Hard-photons in proton-antiproton annihilation to a lepton pair for PANDA experiment

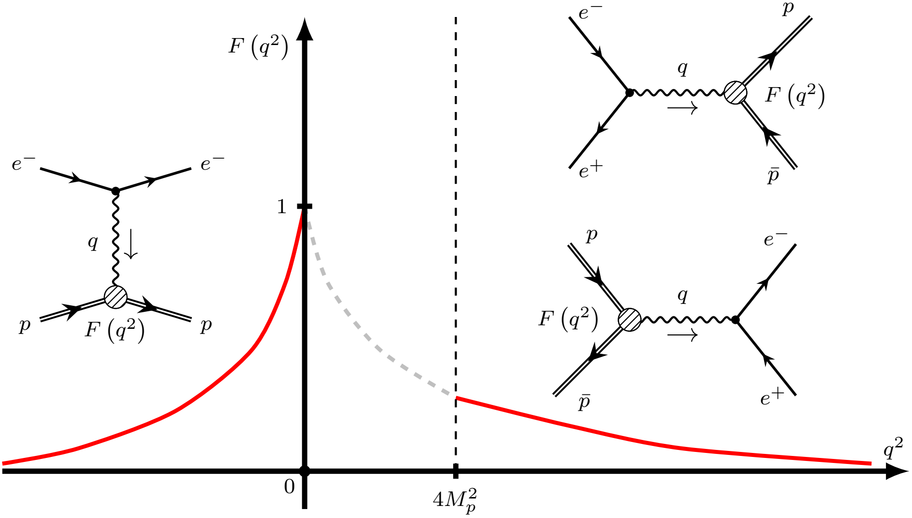
Yu.M. Bystritskiy[1]

In collaboration with Dr. V.A. Zykunov[1,2] and Mainz group: Prof. F. Maas[3,4],
Dr. E. Tomasi-Gustafsson[5], A. Dbeyssi[3,4], M. Zambrana[3,4]

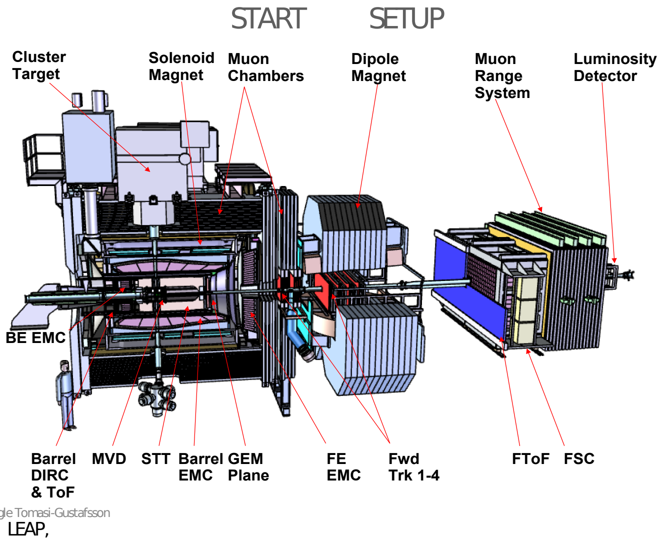
[1] Joint Institute for Nuclear Research, Dubna, Russia
[2] Gomel State University, Belarus
[3] Institut fur Kernphysik, Johannes Gutenberg Univeritat, Mainz, Germany
[4] Helmholtz-Institut Mainz, Germany
[5] CEA, IRFU, SPhN, Saclay, France

CALC-2018
31 July 2018

Proton form factors

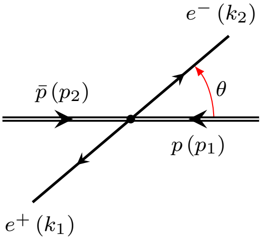


PANDA Setup

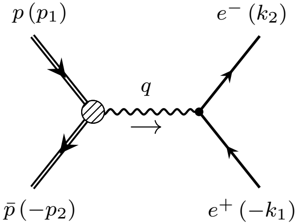


- Anti-proton beam with momentum $1.5 < \sqrt{s} < 15$ GeV/c.
- Target are the frozen Hydrogen microspheres (pellets), vertically traversing the accelerator beam, **or** beam of condensed gas clasters.

