Bremsstrahlung helicity amplitudes with massive fermions

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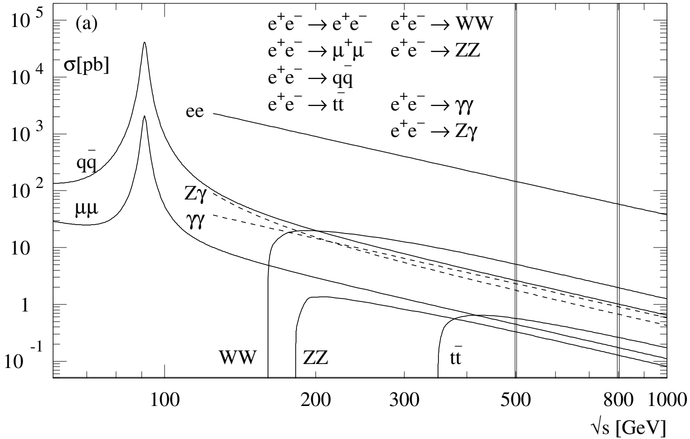
Outline

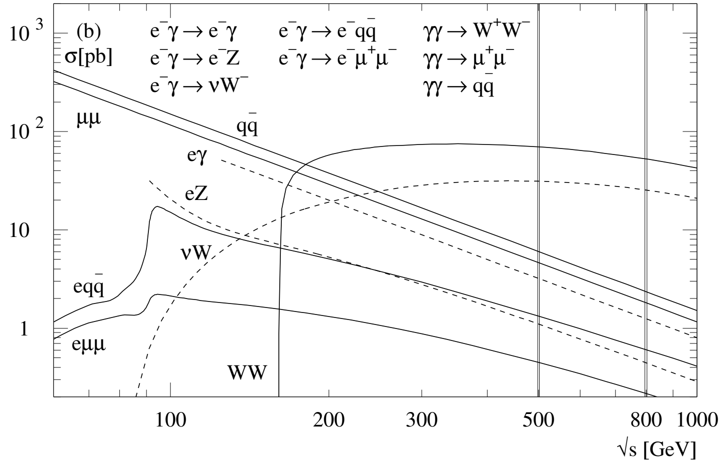
- ARIeL branch for processes with polarized e^+e^- beams
- Preliminary results for polarized $e^+e^- \to \mu^+\mu^-(\tau^+\tau^-)$ at NLO EW
- Numerical results and cross-checks
- Conclusion and plans

Basic processes of SM for e^+e^- annihilation



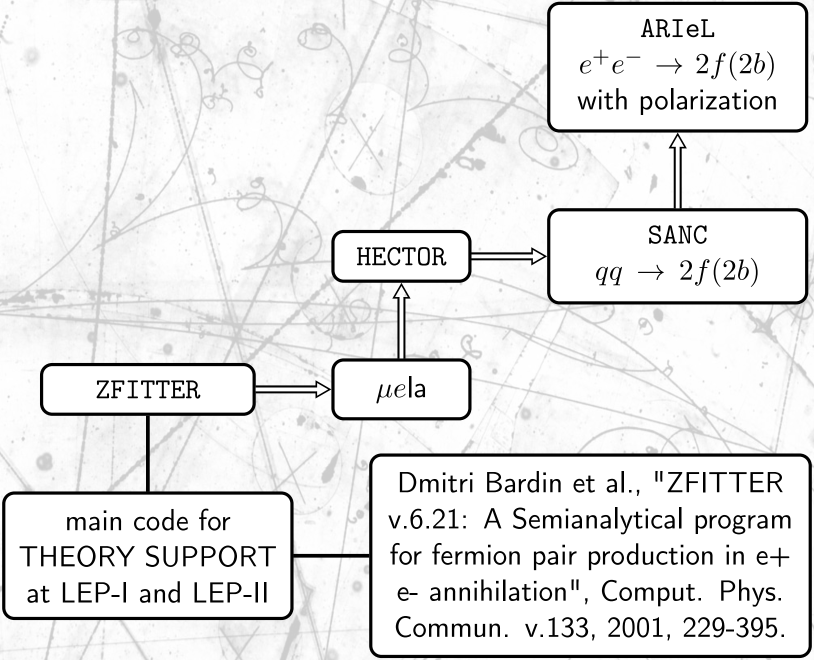
The basic processes of the Standard Model: $e + e-$ annihilation to pairs of fermions and gauge bosons. The cross sections are given for polar angles between $10^o < \theta < 170^o$ in the final state.

