Bardin–Shumeiko's method and its development

Yahor Dydyshka[1,a] Vladimir Zykunov[2,b]

[1]Dzelepov Laboratory of Nuclear Problems JINR

[a]Institute for Nuclear Problems of Belarusian State University

[2]Veksler and Baldin Laboratory of High Energy Physics, JINR

[b]Francisk Skorina Gomel State University

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Outline

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- Mo–Tsai method, the Δ parameter
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Infrared singularity

Calculations during **radiative correction procedure** faces with difficulties, caused by divergences in high-loop diagrams. Infrared singularity (at the region of small momenta) of the additional virtual particles (V-)contribution cancels if corresponding (Real photon) Bremsstrahlung (R-)contribuion is taken into account.

Mathematically Infrared (IR) singularity at the intermediate steps regularized

1. by infinitesimal "photon mass" λ,
2. or by dimensional regularization (Hooft, Veltman, 1972).

Physically both schemes **are equivalent** (Marciano, Sirlin, 1975).

Mo-Tsai method, the Δ parameter

Mo-Tsai method

1. Introduced parameter Δ to separate Bremsstrahlung of low and high energies, i.e. “soft” and “hard” photons.
2. “Soft”-photon contribution can be evaluated analytically. Adding to virtual contribution IR-finite result is obtained.
3. “Hard” part is usually evaluated by numerical methods (Monte Carlo).

Whole phase space of radiated photon

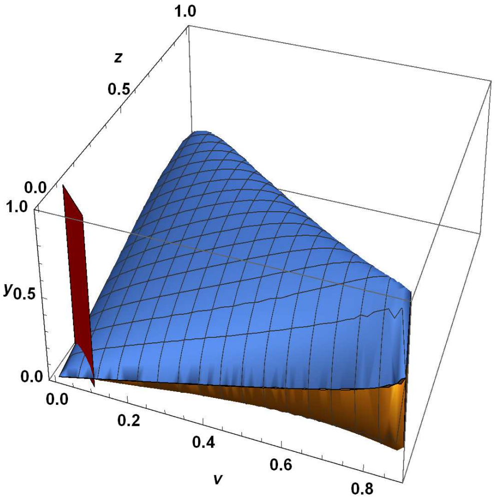

Рис. 1: Phase space volume of radiated photon: red plane separates "soft" and "hard" regions.

Phase space volume with detector acceptance

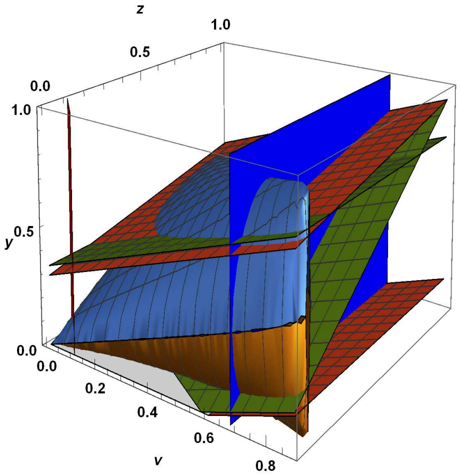

Рис. 2: Phase space volume with detector acceptance. Planes corresponds to cuts on angles and energies of detected particles.

Difficulties and Solutions

Difficulties

For numerical evaluation specific value of parameter Δ should be chosen with contradictory limitations:

1. For high values of Δ we have good numerical convergence of hard part, but soft-photon approximation breaks.
2. For small Δ soft part may become a big negative value leading to precision loss after summation with hard part.
3. Negative values of soft part are unphysical and may be cured with exponentiation, but independence on Δ breaks.

Solutions

1. Integrate analytically not only soft, but also singular terms of hard contribution (Bardin–Shumeiko's method)
2. Exponentiate not only soft, but also part of hard contribution (Kleiss, Krakow MC group)

D. Yu. Bardin and N. M. Shumeiko

Рис. 3: *Dmitry (Dima) Yurievich Bardin (1945–2017)*

Рис. 4: *Nikolai Maximovich Shumeiko (1942–2016)*

Pioneer works on Bardin-Shumeiko method (BSM)

