

Genuine weak corrections and factorizations: from LEP to LHC with Dizet library of Dima Bardin[†]

Z. Was IFJ PAN, 31342 Krakow, Poland

Precision physics as complex system example. Intertwined targets:

- precision measurements at LEP 1,
- precision measurement of Luminosity,
- electroweak measurements at LEP 2,
- electroweak measurements at LHC.

All to confirm Standard Model as fundamental Quantum Field Theory.

Dima Bardin is a central player of that, but always somewhat in background.

Vronsky, Levin, or else ... Life attitude → evening session?

[†] This work is partly supported by the Polish National Science Center grant 2016/23/B/ST2/03927 and the CERN FCC

Design Study Programme.

1. Divide complicated system into manageable parts
 2. Acquire necessary skills, **convince** × **train** partners and oneself
 3. Develop parts
 4. Test parts
 5. Test the whole thing
 6. Enjoy results
 7. Face limitations
 8. Realize that design was too simplistic
 9. Integrate back the parts
 10. **Goto 1**
- My domain: Monte Carlo programs for $e^+e^- \rightarrow l^+l^- n\gamma$, τ decays, bremsstrahlung in decays, also for some LHC signatures, like for Higgs CP measurements or for measurements of W mass.

LEP times legacy for electroweak physics at LHC: is a complicated heritage

- **Precision measurements at LEP 1:** 100 kevt samples: M_Z , N_ν , $\sin^2 \Theta_W$, consistency checks of SM as a field theory. 1989-1995 → QED, lineshape corrections, genuine weak, semianalytic and MC predictions
- **Precision measurement of Luminosity:** Mevt samples. Highest precision of counting experiment ever, → detector geometry details.
- **Electroweak measurements at LEP 2:** 1-10 kevt samples: M_W , triple gauge couplings, New Physics. 1995-2000, → s- t- channel cancellation.

These heritage sectors contradict each other (to a degree).

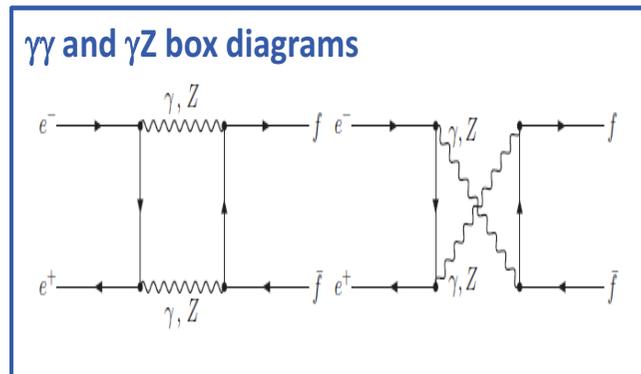
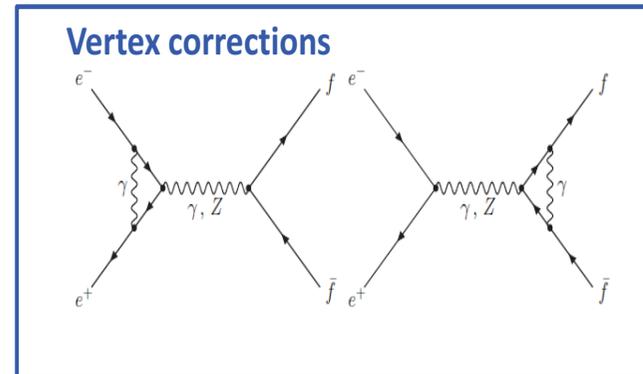
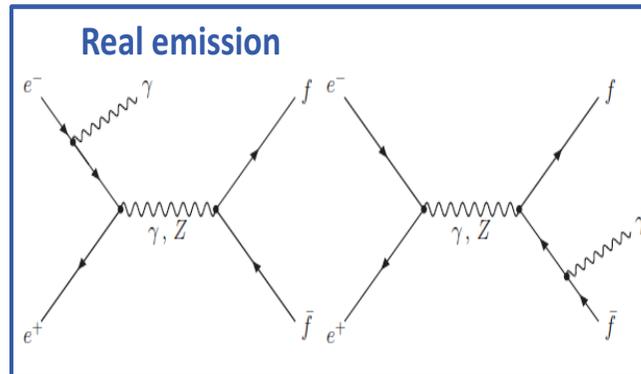
- **Electroweak measurements at LHC:** new frontier.

We want to validate SM as a field theory

- A. Sensitivity to quantum effects was essential.
- B. Formal calculations starting from Lagrangian non negotiable basis.
- C. At the same time it had to be negotiated.
 - * How to bent a rules without breaking them.
 - d. low energy vacuum polarization from $e^+ e^- \rightarrow hadrons$ data and use of dispersion relation.
 - e. partial resummation of higher order corrections
 - f. How to manage activities on partly conflicting topics.
- G. In short: **how not to break the branch one is sitting on.** Who cares about something essential, fragile but assumed granted.
- H. **That is serious fundamental topic for checking/extending applicability boundaries of any theory.**

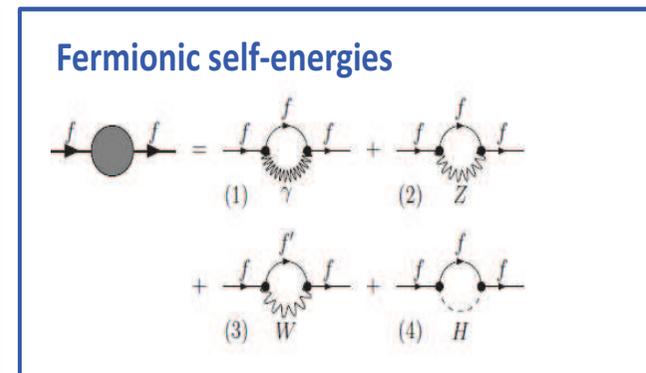
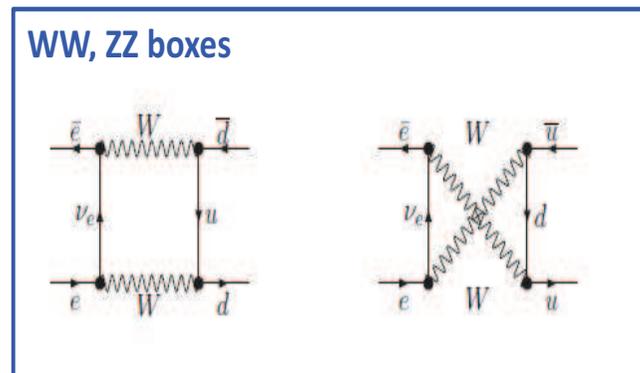
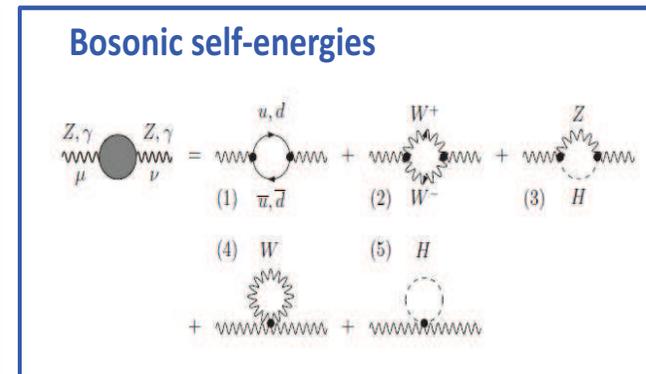
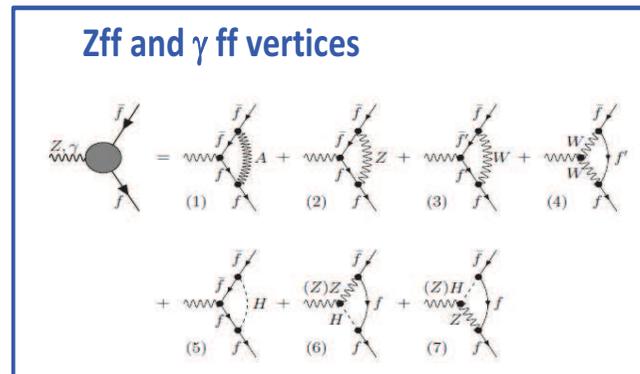
LEP specific $\mathcal{O}(\alpha)$ QED part to be factorized out

- Scheme to be worked out to **higher orders**.
- it is not automatic to identify QED from the rest.
- Efforts...



$\mathcal{O}(\alpha)$ remaining parts \rightarrow genuine weak, lineshape

- Also in this group of contributions one had to separate big **lineshape** terms and take to higher orders. *It is easy to say but ...*

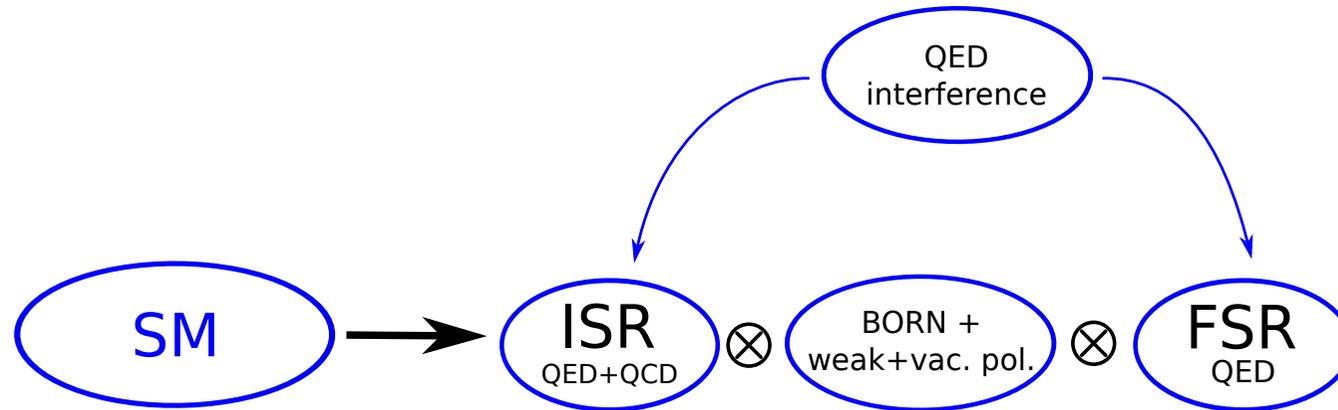


What Z, W, H signatures may mean?

- Once physical gauge is chosen, bosons acquire masses. Still the W, H and Z propagators remain singular $\frac{1}{s-M^2}$. This seems trivial to fix, use $\frac{1}{s-M^2+i\Gamma M}$.
Resummation to all orders of loop correction parts is used for $i\Gamma M$ term!
- Resulting approach, make bosons into physics states of definite properties, including width. Calculations required massive effort at LEP. Fundamental questions had to be solved.
- Results are used by CDF D0 as state-of-art today also. See Arie Bodek talk, CERN Jan 31, 2017, <https://indico.cern.ch/event/571075/>
- **I will not go into all details necessary for fundamentals. I will concentrate on practical aspects/results.**

- **The following observations are of a great importance:**
 - Electroweak Resonance Width is a calculated quantity.
 $\Gamma \sim \alpha_{weak} \sim \alpha_{QED}$ as a consequence
 - at Z peak amplitude for $e^+e^- \rightarrow l\bar{l} \sim \alpha^0$,
 - off the Z peak the same amplitude for $e^+e^- \rightarrow l\bar{l} \sim \alpha^1$.
 - Dominant correction to these purely leptonic amplitudes are α_S at Z peak but α_{QED} off the peak.
 - If lepton pairs of virtualities substantially smaller than Z mass enter the acceptance, then correction from low energy vacuum polarization, taken from low energy $e^+e^- \rightarrow hadrons$ data, must be used for $\alpha_{QED}(s)$.
 - That results come from non-analytic technique, because optical theorem is used. Problems for Z width obtained from analytic continuation arise too, as a consequence of gauge symmetries between Z and γ^* .

*Production and decay of Z/γ^**



- That is the picture which emerged **after lot of pain**.
- Genuine weak corrections were calculated at one loop level, but:
- Separate QED corrections were treated at the second order with exclusive exponentiation taken into account.
- QED ISR with vacuum polarization corrections were called '**lineshape corrections**'. Up to 3 loop QCD contributions for quark loops used. For low energies vacuum polarization was obtained from dispersion relations and $e^+e^- \rightarrow hadrons$ data.

Derived but fundamental and to a point reliable basis.