Basic processes of SM for $e^{\pm}\gamma$ and $\gamma\gamma$ initial state



Elastic/inelastic Compton scattering and $\gamma\gamma$ reactions. $\sqrt{s}$ is the invariant $e\gamma$ and $\gamma\gamma$ energy. The polar angle of the final state particles is restricted as in (a); in addition, the invariant $\mu^+\mu^-$ and $q\bar{q}$ masses in the inelastic Compton processes are restricted to $M_{inv} > 50$ GeV.

HISTORY OVERVIEW



HISTORY OVERVIEW

- μela

QED corrections at one loop level for polarized elastic μe scattering. Research was used for the analysis of μe scattering data from the beam polarimeter of the SMC experiment at CERN.
Dmitri Bardin et al.,hep-ph/9712310

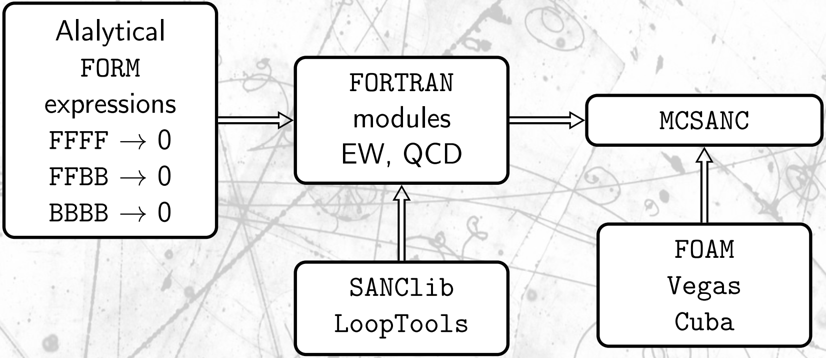
- `HECTOR 1.11`

Polarized ep scattering at HERA
Dmitri Yu. Bardin et al., "QED and electroweak corrections to deep inelastic scattering", Acta Phys. Polon.,v. B28,1997, 511-528.

- SANC

Complete one loop calculation of the EW radiative corrections for scattering e^+e^- polarized beams. The main conclusion of this study is radiative corrections as a function of the angle scattering $\cos\vartheta$ for Super Charm-Tau factory, CLIC, ILC, FCC_{ee} energy.

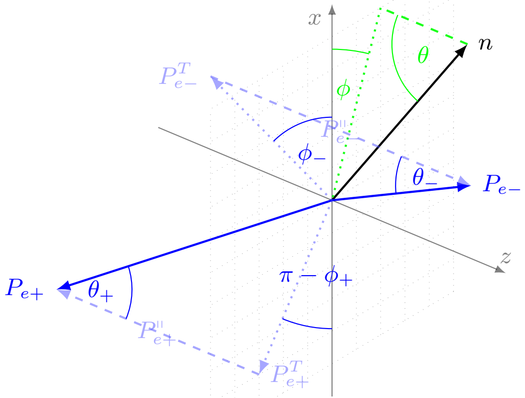
The scheme of calculation



MCSANC for processes with polarized beams

- NLO EW corrections for polarized e^+e^- scattering:
  - Bhabha scattering (`Phys.Rev.D, arXiv:1801.00125`)
  - $e^+e^- \to \mu^+\mu^-$(or $\tau^+\tau^-$) (preliminary)
  - $e^+e^- \to Z\gamma$ (preliminary)
  - $e^+e^- \to t\bar{t}$ (in progress)
  - $e^+e^- \to ZH$ (in progress)
  - $e^+e^- \to \gamma\gamma$ (in progress)
  - $e^+e^- \to ZZ$ (in progress)
  - $e^+e^- \to f\bar{f}\gamma$ (future plans)
  - $e^+e^- \to f\bar{f}H$ (future plans)
- NLO EW corrections for polarized $\gamma\gamma$ scattering:
  - $\gamma\gamma \to \gamma\gamma$ (future plans)
  - $\gamma\gamma \to Z\gamma$ (future plans)
  - $\gamma\gamma \to ZZ$ (future plans)

