиггса и  
ренормгрупповые потоки

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Елисов М.В., Долгополов М.В.

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Физика бозонов Хиггса: ATLAS и CMS:  $m_h = 125.08 \pm 0.12$ . ГэВ

Свойства. Теоретические вопросы:

Интенсивности самодействия (каплинги) +

новые частицы в секторе Хиггса ?

форма потенциала?

Расширенная  $\phi^4$  модель и двухдублетная модель. CP-нарушение.

Ограничения на параметризацию эффективных каплингов.

Критические точки асимптотического поведения, анализ ренормгрупповых потоков.

РГ потоки каплингов для модели с двумя скалярными полями.

Критические явления при эволюции поверхности минимумов потенциала Хиггса в системе с двумя дублетами.

В общей ДДМ<sup>1</sup> вводится система полей:

$$\phi_1 = \begin{pmatrix} -i\omega_1^+ \\ \frac{1}{\sqrt{2}}(v_1 + v_1 + i\chi_1) \end{pmatrix}, \quad \phi_2 = e^{i\xi} \begin{pmatrix} -i\omega_2^+ \\ \frac{1}{\sqrt{2}}(v_2 e^{i\xi} + v_2 + i\chi_2) \end{pmatrix}, \quad (1)$$

$$\langle \Phi_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad \langle \Phi_2 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 \end{pmatrix}. \quad (2)$$

$$v^2 = v_1^2 + v_2^2 = 246^2 \Gamma \Delta B^2$$

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<sup>1</sup>Akhmetzyanova E.N., Dolgoplov M.V, Dubinin M.N. Higgs bosons in the two-doublet model with CP violation // Physical Review D 2005. — Vol. 71. Issue 7. — P. 1-24 & CALC 2003 & SQS03 Proc.

## Мотивация. Эффективный потенциал общей модели

The most general renormalizable hermitian  $SU(2) \otimes U(1)$  invariant potential:

$$\begin{aligned} U(\Phi_1, \Phi_2) = & -\mu_1^2(\Phi_1^\dagger \Phi_1) - \mu_2^2(\Phi_2^\dagger \Phi_2) - \mu_{12}^2(\Phi_1^\dagger \Phi_2) - \mu_{12}^{*2}(\Phi_2^\dagger \Phi_1) + \\ & + \frac{\lambda_1}{2}(\Phi_1^\dagger \Phi_1)^2 + \frac{\lambda_2}{2}(\Phi_2^\dagger \Phi_2)^2 + \lambda_3(\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4(\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \\ & + \frac{\lambda_5}{2}(\Phi_1^\dagger \Phi_2)(\Phi_1^\dagger \Phi_2) + \frac{\lambda_5^*}{2}(\Phi_2^\dagger \Phi_1)(\Phi_2^\dagger \Phi_1) + \lambda_6(\Phi_1^\dagger \Phi_1)(\Phi_1^\dagger \Phi_2) + \\ & + \lambda_6^*(\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_1) + \lambda_7(\Phi_2^\dagger \Phi_2)(\Phi_1^\dagger \Phi_2) + \lambda_7^*(\Phi_2^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) \end{aligned}$$

with effective real parameters  $\mu_1^2, \mu_2^2, \lambda_1, \dots, \lambda_4$  and complex parameters  $\mu_{12}^2, \lambda_5, \lambda_6, \lambda_7$ .

В случае приближенном рассматривают:  $\mu_{12} = \mu_2 = \lambda_6 = \lambda_7 = 0$ , где  $\phi_1, \phi_2$  - скалярные поля

Во общем случае:  $\phi_1, \phi_2$  - дублеты

## Boundary conditions

On the scale of the superpartners  $M_{\text{SUSY}}$



The effective potential method,  
the method of Feynman diagrams  
& finite-temperature corrections

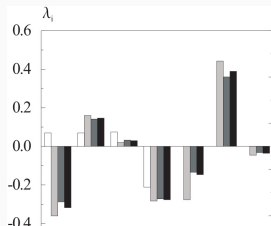
$$\lambda_1^{\text{SUSY}} = \lambda_2^{\text{SUSY}} = \frac{g_1^2 + g_2^2}{8}, \quad \lambda_3^{\text{SUSY}} = \frac{g_2^2 - g_1^2}{4},$$
$$\lambda_4^{\text{SUSY}} = -\frac{g_2^2}{2}, \quad \lambda_5^{\text{SUSY}} = \lambda_6^{\text{SUSY}} = \lambda_7^{\text{SUSY}} = 0.$$

The deviation from the parameters

$$\lambda_i = \lambda_i^{\text{SUSY}} - \Delta\lambda_i$$

Dolgoplov M., Dubinin M., Rykova E.  
potential. Journal of Modern Physics. 2011.