The process $\bar{p}p \to e^+e^-$ overview



$$
\begin{aligned}
s &= (p_1 + p_2)^2 = 2M_p\,(M_p + E)\,, \\
t &= (p_2 - k_2)^2 = \\
&= -\frac{s}{4}\left(1 + \beta^2 - 2\beta\cos\theta\right), \\
u &= (p_1 - k_2)^2 = \\
&= -\frac{s}{4}\left(1 + \beta^2 + 2\beta\cos\theta\right).
\end{aligned}
$$

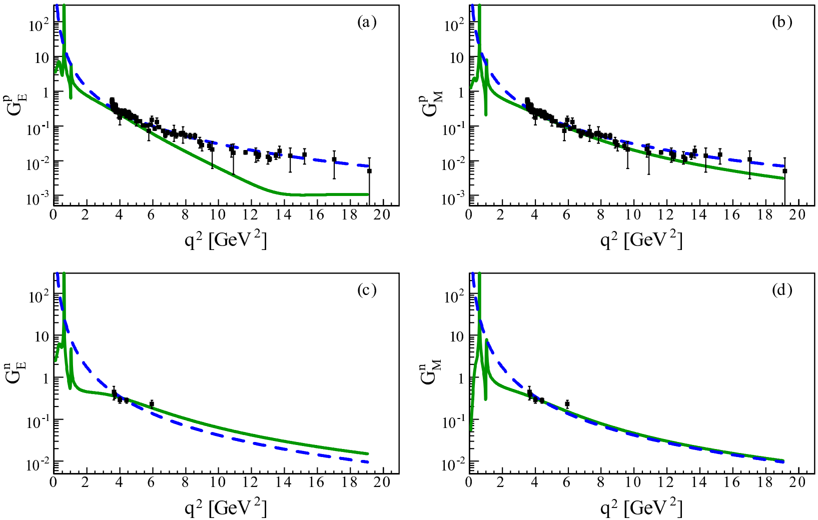


The process of antiproton-proton annihilation into lepton pair is parameterized in terms of two form factors – electric $G_E\left(q^2\right)$ and magnetic $G_M\left(q^2\right)$ – in the following form [A. Zichichi *et al.* Nuovo Cimento **XXIV**, 170 (1962)]:

$$
\frac{d\sigma_B}{d\cos\theta} = \frac{\pi\alpha^2}{2s\beta}\left\{|G_M|^2\left(1 + \cos^2\theta\right) + |G_E|^2\left(1 - \beta^2\right)\left(1 - \cos^2\theta\right)\right\}, \tag{1}
$$

where $\alpha = e^2/4\pi$ and $\beta = \sqrt{1 - \frac{4M_p^2}{s}}$ is the anti-proton velocity.

The process $\bar{p}p \to e^+e^-$ overview: FFs



- Perturbative QCD inspired [D.V. Shirkov, I.L. Solovtsov. Phys.Rev.Lett., **79**, 1209 (1997)].
- Vector meson dominance [F. Iachello, Q. Wan. Phys.Rev., **C69**, 055204 (2004)].
- Data from [B. Aubert et al. (BABAR Collaboration), Phys. Rev. **D73**, 012005 (2006)].

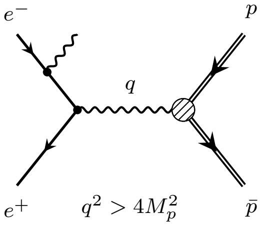
Comment on Radiative Return technic for PANDA

