

Radiative Corrections to DIS at CERN and DESY

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Foreword

I met Dima Bardin in the fall of 1971 when I was a 5th year student of the Moscow State University and we had in Dubna the lectures of D. I. Blokhinzev, B. M. Pontekorvo, V.G. Soloviev, B.M. Barbashev, S.M. Bilenky, N.M. Plakida and the other teachers.

It was the time of the development of the Standard Model when we knew only lights quarks: up, down and strange. I remember the seminars in LTHP in 1974/75 by A.E. Efremov about the discoveries of the J/Psi particles which gave us the c-quark. The quark structure of the nucleons became one of the important task of High Energy Physics.

In 1976 initiated by NA4 experiment at CERN on deep inelastic scattering of muons on nucleus we started a new calculation of RC to deep inelastic IN-scattering at energies SPS:

$$l + N \rightarrow l + \text{hadrons}$$

During 1976-1978 (with D.Yu. Bardin and N.M. Shumeiko) we developed a covariant treatment of the model independent QED RC to deep inelastic lepton-nucleon scattering. We have obtained the new analytical formulae without softness parameter of Mo and Tsai and in Lorenz-invariant form (Yad. Fiz., 26 (1977) p. 1251)

We wrote and tested Fortran code TERAD (RADCOR) for data processing in NA4 (BCDMS) experiment at CERN.

1979-1983 (with D. Yu. Bardin and G. V. Mitselmakher)

First time derived new analytical formulae for RC to the Primakoff process of radiative scattering of charged pions on nucleus:

$$\pi + Z \rightarrow \pi + Z + \gamma$$

(Yad. Fiz., 37 (1983) p. 360).

Wrote and tested Fortran code RCFORGV for analysis of the data in the experiment AJAKS-SIGMA at IHEP (Serpukhov).

1984-1988 (with D. Yu. Bardin, T. Riemann and O. Fedorenko) (Yad. Fiz., 42 (1985) p. 1204).

Developed the algorithms for the exact calculations of the QED RC to $e^+ e^- \rightarrow \mu^+ \mu^-$ using the computer algebra systems such as SCHOONSCHIP (CDC-6500).

1985 (with D. Yu. Bardin and T. Riemann)

Performed the first complete calculations of the EW one-loop corrections to the decay width of the neutral vector boson (Nucl. Phys. B276 (1986) p. 1)

The formulae derived in this paper have been implemented into the weak library DIZET, which became an important ingredient of the well-known codes ZFITTER and TERAD91. (Phys. Lett., B166 (1986) p. 111) (see below).

1985 (with D. Yu. Bardin and N. M. Shumeiko)

Calculated the α^2 -order QED RC to deep inelastic lepton-nucleon scattering at SPS energies.

(Yad. Fiz., 44 (1986) p. 1517).

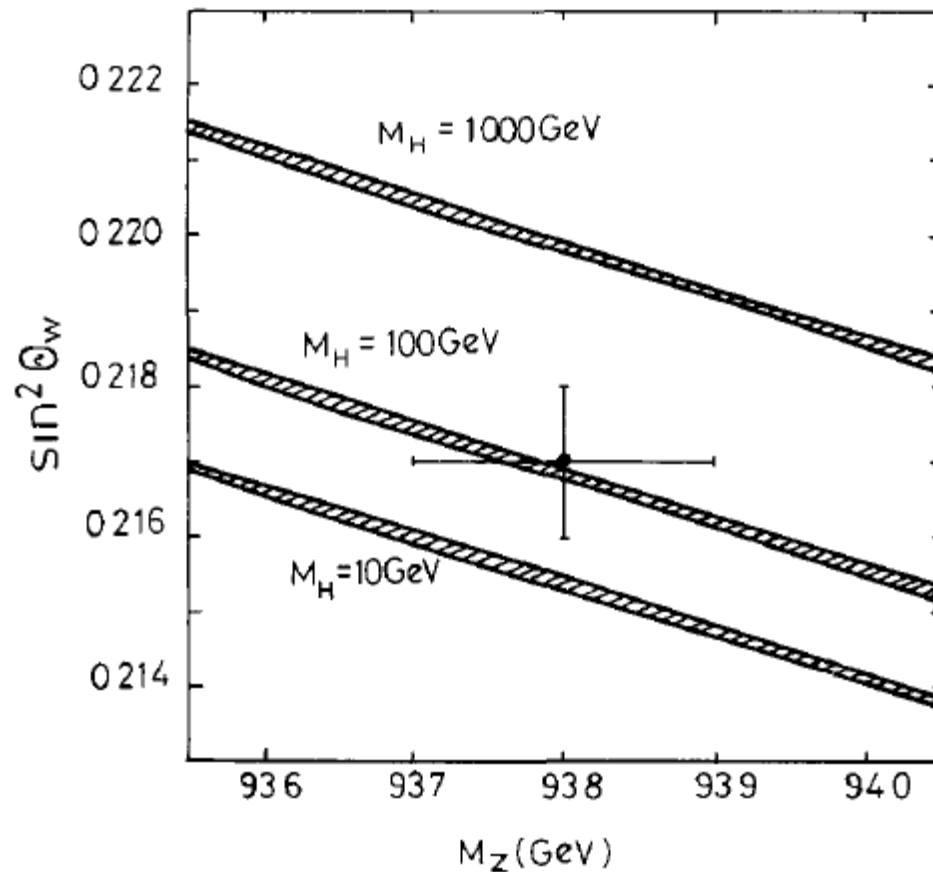


Fig 1 Graph of $\sin^2 \theta_w$ versus M_Z , influenced by M_H through radiative corrections. The thickness corresponds to the range $30 \text{ GeV} \leq m_t \leq 40 \text{ GeV}$, the error bars indicate the accuracy expected at Z boson factories.

1989-1992 (with D. Yu. Bardin and T. Riemann)

Developed a common treatment of the complete model independent QED corrections to the process $e p \rightarrow e X$.

First time derived new analytical formulae for RC in several variables – leptonic, hadronic, mixed and Jaquet-Blondel variables. These formulae have been implemented in the semi-analytical Fortran code TERAD91 for the calculations of RC to deep inelastic $e p$ -scattering at HERA in terms of the various kinematical variables.

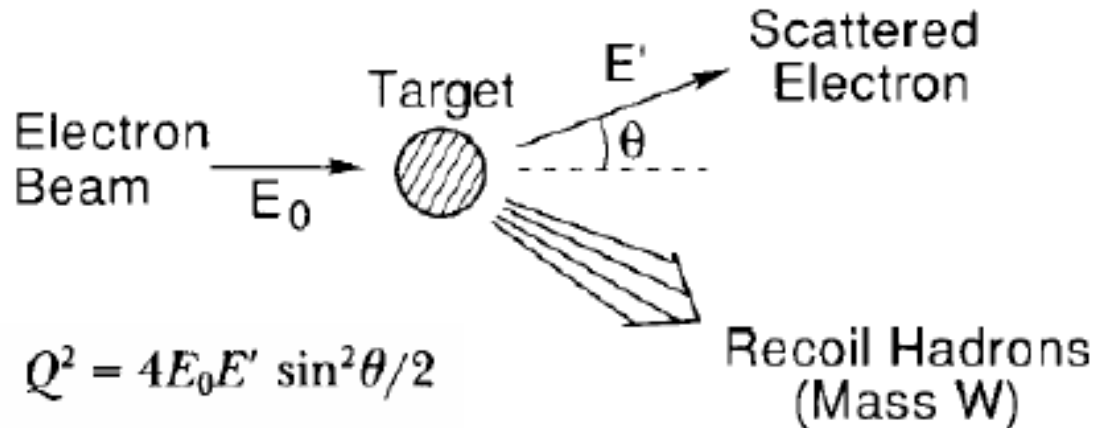
1993-1996 (with D. Yu. Bardin, L. Kalinovskaya and T. Riemann)

Wrote a complete review on the model independent treatment of the QED RC to deep inelastic scattering for fixed target as well as collider experiments at HERA or LEP x LHC (Fortschr. Phys., 44 (1996) p. 373-482).

Radiative Corrections to Deep Inelastic Scattering

- Deep Inelastic Scattering at SLAC
- RC to DIS at SPS (CERN)
- RC to DIS at HERA (DESY)

Kinematics of DIS



$$Q^2 = 4E_0E' \sin^2 \theta/2$$

$$E' = \frac{E_0 - \frac{(W^2 - M^2)}{2M}}{1 + \frac{2E_0}{M} \sin^2 \theta/2}$$

or, since E' and θ are measured:

$$W^2 = M^2 + 2M(E_0 - E') - 4E_0E' \sin^2 \frac{\theta}{2}$$

- Standard variables are:

$$x = \frac{-q^2}{2p \cdot q} = \frac{Q^2}{2M(E - E')}$$
$$y = \frac{q \cdot p}{k \cdot p} = 1 - \frac{E'}{E}$$

where $Q^2 = -q^2 > 0$, $M^2 = p^2$ and energies refer to target rest frame.

- Elastic scattering has $(p + q)^2 = M^2$, i.e. $x = 1$. Hence **deep inelastic** scattering (DIS) means $Q^2 \gg M^2$ and $x < 1$.

- **Structure functions** $F_i(x, Q^2)$ parametrise target structure as 'seen' by virtual photon. Defined in terms of cross section

$$\frac{d^2\sigma}{dxdy} = \frac{4\pi\alpha^2}{Q^2xy} \left[\left(\frac{1 + (1-y)^2}{2} \right) 2xF_1 + (1-y)(F_2 - 2xF_1) - (M/2E)xyF_2 \right].$$

On general grounds the structure functions are functions of two kinematical invariants

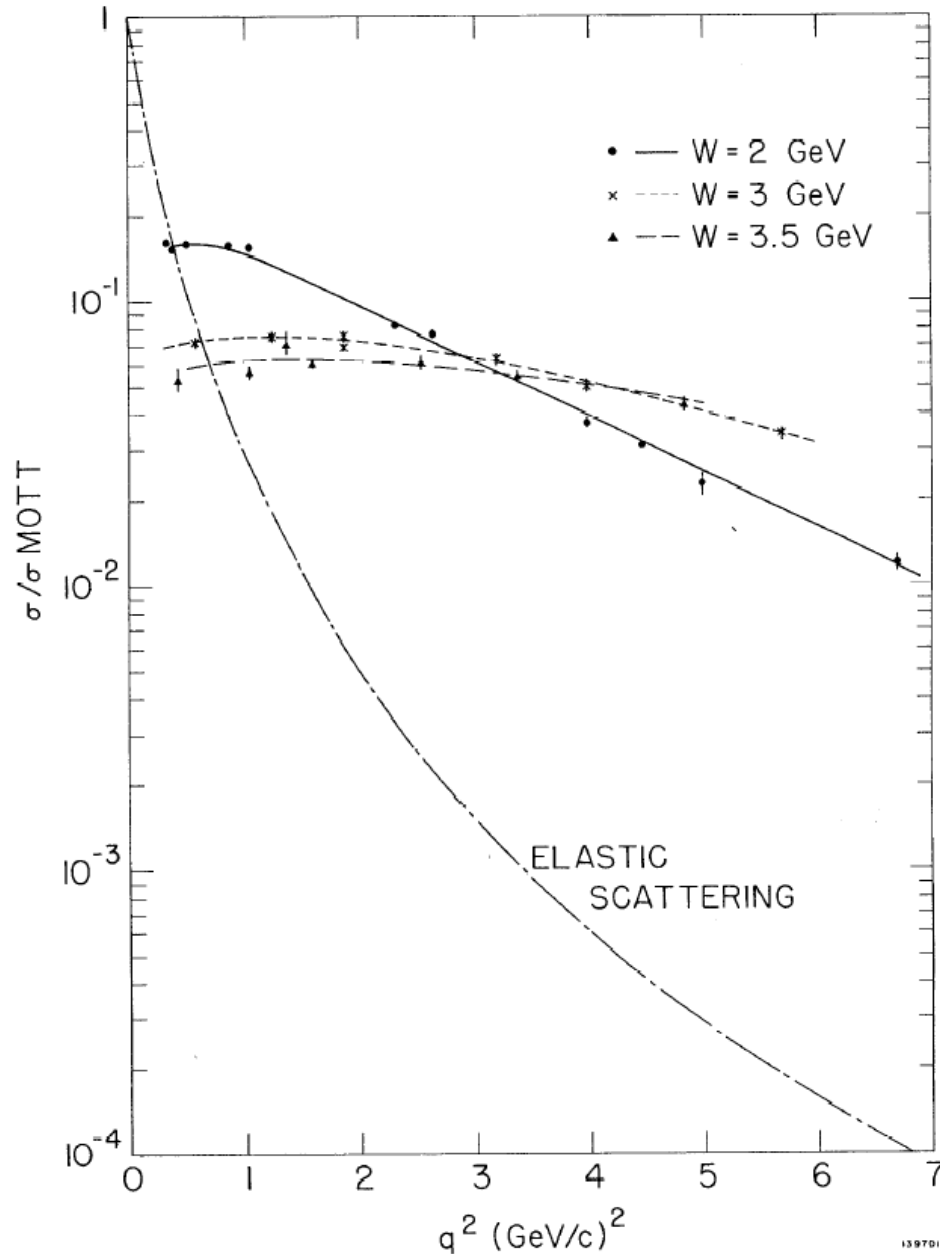
They were expected to drop with increasing Q^2 as rapidly as the form factors

The surprising experimental result was different!