Pp. 301-322



The supersymmetric scalar potential of interaction of Higgs bosons with the third generation quark superpartners on the tree level has the form

$$\mathcal{V}^0 = \mathcal{V}_M + \mathcal{V}_\Gamma + \mathcal{V}_\Lambda + \mathcal{V}_{\tilde{Q}},$$

$$\mathcal{V}_M = (-1)^{i+j} m_{ij}^2 \Phi_i^\dagger \Phi_j + M_{\tilde{Q}}^2 (\tilde{Q}^\dagger \tilde{Q}) + M_{\tilde{U}}^2 \tilde{U}^* \tilde{U} + M_{\tilde{D}}^2 \tilde{D}^* \tilde{D},$$

$$\mathcal{V}_\Gamma = \Gamma_i^D (\Phi_i^\dagger \tilde{Q}) \tilde{D} + \Gamma_i^U (i \Phi_i^T \sigma_2 \tilde{Q}) \tilde{U} + \Gamma_i^D (\tilde{Q}^\dagger \Phi_i) \tilde{D}^* - \Gamma_i^U (i \tilde{Q}^\dagger \sigma_2 \Phi_i^*) \tilde{U}^*,$$

$$\mathcal{V}_\Lambda = \Lambda_{ik}^{jl} (\Phi_i^\dagger \Phi_j) (\Phi_k^\dagger \Phi_l) + (\Phi_i^\dagger \Phi_j) [\Lambda_{ij}^Q (\tilde{Q}^\dagger \tilde{Q}) + \Lambda_{ij}^U \tilde{U}^* \tilde{U} + \Lambda_{ij}^D \tilde{D}^* \tilde{D}] + \bar{\Lambda}_{ij}^Q (\Phi_i^\dagger \tilde{Q}) (\tilde{Q}^\dagger \Phi_j) + \frac{1}{2} [\Lambda \epsilon_{ij} (i \Phi_i^T \sigma_2 \Phi_j) \tilde{D}^* \tilde{U} + \text{h.c.}], \quad i, j, k, l = 1, 2,$$

$\mathcal{V}_{\tilde{Q}}$  denotes the terms of interaction of four scalar quarks.

The most general Hermitian form of the renormalized  $SU(2) \times U(1)$  invariant potential for system of fields has the form:

$$\begin{aligned}
 U(\Phi_1, \Phi_2, S) = & -\mu_1^2(\Phi_1^\dagger \Phi_1) - \mu_2^2(\Phi_2^\dagger \Phi_2) - \mu_3^2 S^* S - (\mu_{12}^2(\Phi_1^\dagger \Phi_2) + \text{h.c.}) \\
 & + \frac{\lambda_1}{2}(\Phi_1^\dagger \Phi_1)^2 + \frac{\lambda_2}{2}(\Phi_2^\dagger \Phi_2)^2 + \lambda_3(\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4(\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \\
 & + \frac{\lambda_5}{2}(\Phi_1^\dagger \Phi_2)(\Phi_1^\dagger \Phi_2) + \frac{\lambda_5^*}{2}(\Phi_2^\dagger \Phi_1)(\Phi_2^\dagger \Phi_1) + \\
 & + \lambda_6(\Phi_1^\dagger \Phi_1)(\Phi_1^\dagger \Phi_2) + \lambda_7(\Phi_2^\dagger \Phi_2)(\Phi_1^\dagger \Phi_2) + \text{h.c.} \\
 & + k_1(\Phi_1^\dagger \Phi_1)S^* S + k_2(\Phi_2^\dagger \Phi_2)S^* S + (k_3(\Phi_1^\dagger \Phi_2)S^* S + \text{h.c.}) \\
 & k_4(S^* S)^2 + k_5(\Phi_1^\dagger \Phi_1)S + k_6(\Phi_2^\dagger \Phi_2)S + k_7(\Phi_1^\dagger \Phi_2)S + k_7^* (\Phi_2^\dagger \Phi_1)S^* + k_8 S^3.
 \end{aligned}$$

The supersymmetric scalar potential of interaction of Higgs bosons with the third generation quark superpartners on the tree level has the form  $V =$

$$\begin{aligned}
 & |y_u(\tilde{Q}\epsilon H_u)|^2 + |y_d(\tilde{Q}\epsilon H_d)|^2 + |y_u\tilde{u}_R^* H_u^0 - y_d\tilde{d}_R^* H_d^-|^2 + |y_d\tilde{d}_R^* H_d^0 - y_u\tilde{u}_R^* H_u^+|^2 - \\
 & -y_u(\tilde{u}_R\tilde{u}_L^* \lambda SH_d^0 + \tilde{u}_R\tilde{d}_L^* \lambda SH_d^- + \text{c.c.}) - y_d(\tilde{d}_R\tilde{d}_L^* \lambda SH_u^0 + \tilde{d}_R\tilde{d}_L^* \lambda SH_u^+ + \text{c.c.}) + \\
 & + \frac{g_2^2}{8}(4|H_d^\dagger \tilde{Q}|^2 - 2(H_d^\dagger H_d)(\tilde{Q}^\dagger \tilde{Q}) + 4|H_u^\dagger \tilde{Q}|^2 - 2(H_u^\dagger H_u)(\tilde{Q}^\dagger \tilde{Q})) + \\
 & + \frac{g_1^2}{2}\left(\frac{1}{6}(\tilde{Q}^\dagger \tilde{Q}) - \frac{2}{3}\tilde{u}_R^* \tilde{u}_R + \frac{1}{3}\tilde{d}_R^* \tilde{d}_R + \frac{1}{2}(H_u^\dagger H_u) - \frac{1}{2}(H_d^\dagger H_d)\right)^2 + \\
 & + (\tilde{u}_R^* y_u A_u (\tilde{Q}^T \epsilon H_u) - \tilde{d}_R y_d A_d (\tilde{Q}^T \epsilon H_d) + \text{c.c.})
 \end{aligned}$$



## Finite temperature corrections of squarks

In the early, high temperature stages of the Universe, the environment had a non-negligible matter and radiation density, making the hypotheses of conventional field theories impracticable. For that reason the methods of conventional field theories are no longer in use, and should be replaced by others, closer to thermodynamics, where the background state is a thermal bath. This field has been called field theory at finite temperature and it is extremely useful to study all phenomena which happened in the early Universe: phase transitions, inflationary cosmology, ...