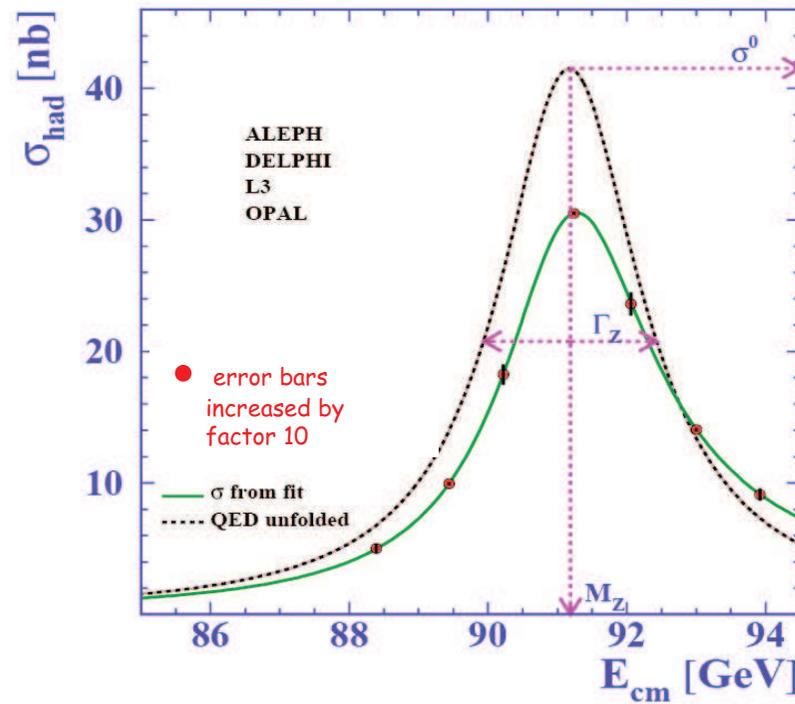
- Separation of amplitudes into parts such as QED/QCD ISR, QED FSR. No loss of precision. See eg. *The standard model in the making: Precision study of the electroweak interactions*, **Dmitri Yu. Bardin**, (Dubna, JINR) , G. Passarino, Oxford, UK: Clarendon (1999) 685 p.
- **What to do e.g. with emissions from intermediate states:** Leading pole approximation as in U. Baur NLO calculation for $W\gamma$ anomalous coupling (Phys. Rev. D 47 (1993) 4889), simplifies the issue of QED FSR separation from rest of EW effects. Nearly direct consequence of formula:

$$\frac{1}{\left((P+k)^2 - M_W^2\right)\left(P^2 - M_W^2\right)} = \left(\frac{1}{P^2 - M_W^2} - \frac{1}{(P+k)^2 - M_W^2}\right) \frac{1}{2Pk}$$
- **We could confront field theory with the data in reliable manner and precision of $\frac{\alpha_{QED}}{\pi}$ level; all necessary resummations included.**

Note: large corrections, small uncertainties.



Z Lineshape



peak X-section
→ N_ν to 2.7‰

$\delta\sigma^0$ is dominated by the uncertainty in absolute L (~1‰)

→ Luminosity measurement

— The final hadronic cross section, measured and QED deconvoluted.

M_Z to 20 ppm → LEP energy calibration

Typically: $3 \cdot 10^5 - 1 \cdot 10^6$ samples

- A. Granularity of the detector was an issue: bare, dressed leptons could be nonetheless used.
- B. Semianalytical calculations could be used for phenomenology.
- C. Picture of logarithm power was used for preliminary estimation of required effects. Cut off induced logarithms were not dominating.

* Techniques:

- d. Approach based on combination of semianalytical and Monte Carlo methods was established.
- e. DIZET and ZFITTER were the semianalytical brands of that time.
- f. Semianalytic calculation mean calculation where at most one (or two) integration are performed numerically.
- g. KKMC KORALZ brands for Monte Carlo.

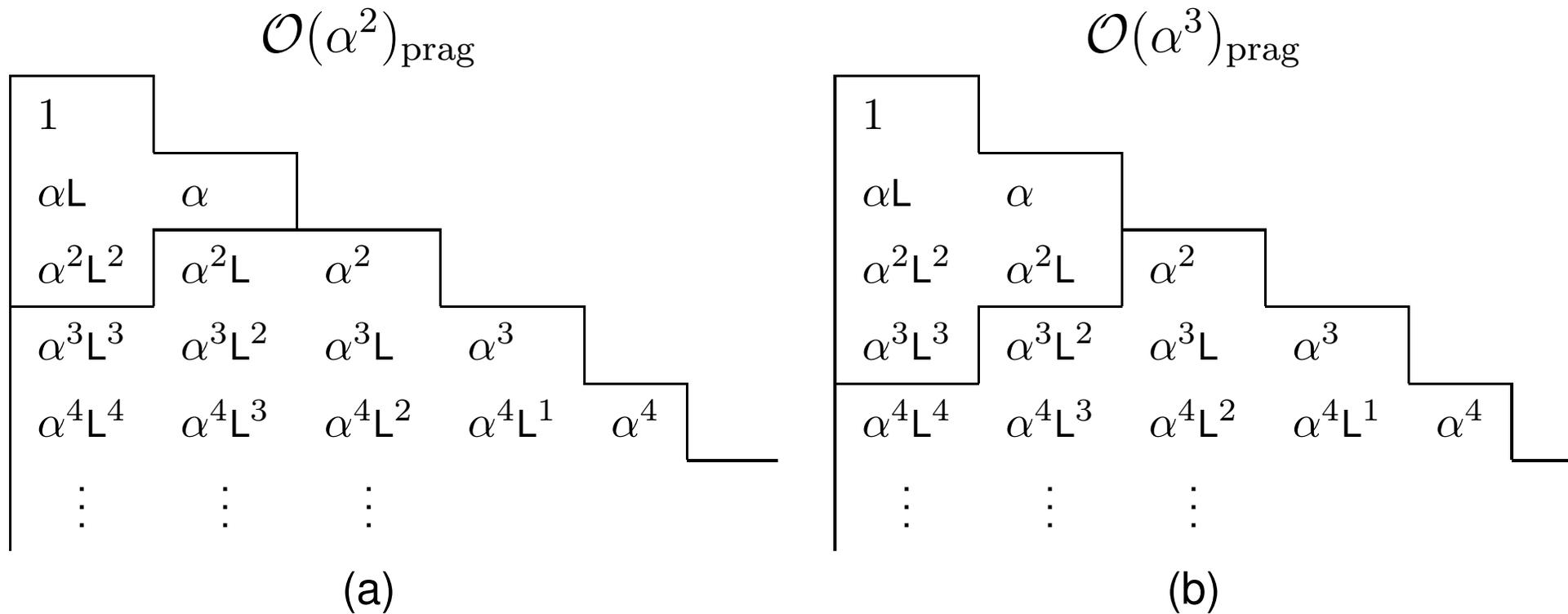


Figure 1: **LEP 1 style, collinear logarithms** $L = \ln \frac{s}{m_l^2}$: QED leading and subleading corrections. The rows for consecutive perturbative orders – the first is the Born contribution. The first column for the leading logarithmic (LL) approximation and the second column the next-to-leading (NLL) approximation. In the Figure, terms selected for (a) second and (b) third order pragmatic expansion (for photon emission from the electron at LEP energies) are limited with the help of an additional line. *Factors like $\ln \frac{M_Z}{\Gamma_Z}$ appeared, but in a controlled manner.*

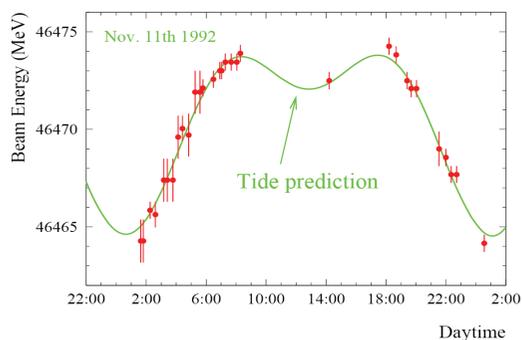
The Instruments/Actors



- **LEP Collider:** clean and stable machine, high luminosity
- **4 Experiments:** well functioning detectors, only upgrades for b-physics (Si vertex detectors) and precision luminosity measurement (SiW calorimeters)
- **Theorists** providing high precision calculations

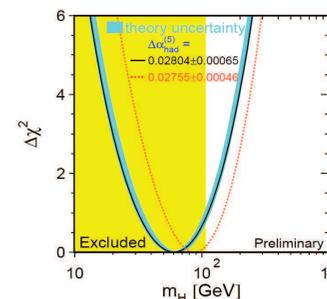
Working groups with members from all three (new style)

LEP Energy WG (LEP E calibration)



4.1.2007

LEP Electroweak-WG
(Combination and SM fits)



Physics Report, 2005, "Precision EW Measurements on the Z Resonance"

D. Schlatter

Typically: $3 \cdot 10^7$ samples

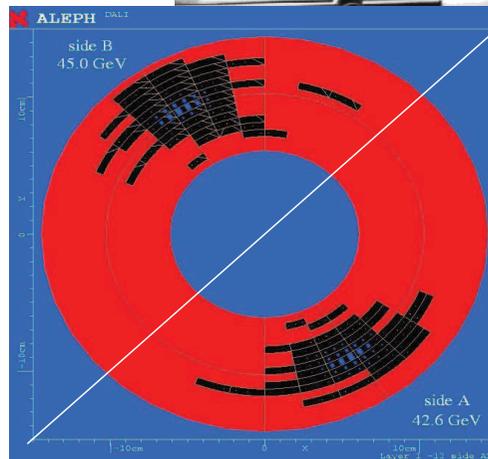
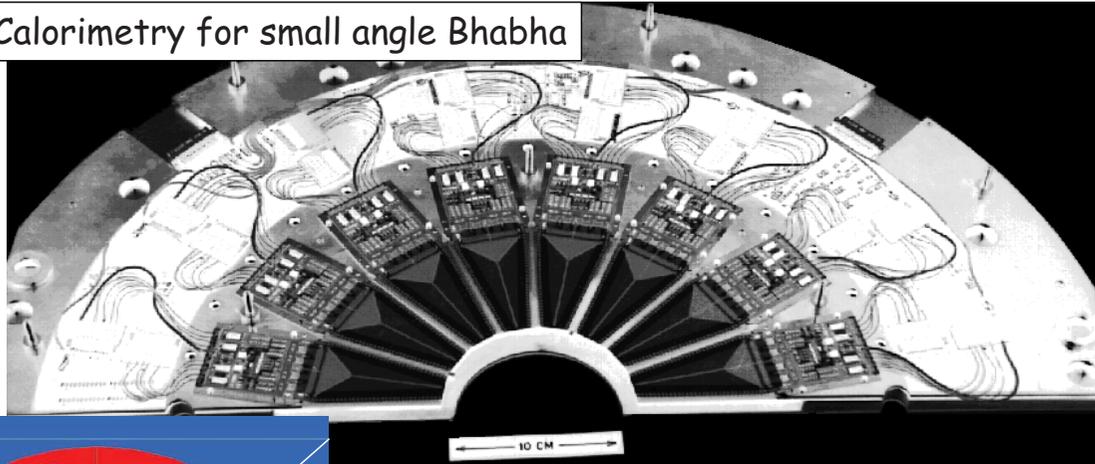
1. From the physics dynamic point of view small angle bhabha scattering $e^+e^- \rightarrow e^+e^-$ is simple. Single t-channel photon exchange dominates.
2. Statistical samples of the order of 10^7 events resulted in sub-permille precision level, see e.g. Eur.Phys.J. C14 (2000) 373 for OPAL measurement.
3. Challenge was rapid variation of the cross section as a function of scattering angle
4. Also lepton directions were affected by bremsstrahlung.
5. Further effects such as photonic vacuum polarization contributed little, but resulting systematic error was dominant.

Note: detector granularity.

