Feynman integral evaluation at supercomputers

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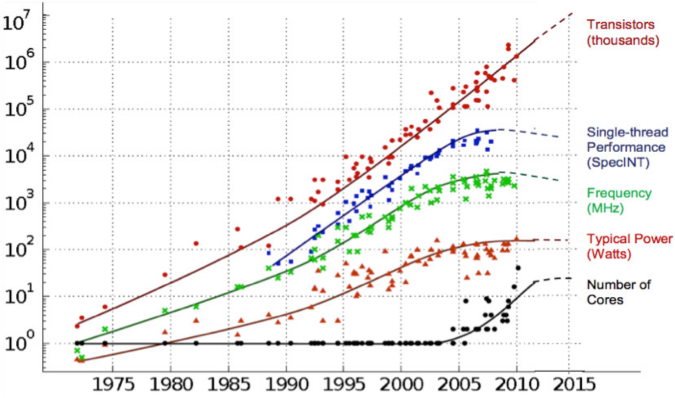
- More or less valid till 2005. And till 2010 counting CPU cores. What now?


10^7
10^6
10^5
10^4
10^3
10^2
10^1
10^0
1975 1980 1985 1990 1995 2000 2005 2010 2015
Transistors (thousands)
Single-thread Performance (SpecINT)
Frequency (MHz)
Typical Power (Watts)
Number of Cores

Feynman integral evaluation

- QCD massless form factors
- Two-loop results G. Kramer and B. Lampe'87 T. Matsuura and W. L. van Neerven'88 T. Matsuura, S. C. van der Marck, and W. L. van Neerven'89
- Three-loop results P. A. Baikov, K. G. Chetyrkin, A. V. Smirnov, V. A. Smirnov and M. Steinhauser'09, T. Gehrmann, E. W. N. Glover, T. Huber, N. Ikizlerli, and C. Studerus'10, R. N. Lee and V. A. Smirnov'10
- Four-loop results J. M. Henn, A. V. Smirnov, V. A. Smirnov and M. Steinhauser'16 J. Henn, A. V. Smirnov, V. A. Smirnov, M. Steinhauser and R. N. Lee'17 R. N. Lee, A. V. Smirnov, V. A. Smirnov and M. Steinhauser'17 A. von Manteuffel and R. M. Schabinger'17

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- Supercomputers

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- One needs a special code structure and special resource for parallelization.

Feynman integrals

- Feynman integrals over loop momenta:

$$\mathcal{F}(a_1,\ldots,a_n) = \int\cdots\int \frac{\mathrm{d}^d k_1 \ldots \mathrm{d}^d k_h}{E_1^{a_1}\ldots E_n^{a_n}}.$$

- Currently one needs to evaluate millions of Feynman integrals with diffeent indices a_i coresponding to a particular diagram, so evaluating each of them analytically turns into an unreal task.

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- evaluation of master integrals.

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- 1) The number of master integrals to be evaluated
- 2) The number of sectors in the sector-decomposition approach.

α-representation

Feynman parametric representation:

$$\mathcal{F}(a_1 \ldots, a_L; d) = \frac{\mathrm{i}^{a+h(1-d/2)} \pi^{hd/2}}{\prod_l \Gamma(a_l)}$$
$$\times \int_0^\infty \ldots \int_0^\infty \prod_l \alpha_l^{a_l - 1} U^{-d/2} e^{\mathrm{i}F/U - \mathrm{i}\sum m_l^2 \alpha_l} \mathrm{d}\alpha_1 \ldots \mathrm{d}\alpha_L \, .$$

where U и F are polynomials α that can be algorithmically determined by the initial diagram.