In the finite temperature field theory Feynman diagrams with boson propagators, containing Matsubara frequencies  $\omega_n = 2\pi nT$  ( $n = 0, \pm 1, \pm 2, \dots$ ), lead to structures of the form

$$I[m_1, m_2, \dots, m_b] = T \sum_{n=-\infty}^{\infty} \int \frac{dk}{(2\pi)^3} \prod_{i=1}^b \frac{(-1)^b}{(k^2 + \omega_n^2 + m_i^2)}, \quad (3)$$

$k$  is the three-dimensional momentum in a system with the temperature  $T$ .

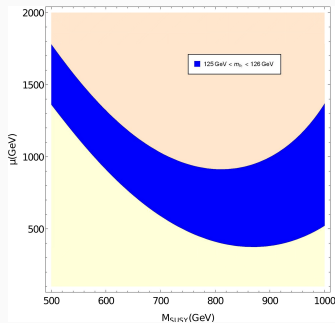
At  $n \neq 0$  the result is

$$I[m_1, m_2, \dots, m_b] = 2T (2\pi T)^{3-2b} \frac{(-1)^b \pi^{3/2}}{(2\pi)^3} \frac{\Gamma(b - 3/2)}{\Gamma(b)} S(M, b - 3/2), \quad (4)$$

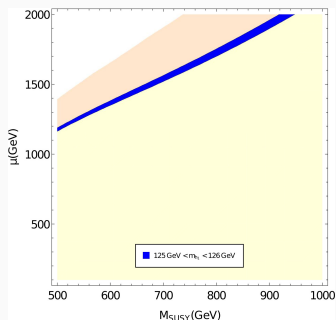
where

$$S(M, b - 3/2) = \int \{dx\} \sum_{n=1}^{\infty} \frac{1}{(n^2 + M^2)^{b-3/2}}, \quad M^2 \equiv \left(\frac{m}{2\pi T}\right)^2.$$

# Restrictions on the parameters of the MSSM



a)  $\tan\beta = 5$



b)  $\tan\beta = 50$

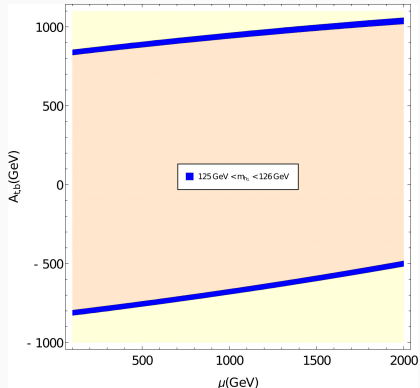
Contour plot in  $\mu - M_{\text{SUSY}}$  plane.

Selected region:  $125 \text{ GeV} < m_{h_1} < 126 \text{ GeV}$  (blue region).

Fixed parameters:

$$A_{t,b} = 1000 \text{ GeV}, m_{H^\pm} = 300 \text{ GeV}, \varphi = \frac{\pi}{3}.$$

# Restrictions on the parameters of the MSSM



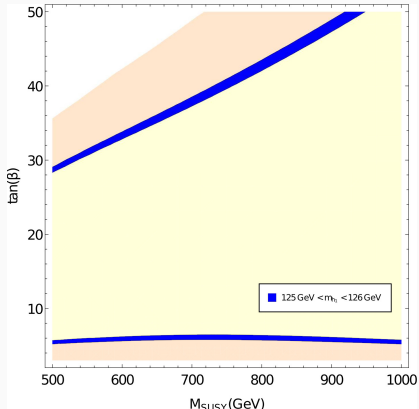
Contour plot in  $\mu - A_{t,b}$  plane.

Selected region:  $125 \text{ GeV} < m_{h_1} < 126 \text{ GeV}$  (blue region).

Fixed parameters:

$M_{\text{SUSY}} = 500 \text{ GeV}$ ,  $m_{H^\pm} = 300 \text{ GeV}$ ,  $\varphi = \frac{\pi}{3}$ ,  $\tan\beta = 5$ .

# Restrictions on the parameters of the MSSM



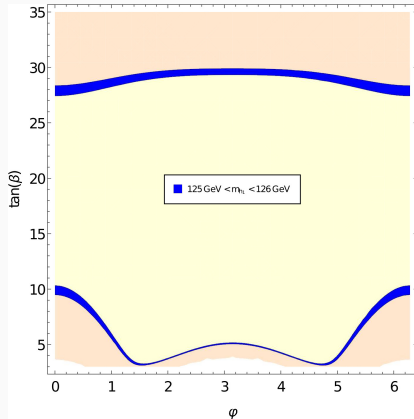
Contour plot in  $M_{\text{SUSY}}$  -  $\tan\beta$  plane.

Selected region:  $125 \text{ GeV} < m_{h_1} < 126 \text{ GeV}$  (blue region).