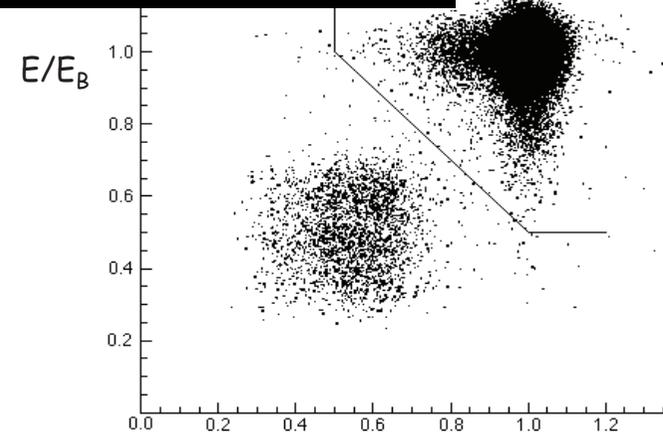
New detector concepts II



SiW Calorimetry for small angle Bhabha



4.1.2007



D. Schlatter

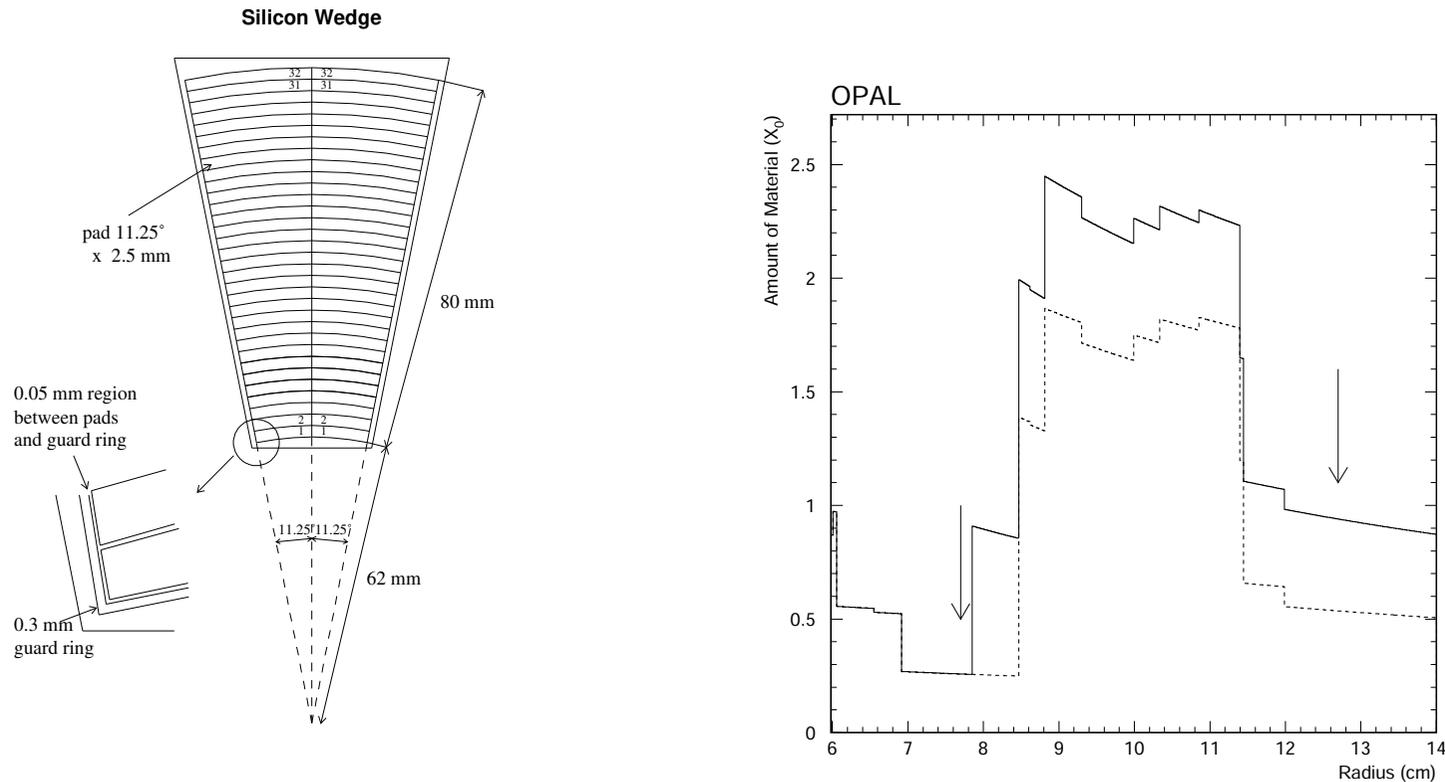
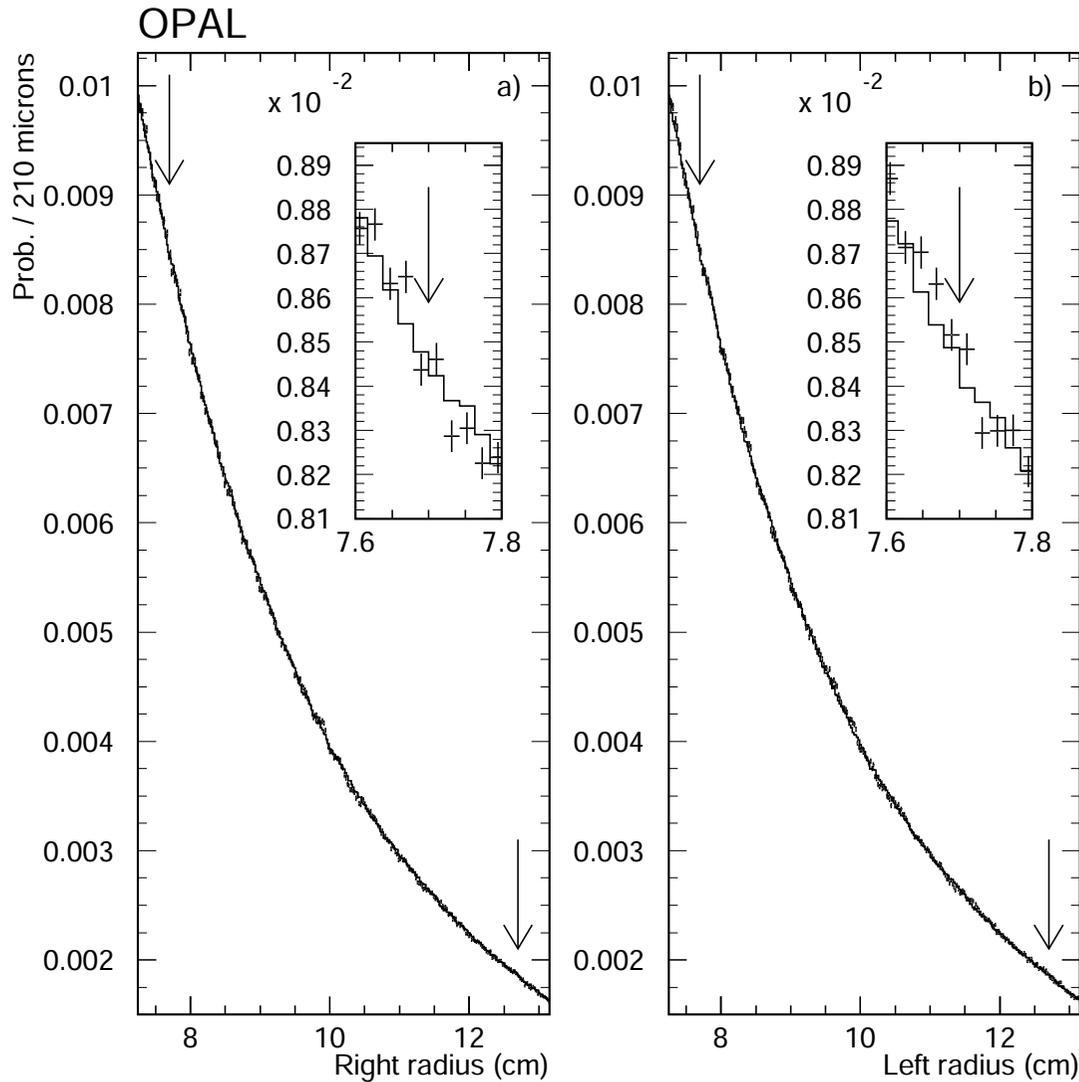


Figure 2: Schematic illustration of the silicon diode pad. The silicon detectors with 2 rows of 32 pads, each covering 11.25° in azimuth. The radial pad segmentation 2.5 mm. Silicon detectors within each layer of the calorimeter are physically overlapped, the azimuthal boundaries of their active regions coincide and no dead or “double counted” regions. The calculated material traversed by particles originating at the interaction point as a function of calorimeter radius, at the reference plane (246 cm) for the detector configuration. The solid curve left, the dotted curve right side. Larger amount of material on the left due to microvertex detector cables. Arrows indicate acceptance cuts on shower radius.



The distributions of R_R after all SWITL selection cuts have been applied (a), and of R_L after all SWITR selection cuts have been applied, (b). The points and error bars are the data and the histogram the Monte Carlo. The vertical dotted lines indicate the radial isolation cuts.

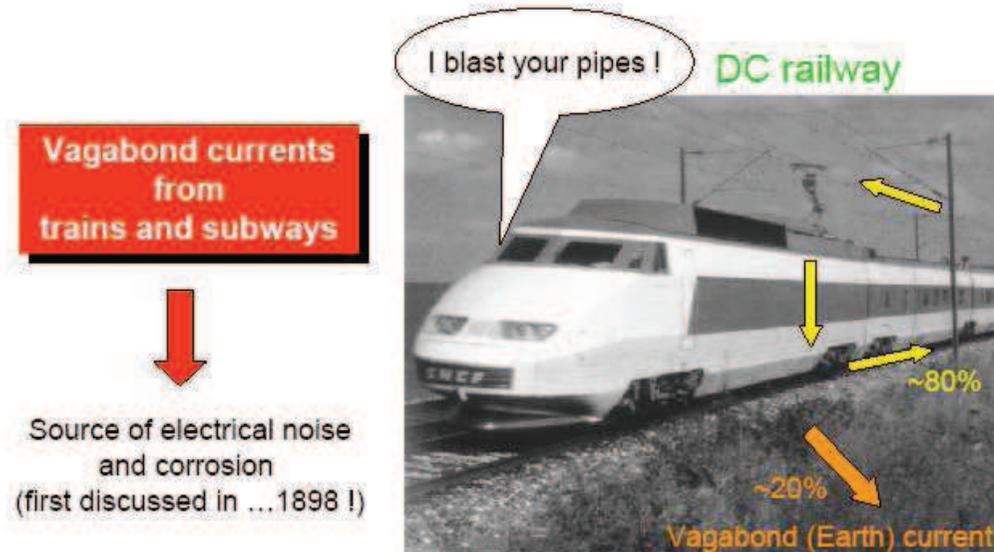
Note: first discussion 1898!

LEP Energy Saga (from J. Wenniger at LEP Fest 2000)



Pipebusters

The explanation was given by the Swiss electricity company EOS...



4.1.2007

D. Schlatter

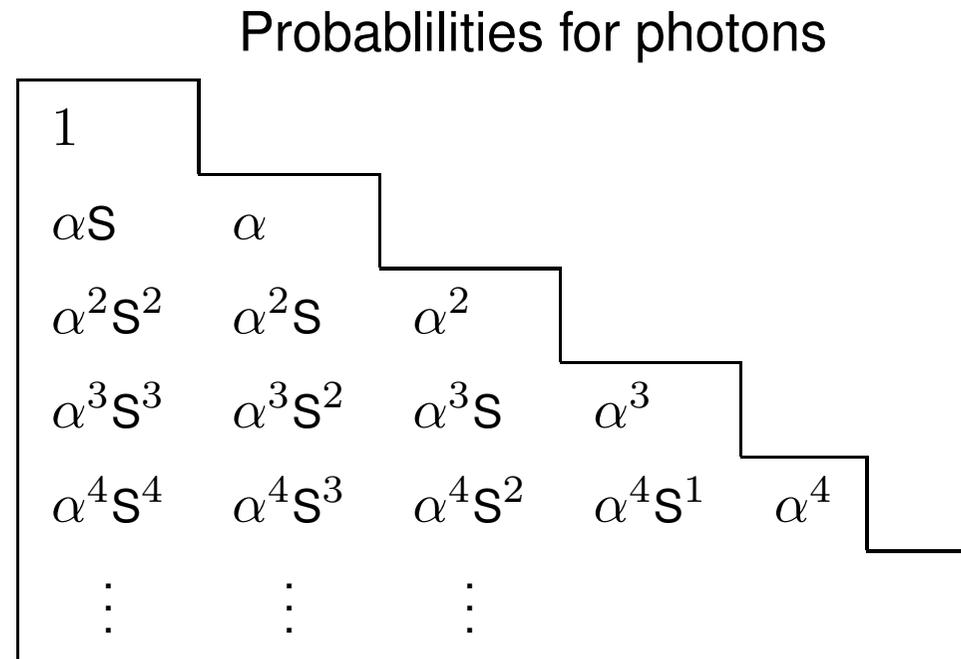


Figure 3: **Attempt to systematize precision requirements:** *Figure entries differ from Fig. 1. The rows again represent corrections in consecutive perturbative orders – the first row is the Born contribution. But the first column represents now the probabilities of a given number of photons which may distort direction or energy of leptons; $S \sim \ln \frac{s}{m_e^2} \ln \frac{E_{max}}{E_{min}}$. The $\alpha^{n+m} S^m$, mean n photons are of specific configuration (e.g. close to the acceptance boundary) and n are integrated over. Probabilities are much larger than terms counted in Fig. 1. In fact picture for counting is more involved.*

Typically: $10^3 - 4 \cdot 10^4$ samples only

- Many refinements, technical complications dropped because much smaller samples. Systematic uncertainties previously studied could be ignored.
- That was important, and also good news, because of necessity to control gauge cancellations between s- and t-channel boson contributions.
- New technical/physics challenges appeared because of
 - WW threshold effects
 - Anomalous triple gauge boson couplings
 - Bose-Einstein correlations between decay products of two W's.
 - Searches for new particles, new physics.
- At LEP2 methods of fits with semi-analytic functions appeared for the first time to be non-feasible.
- But no hints toward new categories of challenges to be of importance for LHC.

1. **Precision measurements at LEP 1:** Once statistical samples are of the order of 10^5 , 10^6 events and precision is better than 0.3 % for phenomenology of electroweak sector (Z couplings to leptons, couplings to invisible sector etc.) resummation of lineshape corrections is necessary.
 - (a) That means in particular 3 loops corrections to vacuum polarization, also from low energy e^+e^- data through dispersion relation.
 - (b) This picture was challenging for theory, massive theoretical effort was needed.
 - (c) If one could ignore contribution from low virtuality lepton pair, life would be easier.
 - (d) Decision depend on experimental condition, background contamination and subtraction, etc...
 - (e) Detector granularity effects were an issue, but could be controlled.
Semi-analytical fitting function were useful. Correction beyond that, could be evaluated by Monte Carlo simulations; then added or even ignored as non contributing.