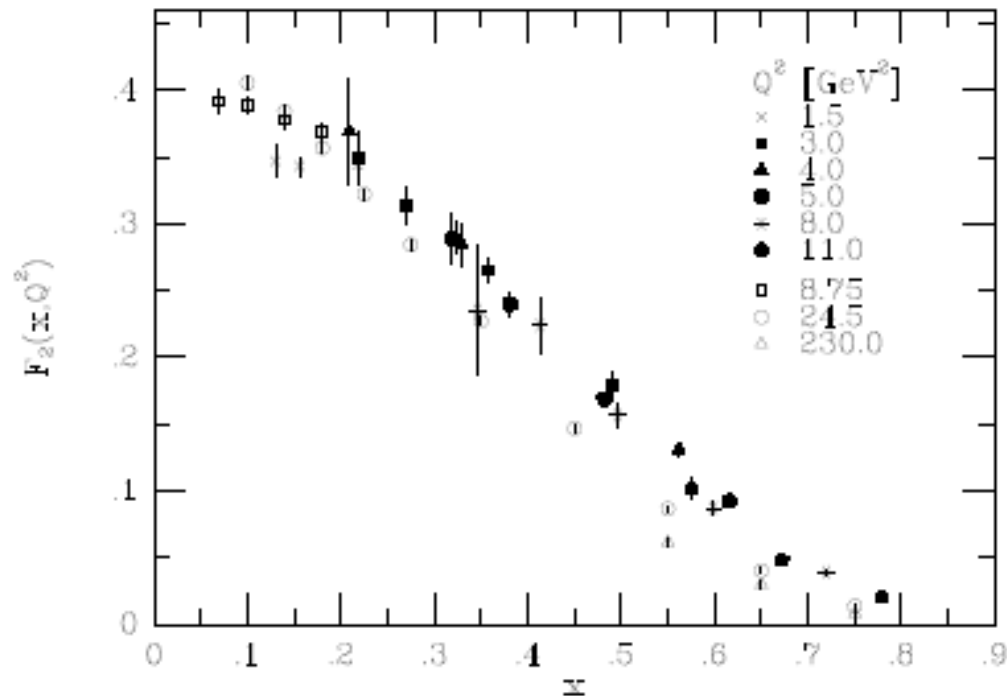
OBSERVED BEHAVIOR OF HIGHLY INELASTIC
ELECTRON-PROTON SCATTERING

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J. Drees, L. W. Mo, R. E. Taylor
Stanford Linear Accelerator Center,† Stanford, California 94305



- Deep inelastic:
 - $W \gg M_p$
- $\sigma/\sigma_{\text{Mott}} \approx \text{const.}$

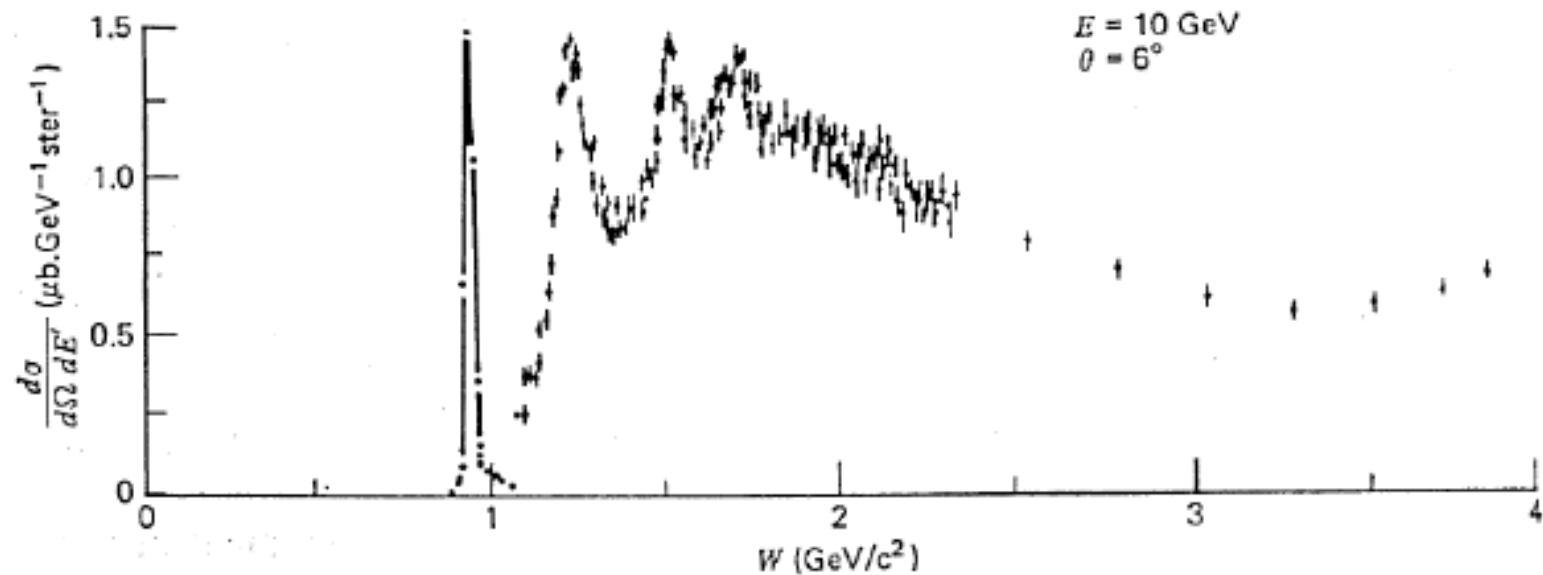


- Figure shows F_2 structure function for proton target. Although Q^2 varies by two orders of magnitude, in first approximation data lie on universal curve.

- **Bjorken limit** is $Q^2, p \cdot q \rightarrow \infty$ with x fixed. In this limit structure functions obey approximate **Bjorken scaling** law, i.e. depend only on dimensionless variable x :

$$\begin{aligned} F_i(x, Q^2) &\longrightarrow F_i(x) \\ \sigma_i(x, Q^2) &= \frac{4\pi^2\alpha}{Q^2} F_i(x, Q^2) \sim 1/Q^2 \end{aligned}$$

- Bjorken scaling implies that virtual photon is scattered by *pointlike constituents* (**partons**) — otherwise structure functions would depend on ratio Q/Q_0 , with $1/Q_0$ a length scale characterizing size of constituents.



up to about $W = 1.8 \text{ GeV}$ there is structure corresponding to the production of **resonances** (excited nucleon states);
there is no structure above 1.8 GeV : this is the region of DIS.

Radiative Corrections to Elastic and Inelastic ep and νp Scattering*

L. W. MO, Y. S. TSAI

Stanford Linear Accelerator Center, Stanford University, Stanford, California

$$(d\sigma_r/d\Omega dp)(E_s, E_p) = (d\sigma/d\Omega dp)(E_s, E_p)[1 + \delta_r(\Delta)] + (d\sigma_r/d\Omega dp)(\omega > \Delta),$$

where $d\sigma/d\Omega dp(E_s, E_p)$ is the continuum nonradiative cross section,

$$\delta_r(\Delta) = \frac{-\alpha}{\pi} \left[\frac{2}{9} - \frac{1}{8} \ln \frac{2(sp)}{m^2} + \left(\ln \frac{E_s}{\Delta} + \ln \frac{E_p}{\Delta} \right) \left(\ln \frac{2(sp)}{m^2} - 1 \right) - \Phi\left(-\frac{E_s - E_p}{E_p}\right) - \Phi\left(\frac{E_s - E_p}{E_s}\right) \right],$$

$\Phi(x)$ is the Spence function, and

$$\frac{d\sigma_r}{d\Omega dp}(\omega > \Delta) = \frac{\alpha^2}{2\pi} \frac{E_p}{ME_s} \int_{-1}^1 d(\cos \theta_k) \int_{\Delta}^{\omega_{\max}(\cos \theta_k)} \frac{\omega d\omega}{q^4} \int_0^{2\pi} B_{\mu\nu} \epsilon T_{\mu\nu} d\phi_k.$$

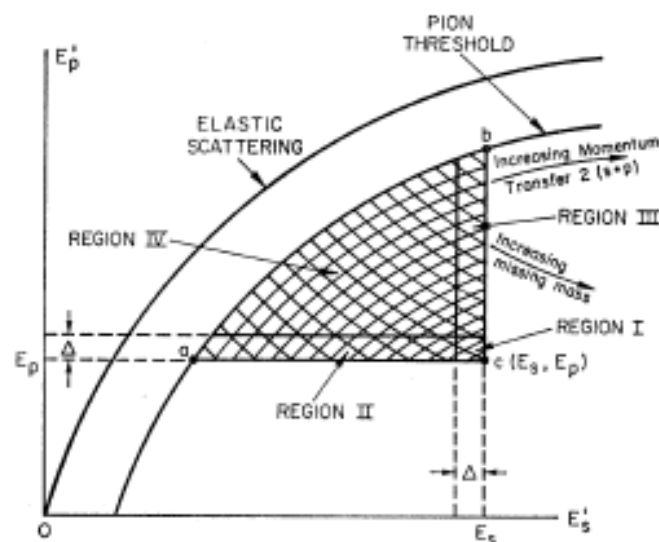
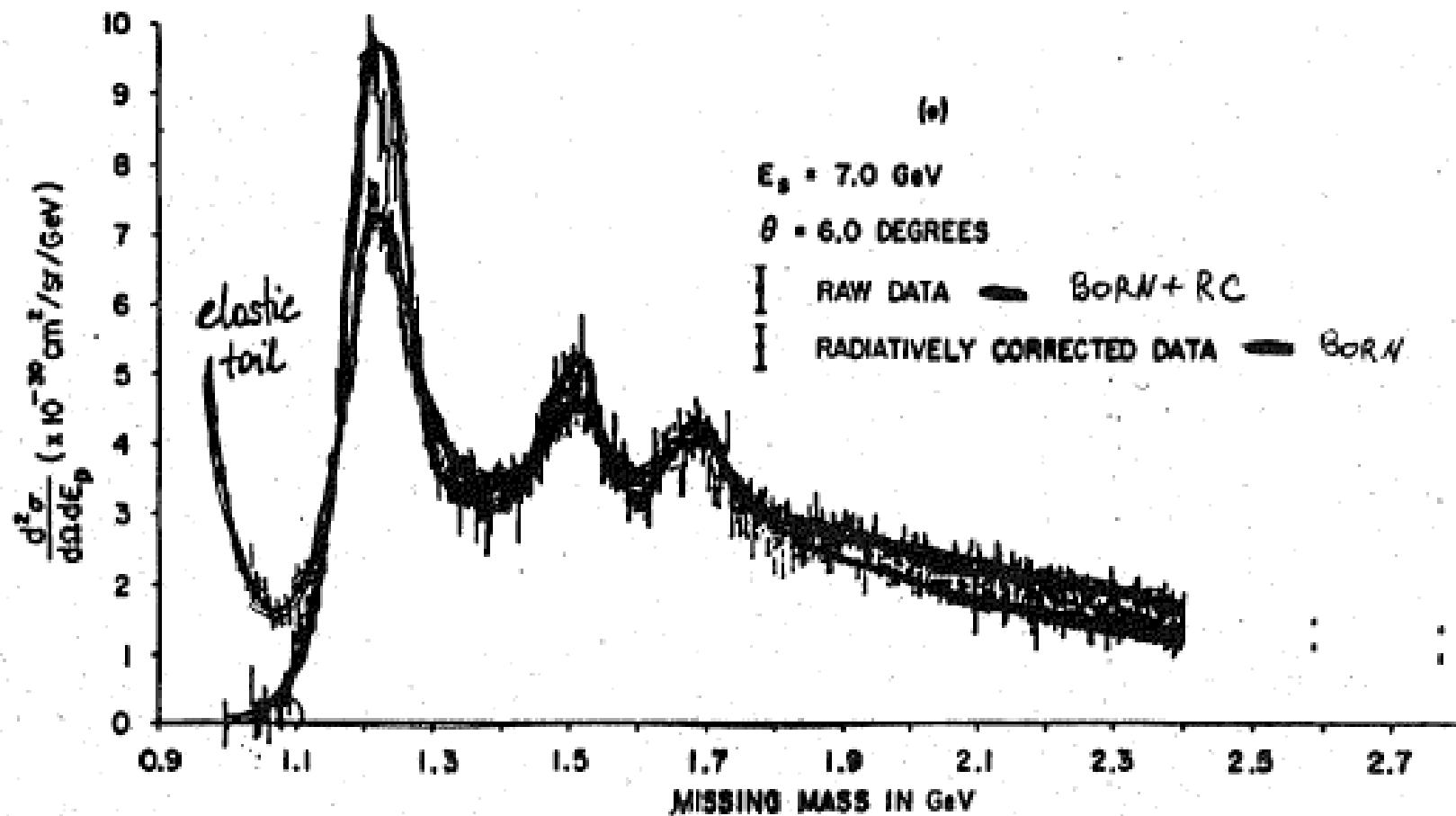


FIG. 3. Kinematic regions necessary for radiative corrections to inelastic electron scattering. E_s' is the incident electron energy and E_p' , the scattered electron energy.



RC to DIS at SPS (CERN)



CALC2018, Dubna, 23.07.2018

of structure functions,

In inclusive-type experiments when only the final lepton is detected the processes

$$l + N \rightarrow l + \gamma + N, \quad (1)$$

$$l + N \rightarrow l + \gamma + N^*, \quad (2)$$

$$l + N \rightarrow l + \gamma + \text{hadrons} \quad (3)$$

(N^* is the nucleon resonance)

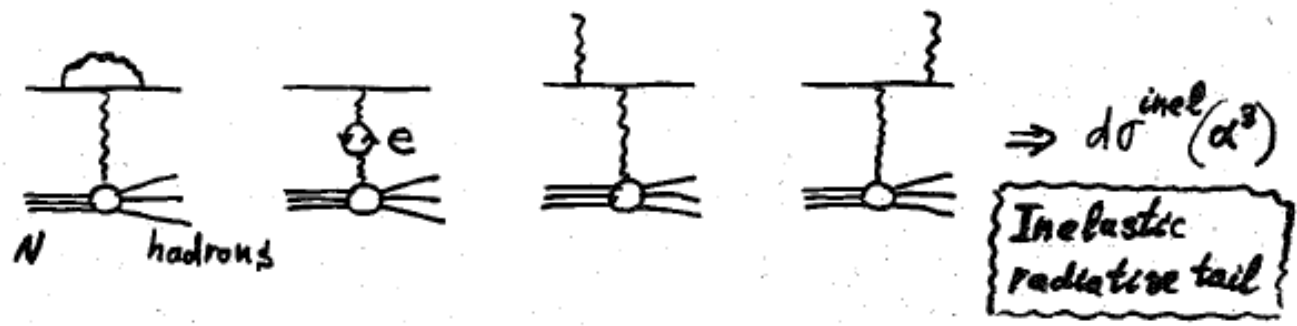
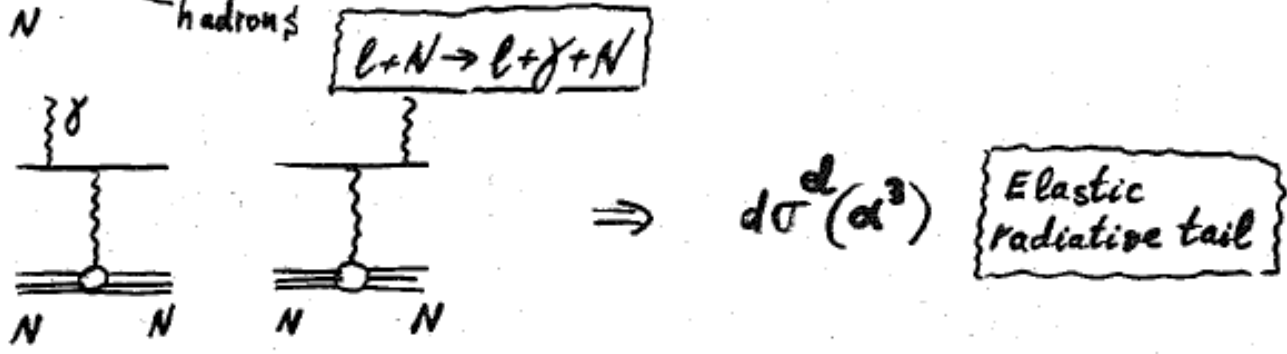
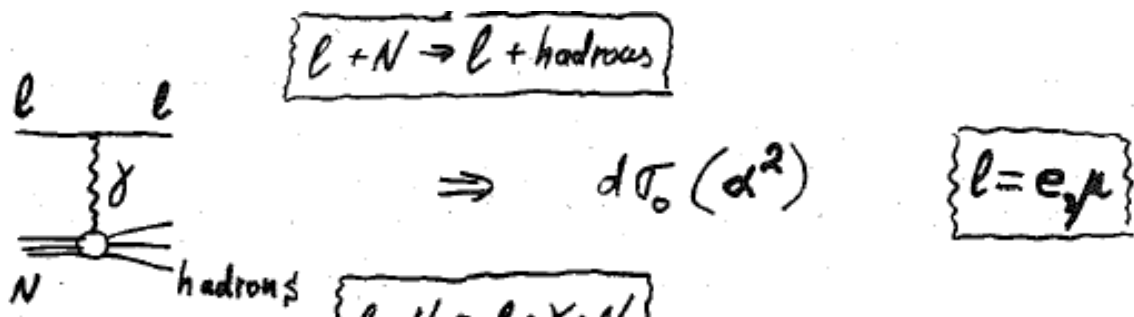
cannot be distinguished from the main reaction

$$l + N \rightarrow l + \text{hadrons}. \quad (4)$$

Thus, the measured cross section of deep inelastic lN -scattering is a sum of inclusive cross sections of processes (1), (2), (3) and main process (4)

$$d^2\sigma_{\text{exp}} = d^2\sigma_{\text{tail}} + \sum d^2\sigma_{\text{Htail}} + d^2\sigma_0 (1+\delta), \quad (5)$$

where $d^2\sigma_0$ is the cross section of reaction (4) in the Born approximation and δ is the EC to the cross section.



$$d\sigma_1(\alpha^3) = d\sigma^d(\alpha^3) + d\sigma^{inel}(\alpha^3)$$

$$\delta_1(\alpha) = d\sigma_1(\alpha^3) / d\sigma_0(\alpha^2) = \delta^d(\alpha) + \delta^{inel}(\alpha)$$

1. Some experiments on μp -scattering^{/7/} are carried out in the region $Q^2 \approx m_\mu^2$ (m_μ - the muon mass). The formulae of Mo and Tsai are inapplicable at such Q^2 , therefore exact formulae are required.

