Renormalization group approach to reggeon model of pomeron and odderon

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Introduction

Effective theory of reggeon interaction, or Gribov model, was introduced as a set of diagram rules[1]. Then it was reimagined[2] as Feynman rules for euclidean field theory with the following Lagrangian density

$$\mathcal{L} = \frac{1}{2}\left(\Phi^+\partial_y\Phi - \Phi\partial_y\Phi^+\right) - \mu\Phi^+\Phi - \alpha'\vec{\nabla}\Phi^+\cdot\vec{\nabla}\Phi + i\lambda\Phi^+\left(\Phi+\Phi^+\right)\Phi.$$

Here $\Phi(y,\vec{b})$ – reggeon complex field, $\Phi^+(y,\vec{b})$ – conjugated field, y – rapidity, $\vec{b}$ – impact parameter vector. Theory parameters $\mu = \alpha(0) - 1$ and $\alpha' = \alpha'(0)$ are determined by pomeron Regge trajectory $j = \alpha(t) + 1$.

[1] V.N. Gribov, Sov. Phys. JETP 26 (1968) 414

[2] A.A. Migdal, A.M. Polyakov, K.A. Ter-Martirosyan, Phys. Lett. 48 B (1974) 239

Model

The model of the pomeron and odderon fields ϕ_1, ϕ_2 in $D = 4 - 2\varepsilon$ transverse dimensions is introduced by the Lagrangian density

$$\mathcal{L} = \sum_{i=1}^{2} \left(\bar{\phi}_i \partial_y \phi_i + \alpha_i' \nabla_{(b)} \bar{\phi}_i \cdot \nabla_{(b)} \phi_i \right) + \frac{i\lambda_1}{2} \left(\bar{\phi}_1 \bar{\phi}_1 \phi_1 + \bar{\phi}_1 \phi_1 \phi_1 \right) + \\ + \frac{i\lambda_2}{2} \left(2\bar{\phi}_2 \phi_2 \phi_1 + 2\bar{\phi}_2 \ \bar{\phi}_1 \phi_2 \right) + \frac{i\lambda_3}{2} \left(-\bar{\phi}_1 \phi_2 \phi_2 + \bar{\phi}_2 \bar{\phi}_2 \phi_1 \right) \quad (1)$$

The theory has two independent scale invariances under $y \to ay$ and $\mathbf{b} \to c\mathbf{b}$. The physical transverse dimension is $D = 2$.

Green functions

We consider one-particle irreducible Green functions in one-loop approximation

$$\Gamma^{n_1,n_2,m_1,m_2}(E, \mathbf{k}, \alpha'_j, \lambda_l), \quad j = 1, 2, \; l = 1, 2, 3, \tag{2}$$

denoting the inverse propagators $\Gamma_1 = \Gamma^{1,0,1,0}$ and $\Gamma_2 = \Gamma^{0,1,0,1}$.
The renormalized functions

$$\Gamma^{R,n_1,n_2,m_1,m_2}(E_i, \mathbf{k}_i, \alpha'_j, \lambda_l, E_N) = Z_1^{\frac{n_1+m_1}{2}} Z_2^{\frac{n_2+m_2}{2}} \Gamma^{n_1,m_1,n_2,m_2}(E_i, \mathbf{k}_i, \alpha'_{j0}, \lambda_{l0}) \tag{3}$$

Physical normalization conditions

$$\begin{aligned}
\frac{\partial}{\partial E}\Gamma_i^R(E,\mathbf{k}^2,E_N)\Big|_{E=-E_N,k^2=0} &= 1,\\
\frac{\partial}{\partial k^2}\Gamma_i^R(E,\mathbf{k}^2,E_N)\Big|_{E=-E_N,k^2=0} &= -\alpha_i', \quad i=1,2,\\
\Gamma^{R,1,0,2,0}\Big|_{r.p.} &= i\lambda_1(2\pi)^{-\frac{D+1}{2}},\\
\Gamma^{R,0,1,1,1}\Big|_{r.p.} &= i\lambda_2(2\pi)^{-\frac{D+1}{2}},\\
\Gamma^{R,1,0,0,2}\Big|_{r.p.} &= i\lambda_3(2\pi)^{-\frac{D+1}{2}}
\end{aligned} \tag{4}$$

with the renormalization point

$$E_1 = 2E_2 = 2E_3 = -E_N, \mathbf{k}_1 = \mathbf{k}_2 = \mathbf{k}_3 = 0 \tag{5}$$