1. О точном вычислении электромагнитной поправки низшего порядка к упругому рассеянию / Д.Ю. Бардин, Н.М. Шумейко. – Дубна, 1976. – 20 с. (Препринт ОИЯИ Р2-10113).
2. Точное вычисление радиационной поправки низшего порядка к процессам рассеяния точечных частиц со спинами 0 и 1/2 / Д.Ю. Бардин, О.М. Федоренко, Н.М. Шумейко. – Дубна, 1976. – 16 с. (Препринт ОИЯИ Р2-10114).
3. On an exact calculation of the lowest - order electromagnetic correction to the point particle elastic scattering / D.Yu. Bardin, N.M. Shumeiko // Nucl. Phys. B. – 1977. – Vol. 127. – P. 242–258.
4. Электромагнитные поправки к глубоконеупругому $\mu - p$ рассеянию / А.А. Ахундов, Д.Ю. Бардин, Н.М. Шумейко // ЯФ. – 1977. – Т. 26. – С. 1251–1257.

Short description of method

$$\sigma^{\mathsf{NLO}} = \int_{m+1} T d\sigma_{\mathsf{R}}(\lambda) + \int_{m} \tilde{T}(d\sigma_{\mathsf{B}} + d\sigma_{\mathsf{V}}(\lambda)) \tag{1}$$

$$d\sigma_{\mathsf{R}}(\lambda) = d\sigma_{\mathsf{R}} - d\sigma_{\mathsf{R}}^{\mathsf{IR}} + d\sigma_{\mathsf{R}}^{\mathsf{IR}}(\lambda) = d\sigma_{\mathsf{F}} + d\sigma_{\mathsf{R}}^{\mathsf{IR}}(\lambda) \tag{2}$$

$$\begin{aligned}\sigma^{\mathsf{NLO}} &= \int_{m+1} T d\sigma_{\mathsf{F}} + \int_{m+1} (T - \tilde{T}) d\sigma_{\mathsf{R}}^{\mathsf{IR}} \\ &+ \int_{m} \tilde{T}(d\sigma_{\mathsf{B}} + d\sigma_{\mathsf{V}}(\lambda) + \int_{1} d\sigma_{\mathsf{R}}^{\mathsf{IR}}(\lambda))\end{aligned} \tag{3}$$

where T is "observable" function including experimental cuts,

$$\tilde{T} = T|_{k\to 0}$$

Short description (2)

$$d\sigma_{\mathrm{R}}^{\mathrm{IR}} = d\sigma_{\mathrm{B}} \times dF^{\mathrm{IR}} \tag{4}$$

$$dF^{\mathrm{IR}} = -\left|\sum_i \eta_i Q(i) \frac{p_i}{p_i k}\right|^2 \frac{d^3 k}{2k_0} \tag{5}$$

$$\int_1 d\sigma_{\mathrm{R}}^{\mathrm{IR}}(\lambda) = d\sigma_{\mathrm{B}}(\delta^{\mathrm{soft}}(\lambda, \Delta) + \bar{\delta}(\Delta) + \delta_1^{\mathrm{H}}) \tag{6}$$

$$\delta_1^{\mathrm{H}} = f\left\{\frac{X_0}{\sqrt{\lambda_X^0}}\left[\Phi\left(\frac{\sqrt{\lambda_X^0} - \sqrt{\lambda_{\max}}}{\sqrt{\lambda_X^0} - X_0}\right) - \Phi\left(\frac{\sqrt{\lambda_X^0} - \sqrt{\lambda_{\max}}}{\sqrt{\lambda_X^0} + X_0}\right)\right.\right.$$

$$+ \Phi\left(\frac{\sqrt{\lambda_X^0} + \sqrt{\lambda_{\max}}}{\sqrt{\lambda_X^0} - X_0}\right) - \Phi\left(\frac{\sqrt{\lambda_X^0} + \sqrt{\lambda_{\max}}}{\sqrt{\lambda_X^0} + X_0}\right) + \Phi\left(\frac{2\sqrt{\lambda_X^0}}{\sqrt{\lambda_X^0} + X_0}\right)$$

$$\left.- \Phi\left(\frac{2\sqrt{\lambda_X^0}}{\sqrt{\lambda_X^0} - X_0}\right)\right] - \frac{1}{2m^2}\left(\lambda_X^0 L_X^0 - \sqrt{\lambda_{\max}} \ln \frac{X_0 + \sqrt{\lambda_{\max}}}{X_0 - \sqrt{\lambda_{\max}}}\right.$$

First applications

- Akhundov, Bardin and Shumeiko evaluated leptonic QED RC to DIS with unpolarized particles. Results are applied in data analysis at CERN.
- In papers by Kukhto and Shumeiko (e. g. Nucl. Phys. B. 1983. V.219. P.412) RC to polarized asymmetry were analyzed, they were applied for RC procedure in expreiment SMC at CERN.
- RC to semi-inclusive DIS (Soroko, Shumeiko Yad. Ph. 1989. T.49. C.1348)
- QED RC to cross section and polarization asymmetry in DIS were calculated (e. g. Shumeiko, Timoshin, J. Phys. G. 1991. V.17. P.1145; Kuzhir, Shumeiko, Yad. Fiz. 1992. V.55. P.1958. Zykunov, Timoshin, Shumeiko, Yad. Fiz. 1995. V.58. P.2021.).