2. Expressions for the EC in ref.^{/4/} include the "softness" parameter Δ dividing the contributions of soft and hard photons thus breaking their Lorentz-invariance. However, in inclusive processes these contributions are not separated physically^{/8/} therefore it would be reasonable to obtain formulae independent of the "softness" parameter and covariant in order to make them be applicable directly to the planned experiments on the colliding $e p(\mu p)$ beams.

A revised calculation of electromagnetic radiative corrections was performed in refs.^{/2,3/}. Completely covariant formulae are obtained which contrary to those given by Mo and Tsai, do not contain the unphysical "soft-photon" parameter.

A.A. Akhundov, D.Yu. Bardin and N.M. Shumeiko, *Electromagnetic corrections to deep inelastic lepton-nucleon scattering at high energies. I. Contribution of the radiative tail of elastic peak*, JINR Report, JINR-E2-10147 (1976), 24 pp.

A.A. Akhundov, D.Yu. Bardin and N.M. Shumeiko, *Electromagnetic corrections to deep inelastic μp scattering at high energies*, JINR Preprint, JINR-E2-10471 (1977), 18 pp. &

Yadernaya Fizika, v.26, p.1251-1257 (1977) (in Russian) &
Sov. J. Nucl. Phys., v.26, 660 (1977) (in English, USA).

RC from Elastic Radiative Tail

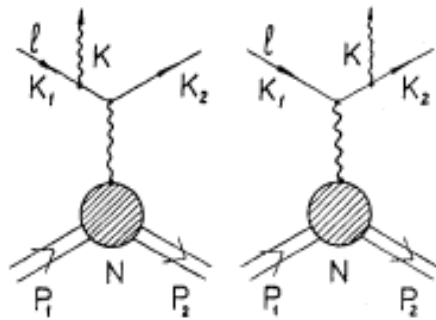


Fig. 1. The diagram giving the dominant contribution to the cross section of the process (1).

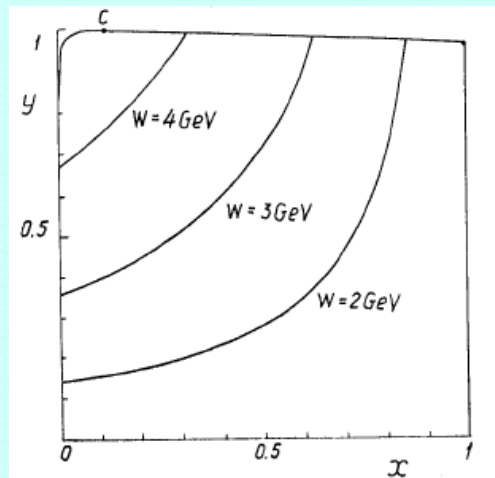
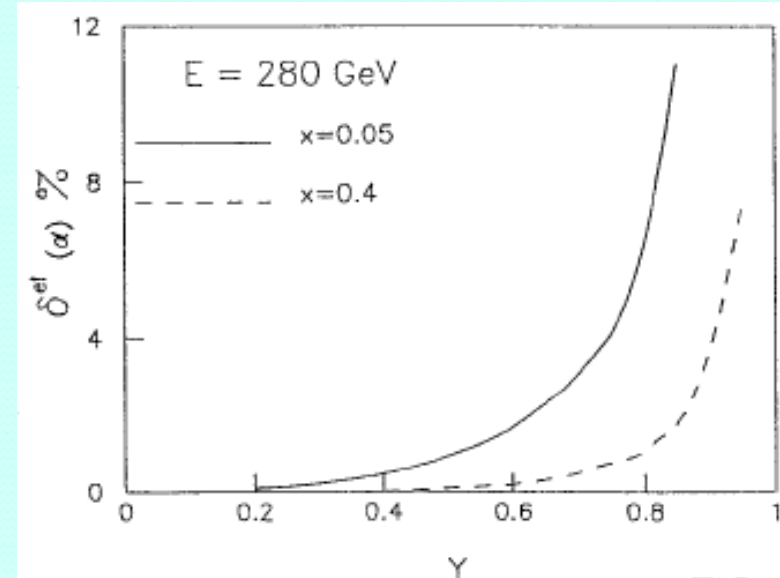


Fig. 3. The physical region of the process (4) in the scaling variables. Numerical values refer to the reaction $\mu p \rightarrow \mu X$ at $E=120\text{ eV}$.

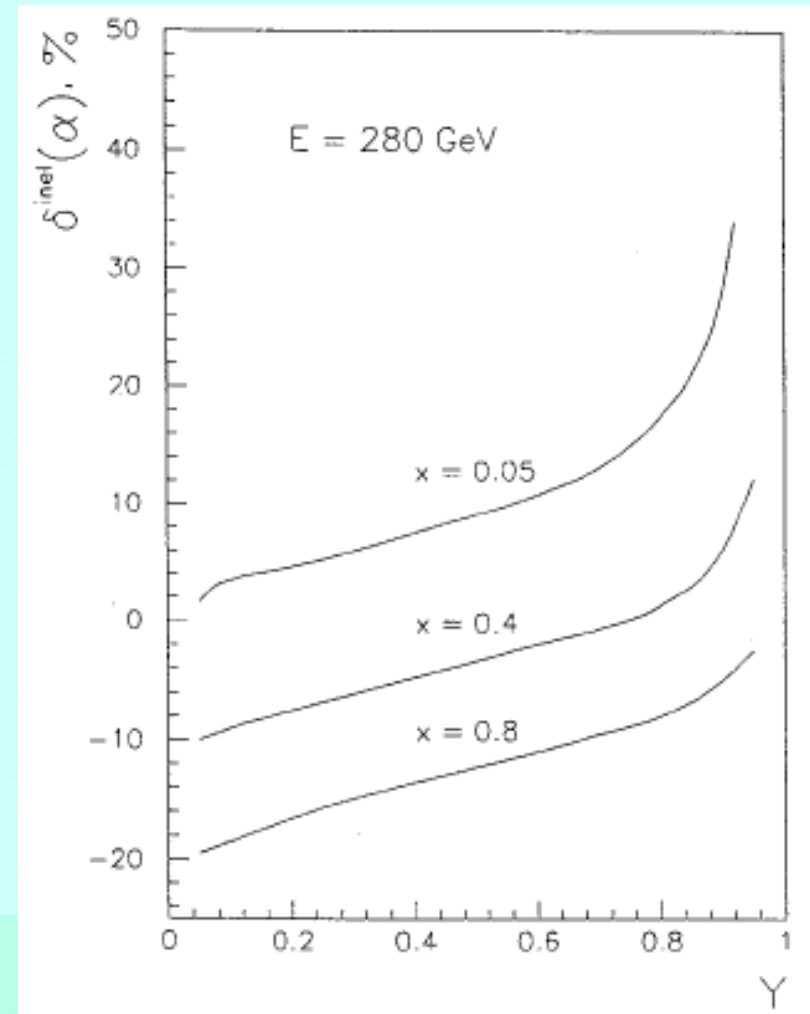


RC from Deep Inelastic Radiative Tail

In the lowest α order this contribution can be represented as follows:

$$2\text{Re} \left[\begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \\ \text{Diagram 3} \\ \text{Diagram 4} \end{array} \right] + \begin{array}{c} \text{Diagram 5} \\ \text{Diagram 6} \\ \text{Diagram 7} \end{array} \quad (3)$$

The diagrams represent various Feynman diagrams for the radiative tail contribution. Diagrams 1-4 are grouped in a bracket with a 2Re factor, and Diagrams 5-7 are added separately. Diagram 2 includes a label P, μ .



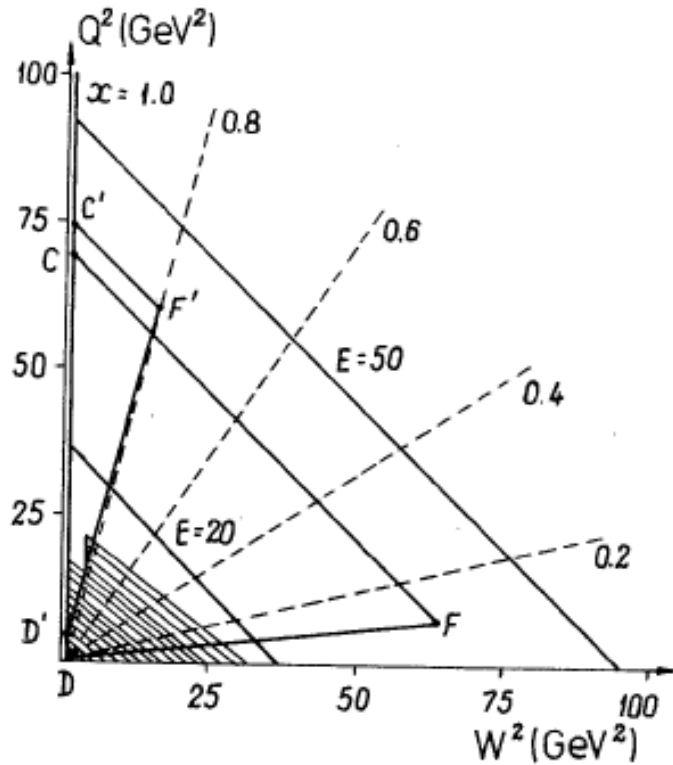
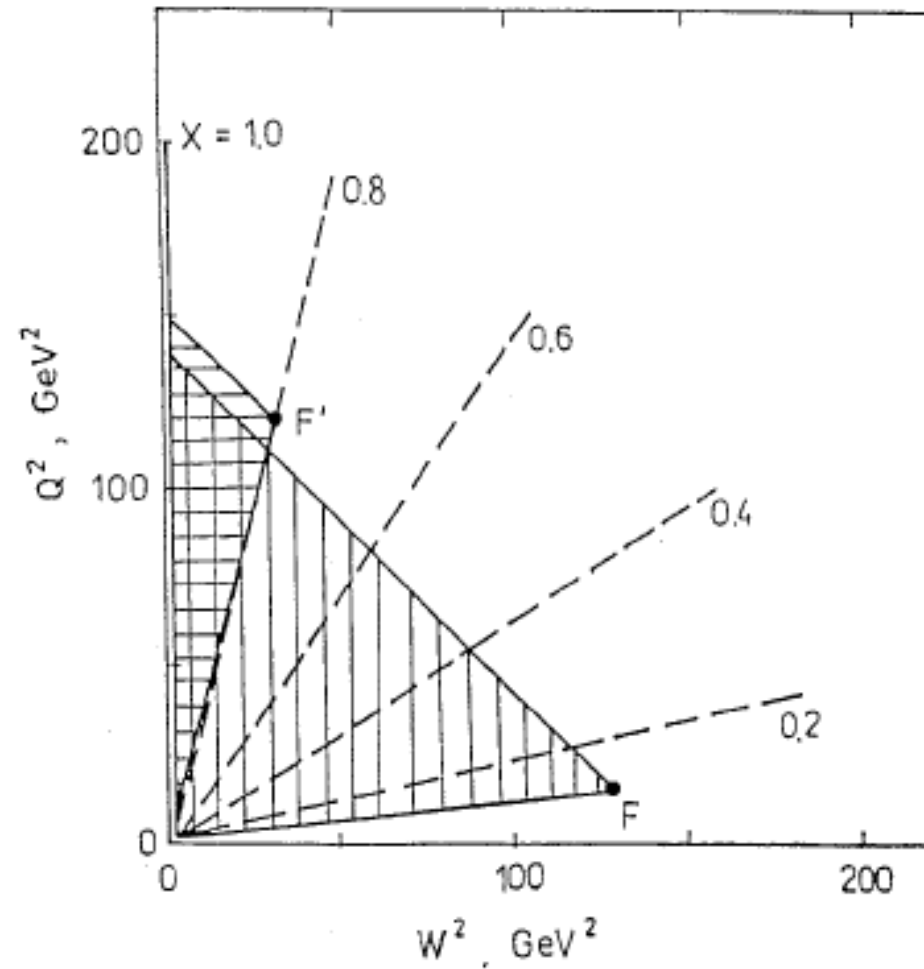
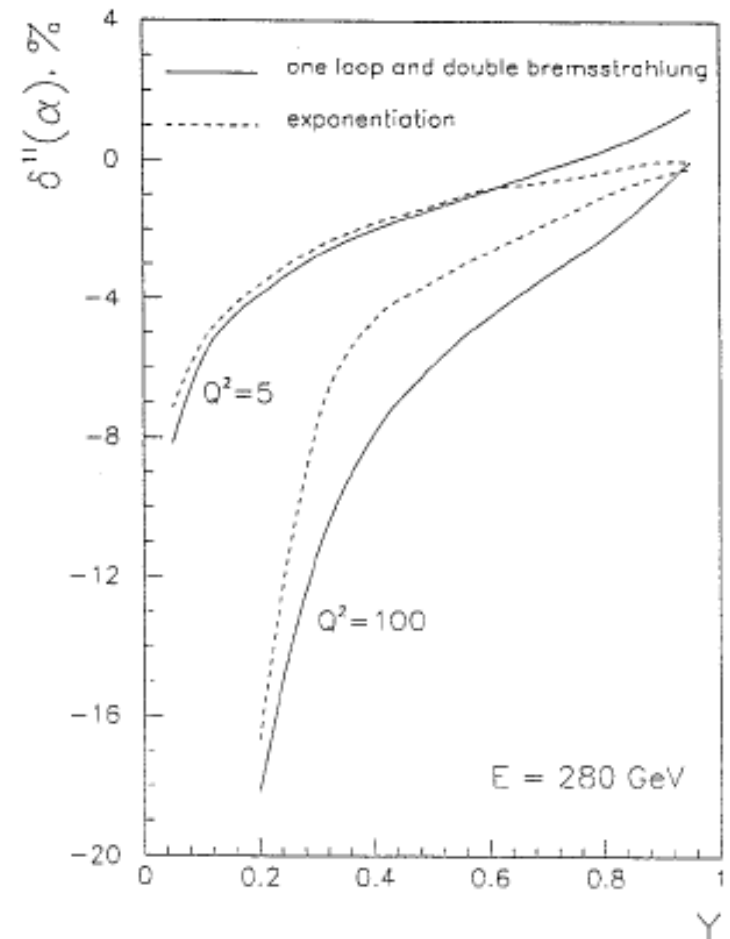
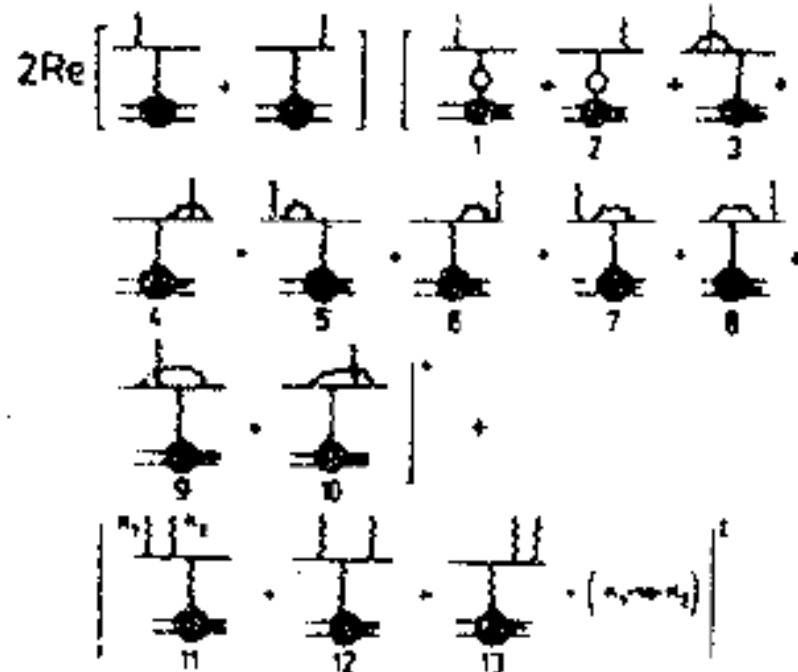