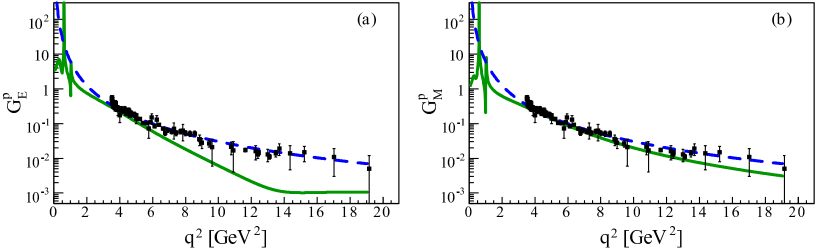
BABAR facility uses "Radiative Return" technic to scan the nucleon form factors in wide range of momentum transfer [B. Aubert et al. (BABAR Collaboration), Phys. Rev. **D73**, 012005 (2006)]:

$$\frac{d\sigma^{e^+e^-\to p\bar{p}\gamma}}{dq^2\, d\cos\theta} = \frac{2q^2}{s} W(s,x,\theta)\, \sigma^{e^+e^-\to p\bar{p}}\left(q^2\right),$$

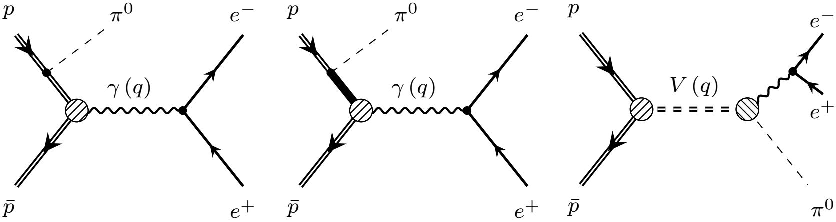
where $x = \frac{2E_\gamma}{\sqrt{s}} = 1 - \frac{q^2}{s}$ and radiative factor W has the following form:

$$W(s,x,\theta) = \frac{\alpha}{\pi x}\left(\frac{2-2x+x^2}{\sin^2\theta} - \frac{x^2}{2}\right), \qquad \theta \gg \frac{m_e}{\sqrt{s}}.$$





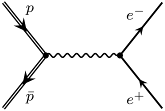
In papers [M.P. Rekalo. Sov. J. Nucl. Phys. **1**, 760 (1965)] and [E. Tomasi-Gustafsson and M.P. Rekalo, arXiv:0810.4245 [hep-ph]] the process $\bar{p}p \to \pi^0\gamma^* \to \pi^0\left(e^+e^-\right)$ was proposed as strong interaction version of previous technic which allows to measure form factors in the range $4m_e^2 < q^2 < 4M_p^2$:



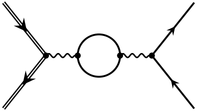
We discussed this possibility in details in [C. Adamuscin, E.A. Kuraev, E. Tomasi-Gustafsson, F.E. Maas. Phys.Rev. **C75**, 045205 (2007)] and [E.A. Kuraev, Yu.M. Bystritskiy, V.V. Bytev, E. Tomasi-Gustafsson, A. Dbeyssi, J.Exp.Theor.Phys. **115**, 93 (2012)].

Radiative corrections

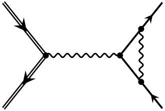
At next to leading orders (NLO) more diagrams contribute to the amplitude:



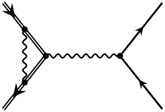
Born $\mathcal{M}_B \sim e^2$

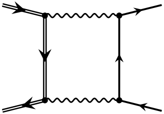


Vacuum polarization

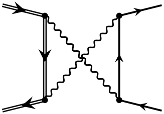


Vertex corrections

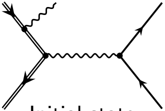




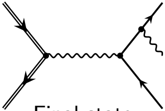
Box corrections



Virtual $\mathcal{M}_V \sim e^4$



Initial state radiation (ISR)



Final state radiation (FSR)

Real photon emission $\mathcal{M}_\gamma \sim e^3$

Radiative corrections

So at NLO we have two different (i.e. non-interfering) final states:

- e^+e^-: Born ($\mathcal{M}_B$) and virtual corrections ($\mathcal{M}_V = \mathcal{M}_{vp} + \mathcal{M}_{ver} + \mathcal{M}_{box}$)
- $e^+e^-\gamma$: Real photon emission ($\mathcal{M}_\gamma = \mathcal{M}_{\text{ISR}} + \mathcal{M}_{\text{FSR}}$)