Fixed parameters:

$$\mu = 2000 \text{ GeV}, A_{t,b} = 1000 \text{ GeV}, m_{H^\pm} = 300 \text{ GeV}, \varphi = \frac{\pi}{3}.$$

# Restrictions on the parameters of the MSSM



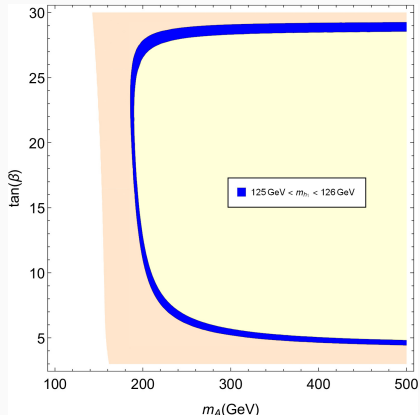
Contour plot in  $\varphi - \tan\beta$  plane.

Selected region:  $125 \text{ GeV} < m_{h_1} < 126 \text{ GeV}$  (blue region).

Fixed parameters:

$\mu = 2000 \text{ GeV}$ ,  $A_{t,b} = 1000 \text{ GeV}$ ,  $M_{\text{SUSY}} = 500 \text{ GeV}$ ,  $m_{H^\pm} = 300 \text{ GeV}$ .

# Restrictions on the parameters of the MSSM



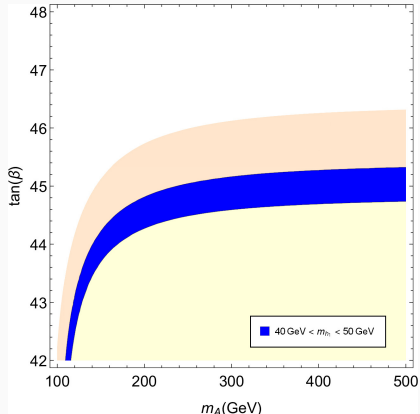
Contour plot in  $m_A - \tan\beta$  plane.

Selected region:  $125 \text{ GeV} < m_{h_1} < 126 \text{ GeV}$  (blue region).

Fixed parameters:

$$\mu = 2000 \text{ GeV}, A_{t,b} = 1000 \text{ GeV}, M_{\text{SUSY}} = 500 \text{ GeV}, \varphi = \frac{\pi}{3}.$$

# Restrictions on the parameters of the MSSM



Contour plot in  $m_A - \tan\beta$  plane.

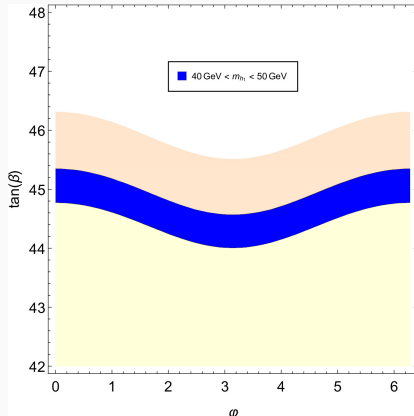
Selected region:  $40 \text{ GeV} < m_{h_1} < 50 \text{ GeV}$  (blue region).

Fixed parameters:

$$\mu = 2000 \text{ GeV}, A_{t,b} = 1000 \text{ GeV}, M_{\text{SUSY}} = 500 \text{ GeV}, \varphi = \frac{\pi}{3}.$$



# Restrictions on the parameters of the MSSM



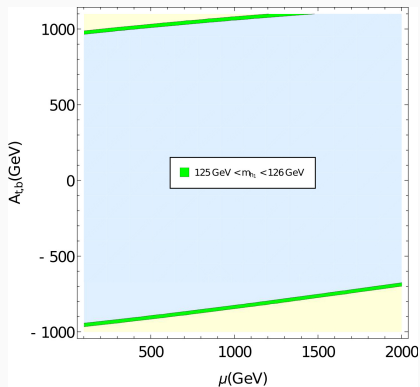
Contour plot in  $\varphi - \tan\beta$  plane.

Selected region:  $40 \text{ GeV} < m_{h_1} < 50 \text{ GeV}$  (blue region).

Fixed parameters:

$\mu = 2000 \text{ GeV}$ ,  $A_{t,b} = 1000 \text{ GeV}$ ,  $M_{\text{SUSY}} = 500 \text{ GeV}$ ,  $m_{H^\pm} = 300 \text{ GeV}$ .

# Restrictions on the parameters of the MSSM



Contour plot in  $m_A - \tan\beta$  plane.

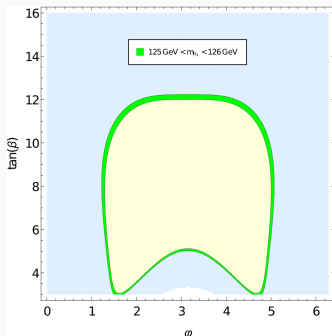
Selected region:  $125 \text{ GeV} < m_{h_1} < 126 \text{ GeV}$  (green region).

Fixed parameters:

$m_Q = 500 \text{ GeV}$ ,  $m_t = 800 \text{ GeV}$ ,  $m_b = 200 \text{ GeV}$ ,  $m_{H^\pm} = 300 \text{ GeV}$ ,  $\varphi = \frac{\pi}{3}$ ,

$\tan\beta = 5$ ,  $T = 500 \text{ GeV}$ .