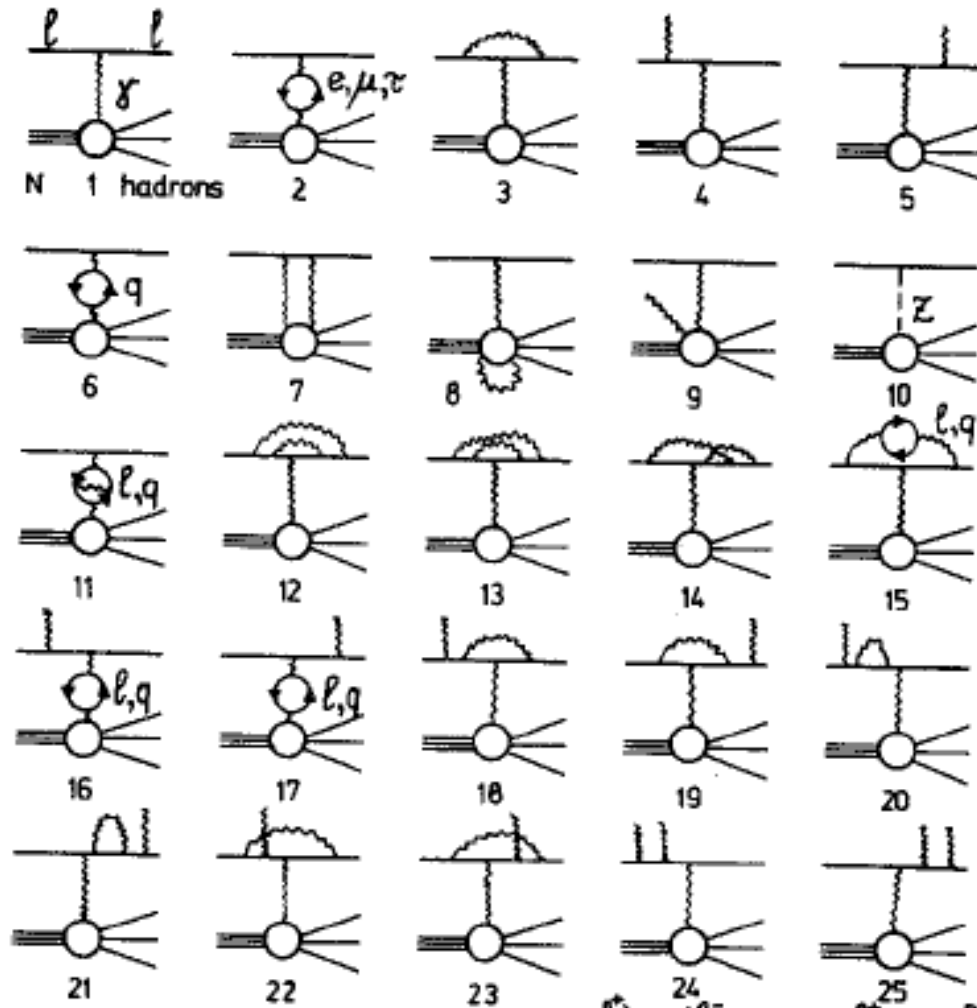
Fig.2. The kinematic region of the deep inelastic μN -scattering in the (W^2, Q^2) plane. The boundary is shown at $E=20$ and 50 GeV. Dotted lines correspond to x constant. The shaded region shows where the structure functions are studied in detail^{13/}.



A.A. Akhundov, D.Yu. Bardin and N.M. Shumeiko,
Electromagnetic corrections to the elastic radiative tail in deep inelastic lepton-nucleon scattering, JINR Preprint, JINR-P2-85-831 (1985) 12 pp. &
Yadernaya Fizika, v.44, p.1517-1526 (1986) (in Russian) &
Sov. J. Nucl. Phys. v.44, p.988 (1986) (in English, USA).



TERAD86 -> BCDMS, NMC



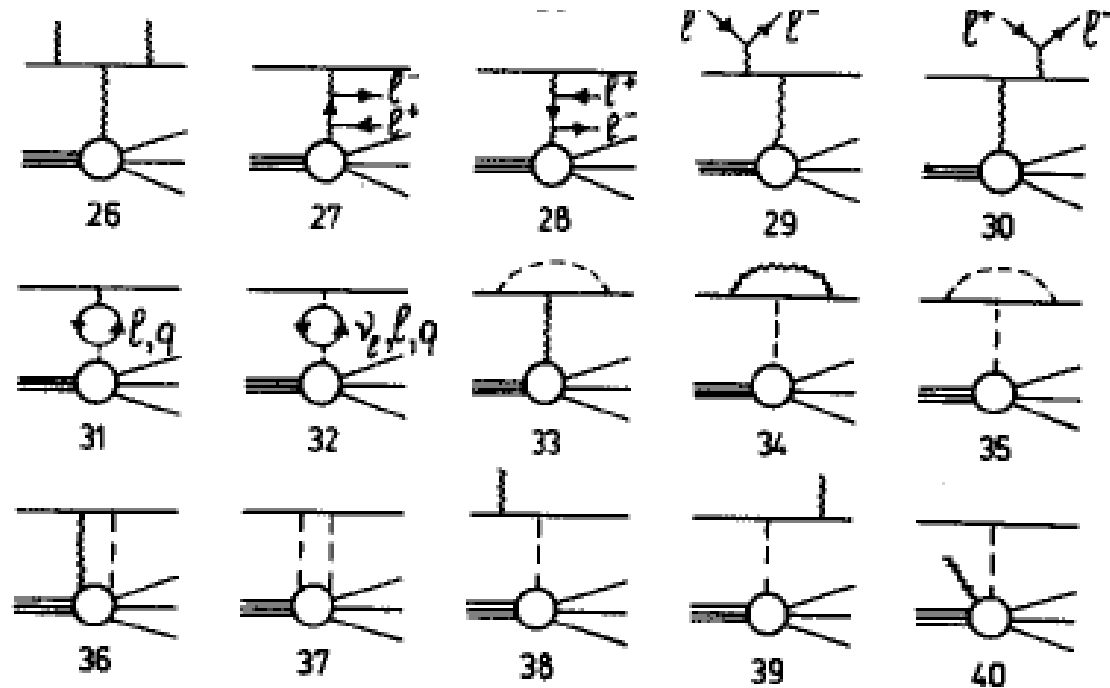


Fig.1. Lowest- and higher-order electroweak radiative processes contributing to the observed deep inelastic cross section.

A. Akhundov, D. Bardin and W. Lohmann, Fortran program TERAD86 and JINR Dubna preprint E2-86-104 (1986);

A. Akhundov and W. Lohmann, IfH Zeuthen preprint PHE 90-32 (1990).

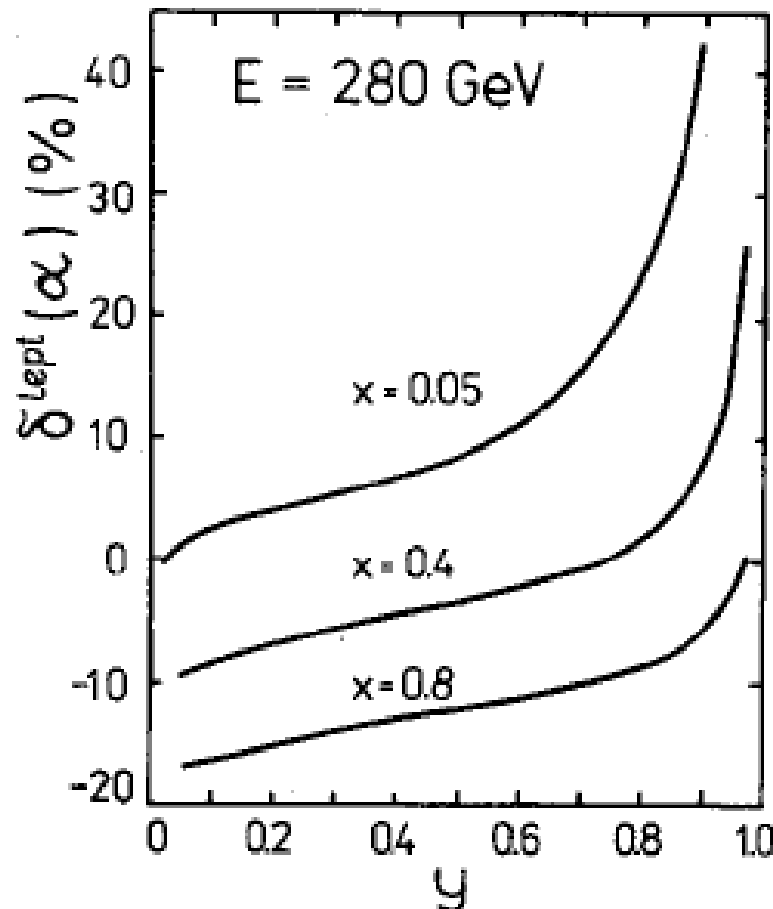
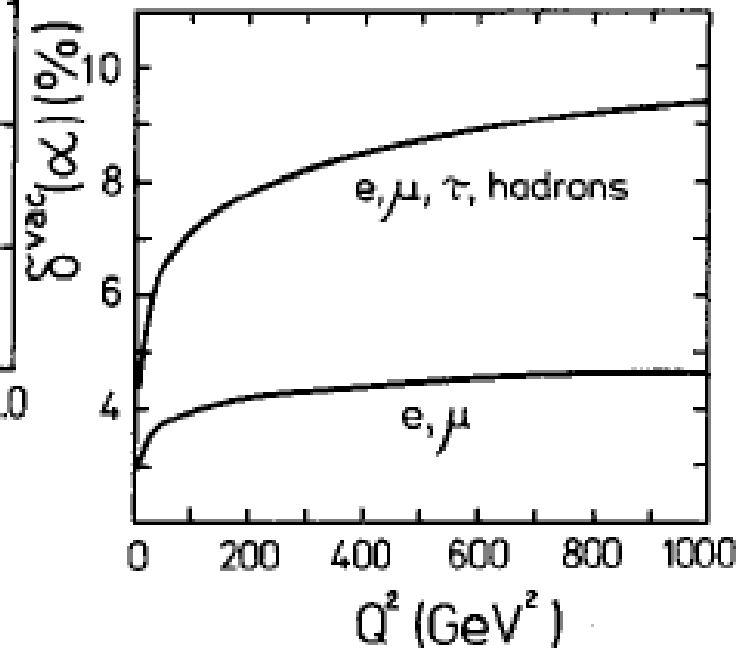


Fig.2. The lepton electromagnetic correction $\delta^{\text{lept}}(\alpha)$ to deep inelastic μp scattering as functions of y for several x at 280 GeV.

Fig.3. The vacuum polarization corrections $\delta^{\text{vac}}(\alpha)$ arising from (e, μ) and $(e, \mu, \tau, \text{hadrons})$ contributions as functions of Q^2 .



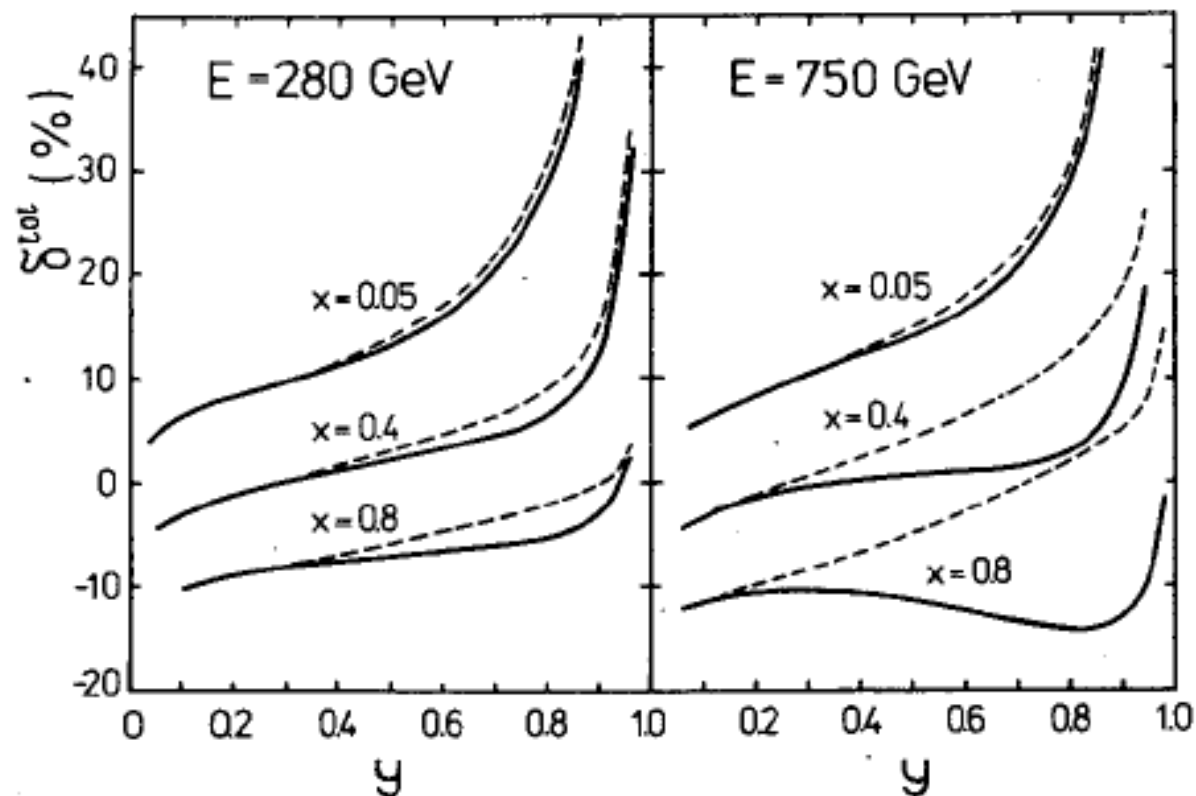
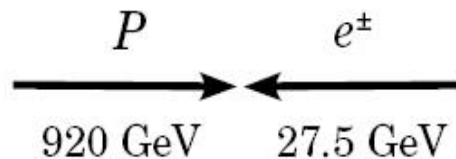
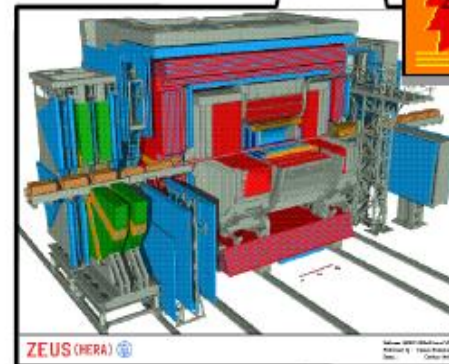
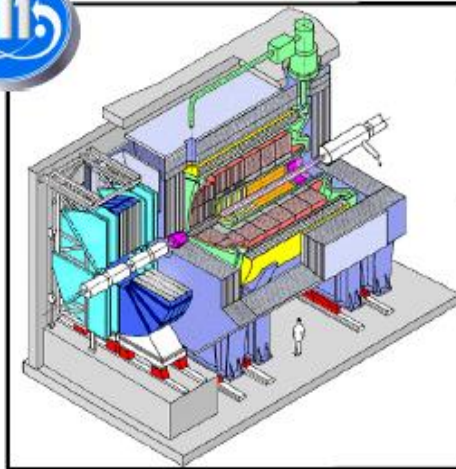


Fig.5. The total electroweak corrections $\delta_{\pm\lambda}^{\text{tot}}$ to deep inelastic μ^*p scattering as functions of y for several x at 280 and 750 GeV. Solid lines correspond to μ^+ ($\lambda = -0.8$) and dotted lines, to μ^- ($\lambda = +0.8$).