And thus cross section with radiative correction has the following structure:

$$d\sigma \sim |\mathcal{M}_B + \mathcal{M}_V|^2 + |\mathcal{M}_\gamma|^2 = \underbrace{|\mathcal{M}_B|^2}_{\alpha^2} + \underbrace{2\,\text{Re}\left(\mathcal{M}_B \mathcal{M}_V^*\right)}_{\alpha^3} + \underbrace{|\mathcal{M}_V|^2}_{\alpha^4} + \underbrace{|\mathcal{M}_\gamma|^2}_{\alpha^3}. \tag{2}$$

If we leave contributions of order $O\left(\alpha^3\right)$ then:

$$\begin{aligned} d\sigma &\sim |\mathcal{M}_B|^2 + 2\,\text{Re}\left(\mathcal{M}_B \mathcal{M}_V^*\right) + |\mathcal{M}_\gamma|^2 = \\ &= |\mathcal{M}_B|^2 \left(1 + \frac{2\,\text{Re}\left(\mathcal{M}_B \mathcal{M}_V^*\right)}{|\mathcal{M}_B|^2} + \frac{|\mathcal{M}_\gamma|^2}{|\mathcal{M}_B|^2}\right) = |\mathcal{M}_B|^2 \left(1 + \delta_V + \delta_\gamma\right). \end{aligned} \tag{3}$$

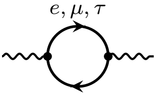
Therefore at $O\left(\alpha^3\right)$ level we write:

$$\boxed{d\sigma = d\sigma_B \left(1 + \delta_V + \delta_\gamma\right).} \tag{4}$$

Virtual corrections: Vacuum polarization

The vacuum polarization operator has few contributions:

$$\Pi(s) = \Pi_e(s) + \Pi_\mu(s) + \Pi_\tau(s) + \Pi_{\text{hadr}}(s),$$



where

$$\Pi_e(s) = \frac{\alpha}{3\pi}\left(L_e - \frac{5}{3}\right) - i\frac{\alpha}{3},$$

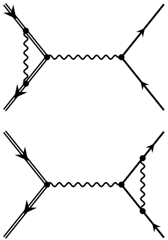
$$\Pi_\mu(s) = -\frac{\alpha}{\pi}\left(\frac{8}{9} - \frac{\beta_\mu^2}{3} - \frac{\beta_\mu}{2}\left(1 - \frac{\beta_\mu^2}{3}\right)L_\mu\right) - i\frac{\alpha}{2}\left(1 - \frac{\beta_\mu^2}{3}\right),$$

where $L_\mu = \ln\frac{1+\beta_\mu}{1-\beta_\mu}$, $\beta_\mu = \sqrt{1 - 4m_\mu^2/s}$. τ-lepton contribution can be found by substitution $\Pi_\tau(s) = \Pi_\mu(s)|_{\mu\to\tau}$.
Hadronic contribution were estimate using dispersion relations method

$$\Pi_{\text{hadr}}(s) = \Pi_{\pi^+\pi^-}(s) = \frac{2\alpha}{\pi}\left(\frac{1}{12}\ln\frac{1+\beta_\pi}{1-\beta_\pi} - \frac{2}{3} - 2\beta_\pi^2 - i\frac{\beta_\pi^3}{12}\right),$$

where $\beta_\pi = \sqrt{1 - 4m_\pi^2/s}$.