Decomposition of the $e^{\pm}$ polarization vectors



Matrix element squared

$$
\begin{aligned}
|\mathcal{M}|^2 = \quad & L^{||}_{e-}R^{||}_{e+}|\mathcal{H}_{-+}|^2 + R^{||}_{e-}L^{||}_{e+}|\mathcal{H}_{+-}|^2 + L^{||}_{e-}L^{||}_{e+}|\mathcal{H}_{--}|^2 + R^{||}_{e-}R^{||}_{e+}|\mathcal{H}_{++}|^2 \\
& -\frac{1}{2}P^T_{e-}P^T_{e+}Re\Big[e^{i(\Phi_+-\Phi_-)}\mathcal{H}_{++}\mathcal{H}^*_{--} + e^{i(\Phi_++\Phi_-)}\mathcal{H}_{+-}\mathcal{H}^*_{-+}\Big] \\
& +P^T_{e-}Re\Big[e^{i\Phi_-}\Big(L^{||}_{e+}\mathcal{H}_{+-}\mathcal{H}^*_{--} + R^{||}_{e+}\mathcal{H}_{++}\mathcal{H}^*_{-+}\Big)\Big] \\
& -P^T_{e+}Re\Big[e^{i\Phi_+}\Big(L^{||}_{e-}\mathcal{H}_{-+}\mathcal{H}^*_{--} + R^{||}_{e-}\mathcal{H}_{++}\mathcal{H}^*_{+-}\Big)\Big],
\end{aligned}
$$

where

$$
L^{||}_{e\pm} = \frac{1}{2}(1-P^{||}_{e\pm}), \quad R^{||}_{e\pm} = \frac{1}{2}(1+P^{||}_{e\pm}), \quad \Phi_\pm = \phi_\pm - \phi,
$$

$\mathcal{H}_{--}$, $\mathcal{H}_{++}$,$\mathcal{H}_{-+}$,$\mathcal{H}_{+-}$ — helicity amplitudes.

Polarized electron-positron annigilation: notations

We consider scattering of two polarized e^+ and e^- beams with 4-momenta of incoming particles p_1 and p_2, outgoing particles p_3 and p_4, at the one-loop EW level

$$e^+(p_1) + e^-(p_2) \longrightarrow f(p_3) + \bar{f}(p_4).$$

The cross-section of this process at one-loop can be devided into four parts:

$$\sigma^{\text{1-loop}} = \sigma^{\text{Born}} + \sigma^{\text{virt}}(\lambda) + \sigma^{\text{soft}}(\lambda, \omega) + \sigma^{\text{hard}}(\omega),$$

where σ^{Born} — Born level cross-section, σ^{virt} — contribution of virtual(loop) corrections, σ^{soft} — contribution due to soft photon emission, σ^{hard} — contribution due to hard photon emission (with energy $E_\gamma > \omega$).
Auxiliary parameters λ ("photon mass") and ω cancel out after summation.

Bremsstrahlung Helicity Amplitudes

$$\mathcal{H}^{\mathsf{hard}} = \mathcal{H}^{\mathsf{isr}} + \mathcal{H}^{\mathsf{fsr}}$$

Crossing symmetry

$$\mathcal{H}^{\mathsf{fsr}}_{\chi_1\chi_2\chi_3\chi_4\chi_5}(p_1, p_2, p_3, p_4) = +\mathcal{H}^{\mathsf{isr}}_{-\chi_4-\chi_3-\chi_2-\chi_1\chi_5}(-p_4, -p_3, -p_2, -p_1)$$

CP-symmetry

$$\mathcal{H}^{\mathsf{hard}}_{\chi_1\chi_2\chi_3\chi_4\chi_5} = -\chi_1\chi_2\chi_3\chi_4\overline{\mathcal{H}}^{\mathsf{hard}}_{-\chi_1-\chi_2-\chi_3-\chi_4-\chi_5}$$

Massless kinematics

Light-cone projection

Spinors originated from the problem of light-like momenta parametrization, so we project all massive momenta with $p_i^2 = m_i^2$ to the light-cone of photon p_5 and introduce associated "momenta":

$$k_i = p_i - \frac{m_i^2}{2p_i \cdot p_5} p_5, \quad k_i^2 = 0, \quad i = 1..4, \tag{1}$$

$$\begin{aligned} k_5 &= -\sum_{i=1}^{4} k_i = K p_5, \quad K = 1 + \sum_{i=1}^{4} \frac{m_i^2}{2p_i \cdot p_5} = 1 + \sum_{i=1}^{4} \frac{m_i^2}{2k_i \cdot p_5} \\ p_5 &= -\sum_{i=1}^{4} p_i = K' k_5, \quad K' = 1 - \sum_{i=1}^{4} \frac{m_i^2}{2p_i \cdot k_5} = 1 - \sum_{i=1}^{4} \frac{m_i^2}{2k_i \cdot k_5} \end{aligned} \tag{2}$$

Vector k_5 appear to be light-like, so we left with "momentum conservation" of associated vectors.