RC to DIS at HERA

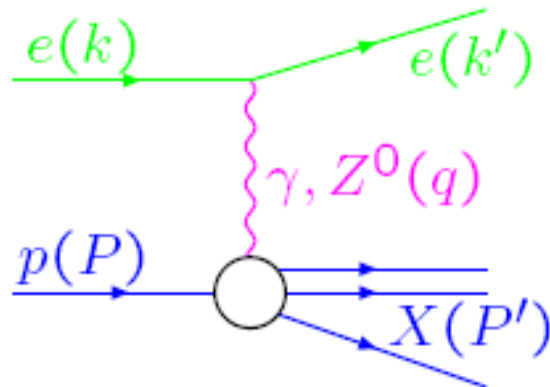
HERA Collider at DESY



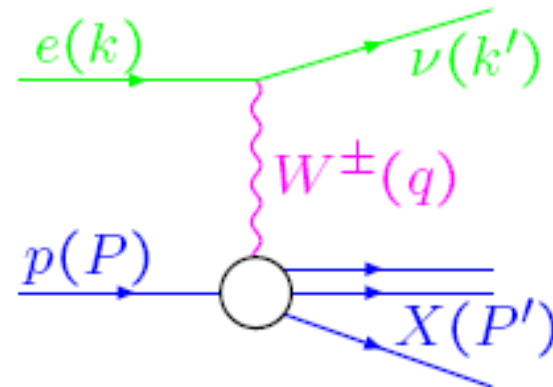
Central mass energy $\sqrt{s} = 318 \text{ GeV}$

DIS at HERA

Neutral Current (NC)



Charged Current (CC)



Invariant kinematic quantities:

$$Q^2 = -q^2 = -(k - k')^2 \quad \text{negative four-momentum transfer squared}$$

$$x = \frac{Q^2}{2P \cdot q} \quad \text{In proton infinite-momentum frame: fraction of proton momentum}$$

$$y = \frac{P \cdot q}{P \cdot k} \quad \text{In proton rest-frame: energy-transfer}$$

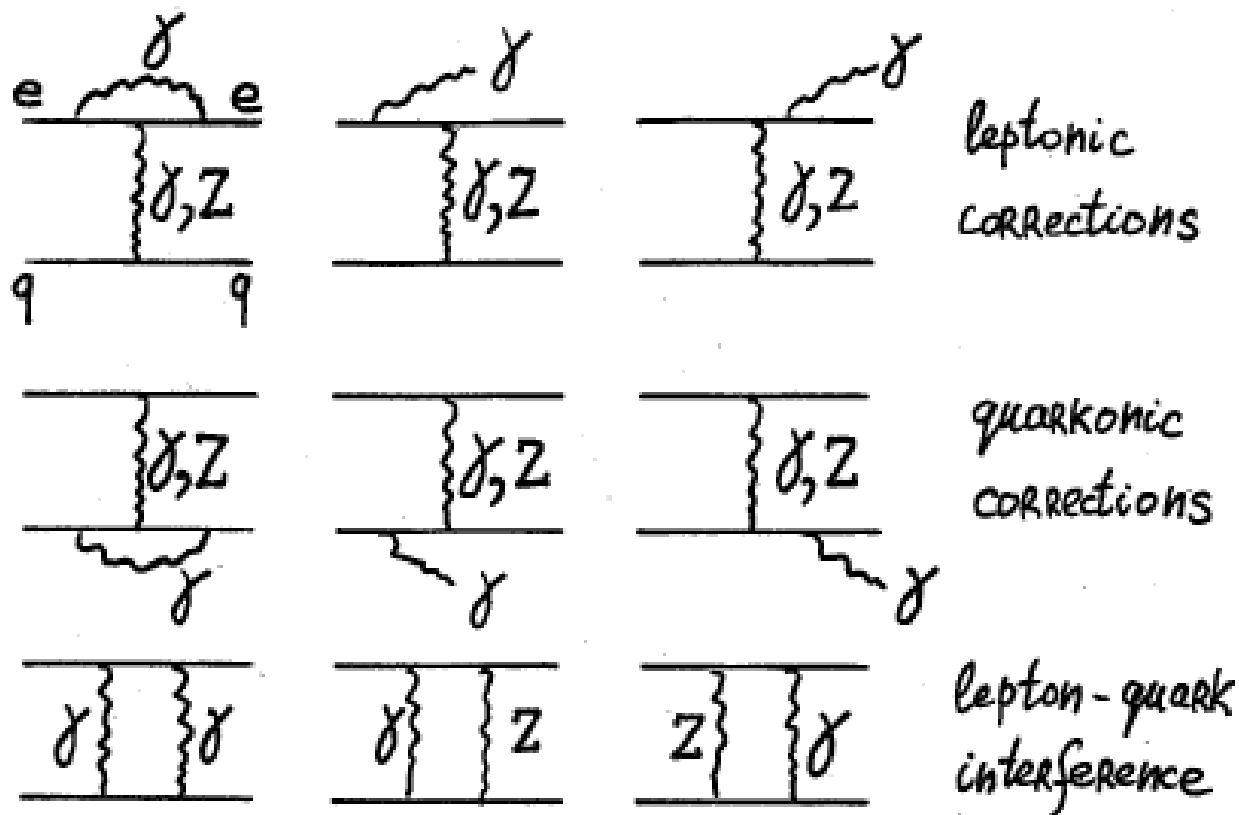
$$s = (k + P)^2 = \frac{Q^2}{xy} \quad \text{squared cms energy}$$

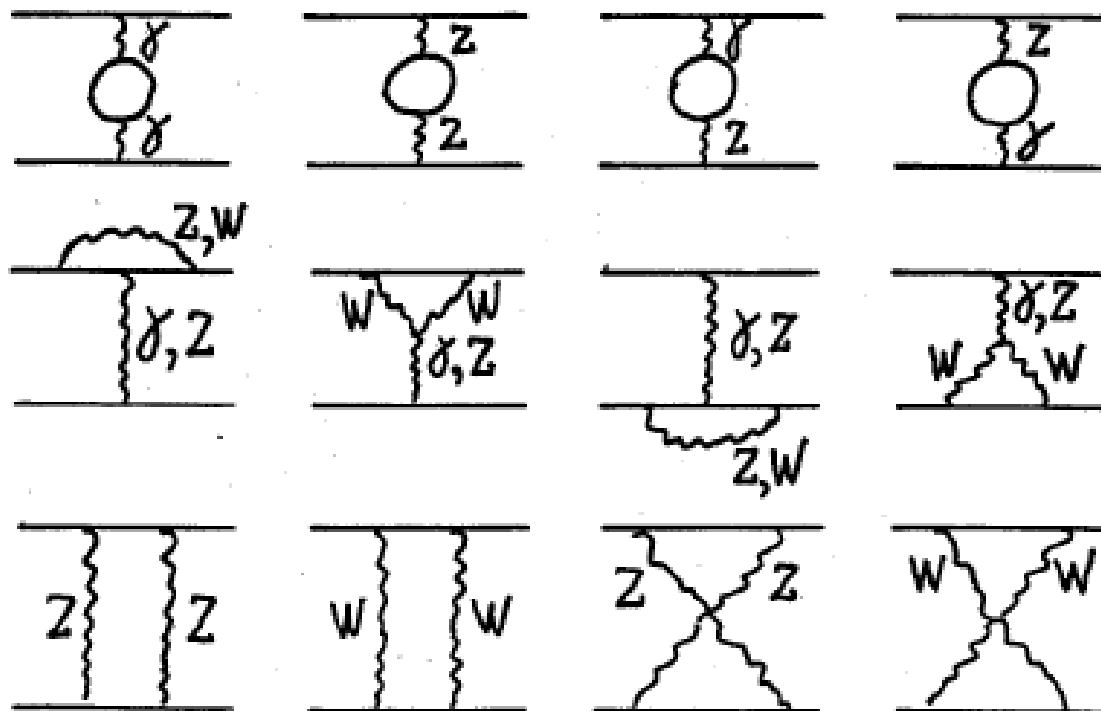
TERAD91: A Program package for the calculation of the cross-sections of deep inelastic N C and C C scattering at HERA.

A. Akhundov, D. Bardin, L. Kalinovskaya , T. Riemann ,
In *Hamburg 1991, Proceedings, Physics at HERA,
vol. 3, p.1285-1293.

TERAD91 is a semi-analytic code for QED and weak corrections to deep-inelastic NC and CC scattering at HERA. Version 2.10 was released on 3 Oct. 1991. The source of TERAD91 originates from four different codes: TERAD, DISEPNC, DISEPCC, and DIZET, which will be discussed in what follows.

Classification of RC





weak corrections

Elastic Radiative Tail at HERA

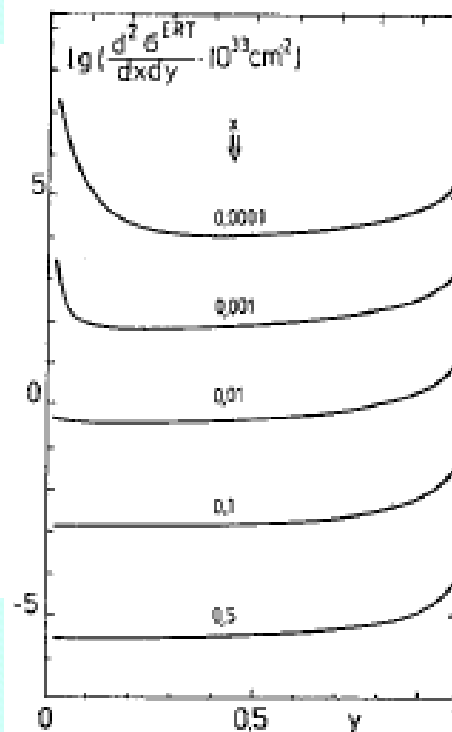
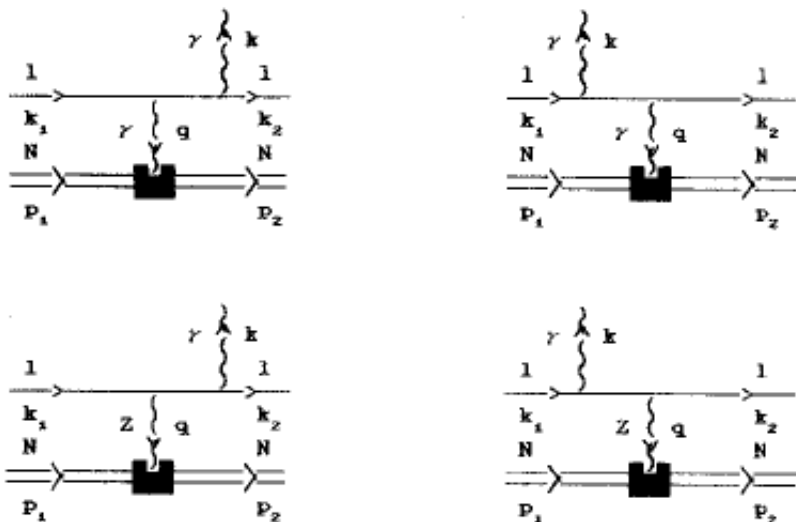


Fig.4. Inclusive cross section of the process $e + p \rightarrow e + \gamma + p$ at $S = 10^5 \text{ GeV}^2$.

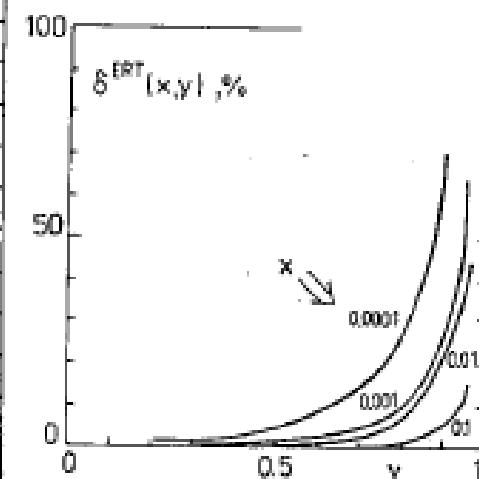


Fig.5. The ERT correction to $e + p \rightarrow e + X$ at $S = 10^5 \text{ GeV}^2$.

Reconstruction of kinematic variables Q^2, x & y

- incoming particles
 - electron E'_e, θ_e
 - hadronic system E_h, γ_h
 - momentum-conservation
- \Rightarrow overconstrained system

\rightarrow choose two quantities
for reconstruction

Electron (EL): E'_e, θ_e

$$Q_{EL}^2 = 2E_e E'_e (1 + \cos \theta_e)$$

$$x_{EL} = \frac{E_e}{E_p} \frac{E'_e (1 + \cos \theta_e)}{2E_e - E'_e (1 - \cos \theta_e)}$$

$$y_{EL} = 1 - \frac{E'_e}{2E_e} (1 - \cos \theta_e)$$

Jacquet-Blondel (JB):
only hadronic energies

$$Q_{JB}^2 = \frac{P_{t,h}^2}{1 - y_{JB}}$$

$$x_{JB} = \frac{Q_{JB}^2}{s \cdot y_{JB}}$$

$$y_{JB} = \frac{(E - p_z)_h}{2E_e}$$

Double-Angle (DA): θ, γ_h

$$Q_{da}^2 = 4E_e^2 \frac{\sin \gamma_h (1 + \cos \theta_e)}{\sin \gamma_h + \sin \theta_e - \sin(\gamma_h + \theta_e)}$$

$$x_{DA} = \frac{E_e \sin \gamma_h + \sin \theta_e + \sin(\gamma_h + \theta_e)}{E_p \sin \gamma_h + \sin \theta_e - \sin(\gamma_h + \theta_e)}$$

$$y_{DA} = \frac{\sin \theta_e (1 - \cos \gamma_h)}{\sin \gamma_h + \sin \theta_e - \sin(\gamma_h + \theta_e)}$$

... and many more mixed methods!