# Restrictions on the parameters of the MSSM



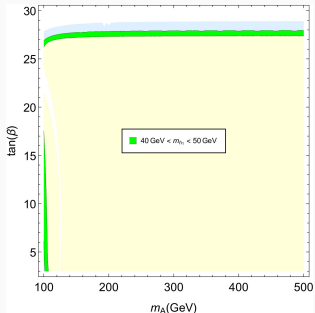
Contour plot in  $\varphi - \tan\beta$  plane.

Selected region:  $125 \text{ GeV} < m_{h_1} < 126 \text{ GeV}$  (green region).

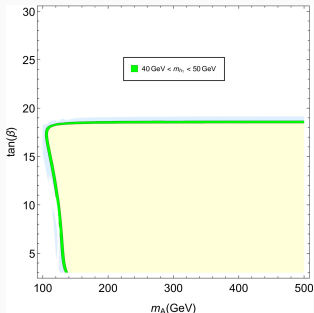
Fixed parameters:

$\mu = 2000 \text{ GeV}$ ,  $A_{t,b} = 1000 \text{ GeV}$ ,  $m_Q = 500 \text{ GeV}$ ,  $m_t = 800 \text{ GeV}$ ,  $m_b = 200 \text{ GeV}$ ,  $m_{H^\pm} = 300 \text{ GeV}$ ,  $T = 500 \text{ GeV}$ .

# Restrictions on the parameters of the MSSM



a)  $T = 10 \text{ GeV}$



b)  $T = 500 \text{ GeV}$

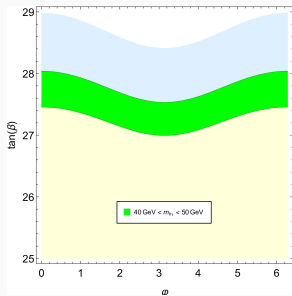
Contour plot in  $\varphi - \tan\beta$  plane.

Selected region:  $40 \text{ GeV} < m_{h_1} < 50 \text{ GeV}$  (green region).

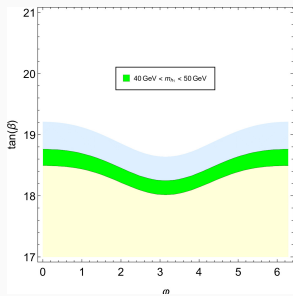
Fixed parameters:

$\mu = 2000 \text{ GeV}$ ,  $A_{t,b} = 1000 \text{ GeV}$ ,  $m_Q = 500 \text{ GeV}$ ,  $m_t = 800 \text{ GeV}$ ,  $m_b = 200 \text{ GeV}$ ,  $m_{H^\pm} = 300 \text{ GeV}$ .

# Restrictions on the parameters of the MSSM



a)  $T = 10 \text{ GeV}$



b)  $T = 500 \text{ GeV}$

Contour plot in  $m_A - \tan\beta$  plane.

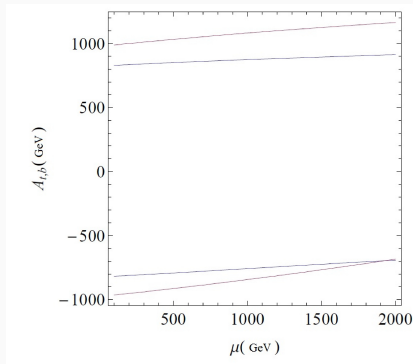
Selected region:  $40 \text{ GeV} < m_{h_1} < 50 \text{ GeV}$  (green region).

Fixed parameters:

$\mu = 2000 \text{ GeV}$ ,  $A_{t,b} = 1000 \text{ GeV}$ ,  $m_Q = 500 \text{ GeV}$ ,  $m_t = 800 \text{ GeV}$ ,  $m_b = 200 \text{ GeV}$ ,

$\varphi = \frac{\pi}{3}$ .

# Restrictions on the parameters of the MSSM



Contour plots in  $\mu - A_{t,b}$  plane.

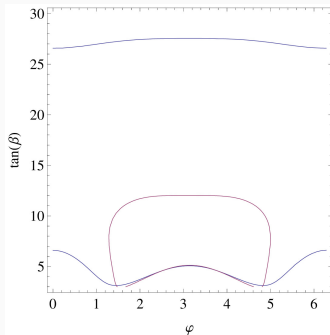
Selected region:  $m_{h_1} = 126$  GeV ( $T = 0$  GeV - blue line;  $T \neq 0$  GeV - purple line).

Fixed parameters:

$m_Q = 500$  GeV,  $m_t = 800$  GeV,  $m_b = 200$  GeV,  $m_{H^\pm} = 300$  GeV,  $\varphi = \frac{\pi}{3}$ ,

$\tan \beta = 5$ .

# Restrictions on the parameters of the MSSM



Contour plots in  $\varphi - \tan\beta$  plane.

Selected region:  $m_{h_1} = 126$  GeV ( $T = 0$  GeV - blue line;  $T \neq 0$  GeV - purple line ).

Fixed parameters:

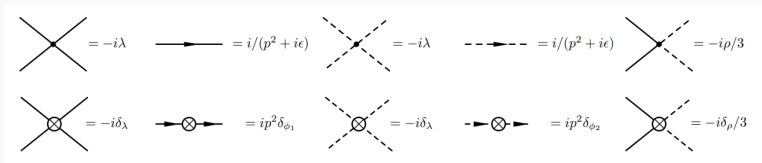
$\mu = 2000$  GeV,  $A_{t,b} = 1000$  GeV,  $m_Q = 500$  GeV,  $m_t = 800$  GeV,  $m_b = 200$  GeV,  $m_{H^\pm} = 300$  GeV.