2. **Precision measurement of Luminosity:** statistical samples of 10^7 , errors of measurements down to 0.041% !! It was possible because:
- (a) low energy e^+e^- data was used to control loop effects
 - (b) Detector granularity and t-channel near singularity angular dependencies were taken into account in great detail.
 - (c) Effects of detector response to configurations were soft collinear protons affected calorimeter response.
 - (d) That is why counting of perturbative powers was essential. Double logarithm (collinear and soft) defined the pattern. Logarithm resulting from assumed acceptance were necessary.
 - (e) Much smaller role of semianalytical calculations. Calculations were useful for benchmarks, but were not as important for phenomenology.
 - (f) Beam geometry was also taken into account in theoretical predictions.
- Important kinematic consequences arised due to t-channel exchange of a photon. Cross section varied a lot over acceptance. Predictions predominantly arised from QED. Genuine weak effects were not that important.

3. **Electroweak measurements at LEP 2**, Much smaller samples of several 10^4 events for WW and just 10^3 for ZZ . As a consequence:
- Detector granularity was not an issue for theoretical predictions.
 - Higher order QCD line shape loops or higher order QED could be simplified and used for tests only. Available LEP 1 results simplified the tasks.
 - All that was beneficial because lowest order gauge dependence cancellations of non resummed propagators could be used.
 - Also effective $\sin^2 \Theta_W$ was not obligatory for the Z couplings. The G_μ scheme could be used.

==*==

4. **These heritage benefit observations are in conflict with each other...**

Having all simultaneously may be still a challenge, despite all past efforts.

SM fits



SM: Each observable calculated as a function of:

$$\Delta\alpha_{\text{had}}, \alpha_s(M_Z), M_Z, M_{\text{top}}, M_{\text{Higgs}}$$

Therefore, the **input parameters** are chosen to be:

$\alpha^{-1}(0) = 137.03599877(40)$	$\approx 3 \times 10^{-9}$
$\alpha_s(M_Z) = 0.118(2)$	$\approx 2 \times 10^{-2}$
$G_\mu(m_\mu) = 1.16637(1) \times 10^{-5} \text{ GeV}^{-2}$	$\approx 9 \times 10^{-6}$
$m_Z = 91.1875(21) \text{ GeV}$	$\approx 2 \times 10^{-5}$

But... the relevant scale is $s \approx M_Z^2 \dots$

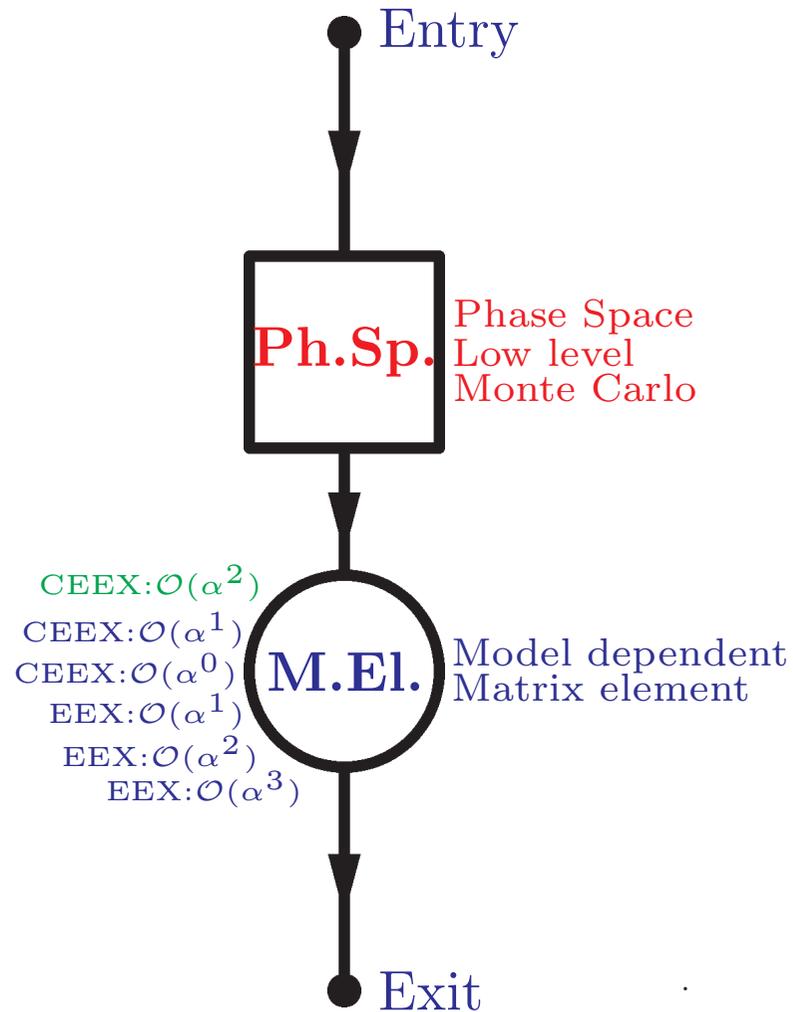
$$\alpha(s) = \alpha(0) / [1 - \Delta\alpha_{e\mu\tau}(s) - \Delta\alpha_{\text{top}}(s) - \Delta\alpha_{\text{had}}(s)]$$

$s = M_Z^2$ 0.0315 small 0.0280 - 0.0275

G_μ scheme could be used, because:

- → 40 kevt of WW
- Cross checks if precision one achieved was sufficient were multiple:
- An example of extensive study of systematic errors for WW can be found in:
S. Jadach et al. Comp. Phys. Commun. 140(2001) 475.
- → Only 1000 events of ZZ
- Large statistical errors.
- Brutal approximation, where $\sin^2 \Theta_W = 1 - M_W^2/M_Z^2$ was sufficient.
- Earlier tests and results of LEP 1 were the source of confidence.

Textbook principle “matrix element \times full phase space” for technical and physics uncertainties



- Phase-space Monte Carlo simulator is a module producing “raw events” (including importance sampling for possible intermediate resonances/singularities)
- Library of Matrix Elements; input for “model weight”; independent module
- This was used extensively for LEP Monte Carlos systematic errors.
- Correlated samples techniques. Variants used for the difference semi-analytical vs. Monte Carlo simulation. Beware: relation of crude level phase-space and semianalytical integration variables, Phys. Rev. D41 (1990) 1425.
- Useful for Mashine Learning input too!

- Clear separation into phase-space \times Matrix elements, enabled frame for discussion of systematic errors;
 - technical uncertainty,
 - statistical uncertainty,
 - physics uncertainty.
- Reweighting at the level of partly integrated phase space is perilous, unless:
 - clear factorization at the level of fully differential distribution is valid,
 - separation of production and decay is possible, like in case of Higgs.
- Angular coefficients are a step toward such goal for Drell Yan at LHC.
- **Comment:** semi-analytical calculations were covered in other talks, that is why I will concentrate on DIZET, not ZFITTER.

Exclusive exponentiation

One of the key technical achievement was application of the concept of exclusive exponentiation for the Monte Carlo algorithms.

This was based on Yennie Frautchi Suura work.

Re-ordering of perturbative expansion, where already at the lowest order eikonal factors for arbitrary number of photons were present was very beneficial.

This was valid all over the phase space.

Full/exact control of the phase space distribution was possible thanks to the conformal symmetry of eikonal factors and of phase space because of massless photon

This was major achievement for Monte Carlo development.

Results are relevant for LHC because of precise measurement of lepton directions.

KKMC will be presented later, let me point here to some features only.

Program is build on exact phase-space generator. This was possible because of additional symmetry for eikonal soft photon amplitudes and phase space (conformal symmetry)

This was valid all over phase space.

Corrections due to matrix elements were introduced up to second order in the scheme of Yennie Frautchi Suura.

KKMC was the Monte Carlo, where *matrix element* \times *phase space* paradigm was applied for the *case of resummation*.

Processes $e^+e^- \rightarrow f\bar{f}(n\gamma)$, $n = 0, 1, 2, 3, 4, \dots$ were covered.

Amount of documentation and tests was extensive, main publications were about 100 pages each: S. Jadach et al. Comput.Phys.Commun. 130 (2000) 260
Phys.Rev. D63 (2001) 113009,

I have to skip details (and part of our life), see talk by S.Yost later.

1. PHOTOS Monte Carlo is for simulation of multiphoton FSR bremsstrahlung.
2. Generates correlated samples: events with and without FSR differ in proportion to photon energy.
3. For processes mediated by Z/γ' and W 's high precision is obtained. Decay dependent matrix elements are used.
4. Important for program construction were studies of spin amplitudes. Their gauge invariant parts are used in definition of photon emission kernel.
5. Remaining parts of amplitudes are needed for discussion of systematic errors, for optimization or for correcting weights.
6. Program version using C++ HepMC event record is available.
7. For us LL means collinear leading logs. PHOTOS NLL is equivalent to NNNLL in double log classification.

- PHOTOS feature complete exact phase space for multiphoton radiation. High precision has to be demonstrated on distributions with experiment like cuts.
- Phase space parametrization is different than for KKMC. One starts with Poissonian distribution based on the eikonal matrix elements for photons candidates distributed over the unphysical *tangent to phase-space*.
- Then iteratively project photon after photon projection on the actual phase-space was performed, often with the help of the explicit matrix element.
- Solution opened the way for generation of extra lepton pair too.
- Comparisons with KKMC, KORALW provide essential test-bed. KKMC is based on exclusive exponentiation and features complete matrix element for **double photon emission**.
- Comparisons with SANC (D. Bardin et al.), for Z and W decay were necessary to understand numerically separation of electroweak corrections into genuine weak and QED. A. Arbuzov et al. JETP Lett. 103 (2016) no.2, 131.
- Number of the own semi-analytical tests could be avoided.

Presentation, practical aspects

- PHOTOS (by E.Barberio, B. van Eijk, Z. W., P.Golonka) is used to simulate the effect of radiative corrections in decays, since 1989.
- Full events combining complicated tree structure of production and subsequent decays are fed into PHOTOS, with the help of HEPEVT event record of F77
- PHOTOS C++ version for HepMC event record: Photospp
- At every branching of event tree, PHOTOS intervene. With certain probability extra photon(s) are added and kinematics of other particles adjusted.
- PHOTOS algorithm is iterative. First over emitters; interference (or matrix element) weight is used. Iteration over consecutive emissions is external.
- Compatibility with exponentiation and resummation of collinear terms at the same time.

Main References

- E. Barberio, B. van Eijk and Z. Was, Comput. Phys. Commun. **66**, 115 (1991): **single emission**
- E. Barberio and Z. Was, Comput. Phys. Commun. **79**, 291 (1994). **double emission introduced, tests with second order matrix elements**
- P. Golonka and Z. Was, EPJC 45 (2006) 97 **multiple photon emission introduced, tests with precision second order exponentiation MC.**
- P. Golonka and Z. Was, EPJC 50 (2007) 53 **complete QED ME in Z decay**
- G. Nanava, Z. Was, Eur.Phys.J.C51:569-583,2007, **best description of phase space**
- G. Nanava, Z. Was, Q. Xu, Eur.Phys.J.C70:673,2010. **complete QED ME in W decay**
- N. Davidson, T. Przedzinski, Z. Was, Comput. Phys. Commun. 199 (2016) 86, **HepMC interface ME in W, Z decays, light lepton pair emission.**
- S. Antropov, A. Arbuzov, et al. Acta Phys.Polon. B48 (2017) 1469 **tests light lepton pair emission.**

Tests with members of D. Bardin group, also web pages:

<http://mc-tester.web.cern.ch/MC-TESTER/>

<http://annapurna.ifj.edu.pl/~wasm/phNLO.htm>

<http://annapurna.ifj.edu.pl/~tprzedzinski/KI3/>

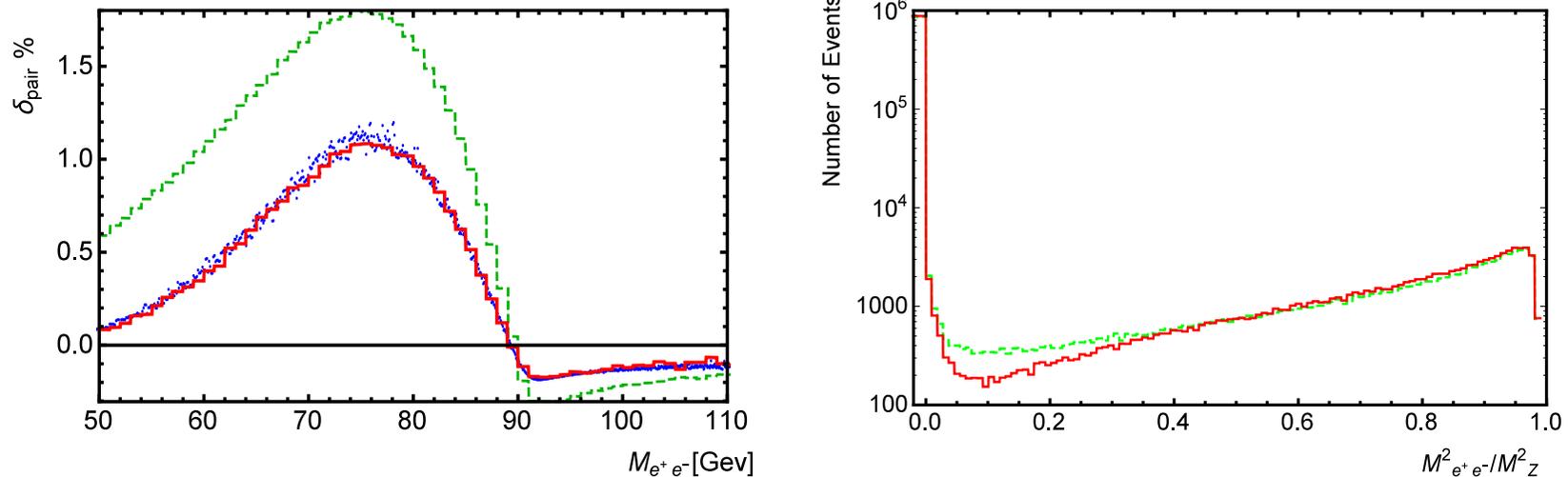
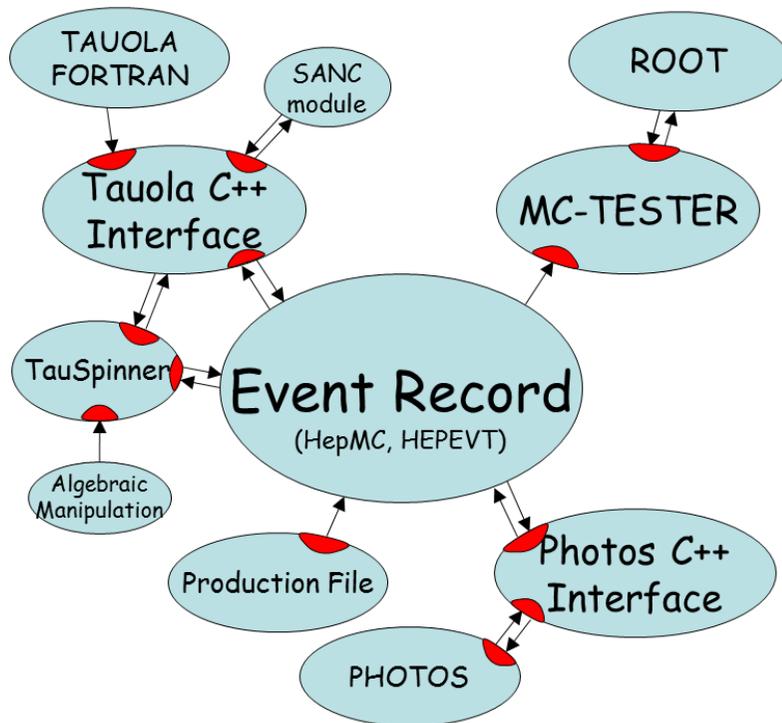


Figure 4: An example from <https://arxiv.org/pdf/1706.05571.pdf> of semianalytical test (left effect of extra ee pair on lepton pair invariant mass spectrum in $pp \rightarrow Z/\gamma \rightarrow ee(ee)$) and test with legacy Monte Carlo KORALW (right side, invariant mass of ee in $Z \rightarrow \mu\mu ee$ decay). Case of pair emission. Agreement is between red line and blue points. **Tests have to be repeated by the user to check data flow from event record.**

Event record communication, reliable, because work of D.B.36

Photos, Tauola++, TauSpinner
communicate through event record:



- Parts:

- hard process: (Born, weak, new physics),
- parton shower, • τ decays
- -QED bremsstrahlung
- Detector studies: acceptance, resolution lepton with or without photon.

Such organization requires:

- Good control of factorization (theory)
- Good understanding of tools on user side.
- For Photospp, (now 100% C++ or C) :

Web page <http://photospp.web.cern.ch/photospp/>

- For Tauolapp:

Web page <http://tauolapp.web.cern.ch/tauolapp/>

- TAUOLA features interface to HepMC and electroweak library of SANC (also DIZET) Useful to re-weight events for weak corrections + new physics.

- Lessons from LEP: with the improved statistical precision, more and more sophistication for theory is needed.
- Not only in the sectors ...
 - QED
 - Genuine weak
 - Strong interaction
 - detector granularity details
 - beam properties
- ... but also in the ways how to combine.
- In particular: t-channel mean not only pressure on gauge cancellation constraints but also enhance importance of detector details, pad geometry vs. QED brem. etc.
- All this was necessary. It makes conditions for central LHC concerns, such as of strong interactions and proton structure, less convenient.

Keynotes

- LEP1: One loop genuine weak corrections, but up to 3 loops QCD effects and multiphoton corrections were necessary at the same time. Semianalytical calculations very useful. Dressed leptons as well.
- Luminosity: Highest precision: Theoretical predictions dependent on detector structure was a must. How (unresolved) photons affect detector response to leptons at the edge of acceptance or edge of detector pad.
- LEP2: Samples were tiny, solution for new problems such as gauge cancellation between s- and t-channel boson exchanges could be assured without worry of theoretical sophistications necessary at LEP 1 or detector granularity as for luminosity.
- **Frame for predictions which was build at LEP time turned out to be robust, survived Tevatron times too.**
- **How it looks now?**

- Constructs of LEP times, can be extended to LHC applications.
- This required effort
- I assume that many aspects of Dima Bardin work will be presented in other talk.
- I will concentrate on this, what was needed for our approach and tools.

TAUSPINNER I will quote from our paper: T. Przedzinski, E. Richter-Was and Z. Was, “Documentation of TauSpinner algorithms – program for simulating spin effects in tau-lepton production at LHC,” arXiv:1802.05459 [hep-ph].

ATLAS-EW Also from last week publication by ATLAS: The ATLAS collaboration [ATLAS Collaboration], “Measurement of the effective leptonic weak mixing angle using electron and muon pairs from Z -boson decay in the ATLAS experiment at $\sqrt{s} = 8$ TeV,” ATLAS-CONF-2018-037.

- I will show some other related results as well.

- The paper ‘Measurement of the effective leptonic weak mixing angle using electron and muon pairs from Z -boson decay in the ATLAS experiment at $\sqrt{s} = 8$ TeV,’ ATLAS-CONF-2018-037, became available 11 days ago.
- Quoted there papers by D.Bardin:
 - Comput. Phys. Commun. 59 (1990) 303.
 - Comput. Phys. Commun. 133 (2001) 229
 - Nucl. Phys.B175(1980) 435.
 - Nucl. Phys. B197 (1982) 1.
- Why His achievements were so fundamental to survive test of time and why survived migration from LEP to LHC.
- There are many reasons,
- I will cover those which I have experienced myself.

Let us start with the lowest order coupling constants (without EW corrections) of the Z boson to fermions, where $s_W^2 = 1 - m_W^2/m_Z^2$ denotes $\sin^2 \theta_W$ in the on-line scheme and T_3^f denotes third component of the isospin.

The vector v_e, v_f and axial a_e, a_f couplings for leptons and quarks respectively are defined with formulas below.

$$\begin{aligned}
 v_e &= (2 \cdot T_3^e - 4 \cdot q_e \cdot s_W^2) / \Delta \\
 v_f &= (2 \cdot T_3^f - 4 \cdot q_f \cdot s_W^2) / \Delta \\
 a_e &= (2 \cdot T_3^e) / \Delta \\
 a_f &= (2 \cdot T_3^f) / \Delta
 \end{aligned} \tag{1}$$

where

$$\Delta = \sqrt{16 \cdot s_W^2 \cdot (1 - s_W^2)} \tag{2}$$

With this notation, matrix element for the $q\bar{q} \rightarrow Z/\gamma^* \rightarrow l^+l^-$, denoted as ME_{Born} , can be written as:

$$\begin{aligned}
 ME_{Born} &= [\bar{u}\gamma^\mu v g_{\mu\nu} \bar{\nu}\gamma^\nu u] \cdot (q_e \cdot q_f) \cdot \frac{\chi_\gamma(s)}{s} \\
 &+ [\bar{u}\gamma^\mu v g_{\mu\nu} \bar{\nu}\gamma^\nu u \cdot (v_e \cdot v_f) + \bar{u}\gamma^\mu v g_{\mu\nu} \bar{\nu}\gamma^\nu \gamma^5 u \cdot (v_e \cdot a_f) \\
 &+ \bar{u}\gamma^\mu \gamma^5 v g_{\mu\nu} \bar{\nu}\gamma^\nu u \cdot (a_e \cdot v_f) + \bar{u}\gamma^\mu \gamma^5 v g_{\mu\nu} \bar{\nu}\gamma^\nu \gamma^5 u \cdot (a_e \cdot a_f)] \cdot \frac{\chi_Z(s)}{s}
 \end{aligned} \tag{3}$$

and Z -boson and photon propagators defined respectively as

$$\chi_\gamma(s) = 1 \tag{4}$$

$$\chi_Z(s) = \frac{G_\mu \dot{M}_Z^2}{\sqrt{2} \cdot 8\pi \cdot \alpha_{QED}(0)} \cdot \Delta^2 \cdot \frac{s}{s - M_Z^2 + i \cdot \Gamma_Z \cdot M_Z} \tag{5}$$

At the peak of resonance $|\chi_Z(s)|(v_e \cdot v_f) > (q_e \cdot q_f)$ and as a consequence angular distribution asymmetries of leptons are proportional to $v_e = (2 \cdot T_3^e - 4 \cdot q_e \cdot s_W^2)$. This gives good sensitivity for s_W^2 measurement.

Above and below resonance we are sensitive to lepton charge instead ...

Born cross-section, for $q\bar{q} \rightarrow Z/\gamma^* \rightarrow \ell^+\ell^-$ can be expressed as:

$$\frac{d\sigma_{Born}^{q\bar{q}}}{d\cos\theta}(s, \cos\theta, p) = (1+\cos^2\theta)F_0(s) + 2\cos\theta F_1(s) - p[(1+\cos^2\theta)F_2(s) + 2\cos\theta F_3(s)] \quad (6)$$

p denotes polarization of the outgoing leptons, and form-factors read:

$$\begin{aligned} F_0(s) &= \frac{\pi\alpha^2}{2s} [q_f^2 q_\ell^2 \cdot \chi_\gamma^2(s) + 2 \cdot \chi_\gamma(s) \text{Re}\chi_Z(s) q_f q_\ell v_f v_\ell + |\chi_Z^2(s)|^2 (v_f^2 + a_f^2)(v_\ell^2 + a_\ell^2)], \\ F_1(s) &= \frac{\pi\alpha^2}{2s} [2\chi_\gamma(s) \text{Re}\chi(s) q_f q_\ell v_f v_\ell + |\chi^2(s)|^2 2v_f a_f 2v_\ell a_\ell], \\ F_2(s) &= \frac{\pi\alpha^2}{2s} [2\chi_\gamma(s) \text{Re}\chi(s) q_f q_\ell v_f v_\ell + |\chi^2(s)|^2 (v_f^2 + a_f^2) 2v_\ell a_\ell], \\ F_3(s) &= \frac{\pi\alpha^2}{2s} [2\chi_\gamma(s) \text{Re}\chi(s) q_f q_\ell v_f v_\ell + |\chi^2(s)|^2 (v_f^2 + a_f^2) 2v_\ell a_\ell], \end{aligned} \quad (7)$$

$\cos\theta$ denotes angle between incoming quark and outgoing lepton in the rest frame of outgoing leptons. That is rather simple spherical harmonics of the second order.

What Changes come with jets. No QED exponentiation, but nonetheless...

- E. Mirkes and J. Ohnemus, “Angular distributions of Drell-Yan lepton pairs at the Tevatron: Order $\alpha - s^2$ corrections and Monte Carlo studies,” Phys. Rev. D **51** (1995) 4891
- R. Kleiss, “Inherent Limitations in the Effective Beam Technique for Algorithmic Solutions to Radiative Corrections,” Nucl. Phys. B **347**, 67 (1990).
- F. A. Berends, R. Kleiss and S. Jadach, “Monte Carlo Simulation of Radiative Corrections to the Processes $e^+ e^- \rightarrow \mu^+ \mu^-$ and $e^+ e^- \rightarrow \text{anti-}q q$ in the Z^0 Region,” Comput. Phys. Commun. **29**, 185 (1983).

General form of Born level distribution is preserved but choice of reference frame for lepton pair usually brings in all coefficients for second order spherical harmonics.

IMPORTANT FOR REWEIGHING: whatever the jets, the second order polynomial factorizes out:

Mustraal frame

[18] F. A. Berends, R. Kleiss, and S. Jadach, *Comput. Phys. Commun.* **29** (1983) 185–200.