Model independent QED corrections to the process $ep \longrightarrow eX^\dagger$

Arif Akhundov ^{1,2}, Dima Bardin ^{3,4}, Lida Kalinovskaya ⁴, Tord Riemann ⁵

ABSTRACT

We give an exhaustive presentation of the semi-analytical approach to the model independent leptonic QED corrections to deep inelastic neutral current lepton-nucleon scattering. These corrections include photonic bremsstrahlung from and vertex corrections to the lepton current of the order $\mathcal{O}(\alpha)$ with soft photon exponentiation. A common treatment of these radiative corrections in several variables – leptonic, hadronic, mixed, Jaquet-Blondel variables – has been developed and double differential cross-sections are calculated. In all sets of variables we use some structure functions, which depend on the hadronic variables and which do not have to be defined in the quark parton model. The remaining numerical integrations are twofold (for leptonic variables) or onefold (for all other variables). For the case of hadronic variables, all phase space integrals have been performed analytically. Numerical results are presented for a large kinematical range, covering fixed target as well as collider experiments at HERA or LEP⊗LHC, with a special emphasis on HERA physics.

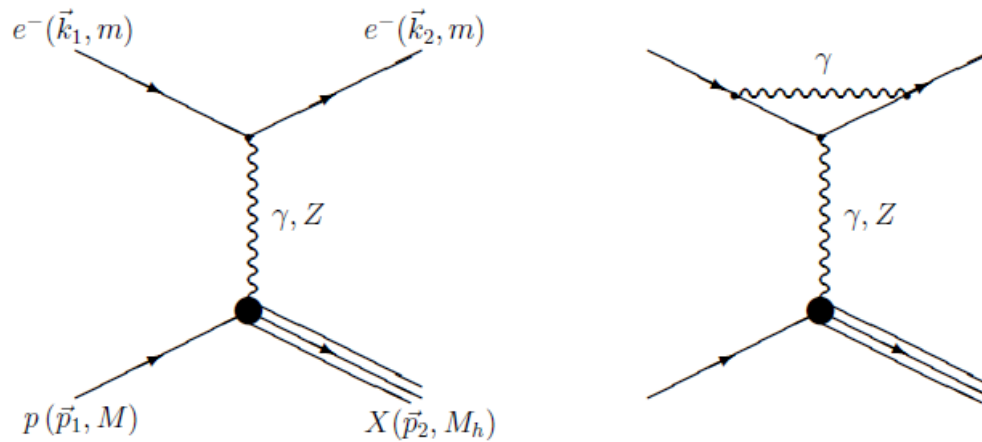


Figure 1: *Deep inelastic scattering of electrons off protons: (a) Born diagram, (b) leptonic vertex correction.*

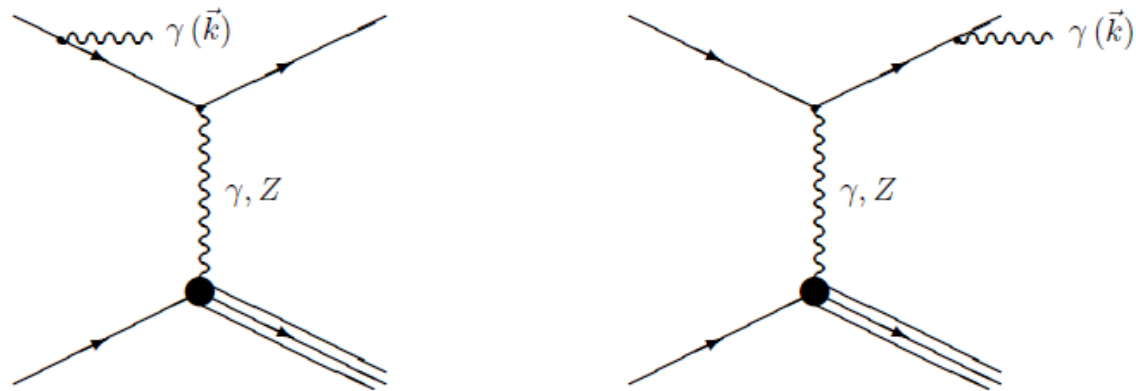


Figure 2: *The two leptonic bremsstrahlung diagrams.*

at HERA one may use not only the familiar electron variables,

$$Q_l^2 = (k_1 - k_2)^2, \quad y_l = \frac{p_1(k_1 - k_2)}{p_1 k_1}, \quad x_l = \frac{Q_l^2}{y_l S}, \quad (1.4)$$

where

$$s = -(k_1 + p_1)^2 = S + m^2 + M^2 \approx 4E_e E_p, \quad (1.5)$$

but also the kinematical variables from the hadron measurement,

$$Q_h^2 = (p_2 - p_1)^2, \quad y_h = \frac{p_1(p_2 - p_1)}{p_1 k_1}, \quad x_h = \frac{Q_h^2}{y_h S}, \quad (1.6)$$

or some composition of both, the so-called mixed variables [28, 30]:

$$Q_m^2 = Q_l^2, \quad y_m = y_h, \quad x_m = \frac{Q_l^2}{y_h S}. \quad (1.7)$$

Here E_e , m and E_p , M are the energies and masses of incident electron and proton (see figure 1).

Another useful set of hadronic variables has been introduced by Jaquet and Blondel [31]:

$$Q_{JB}^2 = \frac{(\vec{p}_2^\perp)^2}{1 - y_h}, \quad y_{JB} = y_h, \quad x_{JB} = \frac{Q_{JB}^2}{y_h S}. \quad (1.8)$$

Different choices of variables make no difference for the determination of the cross section of reaction (1.1) in the Born approximation. Although, there are huge differences in the predictions for the radiative corrections, because the kinematics becomes quite different. This may be seen from the tetrahedron of momenta which is shown in figure 3. For vanishing photon momentum k , the simple Born kinematics is recovered. The differences concern the *calculation* of the corrections, but also, and maybe more important, their *numerical values*.

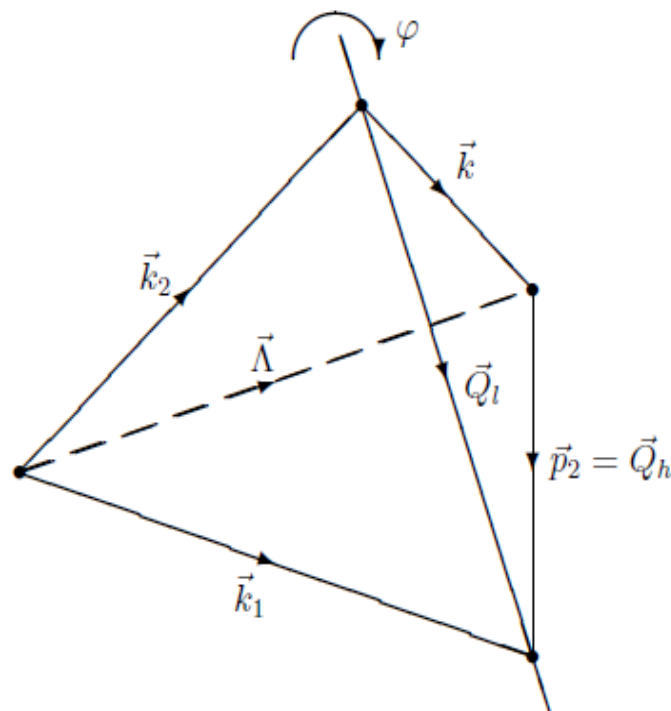


Figure 3: *Spatial configuration of the momenta in reaction (1.2) in the proton rest system.*

Akhundov A.A., Bardin D.Yu.

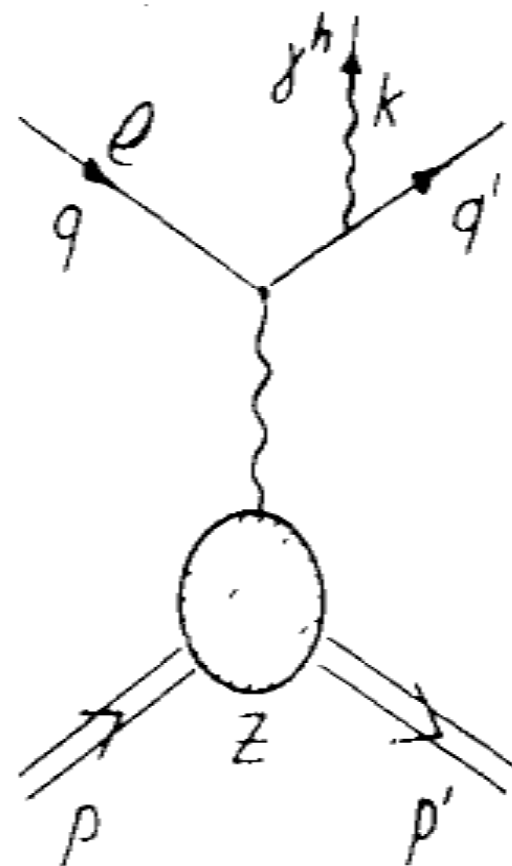
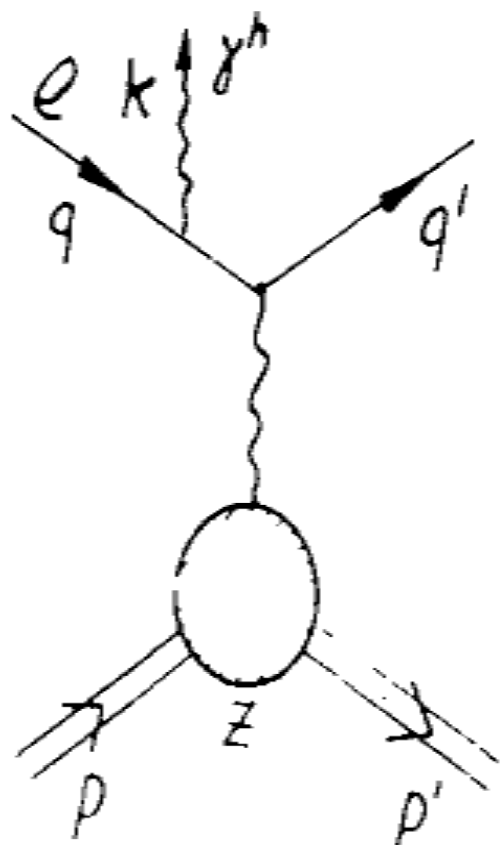
On the "Heavy Photon" Bremsstrahlung

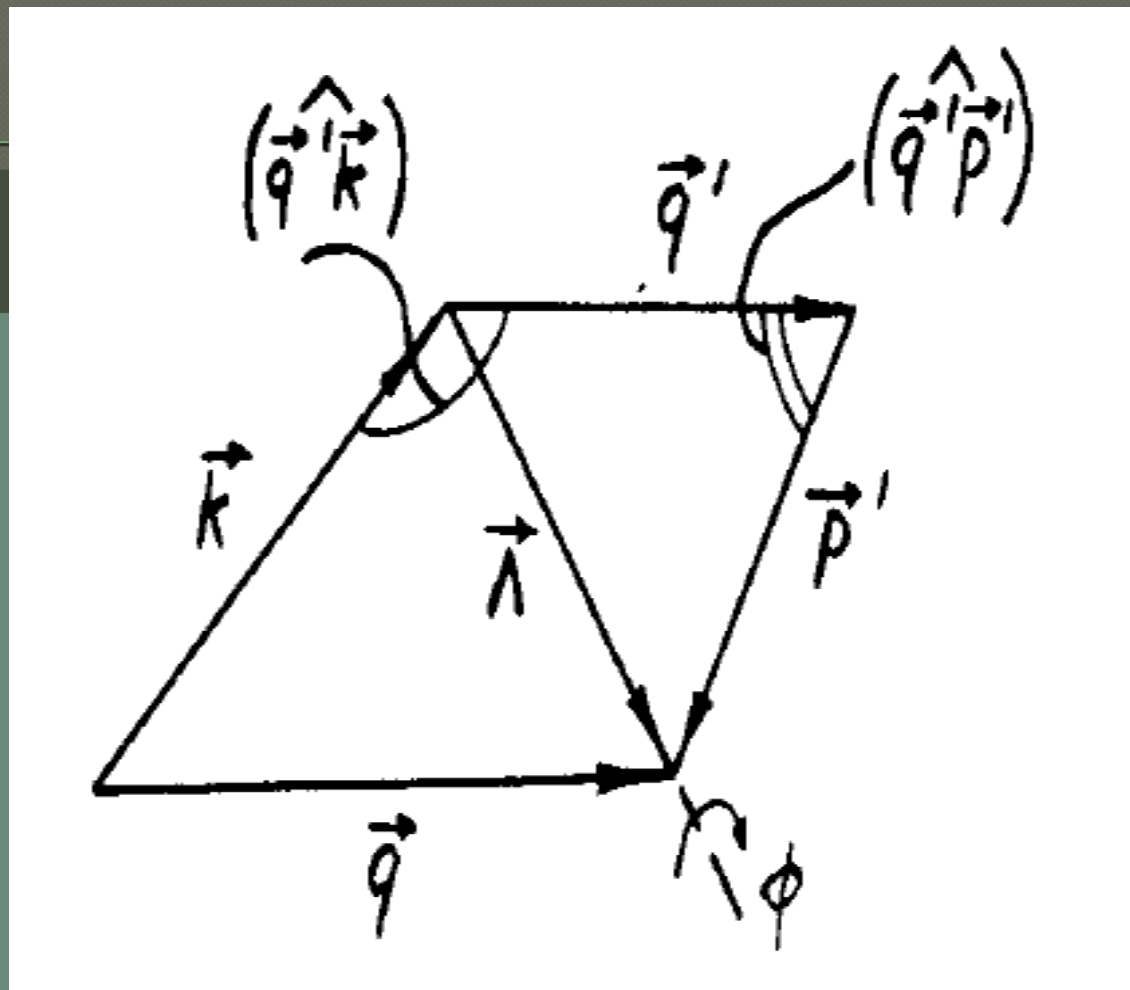
The bremsstrahlung of a "heavy photon" in the lepton-nucleus scattering has been considered. The kinematics of the processes has been studied in detail. The energy spectrum of the "heavy photon" has been calculated.