DIS and POLRAD 2.0

Results of the papers (Akushevich, Shumeiko, J. Phys. G. 1994. V.20. P.513) were used as a background for POLRAD 2.0 code (Akushevich, Ilyichev, Soroko, Shumeiko, Tolkachov), which was in use at DESY.

- Decomposition of polarization vectors in 4-momenta are applied. It allows avoid complex tensor integrals.
- RC to DIS with polarized targets with spin 1/2 and 1 are calculated.
- RC to quadrupole asymmetry in DIS with targets of spin 1 are evaluated.
- RC to semi-inclusive DIS are calculated.
- Iterational procedure for application of RC to experimental data is developed.

Codes and MC Generators

- `RADGEN` MC of radiative events in DIS with (non)polarized targets: (semi)inclusive, exclusive case.
- `DIFFRAD` `FORTRAN` code for RC in processes of electroproduction of vector mesons.
- `HAPRAD` `FORTRAN` code for RC in semi-inclusive electroproduction of hadrons.
- `ESFRAD` *(Afanasev, Akushevich, Merenkov)* `FORTRAN` code for RC at elastic, inelastic and DIS with structure functions.
- `ELARADGEN` *(Afanasev, Akushevich, Ilyichev, Niczyporuk)* MC generator for elastic *ep*-scattering.
- `MASCARAD` `FORTRAN` code for RC calculation in polarized elastic *eN*-scattering
- `EXCLURAD` *(Afanasev, Akushevich, Burkert, Joo)* RC to exclusive electroproduction of pions.

Another applications of BSM

- RC to diffractive electroproduction of vector mesons (Akushevich)
- RC to exclusive production of vector mesons (Akushevich, Kuzhir)
- RC to W-boson production asymmetry in polarized hadrons collision (Zykunov)
- QED RC to Møller scattering (Suarez, Shumeiko)
- Results from series of works (Kolomensky, Shumeiko, Suarez, Zykunov, 2003–2005) were applied to data analysis at SLAC experiment E-158.
- Numeric code `MERA` (Zykunov, Ilyichev) and MC `MERADGEN` (Afanasiev, Chudakov, Ilyichev, Zykunov) for Møller scattering
- RC to Bethe–Heitler process (Akushevich, Ilyichev, Shumeiko)

Generalizations and extensions

Dipole Subtraction Formalism (DSF)

DSF can be easily recognized as a generalization of BSM by including collinear singularities to subtract.

Catani–Seymour Algorithm

$$\sigma^{\mathrm{NLO}} = \int_{m+1} T d\sigma_{\mathrm{R}}(\epsilon) + \int_{m} \tilde{T}(d\sigma_{\mathrm{B}} + d\sigma_{\mathrm{V}}(\epsilon)) \tag{7}$$

$$d\sigma_{\mathrm{R}}(\epsilon) = d\sigma_{\mathrm{R}} - d\sigma_{\mathrm{A}} + d\sigma_{\mathrm{A}}(\epsilon) = d\sigma_{\mathrm{F}} + d\sigma_{\mathrm{A}}(\epsilon) \tag{8}$$

$$\begin{aligned}\sigma^{\mathrm{NLO}} &= \int_{m+1} J d\sigma_{\mathrm{F}} + \int_{m+1} (J - \tilde{J}) d\sigma_{\mathrm{A}} \\ &+ \int_{m} \tilde{J}[d\sigma_{\mathrm{B}} + d\sigma_{\mathrm{V}}(\epsilon) + \int_{1} d\sigma_{\mathrm{A}}(\epsilon)]\end{aligned} \tag{9}$$

where J is “jet”-function to describe IR-safe observable, $\tilde{J} = J|_{k\to 0}$