Mustraal: Monte Carlo for $e^+ e^- \rightarrow \mu^+ \mu^- (\gamma)$

$$s = 2p_+ \cdot p_-, \quad t = 2p_+ \cdot q_+, \quad u = 2p_+ \cdot q_- \\ s' = 2q_+ \cdot q_-, \quad t' = 2p_- \cdot q_-, \quad u' = 2p_- \cdot q_+$$

$$\sigma_{\text{hard}} = \int d\tau (X_i + X_f + X_{\text{int}}),$$

The explicit forms of the three terms in σ_{hard} read:

$$X_i = \frac{Q^2 \alpha}{4\pi^2 s} \frac{1 - \Delta}{k_+ k_-} s'^2 \left[\frac{d\sigma^B}{d\Omega}(s', t, u) + \frac{d\sigma^B}{d\Omega}(s', t', u') \right], \quad (3.4)$$

$$X_f = \frac{Q'^2 \alpha}{4\pi^2 s} \frac{1 - \Delta'}{k'_+ k'_-} s^2 \left[\frac{d\sigma^B}{d\Omega}(s, t, u') + \frac{d\sigma^B}{d\Omega}(s, t', u) \right], \quad (3.5)$$

$$X_{\text{int}} = \frac{QQ'\alpha}{4\pi^2 s} W \frac{\alpha^2}{2ss'} \left[(u^2 + u'^2 + t^2 + t'^2) \tilde{f}(s, s') + \frac{1}{2}(u^2 + u'^2 - t^2 - t'^2) \tilde{g}(s, s') \right] \\ + \frac{QQ'\alpha^3}{4\pi^2 s} \frac{(s - s') M \Gamma}{k_+ k_- k'_+ k'_-} \epsilon_{\mu\nu\rho\sigma} p_+^\mu p_-^\nu q_+^\rho q_-^\sigma \left[\tilde{E}(s, s')(t^2 - t'^2) + \tilde{F}(s, s')(u^2 - u'^2) \right], \quad (3.6)$$

Resulting optimal frame used to minimise higher order corrections from initial state radiation in $e^+e^- \rightarrow Z/\gamma^* \rightarrow \mu \mu$ for algorithms of genuine EW corrections implementation in LEP time Monte Carlo's like Koral Z.

Extending definition of Mustraal frame

- We extended this frame to $pp \rightarrow l^+ l^- j (j)$ case
 - reconstruct x_1, x_2 of incoming partons from final state kinematics (information on jets used)
 - assume the quark is following x_1 direction (equivalent to what done in CS frame)
 - calculate $(\theta_1, \phi_1), (\theta_2, \phi_2)$ of two Born's, weight with probability calculated not using couplings

$$wt_1 = \frac{E_{p1}^2(1 + \cos \theta_1^2)}{E_{p1}^2(1 + \cos \theta_1^2) + E_{p2}^2(1 + \cos \theta_2^2)}, \quad wt_2 = \frac{E_{p2}^2(1 + \cos \theta_2^2)}{E_{p1}^2(1 + \cos \theta_1^2) + E_{p2}^2(1 + \cos \theta_2^2)}$$

3

Instead of Born we get (with $\alpha_s^2 \sim 0.01$ corrections only) for the case when Jets are present:

$$\frac{d\sigma}{dp_T^2 dY d\cos\theta d\phi} = \frac{3}{16\pi} \frac{d\sigma^{U+L}}{dp_T^2 dY} [(1 + \cos^2\theta) + 1/2 A_0(1 - 3\cos^2\theta) + A_1 \sin(2\theta) \cos\phi + 1/2 A_2 \sin^2\theta \cos(2\phi) + A_3 \sin\theta \cos\phi + A_4 \cos\theta + A_5 \sin^2\theta \sin(2\phi) + A_6 \sin(2\theta) \sin\phi + A_7 \sin\theta \sin\phi]$$

Collins-Soper: the polar θ and azimuthal ϕ angles are constructed in lepton pair rest-frame. Since the Z -boson has usually a transverse momentum, the directions of initial protons are not collinear. The polar axis (z-axis) is bisecting the angle between the momentum of one of the proton and inverse of the momentum of the other one. The sign of the z-axis is defined by the sign of the lepton-pair momentum with respect to z-axis in the laboratory frame. The y-axis is defined as the normal vector to the plane spanned by the two incoming proton momenta.

- Mustraal:**
- **Definition below is for reference. It is important that every event may contribute with one of two configurations, defined either with the help of first or second beam (reconstructed parton) as seen in the rest frame of lepton pair. The final choice is made with probability independent of any couplings or PDFs.**
 - We start from the following information, which turns out to be sufficient: (i) The 4-momenta and charges of outgoing leptons τ_1, τ_2 . (ii) The sum of 4-momenta of all outgoing partons.
 - The orientation of incoming beams b_1, b_2 is fixed as follows: b_1 is chosen to be always along positive z -axis of the laboratory frame and b_2 is anti-parallel to z axis. The information on incoming partons of p_1, p_2 is

not taken from the event record. It is recalculated from kinematics of outgoing particles and knowledge of the center of mass energy of colliding protons. In this convention the energy fractions x_1 and x_2 of p_1, p_2 carried by colliding partons, define also the 3-momenta which are along b_1, b_2 respectively.

- The flavour of incoming partons (quark or antiquark) is attributed as follows: incoming parton of larger x_1 (x_2) is assumed to be the quark. This is equivalent to choice that the quark follow direction of the outgoing $\ell\ell$ system, similarly as it is defined for the Collins-Soper frame. This choice is necessary to fix sign of $\cos \theta_{1,2}$ defined later.
- The 4-vectors of incoming partons and outgoing leptons are boosted into lepton-pair rest frame.
- To fix orientation of the event we use versor \hat{x}_{lab} of the laboratory reference frame. It is boosted into lepton-pair rest frame as well. It will be used in definition of azimuthal angle ϕ , which has to extend over the range $(0, 2\pi)$.
- We first calculate $\cos \theta_1$ (and $\cos \theta_2$) of the angle between the outgoing lepton and incoming quark (outgoing anti-lepton and incoming anti-quark) directions.

$$\cos \theta_1 = \frac{\vec{\tau}_1 \cdot \vec{p}_1}{|\vec{\tau}_1| |\vec{p}_1|}, \quad \cos \theta_2 = \frac{\vec{\tau}_2 \cdot \vec{p}_2}{|\vec{\tau}_1| |\vec{p}_2|} \quad (8)$$

- The azimuthal angles ϕ_1 and ϕ_2 corresponding to θ_1 and θ_2 are defined as follows. We first define $e_{y_{1,2}}^{\vec{}}$ versors and with their help later $\phi_{1,2}$ as:

$$\vec{e}_y = \frac{x_{lab}^{\vec{}} \times \vec{p}_2}{|e_y^{\vec{}}|}, \quad \vec{e}_x = \frac{\vec{e}_y \times \vec{p}_2}{|e_x^{\vec{}}|}$$

$$\begin{aligned}\cos \phi_1 &= \frac{e_x \cdot \tau_1}{\sqrt{(e_x \cdot \tau_1)^2 + (e_y \cdot \tau_1)^2}} \\ \sin \phi_1 &= \frac{e_y \cdot \tau_1}{\sqrt{(e_x \cdot \tau_1)^2 + (e_y \cdot \tau_1)^2}}\end{aligned}\quad (9)$$

and similarly for ϕ_2 :

$$\begin{aligned}e_y &= \frac{x_{lab} \times p_1}{|e_y|}, \quad e_x = \frac{e_y \times p_1}{|e_x|} \\ \cos \phi_2 &= \frac{e_x \cdot \tau_2}{\sqrt{(e_x \cdot \tau_2)^2 + (e_y \cdot \tau_2)^2}} \\ \sin \phi_2 &= \frac{e_y \cdot \tau_2}{\sqrt{(e_x \cdot \tau_2)^2 + (e_y \cdot \tau_2)^2}}.\end{aligned}\quad (10)$$

- Each event contributes with two Born-like kinematics configurations $\theta_1 \phi_1, (\theta_2 \phi_2)$, respectively with wt_1 (and wt_2) weights; $wt_1 + wt_2 = 1$ where

$$\begin{aligned}wt_1 &= \frac{E_{p1}^2 (1 + \cos^2 \theta_1)}{E_{p1}^2 (1 + \cos^2 \theta_1) + E_{p2}^2 (1 + \cos^2 \theta_2)}, \\ wt_2 &= \frac{E_{p2}^2 (1 + \cos^2 \theta_2)}{E_{p1}^2 (1 + \cos^2 \theta_1) + E_{p2}^2 (1 + \cos^2 \theta_2)}.\end{aligned}\quad (11)$$

In the calculation of the weight, incoming partons energies E_{p1}, E_{p2} in the rest frame of lepton pair are used, but not their couplings or flavours. That is also why, instead of $\sigma_B(s, \cos \theta)$ the simplification $(1 + \cos^2 \theta)$ is used in Eq. (11).

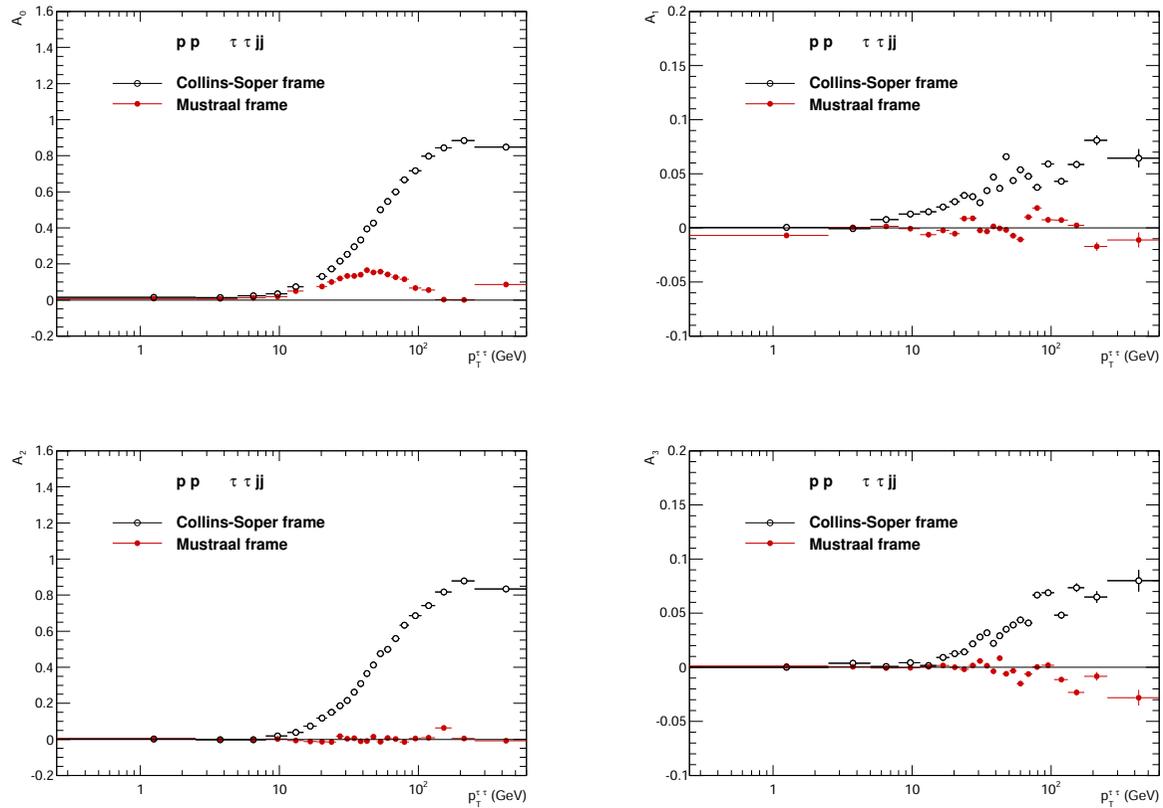


Figure 5: arXiv:1605.05450: The A_i coefficients of Eq. (8) calculated in Collins-Soper (black) and in Mustraal (red) frames for $pp \rightarrow \tau\tau jj$ process generated with MadGraph. Details of initialization are given in the reference.