The renormalization constants Z, U, W defined by

$$\begin{gathered} Z_i^{1/2}\phi_i^R = \phi_i, \quad U_i Z_i^{-1}\alpha'_i = \alpha'_{i0}, \ \ i = 1, 2, \\ W_1 Z_1^{-3/2}\lambda_1 = \lambda_{10}, \\ W_{(2,3)} Z_1^{-1/2} Z_2^{-1}\lambda_{(2,3)} = \lambda_{(2,3)0} \end{gathered} \tag{6}$$

depend on four dimensionless combinations of parameters (charges)

$$g_i = \frac{\lambda_i}{(8\pi\alpha'_1)^{D/4}\, E_N^{(4-D)/4}}, \quad i = 1, 2, 3 \ \ \text{and} \ \ g_4 \equiv u = \frac{\alpha'_2}{\alpha'_1}. \tag{7}$$

Note that $g_4 = u$ is not a small parameter of the perturbation theory.

Renormalization group equation

From the dimensional analysis

$$\Gamma^R(E_i, \mathbf{k}_i, \alpha_1', g, E_N) = E_N \left(\frac{E_N}{\alpha_1'}\right)^{(2-n-m)D/4} \Phi\left(\frac{E_i}{E_N}, \frac{\alpha_1'}{E_N}\mathbf{k}_i\mathbf{k}_j, g\right). \qquad (8)$$

Then the renormalization group (RG) equation is

$$\left\{\xi\frac{\partial}{\partial\xi} - \sum_{i=1}^{4}\beta_i(g)\frac{\partial}{\partial g_i} + [1-\tau_1(g)]\alpha_1'\frac{\partial}{\partial\alpha_1'} + \left[\sum_{i=1}^{2}\frac{1}{2}(n_i+m_i)\gamma(g)\right] - 1\right\}\cdot$$
$$\cdot\,\Gamma^R(\xi E_i, \mathbf{k}_i, \alpha_1', g, E_N) = 0. \qquad (9)$$

The RG functions are defined by

$$\beta_i(g) = E_N \frac{\partial g_i}{\partial E_N}, \quad i = 1, ..., 4,$$
$$\gamma_i(g) = E_N \frac{\partial \ln Z_i}{\partial E_N}, \quad i = 1, 2, \tag{10}$$
$$\tau_1(g) = E_N \frac{\partial}{\partial E_N} \ln\Big(U_1^{-1} Z_1\Big),$$

The asymptotical behaviour at $E \to 0$ is determined by the IR-attractive fixed point g_* of the RG flow.

$$\Gamma_i^R(E, \mathbf{k}^2, \alpha_1', g, E_N) = E_N \left(\frac{-E}{E_N}\right)^{1-\gamma_i(g_*)} \Phi_i \left(\left(\frac{-E}{E_N}\right)^{-z} \frac{\alpha_1' \mathbf{k}^2}{E_N}, g_*\right), \quad i = 1, 2,$$

$$P_p(s, \mathbf{k}^2 = 0) \sim s(\ln s)^{-\gamma_1(g_*)}, \quad P_o(s, \mathbf{k}^2 = 0) \sim s(\ln s)^{-\gamma_2(g_*)}. \tag{11}$$

where $z = 1 - \tau_1(g_*)$.

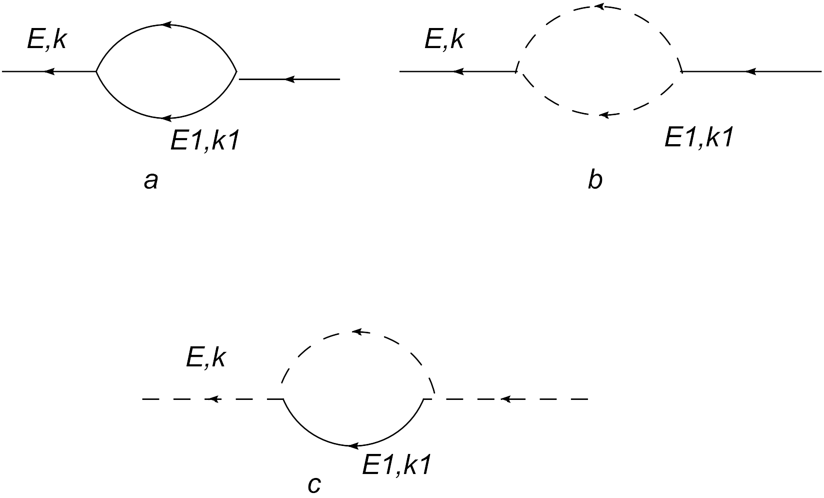

Figure: Self-masses for Γ_1 (a, b) and Γ_2 (c).