Phase space variables

Phase space volume

$$dR_3 = d^4p_3\delta(p_3^2 - m_3^2)d^4p_4\delta(p_4^2 - m_4^2)d^4p_5\delta(p_5^2)\delta^4(\sum_{i=1}^{5} p_i)$$

after change of variables:

$$dR_3 = d^4k_3\delta(k_3^2)d^4k_4\delta(k_4^2)d^4k_5\delta(k_5^2)\delta^4(\sum_{i=1}^{5} k_i)K' =$$
$$= \frac{\pi}{8\sqrt{\lambda_{12}}}(s_{12} - s_{34})d(s_{12} - s_{34})d(\cos^2\frac{\theta_{15}}{2})d(\cos^2\frac{\theta_{35}}{2})d\phi$$

Some notations

$$p_{i\ldots j} = p_i + \cdots + p_j \quad s_{i\ldots j} = p_{i\ldots j}^2$$
$$k_{i\ldots j} = k_i + \cdots + k_j \quad z_{i\ldots j} = k_{i\ldots j}^2$$

Phase space volume

$$s_{12} - s_{34} = s_{35} + s_{45} = -(s_{15} + s_{25})$$

$$z_{12} - z_{34} = z_{35} + z_{45} = -(z_{15} + z_{25}) \tag{3}$$

$$s_{12} = z_{12} - \left(\frac{m_1^2}{z_{15}} + \frac{m_2^2}{z_{25}}\right)(z_{12} - z_{34})$$

$$s_{34} = z_{34} + \left(\frac{m_3^2}{z_{35}} + \frac{m_4^2}{z_{45}}\right)(z_{12} - z_{34}) \tag{4}$$

$$s_{12} - s_{34} = (z_{12} - z_{34})K'$$

$$z_{15} = -(z_{12} - z_{34})\sin^2\tfrac{\theta_{15}}{2} \qquad \sin^2\tfrac{\theta_{15}}{2} = \frac{m_1 e^{\eta_{15}}}{m_1 e^{\eta_{15}} + m_2}$$

$$s_{12} = z_{12} + \frac{m_1^2}{\sin^2\frac{\theta_{15}}{2}} + \frac{m_2^2}{1 - \sin^2\frac{\theta_{15}}{2}} \qquad \lambda_{12} = 2m_1 m_2 \sinh\varsigma_{12}$$

$$s_{12} = z_{12} + m_1^2 + m_2^2 + 2m_1 m_2\cosh\eta_{15} \quad z_{12} > 0 \Rightarrow -\varsigma_{12} < \eta_{15} < \varsigma_{12}$$

$$\varsigma_{12} = \operatorname{acosh}\frac{s_{12} - m_1^2 - m_2^2}{2m_1 m_2} \quad d(\cos^2\tfrac{\theta_{15}}{2}) = \sin^2\tfrac{\theta_{15}}{2}\cos^2\tfrac{\theta_{15}}{2}\,d\eta_{15}$$

Spinor label notation

Phase fixing and notation

For any massless momentum $k_1^2 = 0$ we can solve Dirac equation $\hat{k_1} u(k_1) = 0$ and obtain two solutions:

$$
\begin{aligned}
|1\rangle = u(k_1,+) = v(k_1,-) \quad [1| = \bar{u}(k_1,+) = \bar{v}(k_1,-) \\
|1] = u(k_1,-) = v(k_1,+) \quad \langle 1| = \bar{u}(k_1,-) = \bar{v}(k_1,+)
\end{aligned}
\tag{5}
$$

Spinor products

Inner products of spinors are complex Lorentz invariants

$$
\begin{aligned}
\langle a|b\rangle = -\langle b|a\rangle \quad \langle a|a\rangle = 0 \\
[b|a] = -[a|b] \quad [a|a] = 0
\end{aligned}
\tag{6}
$$

$$[b|a] = \overline{\langle a|b\rangle} \tag{7}$$

$$\langle a|b\rangle [b|a] = \left|\langle a|b\rangle\right|^2 = 2k_a \cdot k_b = (k_a + k_b)^2 = z_{ab} \tag{8}$$

Polarization vectors

For massless vector boson with momentum k_5 in axial gauge (fixed by light-like vector q) we can construct polarization vectors explicitly in terms of spinors. Gauge-fixing vector q disappears in final result.