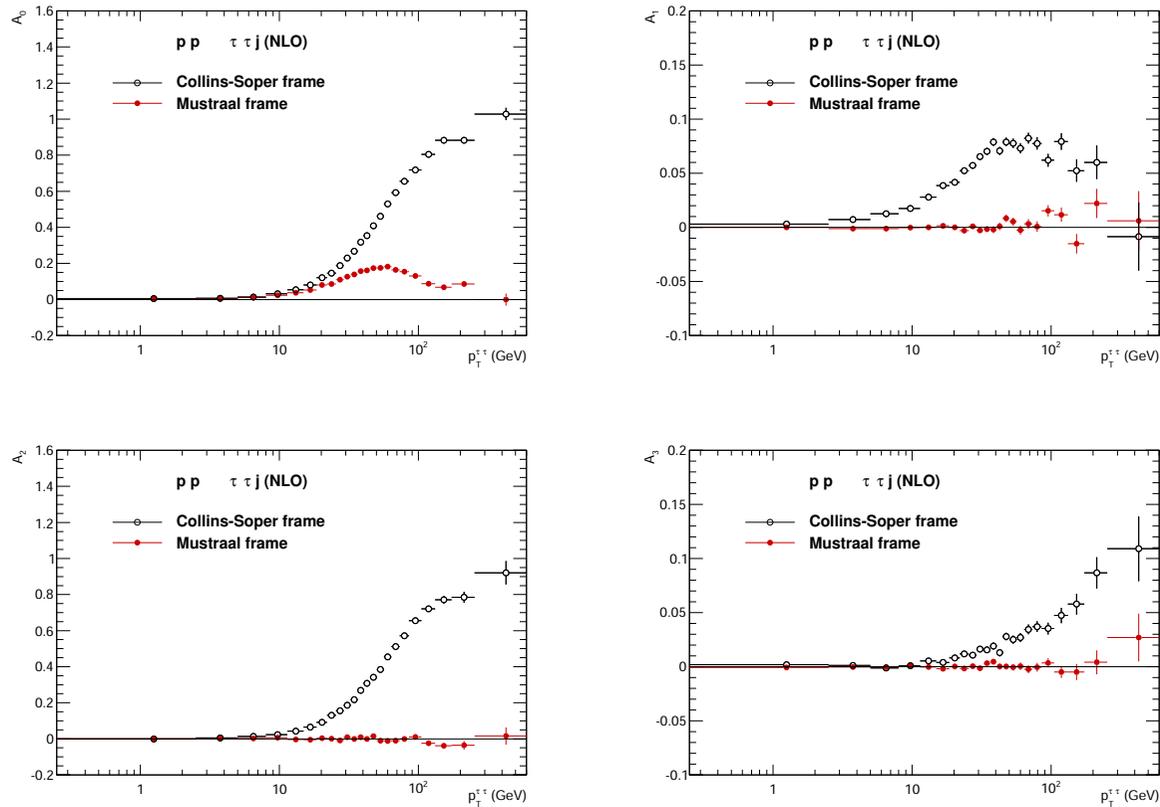


Figure 6: arXiv:1605.05450: The A_i coefficients of Eq. (8)) calculated in Collins-Soper (black) and in Mustraal (red) frames for $pp \rightarrow \tau\tau j$ (NLO) process generated with Powheg+MiNLO. Details of initialization are given in the reference.

- The choice of Mustraal frame is result of careful study of single photon (gluon emission)
- In Ref of 1982 it was shown, that differential distribution is a sum of two born-like distributions convoluted with emission factors.
- This is a consequence of Lorentz group representation and that is why it generalizes to the case of double gluon or even double parton emissions.
- **Presence of jets is like change of orientation of frames.**
- That is why use of electroweak Borns I will discuss later is justified.
- More elegant proof may come from common work with SANC team.
- For the moment figures demonstrating how proper choice of frames can turn high p_T events into electroweak Born must be sufficient.
- Far more detailed studies were performed for the purpose of LEP Monte Carlo programs and matching of genuine weak corrections with QED bremsstrahlung.

We can write amplitude for Born with EW loop corrections, $ME_{Born+EW}$, as:

$$\begin{aligned}
 ME_{Born+EW} &= [\bar{u}\gamma^\mu v g_{\mu\nu} \bar{v}\gamma^\nu u] \cdot (q_e \cdot q_f) \cdot \Gamma_{V\Pi} \cdot \frac{\chi_\gamma(s)}{s} \\
 &+ [\bar{u}\gamma^\mu v g_{\mu\nu} \bar{v}\gamma^\nu u \cdot (v_e \cdot v_f \cdot vv_{ef}) + \bar{u}\gamma^\mu v g_{\mu\nu} \bar{v}\gamma^\nu \gamma^5 u \cdot (v_e \cdot a_f) \\
 &+ \bar{u}\gamma^\mu \gamma^5 v g_{\mu\nu} \bar{v}\gamma^\nu u \cdot (a_e \cdot v_f) + \bar{u}\gamma^\mu \gamma^5 v g_{\mu\nu} \bar{v}\gamma^\nu \gamma^5 u \cdot (a_e \cdot a_f)] \frac{\chi_Z(s) Z_{V\Pi}}{s}
 \end{aligned} \tag{12}$$

One has to take into account, the angle dependent double-vector coupling extra correction, which breaks structure of the couplings into ones associated with Z boson production and decay:

$$\begin{aligned}
 vv_{ef} = & \frac{1}{v_e \cdot v_f} [(2 \cdot T_3^e)(2 \cdot T_3^f) - 4 \cdot q_e \cdot s_W^2 \cdot K_f(s, t) - 4 \cdot q_f \cdot s_W^2 \cdot K_e(s, t) \\
 & + (4 \cdot q_e \cdot s_W^2)(4 \cdot q_f \cdot s_W^2) K_{ef}(s, t)] \frac{1}{\Delta^2}
 \end{aligned} \tag{13}$$

further terms are straightforward:

$$\begin{aligned}
v_e &= (2 \cdot T_3^e - 4 \cdot q_e \cdot s_W^2 \cdot K_e(s, t)) / \Delta \\
v_f &= (2 \cdot T_3^f - 4 \cdot q_f \cdot s_W^2 \cdot K_f(s, t)) / \Delta \\
a_e &= (2 \cdot T_3^e) / \Delta \\
a_f &= (2 \cdot T_3^f) / \Delta
\end{aligned} \tag{14}$$

The form-factors $K_e(s, t)$, $K_f(s, t)$ are functions of two Mandelstam invariants (s, t) due to the WW and ZZ box contributions.

Vacuum polarisation corrections $\Gamma_{V\Pi}$ to γ propagator are expressed as:

$$\Gamma_{V\Pi} = \frac{1}{2 - (1 + \Pi_{\gamma\gamma})} \tag{15}$$

Normalisation correction $Z_{V\Pi}$ to Z-boson propagator is expressed as

$$Z_{V\Pi} = \rho_{e,f}(s, t) \tag{16}$$

From D. Bardin *Comput.Phys.Commun.* 133 (2001) 229

$$\boxed{\rho_{ef}} = 1 + \frac{g^2}{16\pi^2} \left\{ -\Delta\rho_Z^F + \mathcal{D}_Z^F(s) + \frac{5}{3}B_0^F(-s; M_W, M_W) - \frac{9}{4} \frac{c_w^2}{s_w^2} \ln c_w^2 - 6 \right. \\ \left. + \frac{5}{8}c_w^2(1+c_w^2) + \frac{1}{4c_w^2}(3v_e^2+a_e^2+3v_f^2+a_f^2)\mathcal{F}_Z(s) + \mathcal{F}_W^0(s) + \mathcal{F}_W(s) \right. \\ \left. - \frac{\tau_1}{4}[B_0^F(-s; M_W, M_W) + 1] - c_w^2(R_Z - 1)s\mathcal{B}_{WW}^d(s,t) \right\}, \quad (\text{A.4.80})$$

$$\boxed{\kappa_e} = 1 + \frac{g^2}{16\pi^2} \left\{ -\frac{c_w^2}{s_w^2}\Delta\rho^F - \Pi_{Z\gamma}^F(s) - \frac{1}{6}B_0^F(-s; M_W, M_W) - \frac{1}{9} - \frac{v_e\sigma_e}{2c_w^2}\mathcal{F}_Z(s) \right. \\ \left. - \mathcal{F}_W^0(s) + (R_Z - 1) \left[\frac{|Q_f|}{2}(1-4|Q_f|s_w^2)\mathcal{F}_Z(s) + c_w^2[\mathcal{F}_{W_n}^0(s) \right. \right. \\ \left. \left. - |Q_f|\mathcal{F}_{W_n}(s) + s\mathcal{B}_{WW}^d(s,t)] \right] \right\}, \quad (\text{A.4.81})$$

$$\boxed{\kappa_f} = 1 + \frac{g^2}{16\pi^2} \left\{ -\frac{c_w^2}{s_w^2}\Delta\rho^F - \Pi_{Z\gamma}^F(s) - \frac{1}{6}B_0^F(-s; M_W, M_W) - \frac{1}{9} - \frac{v_f\sigma_f}{2c_w^2}\mathcal{F}_Z(s) \right. \\ \left. - \mathcal{F}_W^0(s) + (R_Z - 1) \left[\frac{|Q_e|}{2}(1-4|Q_e|s_w^2)\mathcal{F}_Z(s) + c_w^2[\mathcal{F}_{W_n}^0(s) \right. \right. \\ \left. \left. - |Q_e|\mathcal{F}_{W_n}(s) + s\mathcal{B}_{WW}^d(s,t)] \right] - \frac{\tau_1}{4}[B_0^F(-s; M_W, M_W) + 1] \right\}, \quad (\text{A.4.82})$$

interference

$$\boxed{\kappa_{ef}} = 1 + \frac{g^2}{16\pi^2} \left\{ -2\frac{c_w^2}{s_w^2}\Delta\rho^F - 2\Pi_{Z\gamma}^F(s) - \frac{1}{3}B_0^F(-s; M_W, M_W) - \frac{2}{9} \right. \\ \left. - \frac{1}{4c_w^2} \left[\frac{\delta_e^2 + \delta_f^2}{s_w^2}(R_W - 1) + 3v_e^2 + a_e^2 + 3v_f^2 + a_f^2 \right] \mathcal{F}_Z(s) \right. \\ \left. - \mathcal{F}_W^0(s) - \mathcal{F}_W(s) - \frac{\tau_1}{4}[B_0^F(-s; M_W, M_W) + 1] \right. \\ \left. + c_w^2(R_Z - 1) \left[\frac{2}{3} - \hat{\Pi}_{\gamma\gamma}^{\text{bos},F}(s) + s\mathcal{B}_{WW}^d(s,t) \right] \right\}. \quad (\text{A.4.83})$$

Fermionic loops in γ propagator

BOX

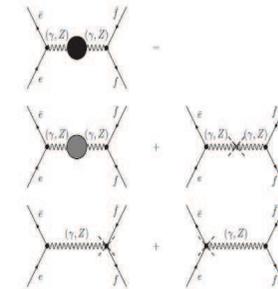


Figure A.11. Bosonic self-energies and bosonic counter-terms for $ee \rightarrow (Z, \gamma) \rightarrow ff$

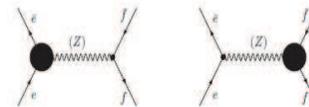


Figure A.10. Electron (a) and final fermion (b) vertices in $ee \rightarrow (Z) \rightarrow ff$

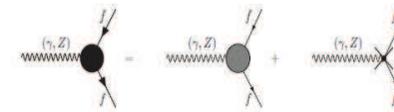


Figure A.6. Off-shell Zff and γff vertices



Figure A.7. The WW boxes

etc. etc.

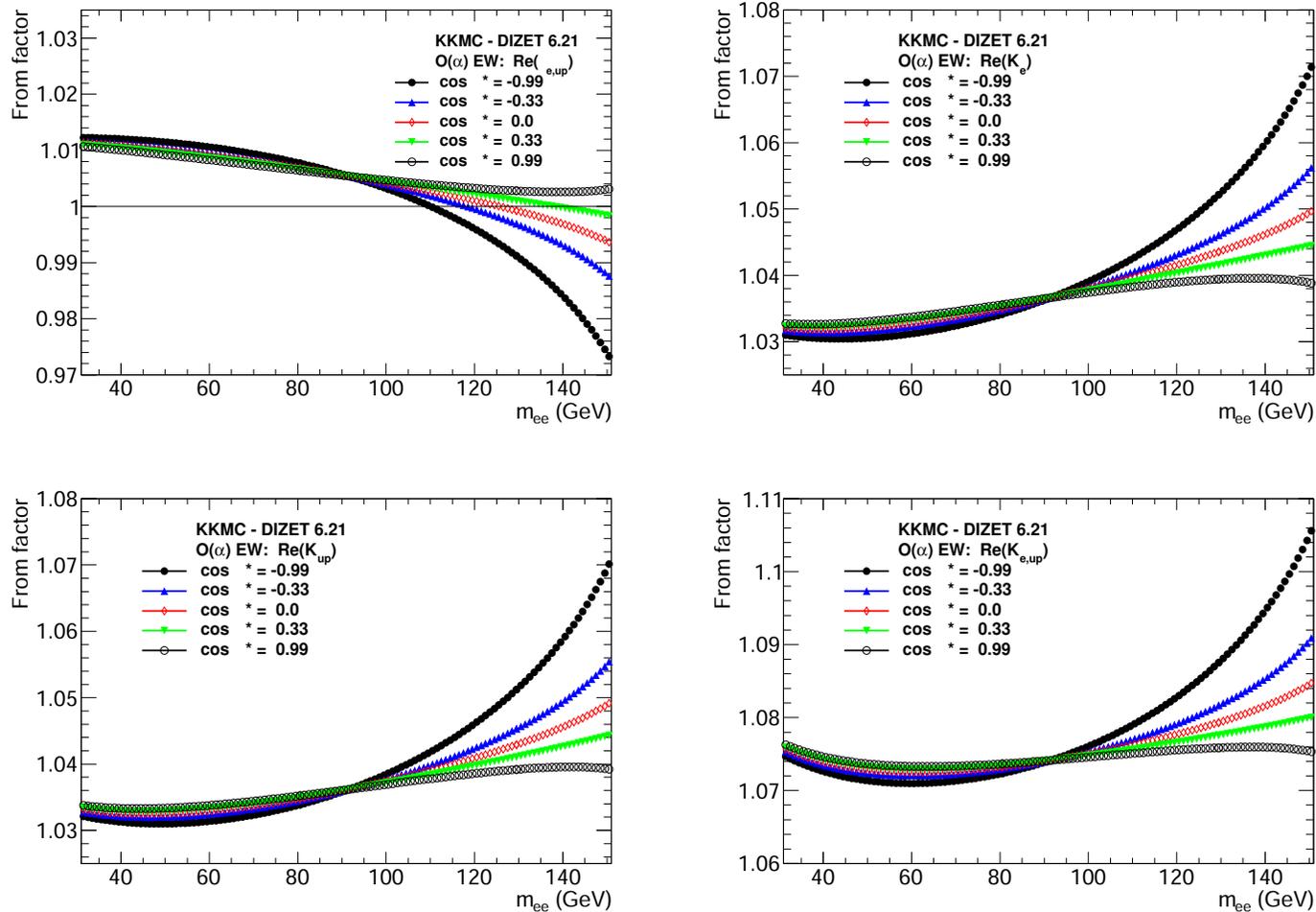


Figure 7: Real part of $\rho_{e,up}$, K_e , K_{up} and $K_{e,up}$ EW form factors as a function of m_{ee} for few values of $\cos \theta^*$ and u-type quark flavour. Note that close to the Z peak angular dependence is minimal. For lower virtualities photon exchange dominates. Electroweak effects do not damage picture of spherical harmonics.