Generalizations and extensions (2)

$$d\sigma_{\mathrm{A}} = \sum_{\mathrm{dipoles}} d\sigma_{\mathrm{B}} \otimes dV^{\mathrm{dipole}} \tag{10}$$

$$dV^{\mathrm{dipole}} = \mathcal{D}_{ij}^{a}(\{p\}) \frac{d^3k}{2k_0} \tag{11}$$

$$\begin{gathered} \int_1 d\sigma_{\mathrm{A}}(\epsilon) = d\sigma_{\mathrm{B}} \otimes \mathbf{I} \\ \mathbf{I} = \sum_{\mathrm{dipoles}} \int_1 dV_{\mathrm{dipoles}} \end{gathered} \tag{12}$$

$$\boldsymbol{I}(\{p\};\epsilon) = -\frac{\alpha_S}{2\pi}\frac{1}{\Gamma(1-\epsilon)}\sum_I \frac{1}{\boldsymbol{T}_I^2}\mathcal{V}_I(\epsilon) \sum_{J\neq I}\boldsymbol{T}_I\cdot\boldsymbol{T}_J \left(\frac{4\pi\mu^2}{2p_I\cdot p_J}\right)^{\epsilon}$$

Using for process $\mathbf{k_1} + \mathbf{p_1} \to \mathbf{k_2} + \mathbf{p_2} + \mathbf{k}$

- c.m.s. of initial particles,
- auxiliary vector
$$\vec{\mathbf{p}}_5 = -\vec{\mathbf{k}}$$
- suitable simple angles

$$\theta_{\mathbf{k}} = \pi - \theta_{\mathbf{5}},\ \ \varphi_{\mathbf{k}} = \pi + \varphi_{\mathbf{5}}.$$

we can transform phase space of brem photon as

$$\int \mathbf{d\Phi} = \int\limits_{\mathbf{\Delta}}^{\mathbf{\Omega}} \mathbf{k_0 dk_0} \int\limits_{-1}^{1} \mathbf{d}\cos\theta \int\limits_{-1}^{1} \mathbf{d}\cos\theta_{\mathbf{k}} \int\limits_{0}^{2\pi} \mathbf{d}\varphi_{\mathbf{k}} \frac{\pi|\vec{\mathbf{k}}_2|}{\mathbf{4p_{20}K_A(k_{20})}}. \quad (13)$$

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Some details of G/N-method

The factor in PS volume

$$\mathbf{K_A(x) = 1 + \frac{x(1 - k_0 A/\sqrt{x^2 - m^2})}{\sqrt{x^2 - 2k_0 A\sqrt{x^2 - m^2} + k_0^2}}}, \tag{14}$$

where $\mathbf{A}$ – cosine of angle between $\vec{\mathbf{k}}_2$ and $\vec{\mathbf{p}}_5$:

$$\mathbf{A} = \sin\theta \sin\theta_\mathbf{5} \cos\varphi_\mathbf{5} + \cos\theta\cos\theta_\mathbf{5}. \tag{15}$$

Energy of final particle depends non trivially on sign of $\mathbf{A}$:

$$\mathbf{k_{20} = \frac{BC \pm \sqrt{C^2 + m^2(1 - B^2)}}{1 - B^2}}, \tag{16}$$

where

$$\mathbf{B = \frac{\sqrt{s} - k_0}{Ak_0}},\ \mathbf{C = \frac{(2k_0 - \sqrt{s})\sqrt{s}}{2Ak_0}}. \tag{17}$$

No Δ-dependency

With G/N-method it is possible to unite "soft" and "hard" parts:

$$\mathbf{soft} + \mathbf{hard} \; = \int\limits_{\lambda}^{\boldsymbol{\Delta}} \mathbf{dk_0} ... + \int\limits_{\boldsymbol{\Delta}}^{\boldsymbol{\Omega}} \mathbf{dk_0} ... = \int\limits_{\lambda}^{\boldsymbol{\Omega}} \mathbf{dk_0} ... \; .$$

- It allows to avoid Δ-dependency in principle.
- An analytical (and numerical) evaluation of "soft" part is unnecessary.
- We have an additional possibility to test the result.

Conclusions

- Dmitry Yurievich Bardin and Nikolai Maximovich Shumeiko were respected as leaders of their communities of high energy physicists, in Dubna, in Minsk, in CERN, etc...
- Excellent results in theoretical and applied areas were obtained under their supervision. Their enthusiasm and energy allowed them to create their scientific schools which exist and work actively up to now.
- Personally and on behalf of Laboratory of High Energy Physics and Laboratory of Nuclear Problems JINR we would like to thank the Organizers of this Conference