$$\epsilon_\mu(k_5,+,q) = \frac{\langle q|\gamma_\mu|5]}{\sqrt{2}\,\langle q|5\rangle}$$

$$\epsilon_\mu(k_5,-,q) = \frac{[q|\gamma_\mu|5\rangle}{\sqrt{2}\,[q|5]}$$

Abbriviations

$$D_{\chi_1,\chi_3}(s) = 2\sqrt{2}e^3 K \left[\frac{Q_e Q_t}{s} + \frac{(v_e+\chi_1 a_e)(v_\tau+\chi_3 a_\tau)}{s-M_Z^2+M_Z\Gamma_Z}\right], \quad \chi_1,\chi_3 = \pm 1$$

$$\kappa = \frac{K-1}{K}$$

$$
\begin{aligned}
\mathcal{M}^{\textsf{isr}}_{+-+-+} &= \; m_f^2 D^{\rm isr}_{+-}\mathcal{A}_{0M}\left[{}^{135}_{24}\right] + D^{\rm isr}_{++}\mathcal{A}_{0}\left[{}^{135}_{24}\right] + m_{f_1}^2 D^{\rm isr}_{-+}\mathcal{A}_{4}\left[{}^{135}_{24}\right] \\
\mathcal{M}^{\textsf{isr}}_{+++++} &= m_{f_1} m_f \Big[\; D^{\textsf{isr}}_{++}\mathcal{A}_{7}\left[{}^{135}_{24}\right] + D^{\rm isr}_{+-}\mathcal{A}_{7}\left[{}^{145}_{23}\right] \\
&\qquad - D^{\rm isr}_{-+}\mathcal{A}_{7}\left[{}^{235}_{14}\right] - D^{\rm isr}_{--}\mathcal{A}_{7}\left[{}^{245}_{13}\right] \Big] \\
\mathcal{M}^{\textsf{isr}}_{----+} &= m_{f_1} m_f \Big[\; D^{\rm isr}_{-+}\mathcal{A}_{1}\left[{}^{145}_{23}\right] + D^{\rm isr}_{--}\mathcal{A}_{1}\left[{}^{135}_{24}\right] \\
&\qquad - D^{\rm isr}_{++}\mathcal{A}_{1}\left[{}^{245}_{13}\right] - D^{\rm isr}_{+-}\mathcal{A}_{1}\left[{}^{235}_{14}\right] \Big] \\
\mathcal{M}^{\textsf{isr}}_{+++-+} &= -m_{f_1}\Big[D^{\rm isr}_{-+}\mathcal{A}_{2}\left[{}^{235}_{14}\right] + D^{\rm isr}_{++}\mathcal{A}_{2}\left[{}^{135}_{24}\right] \Big] \\
\mathcal{M}^{\textsf{isr}}_{-++++} &= -m_{f}\Big[D^{\rm isr}_{-+}\mathcal{A}_{5}\left[{}^{245}_{13}\right] + D^{\rm isr}_{--}\mathcal{A}_{5}\left[{}^{235}_{14}\right] \Big] \\
\mathcal{M}^{\textsf{isr}}_{+---+} &= -m_{f}\Big[D^{\rm isr}_{++}\mathcal{A}_{6}\left[{}^{135}_{24}\right] + D^{\rm isr}_{+-}\mathcal{A}_{6}\left[{}^{145}_{23}\right] \Big] \\
\mathcal{M}^{\textsf{isr}}_{--+-+} &= \; m_{f_1}\Big[D^{\rm isr}_{++}\mathcal{A}_{3}\left[{}^{245}_{13}\right] - D^{\rm isr}_{-+}\mathcal{A}_{3}\left[{}^{145}_{23}\right] \Big]
\end{aligned}
$$

Reduced amplitudes

$$\mathcal{A}_0\left[\begin{smallmatrix}135\\24\end{smallmatrix}\right] = \frac{\langle 1|4\rangle^2\,[4|3]}{\langle 1|5\rangle\,\langle 2|5\rangle} - \kappa\frac{\langle 1|4\rangle\,[5|3]}{\langle 2|5\rangle}$$

$$\mathcal{A}_3\left[\begin{smallmatrix}135\\24\end{smallmatrix}\right] = \frac{\langle 2|3\rangle\,\langle 3|5\rangle\,[4|3]}{\langle 1|5\rangle\,\langle 2|5\rangle^2} - \kappa\frac{\langle 3|5\rangle\,[5|4]}{\langle 1|5\rangle\,\langle 2|5\rangle}$$