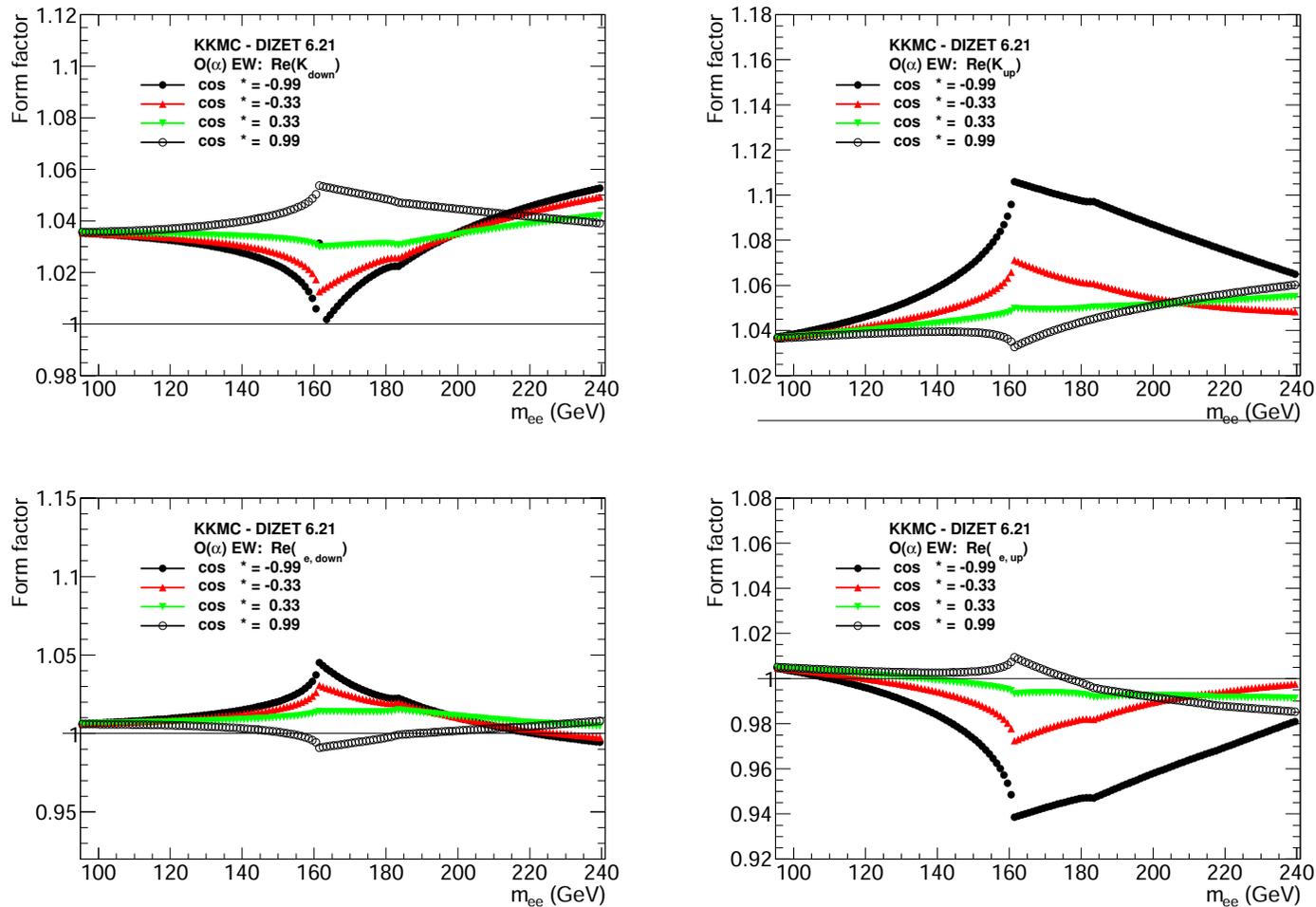
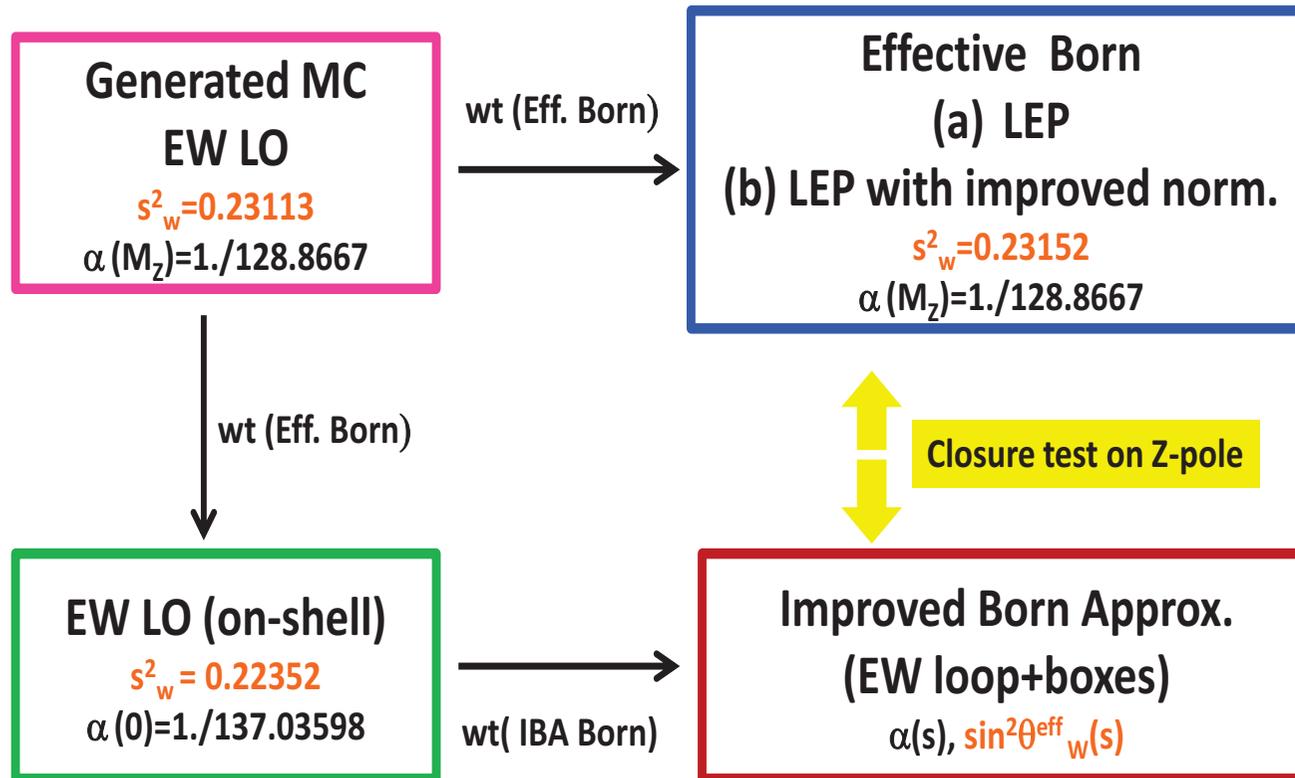


Figure 8: Real part of K_{down} , K_{up} , $\rho_{e,down}$ and $\rho_{e,up}$ as a function of m_{ee} for few values of $\cos \theta^*$. Note the WW and ZZ threshold effects which exhibits as discontinuity. Electroweak effects could complicate picture of spherical harmonics at virtualities above WW threshold. They are important part of LHC-time reweighting. In the past they were important for KKMC.

Observations

- Formfactors break, but in numerically not significant manner, the lepton angular distributions, which are not anymore spherical harmonics of second order.
- This is a constraint for the re-weight algorithm if used at histogram level.
- We need to explore Mustraal frames for reweighting algorithms, which can then be used to install better genuine weak effects into ‘any’ MC sample, provided in its generation known (constant) couplings of Z bosons were used.
- ● EW loop+boxes means Dizet. Later slide of numerical results:



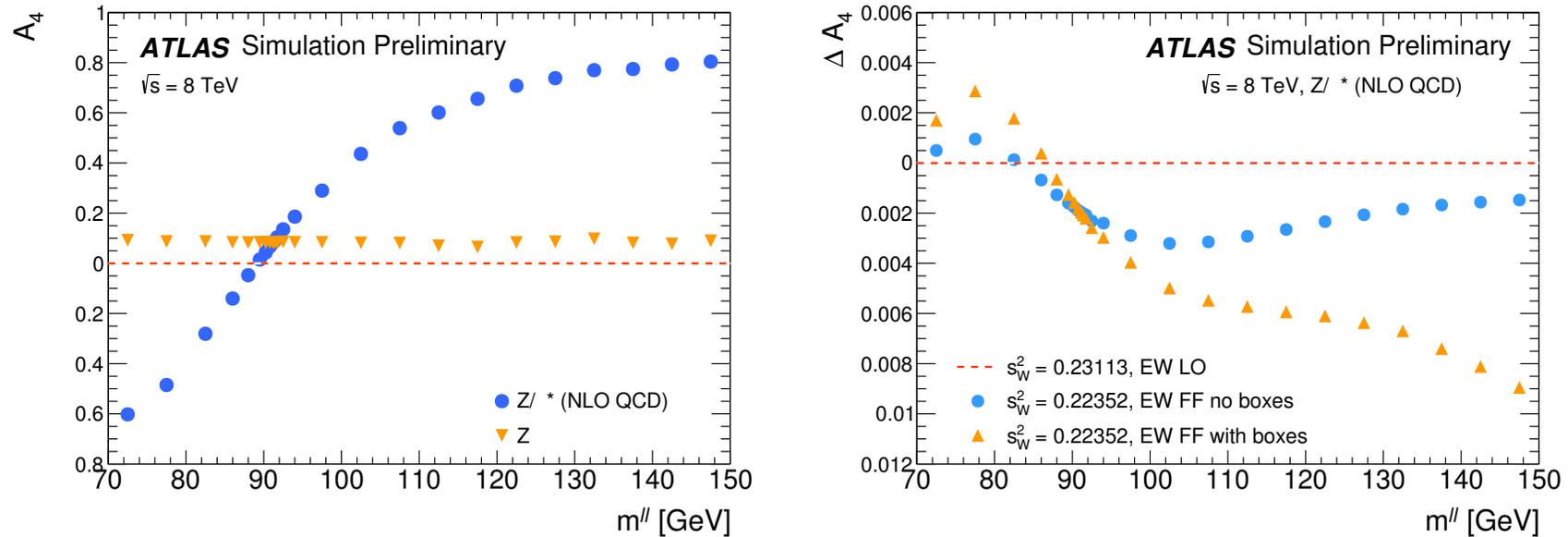


Figure 9: Predicted values at NLO in QCD for the angular coefficient A_4 as a function of m_{ll} , shown for full Z/γ^* production and for only pure Z-boson production (left), for which the small non-zero value of the angular coefficient displays the contribution from the weak mixing angle to the asymmetry. The difference ΔA_4 in the predictions is shown (right) where the reference is taken to be the LO EW predictions from the POWHEGBOX event generator with $\sin^2 \theta_W = 0.23113$. These predictions are compared to those obtained using the same generated sample, reweighted as explained in the text to predictions in the EW $\alpha(0)$ scheme including the EW form factor (FF) corrections. These IBA predictions are shown in the two cases where the calculations include or not the EW box diagrams which break the polynomial decomposition.

1. **Electroweak corrections, their separation into parts was essential for the phenomenology picture enabling verification that Standard Model is Quantum Field theory describing essentially all phenomena we observe.**
2. Matching Lagrangian approach, gauge invariance with the need of resummation of some corrections to higher orders was not easy.
3. Particular difficulty was due to dispersion relations used for low energy $\Pi_{\gamma\gamma}(s)$. This anti analytic constraint makes other techniques like analytic continuation to introduce Z or W width perilous.
4. Effort of separating complexity into parts and later integrating it again,
5. it serves now for LHC and hopefully will continue for FCC.
6. Dima Bardin heritage and team may carry that on to the next precision regime.
7. Hopefully Dima Bardin spirit will guide.



...of the day Dima Bardin left us...