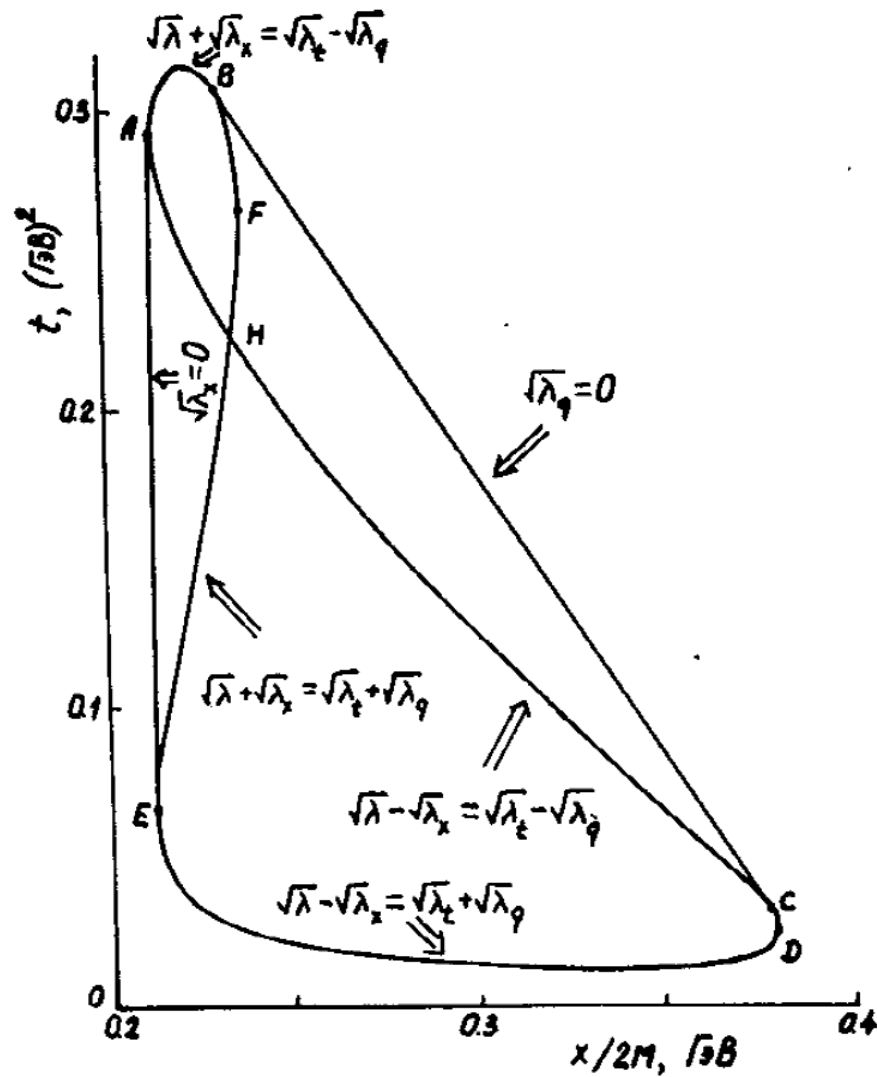
The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Communication of the Joint Institute for Nuclear Research

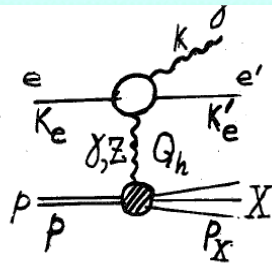
Dubna 1976







Physics region of the variables x and t for the process
 $\mu p \rightarrow \mu p \gamma$
 at $E = 0.5$ GeV and mass of heavy photon 0.2 GeV



$$s = (k_e + p)^2 = 4E_e E_\nu$$

- electron variables

$$Q_e^2 = -(k_e - k_e')^2, \quad y_e = \frac{p(k_e - k_e')}{p k_e}, \quad x_e = \frac{Q_e^2}{s y_e}$$

- hadron variables

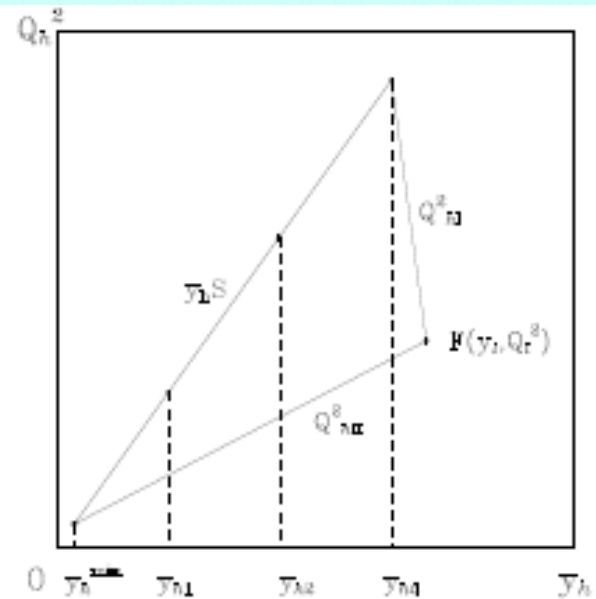
$$Q_h^2 = -(p_h - p_X)^2, \quad y_h = \frac{p(p_X - p)}{p k_e}, \quad x_h = \frac{Q_h^2}{s y_h}$$

- mixed variables

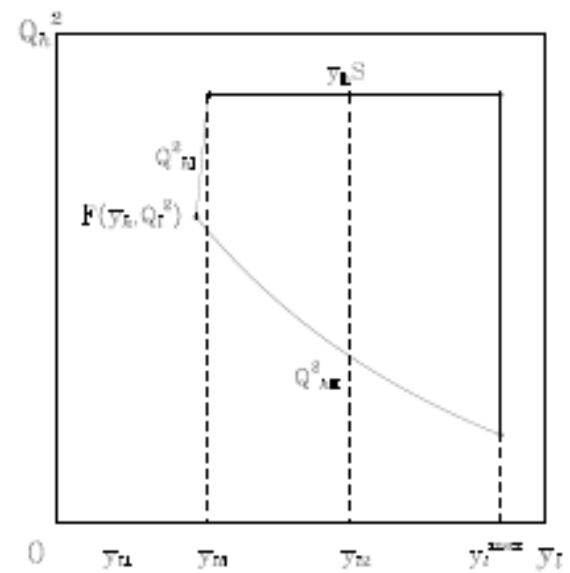
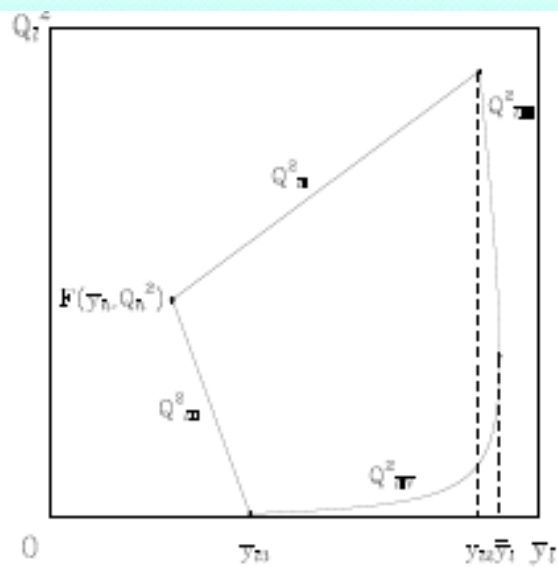
$$Q_m^2 \equiv Q_e^2, \quad y_m = y_h, \quad x_m = \frac{Q_e^2}{s y_h}$$

- Jacquet-Blondel variables

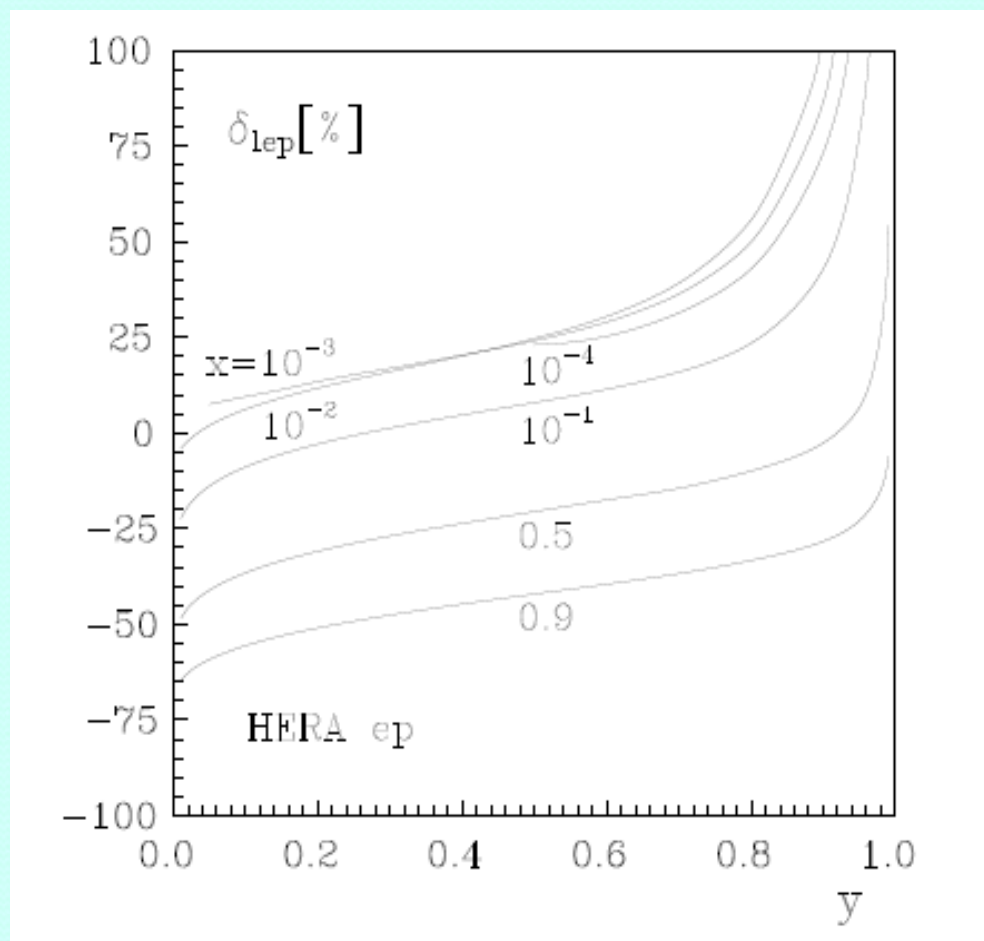
$$Q_{JB}^2 = \frac{(\sum \vec{p}_{\perp, h})^2}{1 - y_{JB}}, \quad y_{JB} = \frac{\sum (E_h - p_{z, h})}{2E_e} \approx y_h, \\ x_{JB} = \frac{Q_{JB}^2}{s y_{JB}}$$

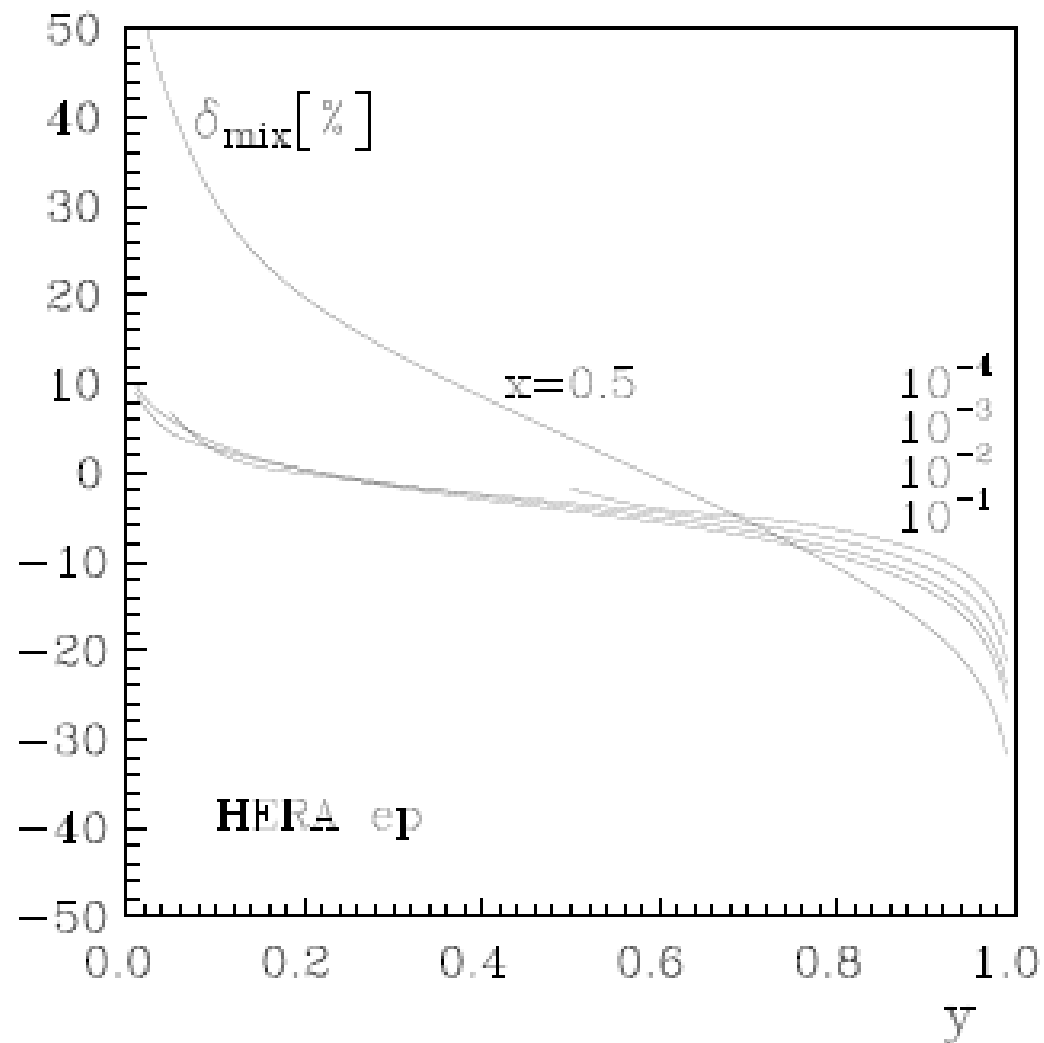


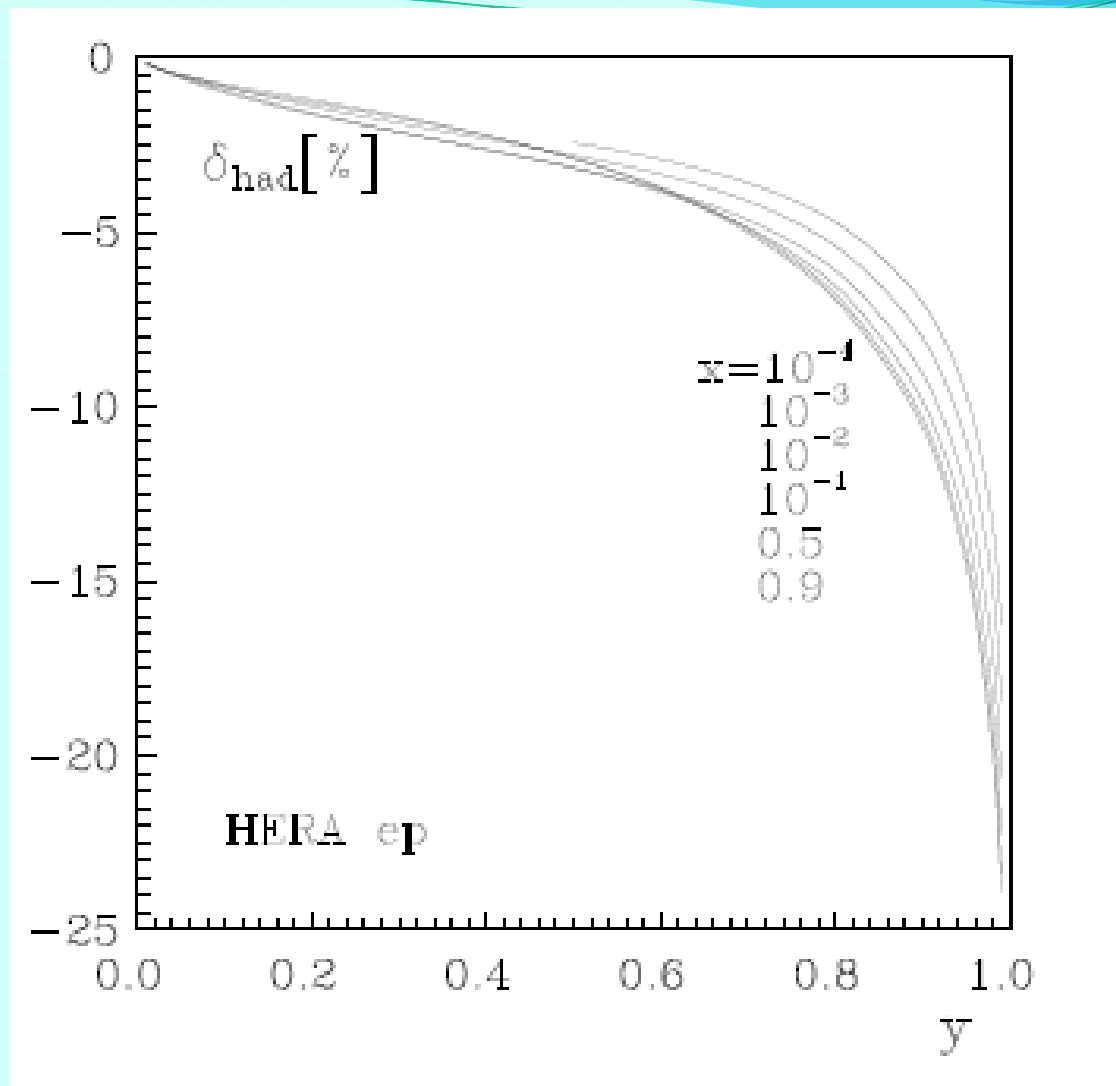
Integration region (y_h, Q_h^2) for the cross-section in leptonic variables.