## Модель с двумя скалярными полями

Рассмотрим лагранжиан с двумя скалярными полями  $\phi_1$  и  $\phi_2$ , при этом тут два каплинга <sup>2</sup>, <sup>3</sup>:

$$L = \frac{1}{2}((\partial_\mu \phi_1)^2 + (\partial_\mu \phi_2)^2) - \frac{\lambda}{4!}(\phi_1^4 + \phi_2^4) - \frac{2\rho}{4!}(\phi_1^2 \phi_2^2). \quad (5)$$

Тогда правила Фейнмана для нашего случая:



<sup>2</sup>Боголюбов Н.Н., Ширков Д.В. Введение в теорию квантованных полей. // 4-е изд. М.: Наука 1984.

<sup>3</sup>Пескин М., Шредер Д. Введение в квантовую теорию поля. // Ижевск: НИЦ «Регулярная и хаотическая динамика», 2001, 784 стр.



$\beta$ -функции для  $\lambda$  и  $\rho$ . Контрчлены перенормировки  $\delta_\lambda$  и  $\delta_\rho$

В однопетлевом порядке:



$$\begin{aligned}
 &= -i\lambda + (-i\lambda)^2(V(t) + V(s) + V(u)) + \left(-i\frac{\rho}{3}\right)^2 (V(t) + V(s) + V(u)) - i\delta_\lambda = \\
 &= -i\lambda - \left(\lambda^2 + \frac{\rho^2}{9}\right)(V(t) + V(s) + V(u)) - i\delta_\lambda.
 \end{aligned}$$

Для  $\rho$  в однопетлевом порядке:



$$\delta_\rho = \frac{1}{16\pi^2} [\lambda\rho + 2\rho^2/3] \log \frac{\Lambda^2}{M^2}$$

$$\beta_\rho = \frac{1}{8\pi^2} [\lambda\rho + 2\rho^2/3]$$

Уравнение Каллана-Симанчика для двухточечной функции:

$$\left[ M \frac{\partial}{\partial M} + \beta(\lambda) \frac{\partial}{\partial \lambda} + 2\gamma(\lambda) \right] G^{(2)}(p) = 0$$

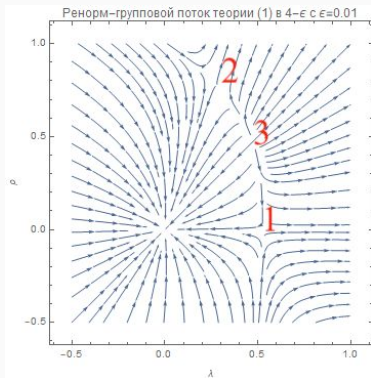
$$\beta_{\rho/\lambda} = \frac{1}{\lambda^2} [\beta_{\rho} \lambda - \beta_{\lambda} \rho] = -\frac{\rho}{48\pi^2} (\rho/\lambda - 3)(\rho/\lambda - 1)$$

Функция имеет нули в точке  $\rho/\lambda = 0, 1, 3$ .

В 4 -  $\epsilon$ :

$$\beta_{\lambda} = -\epsilon\lambda + \frac{3\lambda^2 + \rho^2/3}{(4\pi)^2}; \quad (6)$$

$$\beta_{\rho} = -\epsilon\rho + \frac{2\lambda\rho + 4\rho^2/3}{(4\pi)^2}. \quad (7)$$



Три нетривиальные неподвижные точки на графике получаются из нулей  $\beta$ -функции:

$$1. \lambda_B = \frac{16\pi^2}{3}\epsilon, \quad \rho_B = 0,$$

Седло

$$2. \lambda_C = \frac{8\pi^2}{3}\epsilon, \quad \rho_C = 8\pi^2\epsilon,$$

Седло

$$3. \lambda_D = \frac{24\pi^2}{5}\epsilon, \quad \rho_D = \frac{24\pi^2}{5}\epsilon,$$

Неустойчивый узел

В отличие

от этих трех точек, четвертая точка  $(0,0)$  является ИК стабильной. Это вырожденный устойчивый узел. ( $\lambda_A = \rho_A = 0$ ).

Для целей наших исследований РГ удобно определить <sup>4</sup>.

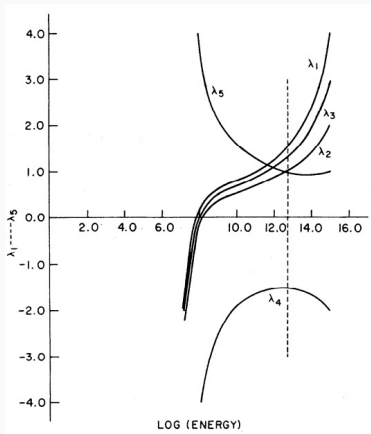
$$\rho_i(\mu) = \frac{\lambda_i(\mu)}{g_3^2(\mu)}, \quad \rho_t(\mu) = \frac{h_t(\mu)}{g_3^2(\mu)}, \quad R_i(\mu) = \frac{\rho_i(\mu)}{\rho_t(\mu)} = \frac{\lambda_i(\mu)}{h_t^2(\mu)} \quad (8)$$

Взаимодействия собственных состояний Хиггса с топ-кварком  $g_{t\bar{t}h_i}$  также могут быть представлены как произведение соответствующего взаимодействия СМ и R-взаимодействия  $R_{t\bar{t}h_i}$  :

$$R_{t\bar{t}h_1} = \frac{\cos \alpha}{\sin \beta}, \quad R_{t\bar{t}h_2} = \frac{\sin \alpha}{\cos \beta} \quad (9)$$

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<sup>4</sup>Froggatt C.D., Nevzorov R., Nielsen H.B., Thompson D. Fixed point scenario in the Two Higgs Doublet Model inspired by degenerate vacua // Phys.Lett. - 2007. - В 657. - P.95-102.