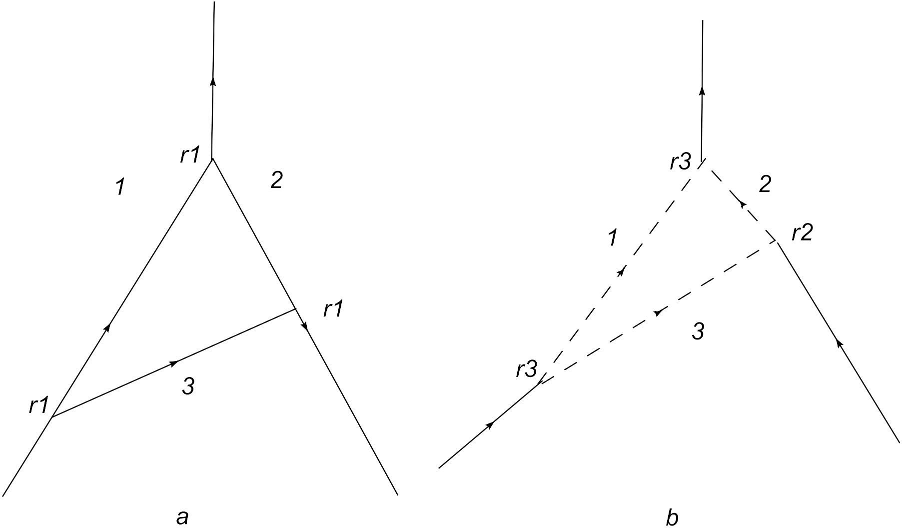

Figure: Diagrams for $\Gamma^{1,0,2,0}$. Inverse diagrams are identical.

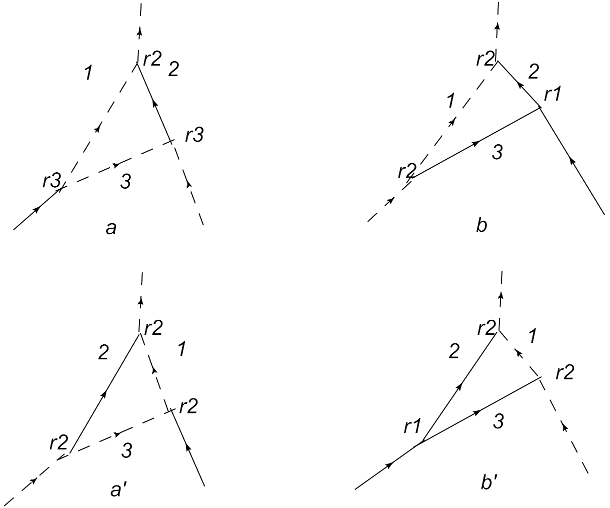

Figure: Diagrams for $\Gamma^{0,1,1,1}$. Inverse diagrams are shown below as (a') and (b').

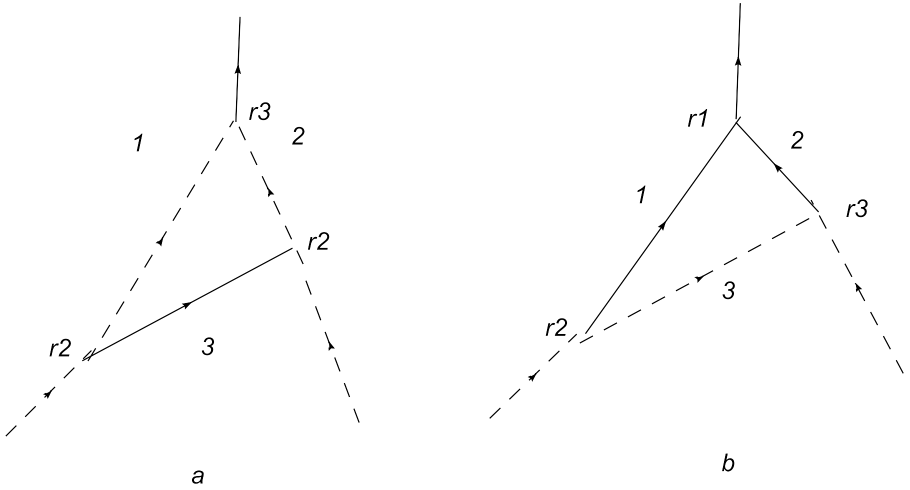

Figure: Diagrams for $\Gamma^{1,0,0,2}$. Inverse diagrams are identical.