$$\mathcal{A}_{0M}\left[\begin{smallmatrix}135\\24\end{smallmatrix}\right] = \frac{\langle 1|2\rangle\,[5|2]}{\langle 2|5\rangle\,\langle 3|5\rangle\,[5|4]} \qquad \mathcal{A}_1\left[\begin{smallmatrix}135\\24\end{smallmatrix}\right] = \frac{\langle 1|2\rangle\,\langle 3|5\rangle\,[5|1]}{\langle 1|5\rangle\,\langle 2|5\rangle^2\,[5|4]}$$

$$\mathcal{A}_2\left[\begin{smallmatrix}135\\24\end{smallmatrix}\right] = \frac{\langle 1|2\rangle\,\langle 1|4\rangle\,[5|3]}{z_{25}\,\langle 1|5\rangle} \qquad \mathcal{A}_4\left[\begin{smallmatrix}135\\24\end{smallmatrix}\right] = \frac{\langle 1|2\rangle\,\langle 4|5\rangle\,[5|3]}{z_{15}\,\langle 2|5\rangle^2}$$

$$\mathcal{A}_5\left[\begin{smallmatrix}135\\24\end{smallmatrix}\right] = \frac{\langle 1|3\rangle\,[4|3]}{\langle 2|5\rangle\,\langle 3|5\rangle} \qquad \mathcal{A}_6\left[\begin{smallmatrix}135\\24\end{smallmatrix}\right] = \frac{\langle 1|2\rangle\,\langle 1|4\rangle\,[5|2]}{\langle 1|5\rangle\,\langle 2|5\rangle\,[5|3]}$$

$$\mathcal{A}_7\left[\begin{smallmatrix}135\\24\end{smallmatrix}\right] = \frac{\langle 1|2\rangle\,[5|3]}{z_{25}\,\langle 4|5\rangle}$$

Spinor reference system

$$\langle 3|4\rangle = e^{-i\phi}\left\langle \mathring{3}|\mathring{4}\right\rangle, \quad \left\langle \mathring{3}|\mathring{4}\right\rangle = \langle 3|4\rangle\,|_{\phi\to 0}$$

$$\langle 1|3\rangle = \frac{\langle 1|2|5|3\rangle + \langle 1|5|4|3\rangle}{z_{12}-z_{34}} = \frac{\langle 1|2|5|3\rangle + \left\langle 1|5|\mathring{4}|\mathring{3}\right\rangle e^{-i\phi}}{z_{12}-z_{34}}$$

$$\langle 1|4\rangle = \frac{\langle 1|2|5|4\rangle + \langle 1|5|3|4\rangle}{z_{12}-z_{34}} = \frac{\langle 1|2|5|4\rangle + \left\langle 1|5|\mathring{3}|\mathring{4}\right\rangle e^{-i\phi}}{z_{12}-z_{34}}$$

$$\langle 2|3\rangle = \frac{\langle 2|1|5|3\rangle + \langle 2|5|4|3\rangle}{z_{12}-z_{34}} = \frac{\langle 2|1|5|3\rangle + \left\langle 2|5|\mathring{4}|\mathring{3}\right\rangle e^{-i\phi}}{z_{12}-z_{34}}$$

$$\langle 2|4\rangle = \frac{\langle 2|1|5|4\rangle + \langle 2|5|3|4\rangle}{z_{12}-z_{34}} = \frac{\langle 2|1|5|4\rangle + \left\langle 2|5|\mathring{3}|\mathring{4}\right\rangle e^{-i\phi}}{z_{12}-z_{34}}$$

$e^+e^- \to H + Z + \gamma$

Common factor

$$\mathcal{M}_{.....} = \frac{\sqrt{2}eQ_1 g_1^- M_Z}{s_w c_w (p_{34}^2 - M_Z^2)} \mathcal{A}_{.....} \tag{29}$$