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# Потоки $U(\phi_1, \phi_2)$ от каплингов $\lambda_1, \lambda_2, \lambda_3, \lambda_4$

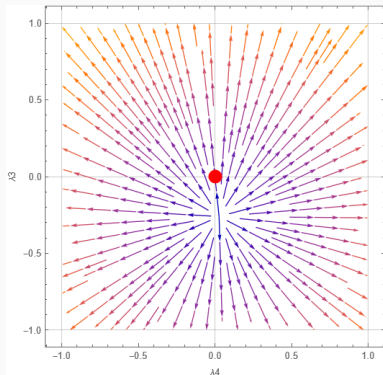
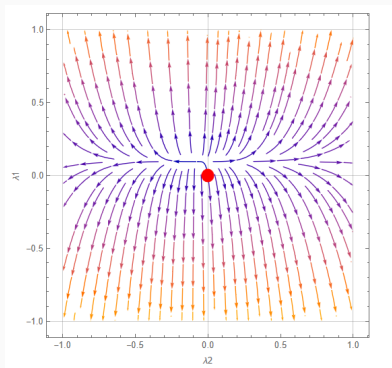
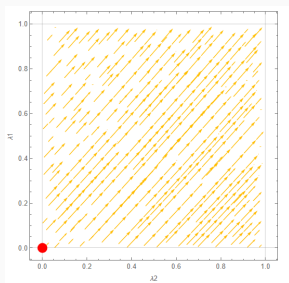
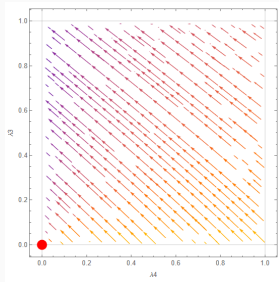
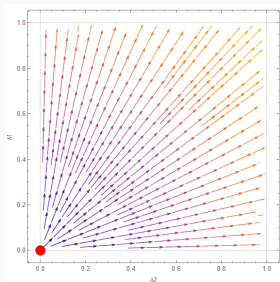


Figure 1: Ренормгрупповые потоки для каплингов 1,2 (слева) и 3,4 (справа)

Результаты: потоки  $R_i(\mu)$  от каплингов  $\lambda_1, \lambda_2, \lambda_3, \lambda_4$  и  $R_{\text{tth}_i}$  от каплингов  $\lambda_1, \lambda_2$





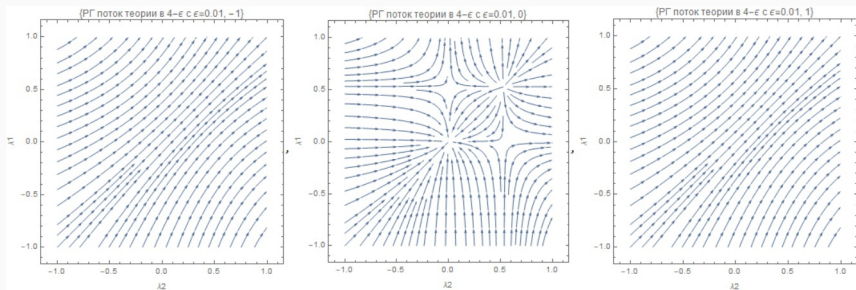


Figure 2: Ренормгрупповые потоки для каплингов 1,2 от 3 слева  $\lambda_3 = -1$ , по центру  $\lambda_3 = 0$ , справа  $\lambda_3 = 1$

# Потоки каплингов $\lambda_1, \lambda_2$ при изменении $\lambda_5$ в пробной модели

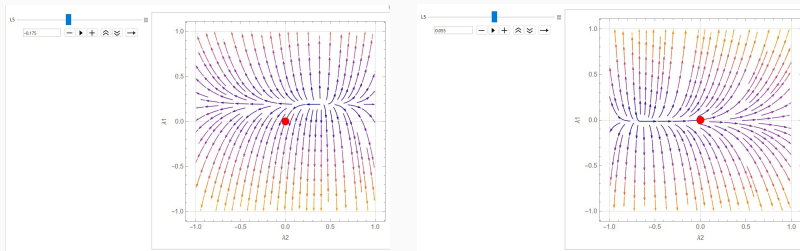


Figure 3: Ренормгрупповые потоки для каплингов 1,2 от 5 слева  $\lambda_5 = -0.175$  справа  $\lambda_5 = 0.055$