Sector decomposition

$$\int_0^1 \int_0^1 \frac{1}{(x+y)^{2-\varepsilon}} dydx$$

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$$2 \int_0^1 \int_0^1 \frac{x}{(x+xz)^{2-\varepsilon}} dzdx = 2 \int_0^1 \int_0^1 x^{-1+\varepsilon} \frac{1}{(1+z)^{2-\varepsilon}} dzdx$$

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- `SDEvaluate[{U,F,l},indices,order]`

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- *Example*:
  SDEvaluate[UF[{k},{-k^2,-$(k+p_1)^2$,-$(k+p_1+p_2)^2$,
  -$(k+p_1+p_2+p_4)^2$}, {$p_1^2 \to 0$,$p_2^2 \to 0$,$p_4^2 \to 0$,
  p_1 p_2 $\to$-S/2,p_2 p_4 $\to$-T/2,p_1 p_4 $\to$(S+T)/2,
  S$\to$3,T$\to$ 1}], {1,1,1,1},0]

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- *Example*:
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- *Answer*: -4.38658 + 1.3333/ep^2 - 0.732466/ep + 0.001 pm9

Classical usage of FIESTA

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- Integration performed by a c++ program (called from Mathematica);
- Mathematica gathers results from the database.

Use NumberOfSubkernels and NumberOfLinks to turn on internal parallelization (by Mathematica and by threads for the c++ part);

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- Run the integrarion separately (this part does not require Mathematica!) with the use of MPI
- Analyze the results with Mathematica

Also use the GPU acceleration if GPU nodes are available at the cluster.

Some results obtained on supercomputers evaluating master integrals with FIESTA.

- Corrections to the muon anomalous magnetic moment at four-loop order A. Kurz, T. Liu, P. Marquard, A. V. Smirnov, V. A. Smirnov and M. Steinhauser'15
- Quark Mass Relations to Four-Loop Order in Perturbative QCD P. Marquard, A. V. Smirnov, V. A. Smirnov and M. Steinhauser'15 P. Marquard, A. V. Smirnov, V. A. Smirnov, M. Steinhauser and D. Wellmann'17

Multiple programs for Feynman integral reduction

- AIR
- FIRE
- Reduze
- LiteRed
- Kira
- different private implementations
- more public algorithms going to appear?

Parallel approach to reduction

Reduction is solving a huge sparse matrix with polynomial coefficients
Current diagrams need (A LOT OF RAM) and (A LOT OF TIME)!

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- Parallel reduction in sectors of same level
- Multiple fermat workers (GCD application)
- Prime field approach (Manteuffel, Panzer, Schabinger)
- Separate evaluation of coefficients at different masters (Chawdhry, Lim, Mitov)

Prime field approach

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- Take different large prime numbers, move from Z to Zp
- Run MANY reductions that are much more simple than the original one
- Reconstruct the coefficients

100 values of d, 100 values of x, 20 prime numbers -> 20000 reductions, each of those takes time and use threads -> fits for a super computer.

Rational reconstruction

- An integer is unequely reconstructed by enough of its projections to Zp
- When reconstructing a rational number, we look for smallest possible numerator and denominator
- A few extra prime numbers are for checks

Polynomial reconstruction

- Newton approach

$f(x) = c0 + (x - x_0)(c_1 + (x - x_1)(c_2 + \ldots) \ldots)$
coefficients c_i are algorithmically evaluated from the values $f(x_i)$.

Rational reconstruction

- Thiele approach

$f(x) = c0 + (x - x_0)/(c_1 + (x - x_1)/(c_2 + \ldots) \ldots)$
coefficients c_i are algorithmically evaluated from the values $f(x_i)$.

Rational reconstruction (multiple variables)

Combine two approaches (when coefficients are again functions)

- Newton-Newton (for polynomials)
- Newton-Thiele (when polynomial in one variable)
- Thiele-Newton (something in between)
- Thiele-Thiele (universal but too complex)

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- In this case first the x denominators are recovered for a given d.
- Then the results are multiplied by the worst denominator and Newton-Thiele is used.
- Can we have such a basis with proper coefficients? We beleive that YES!