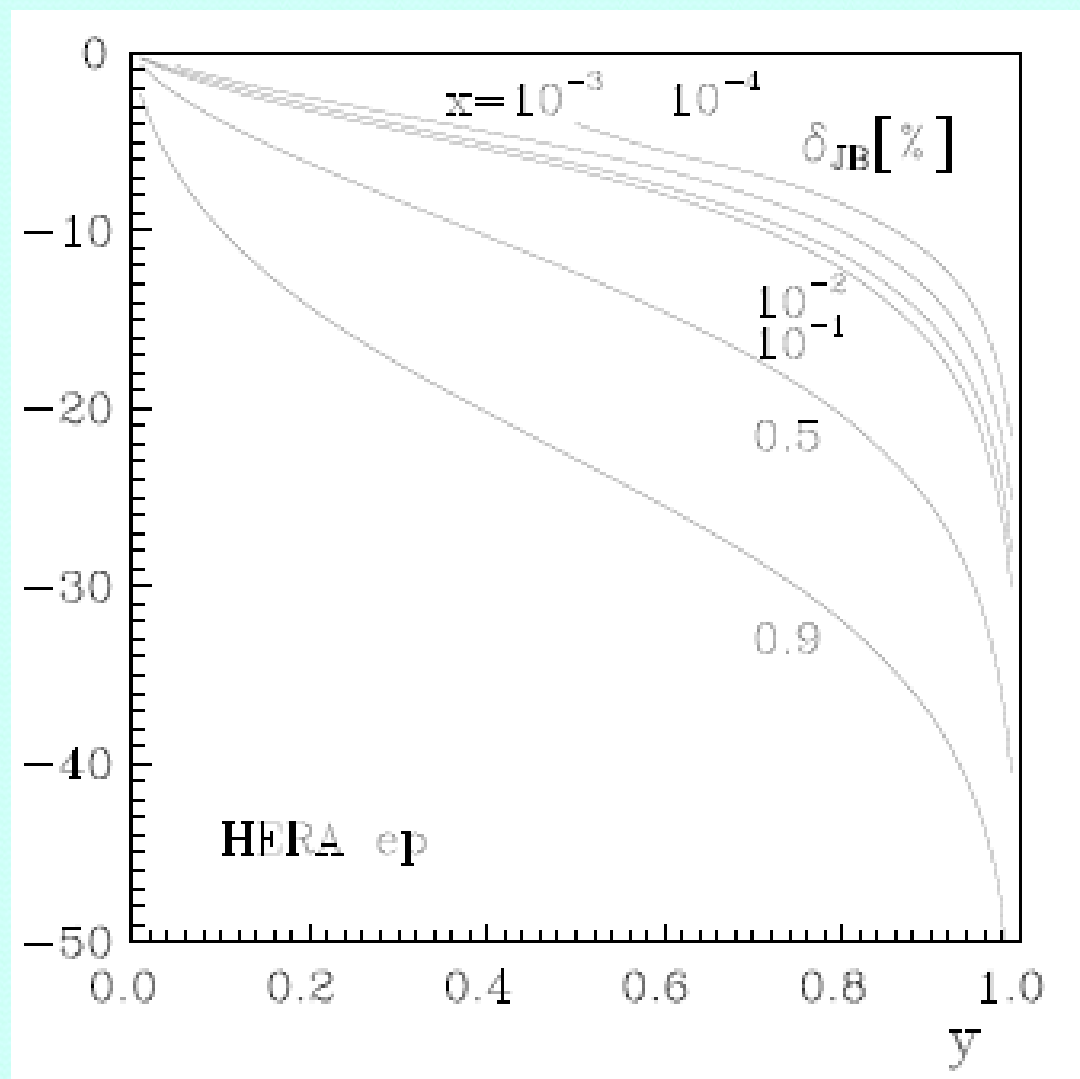


Integration region (y_1, Q_1^2) for the cross-section in hadronic variables. Integration region (y_1, Q_1^2) for the cross-section in mixed variables, $x_m \leq 1$.









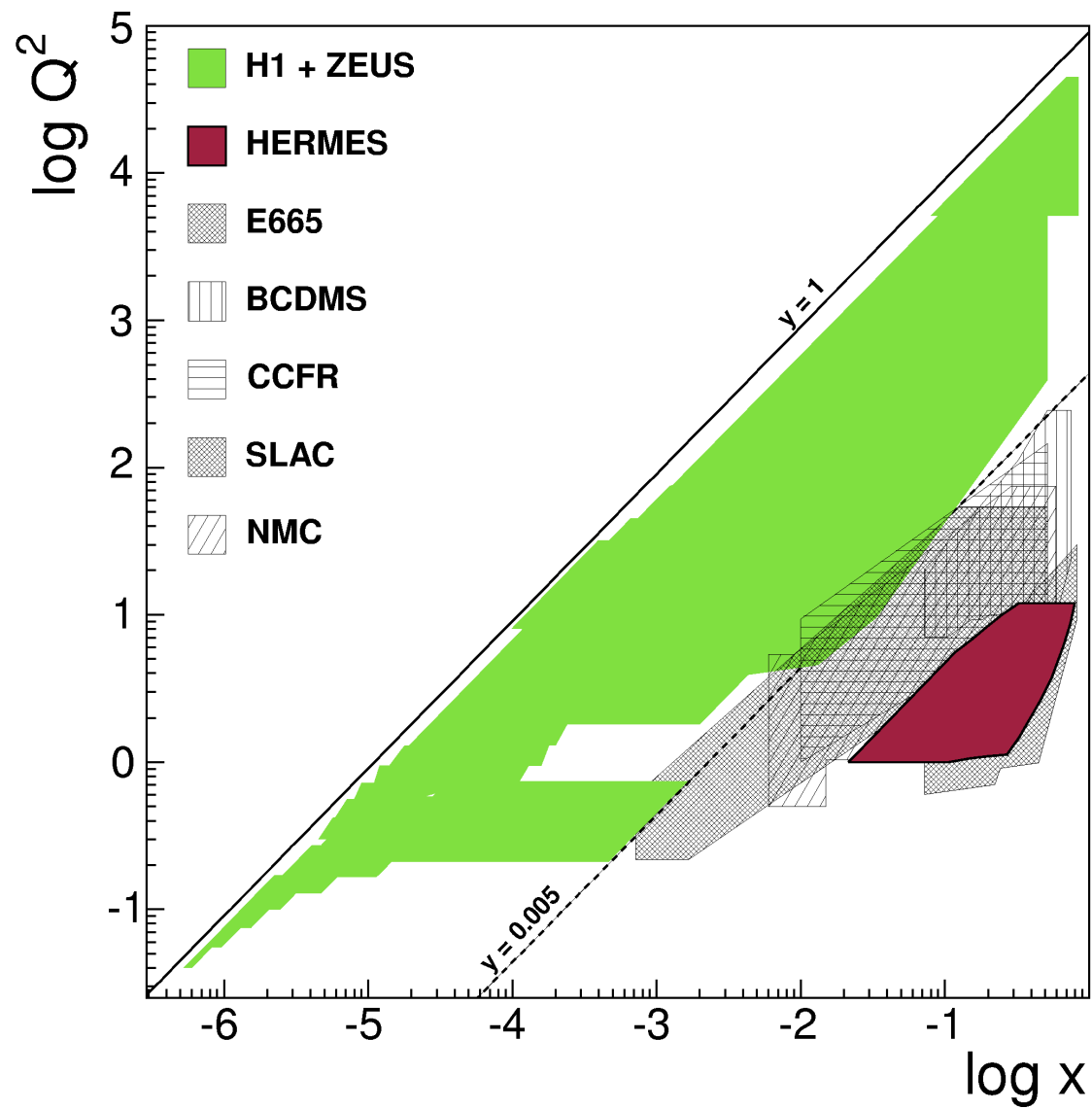
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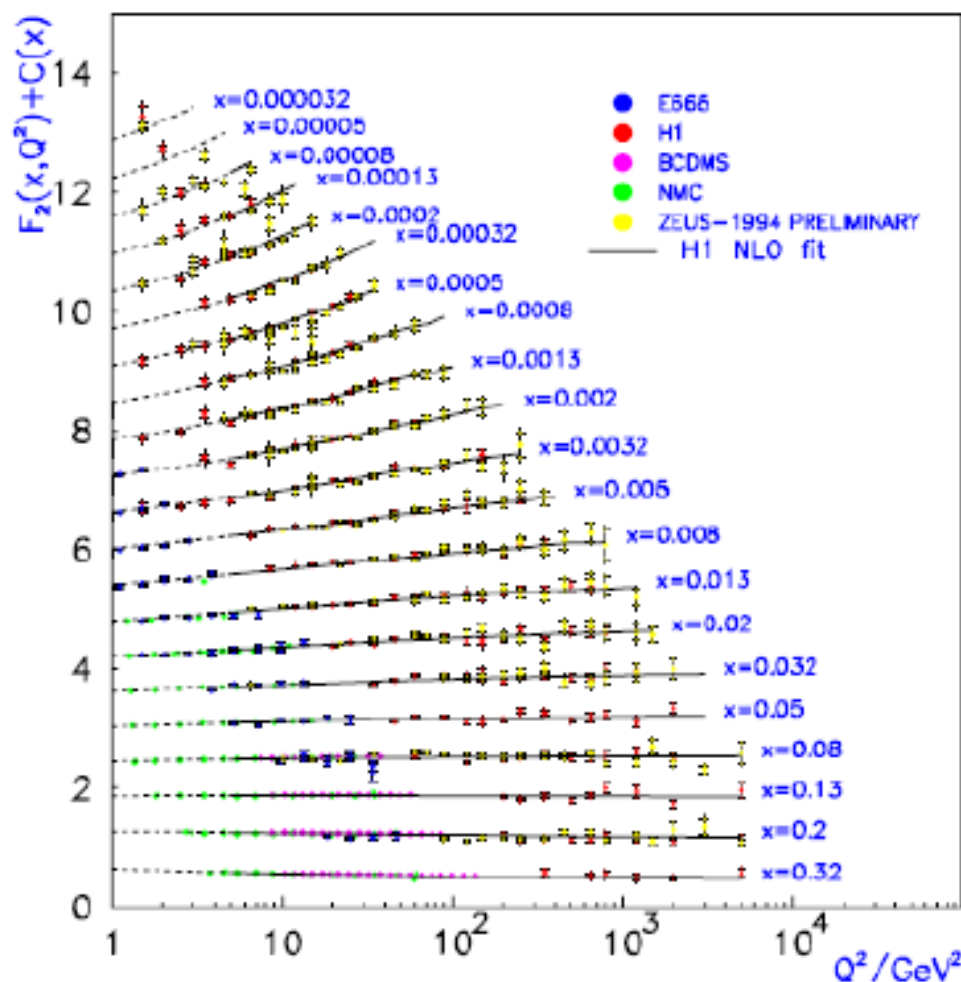
A program for the calculation of QED, QCD and electroweak corrections to ep and $l^\pm N$ deep inelastic neutral and charged current scattering *

A. Arbuzov^{1#}, D. Bardin^{1,2}, J. Blümlein², L. Kalinovskaya¹, T. Riemann²

ABSTRACT

A description of the Fortran program HECTOR for a variety of semi-analytical calculations of radiative QED, QCD, and electroweak corrections to the double-differential cross sections of NC and CC deep inelastic charged lepton proton (or lepton deuteron) scattering is presented. HECTOR originates from the substantially improved and extended earlier programs HELIOS and TERAD91. It is mainly intended for applications at HERA or LEP⊗LHC, but may be used also for μN scattering in fixed target experiments. The QED corrections may be calculated in different sets of variables: leptonic, hadronic, mixed, Jaquet-Blondel, double angle etc. Besides the leading logarithmic approximation up to order $\mathcal{O}(\alpha^2)$, exact $\mathcal{O}(\alpha)$ corrections and inclusive soft photon exponentiation are taken into account. The photoproduction region is also covered.





Compilation of data on the
proton structure function
 $F_2^p(x, Q^2)$

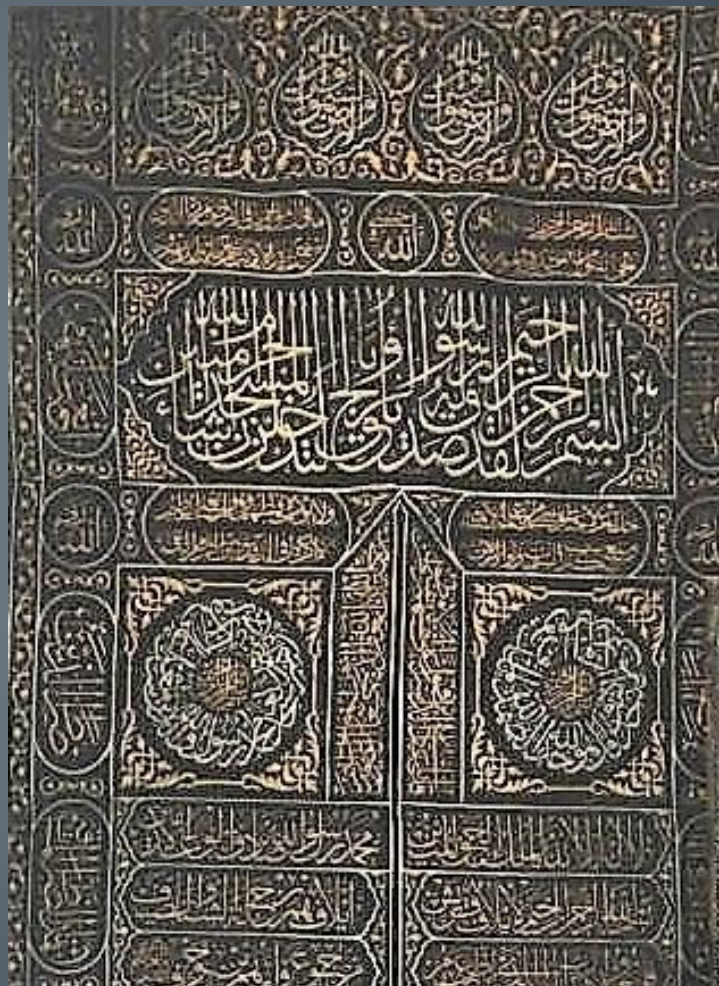
Experiments:

E666	(Fermilab)
H1	(DESY)
BCDMS	(CERN)
NMC	(CERN)
ZEUS	(DESY)

To F_2 of each x range
an amount $C(x)$ is added
“by hand” to separate the
data sets.

Note the scaling violation
at the lowest values of x

"When the son of Adam dies, his deeds are broken, except for three:
an uninterrupted alms; **knowledge from which others benefit**;
a righteous son who will pray for him. "



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- ❑ My special thanks to Andrej Arbuzov and Alexander Bednyakov.