## Conclusion

- Our analysis of the effective MSSM and NMSSM finite-temperature potentials is based on the diagram calculation of various one-loop temperature corrections for the case of nonzero trilinear parameters  $A_t$ ,  $A_b$  and Higgs superfield parameter  $\mu$ .
  - Quantum corrections are incorporated in control parameters  $\lambda_{1,\dots,7}\dots(T)$  of the effective two-doublet (+singlet) potential, which is then explicitly rewritten in terms of Higgs boson mass eigenstates.
  - Types bifurcation sets for the two-Higgs-doublet(+singlet) potential  $U_{\text{eff}}(v_1, v_2)$  are determined.
  - Bifurcation sets for Higgs potential in the case of Peccei–Quinn symmetry are obtained. These sets always describe the system at the **local minimum** (**critical morse point**).
- 
- Constrains on MSSM and NMSSM **allowed parameter space** are evaluated

1. Были получены и исследованы ренормгрупповые уравнения для моделей с двумя скалярными полями и двумя дублетами комплексных скалярных полей.
2. Для ряда каплингов наблюдается смещение РГ потоков, что связано с наличием дополнительной фазы или количеством каплингов. Определено инфракрасное асимптотическое поведение критической точки.

Дальнейшая перспектива: Интерфейс РГ потоков для индуцированных переходов и сравнение с экспериментальными данными по самодействию при энергиях СМ.

We calculate the integral

$$J_0[a_1, a_2] = \int \frac{dk}{(2\pi)^3} \frac{1}{(k^2 + a_1^2)(k^2 + a_2^2)} = \frac{1}{4\pi(a_1 + a_2)},$$

taking a residue in the spherical coordinate system. Dolgoplov M., Dubinin M., Rykova E., Journal of Modern Physics. Vol. 2. No. 5. P. 301-322. (2011).

$a_{1,2}^2$  are the sums of squared frequency and squared mass. Derivatives of  $J_0$  with respect to  $a_1$  and  $a_2$  can be used for calculation of integrals

$$J_1[a_1, a_2] = \int \frac{dk}{(2\pi)^3} \frac{1}{(k^2 + a_1^2)^2(k^2 + a_2^2)} = \frac{1}{8\pi a_1(a_1 + a_2)^2},$$

$$J_2[a_1, a_2] = \int \frac{dk}{(2\pi)^3} \frac{1}{(k^2 + a_1^2)^2(k^2 + a_2^2)^2} = \frac{1}{8\pi a_1 a_2 (a_1 + a_2)^3}.$$

$$\begin{aligned}\Delta\lambda_1 &= C_{31}^4 I_2[m_Q, m_U] + C_{32}^4 I_2[m_Q, m_D] + \\ &+ C_{31}^2 (C_{41} I_1[m_Q, m_U] + C_{43} I_1[m_U, m_Q]) + \\ &+ C_{32}^2 (C_{42} I_1[m_Q, m_D] + C_{44} I_1[m_D, m_Q]).\end{aligned}$$

$$\begin{aligned}\Delta\lambda_2 &= C_{33}^4 I_2[m_Q, m_U] + C_{34}^4 I_2[m_Q, m_D] + \\ &+ C_{33}^2 (C_{45} I_1[m_Q, m_U] + C_{47} I_1[m_U, m_Q]) + \\ &+ C_{34}^2 (C_{46} I_1[m_Q, m_D] + C_{48} I_1[m_D, m_Q]).\end{aligned}$$

$$\begin{aligned}\Delta(\lambda_3 + \lambda_4) &= C_{31}^2 C_{33}^2 I_2[m_Q, m_U] + C_{32}^2 C_{34}^2 I_2[m_Q, m_D] + \\ &+ (C_{31}^2 C_{45} + C_{33}^2 C_{41}) I_1[m_Q, m_U] + (C_{31}^2 C_{47} + C_{33}^2 C_{43}) I_1[m_U, m_Q] + \\ &+ (C_{32}^2 C_{46} + C_{34}^2 C_{42}) I_1[m_Q, m_D] + (C_{32}^2 C_{48} + C_{34}^2 C_{44}) I_1[m_D, m_Q].\end{aligned}$$

$$\Delta\lambda_5 = C_{31}^2 C_{33}^2 I_2[m_Q, m_U] + C_{32}^2 C_{34}^2 I_2[m_Q, m_D].$$

СПАСИБО ЗА ВНИМАНИЕ!

Интеграл:

$$V(k) = \frac{1}{2} \int \frac{d^d k}{(4\pi)^d} \frac{i}{(k^2 + i\epsilon)} \frac{i}{((p-k)^2 + i\epsilon)} = -\frac{1}{2} \int_0^1 dx \frac{i}{(4\pi)^{d/2}} \frac{\Gamma(2-d/2)}{\Lambda^{2-d/2}}$$
$$\underset{d \rightarrow 4}{\sim} -\frac{i}{32\pi^2} \frac{2}{\epsilon} \rightarrow -\frac{i}{32\pi^2} \log \frac{\Lambda^2}{M^2}$$

При этом мы сделали замены  $\Lambda = x(1-x)$ ,  $p^2 = -M^2$ . Применяя условия перенормировки, мы видим, что

$$i\delta_\lambda = \frac{3}{32\pi^2} [\lambda^2 + (\rho/3)^2] \log \frac{\Lambda^2}{M^2}$$

Поскольку в этой теории нет расходящихся диаграмм собственной энергии до однопетлевого порядка, мы можем записать  $\beta$ -функцию:

$$\beta_\lambda = \frac{3}{16\pi^2} [\lambda^2 + (\rho/3)^2]$$



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