Anomalous dimensions and β-functions

Our calculation gives in the lowest order in ε the following anomalous dimensions

$$\gamma_1 = -\frac{1}{2}g_1^2 + \frac{g_3^2}{2u^2}, \quad \gamma_2 = -\frac{4g_2^2}{(1+u)^2}, \quad \tau_1 = -\frac{1}{4}g_1^2 - \frac{(u-2)g_3^2}{4u^2}, \tag{12}$$

and beta-functions

$$\beta_1 = -\frac{1}{2}\varepsilon g_1 + \frac{3}{2}g_1^3 - g_2 g_3^2 \frac{2}{u^2} + g_1 g_3^2 \frac{1+u}{4u^2}, \tag{13}$$

$$\beta_2 = -\frac{1}{2}\varepsilon g_2 + g_1 g_2^2 \frac{6+2u}{(1+u)^2} - g_2 g_3^2 \frac{1+8u-u^2}{4u^2(1+u)}, \tag{14}$$

$$\beta_3 = -\frac{1}{2}\varepsilon g_3 + g_1 g_2 g_3 \frac{4}{1+u} + g_2^2 g_3 \frac{4}{u(1+u)^2} + g_3^3 \frac{u-1}{4u^2}, \tag{15}$$

$$\beta_4 = g_1^2 \frac{u}{4} - g_2^2 \frac{4u^2}{(1+u)^3} + g_3^2 \frac{u-2}{4u}. \tag{16}$$

Fixed points

$$g_*^{(0)} = \{0,\ 0,\ 0,\ 0\},\ \ r = g_3/g_4 \to \sqrt{2}\sqrt{\varepsilon},\quad \gamma_1 = \varepsilon,\ \ \gamma_2 = 0,\ \ z = 1+\varepsilon,$$
$$P_p(s,\mathbf{k}^2 = 0) \sim s(\ln s)^{-\varepsilon},\ \ P_o(s,\mathbf{k}^2 = 0) \sim s. \tag{17}$$

$$g_*^{(1)} = \{\frac{\sqrt{\varepsilon}}{\sqrt{3}}, 0, 0, 0\},\ \ r = g_3/g_4 \to 0,\quad \gamma_1 = -\frac{\varepsilon}{6},\ \ \gamma_2 = 0,\ \ z = 1-\frac{\varepsilon}{12},$$
$$P_p(s,\mathbf{k}^2 = 0) \sim s(\ln s)^{\varepsilon/6},\ \ P_o(s,\mathbf{k}^2 = 0) \sim s. \tag{18}$$

$$g_*^{(2)} = \{\frac{\sqrt{\epsilon}}{\sqrt{3}},\ \frac{233\sqrt{3} + 117\sqrt{11}}{96(\sqrt{33}+15)}\sqrt{\epsilon},\ 0,\ \frac{3(\sqrt{33}-1)}{16}\},$$
$$\gamma_1 = -\frac{\varepsilon}{6}, \gamma_2 = -\frac{2\varepsilon}{11.30},\ \ z = 1-\frac{\varepsilon}{12},$$
$$P_p(s,\mathbf{k}^2 = 0) \sim s(\ln s)^{\varepsilon/6},\ \ P_o(s,\mathbf{k}^2 = 0) \sim s(\ln s)^{2\varepsilon/11.30}. \tag{19}$$

$$g_*^{(3)} = \{\frac{\sqrt{\varepsilon}}{\sqrt{3}},\ \frac{\sqrt{\varepsilon}}{\sqrt{48}},\ 0,\ 0\},\quad r = g_3/g_4 \to 0,$$
$$\gamma_1 = -\frac{\varepsilon}{6},\quad \gamma_2 = -\frac{\varepsilon}{12},\quad z = 1 - \frac{\varepsilon}{12},$$
$$P_p(s, \mathbf{k}^2 = 0) \sim s(\ln s)^{\varepsilon/6},\quad P_o(s, \mathbf{k}^2 = 0) \sim s(\ln s)^{\varepsilon/12}. \tag{20}$$

$$g_*^{(4)} = \{0,\ 0,\ 2\sqrt{2}\sqrt{\varepsilon},\ 2\}, \gamma_1 = \varepsilon,\ \gamma_2 = 0,\quad z = 1,$$
$$P_p(s, \mathbf{k}^2 = 0) \sim s(\ln s)^{-\varepsilon},\quad P_o(s, \mathbf{k}^2 = 0) \sim s. \tag{21}$$

Conclusion

- Anomalous dimensions, β-functions and fixed points were found
- There are two variants of the cross-section behaviour: slowly growing as $(\ln s)^{1/6}$ and slowly decreasing as $(\ln s)^{-1}$
- Numerical analysis of the RG flow (185200 trajectories) was performed. Trajectories running into the fixed points $g_*^{(1)}$, $g_*^{(3)}$ and $g_*^{(4)}$ were found, but the dependence of the choice of the ending point on initial parameters is unknown.

Thanks for your attention!