Symmetrization operator

$$\mathbb{C}_{12}\mathcal{M} = \mathcal{M}(k_1 \leftrightarrow k_2, g_1^h \leftrightarrow g_2^{-h}) \tag{30}$$

$$\mathbb{A}_{12}^{\pm} = \frac{1 \pm \mathbb{C}_{12}}{2} \tag{31}$$

$$\mathcal{A}_{--0+-} = \mathbb{A}_{12}^{+} \frac{-m_1 g_1^- \sqrt{2} \langle 2|1|\mathbf{5}|4\rangle}{\langle \mathbf{5}|2\rangle^2 [4|\mathbf{5}|1\rangle} \quad \mathcal{A}_{--0++} = \mathbb{A}_{12}^{+} \frac{-m_1 g_1^- \sqrt{2} \langle \mathbf{5}|4\rangle [2|1]}{s_{52} [4|\mathbf{5}]}$$

$$\mathcal{A}_{--00-} = \mathbb{A}_{12}^{+} \frac{m_1 g_1^- \langle 2|1\mathbf{5}|4|\mathbf{5}\rangle}{M_Z \langle \mathbf{5}|2\rangle^2 \langle \mathbf{5}|1\rangle} \quad \mathcal{A}_{--00+} = \mathbb{A}_{12}^{+} \frac{m_1 g_1^- \langle \mathbf{5}|4|1] [2|1]}{M_Z s_{52} [1|\mathbf{5}]}$$

$$\mathcal{A}_{-+0--} = \frac{g_1^- \langle 2|1\mathbf{5}|4]}{\sqrt{2} \langle \mathbf{5}|1\rangle \langle \mathbf{5}|4\rangle} \quad \mathcal{A}_{-+0-+} = \frac{-g_1^- \langle \mathbf{5}|2|1] [4|1]}{\sqrt{2} [1|\mathbf{5}|4\rangle [2|\mathbf{5}]}$$

$$\mathcal{A}_{-+0+-} = \frac{g_1^- \langle 2|4\rangle \langle 2|1|\mathbf{5}]}{\sqrt{2} \langle \mathbf{5}|2\rangle [4|\mathbf{5}|1\rangle} \quad \mathcal{A}_{-+0++} = \frac{-g_1^- \langle 4|2\mathbf{5}|1]}{\sqrt{2} [2|\mathbf{5}] [4|\mathbf{5}]}$$

$$\mathcal{A}_{-+00-} = \frac{-g_1^- \langle 2|1\mathbf{5}|4|2\rangle}{2M_Z \langle \mathbf{5}|1\rangle \langle \mathbf{5}|2\rangle} + \frac{-g_1^+ m_1^2 s_{54} \langle 1|2\rangle}{2M_Z \langle \mathbf{5}|1\rangle^2 s_{52}} + \frac{M_Z g_1^- \langle 2|1|\mathbf{5}]}{2 s_{54} \langle \mathbf{5}|1\rangle}$$

$$\mathcal{A}_{-+00+} = \frac{-g_1^- [1|4|2\mathbf{5}|1]}{2M_Z [1|\mathbf{5}] [2|\mathbf{5}]} + \frac{M_Z g_1^- \langle \mathbf{5}|2|1]}{2 s_{54} [2|\mathbf{5}]} + \frac{g_1^+ m_1^2 s_{54} [2|1]}{2M_Z [2|\mathbf{5}]^2 s_{51}}$$

$$\mathcal{A}_{+-0--} = \frac{-g_1^+ \langle 1|2\mathbf{5}|4]}{\sqrt{2} \langle \mathbf{5}|2\rangle \langle \mathbf{5}|4\rangle} \quad \mathcal{A}_{+-0-+} = \frac{g_1^+ \langle \mathbf{5}|1|2] [4|2]}{\sqrt{2} [1|\mathbf{5}|4\rangle [2|\mathbf{5}]}$$

Spin quantization axis

Freedom in the light-cone projection choice corresponds to arbitrariness of spin quantization direction. We exploit it to make expressions compact. To obtain amplitudes for specified direction of polarization spin-rotation matrices should be applied.

Transformation to helicity basis

$$\mathcal{H}_{a_i} = C_{a_i}{}^{b_i} \mathcal{M}_{b_i}$$

$$C_{a_i}{}^{b_i} = \begin{bmatrix} \frac{\left[i^\flat|5\right]}{[i|5]} & \frac{m_i \langle i^*|5\rangle}{\left\langle i^*|i^\flat\right\rangle \langle i|5\rangle} \\ \frac{m_i [i^*|5]}{\left[i^*|i^\flat\right][i|5]} & \frac{\left\langle i^\flat|5\right\rangle}{\langle i|5\rangle} \end{bmatrix} = \begin{bmatrix} \frac{\langle i^*|i\rangle}{\left\langle i^*|i^\flat\right\rangle} & \frac{m_i \langle i^*|5\rangle}{\left\langle i^*|i^\flat\right\rangle \langle i|5\rangle} \\ \frac{m_i [i^*|5]}{\left[i^*|i^\flat\right][i|5]} & \frac{[i^*|i]}{\left[i^*|i^\flat\right]} \end{bmatrix}$$

$$p_i = \{E_i, p_i^x, p_i^y, p_i^z\} \qquad p_i^2 = m_i^2$$

$$k_{i^*} = \{|\vec{p}_i|, -p_i^x, -p_i^y, -p_i^z\} \qquad k_{i^*}^2 = 0$$

$$k_{i^\flat} = p_i - \frac{m_i^2}{2p_i \cdot k_{i^*}} k_{i^*} \qquad k_{i^\flat}^2 = 0$$

MCSANC Monte-Carlo generator for $e^+e^- \to \mu^+\mu^-(\tau^+\tau^-)$ process

We created Monte Carlo generator of unweighted events for the polarized annihilation $e^+e^- \to \mu^+\mu^-(\tau^+\tau^-)$ with complete one-loop EW corrections and with possibility to produce events in standard Les Houches format.

This generator uses adaptive algorithm `mFOAM` (CPC 177:441-458,2007) which is part of `ROOT` program.

Setup for tuned comparison

We performed a tuned comparison of our results for polarized Born and hard Bremsstrahlung with the results WHIZARD program.

Initial parameters

$$\alpha^{-1}(0) = 137.03599976,$$

$$M_W = 80.451495 \text{ GeV}, \quad M_Z = 91.1867 \text{ GeV} \quad \Gamma_Z = 2.49977 \text{ GeV},$$
$$m_e = 0.5109990 \text{ MeV}, \quad m_\mu = 0.105658 \text{ GeV}, \quad m_\tau = 1.77705 \text{ GeV},$$
$$m_d = 0.083 \text{ GeV}, \quad m_s = 0.215 \text{ GeV}, \quad m_b = 4.7 \text{ GeV},$$
$$m_u = 0.062 \text{ GeV}, \quad m_c = 1.5 \text{ GeV}, \quad m_t = 173.8 \text{ GeV}.$$

Cuts

$$|\cos\theta| < 0.9,$$
$$E_\gamma > 1 \text{ GeV} \quad \text{(for comparison of hard Bremsstrahlung)}.$$

$e^+e^- \to \ell^+\ell^-$: WHIZARD vs MCSANC

P_{e^-}, P_{e^+}	0, 0	-0.8, 0	-0.8, -0.6	-0.8, 0.6
$\sqrt{s} = 250$ GeV				
$e^- + e^+ \to e^- + e^+$ (Bhabha)				
σ^{Born}, pb	56.677(1)	57.774(1)	56.272(1)	59.276(1)
σ^{Born}, pb	56.677(1)	57.775(1)	56.272(1)	59.275(1)
σ^{hard}, pb	48.62(1)	49.58(1)	48.74(1)	50.40(1)
σ^{hard}, pb	48.65(1)	49.56(1)	48.78(1)	50.44(1)
$e^- + e^+ \to \mu^- + \mu^+$				
σ^{Born}, pb	1.4174(1)	1.5462(1)	0.7690(2)	2.3231(2)
σ^{Born}, pb	1.4177(1)	1.5464(1)	0.7693(1)	2.3235(1)
σ^{hard}, pb	1.822(1)	2.035(1)	1.000(1)	3.069(1)
σ^{hard}, pb	1.821(1)	2.034(1)	1.016(1)	3.049(1)
$e^- + e^+ \to \tau^- + \tau^+$				
σ^{Born}, pb	1.4174(1)	1.5461(1)	0.7692(1)	2.3230(1)
σ^{Born}, pb	1.4176(1)	1.5462(1)	0.7693(1)	2.3233(1)
σ^{hard}, pb	1.632(1)	1.831(1)	0.898(1)	2.763(1)
σ^{hard}, pb	1.633(1)	1.828(1)	0.915(1)	2.741(1)

Conclusion

- The background for complete one loop calculation of the EW radiative corrections for processes with polarized e^+e^- beams (longitudinal and transversal) is created: HA for virtual part & HA for Bremsstrahlung
- Complete $O(\alpha)$ EW corrections to polarized
  a) Bhabha scattering
  b) $e^+e^- \to \mu^+\mu^-$ $(\tau^+\tau^-)$ are computed for the first time
- Physical program of future e^+e^- colliders is under development. Many new tasks for theoreticians are there: Monte Carlo event generator(s) for experimentalists