Virtual corrections generate contributions to the real parts of Dirac and Pauli proton form factors:



$$\text{Re}\, F_1^{(2)} = \left(\ln\frac{M_p}{\lambda} - 1\right)\left(1 - \frac{1+\beta^2}{2\beta}L_\beta\right) + \\ + \frac{1+\beta^2}{2\beta}\left(\frac{\pi^2}{3} + \text{Li}_2\left(\frac{1-\beta}{1+\beta}\right) - \frac{L_\beta^2}{4} - L_\beta \ln\frac{2\beta}{1+\beta}\right),$$

$$\text{Re}\, F_2^{(2)} = -\frac{1-\beta^2}{4\beta}L_\beta, \qquad \text{where} \quad L_\beta \equiv \ln\frac{1+\beta}{1-\beta}.$$

For the lepton vertex only the Dirac form factor gains radiative contribution:

$$\text{Re}\, F_e^{(2)} = \left(\ln\frac{m_e}{\lambda} - 1\right)(1 - L_e) - \frac{L_e^2}{4} - \frac{L_e}{4} + 2\zeta_2,$$

where $L_e = \ln\frac{s}{m_e^2}$.

Virtual corrections: Box corrections

The contribution of the box diagrams is described by the two virtual photons exchange mechanism. The interference of Born and box-type amplitudes contributes to the differential cross section as:

$$I(t,u,s) = (u-t)\left[\left(\frac{2M_p^2}{\beta^2}+t+u\right)I_{0qp} - \frac{\pi^2}{6} + \frac{1}{2}L_\beta^2 - \frac{1}{\beta^2}L_\beta\right] + (2t+s)\left[\frac{1}{2}L_{ts}^2 - \mathrm{Li}_2\left(\frac{-t}{M_p^2-t}\right)\right] - (2u+s)\left[\frac{1}{2}L_{us}^2 - \mathrm{Li}_2\left(\frac{-u}{M_p^2-u}\right)\right] + (ut - M_p^2(s+M_p^2))\left[\frac{1}{t}L_{ts} - \frac{1}{u}L_{us} + \frac{u-t}{ut}L_s\right] + I_0 L_{tu}(L_{M\lambda}+L_s), \quad (5)$$

with

$$I_{0qp} = \frac{1}{s\beta}\left(L_s L_\beta - \frac{1}{2}L_\beta^2 - \frac{\pi^2}{6} + 2\mathrm{Li}_2\left(\frac{1+\beta}{2}\right) - 2\mathrm{Li}_2\left(\frac{1-\beta}{2}\right) - 2\mathrm{Li}_2\left(\frac{\beta-1}{\beta+1}\right)\right)$$

and logarithms

$$L_{ts} = \ln\frac{M_p^2-t}{s},\; L_{us} = \ln\frac{M_p^2-u}{s},\; L_s = \ln\frac{s}{M_p^2},\; L_{tu} = \ln\frac{M_p^2-t}{M_p^2-u},\; L_{M\lambda} = \ln\frac{M_p^2}{\lambda^2}.$$

It is useful to note that the coefficient at $L_{M\lambda}$ in Eq. (5)

$$I_0 = \frac{2}{s}\left(t^2+u^2-4M_p^2(t+u)+6M_p^4\right) = s\left(2-\beta^2\sin^2\theta\right)$$

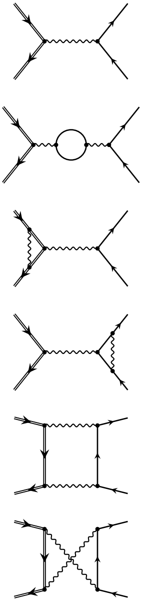
is proportional to the Born matrix element squared.

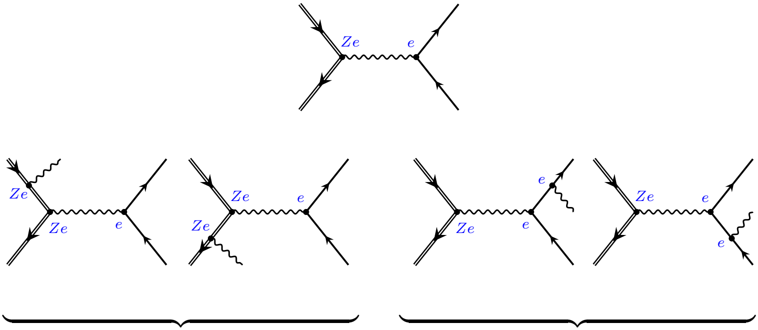
Collect virtual corrections together

Summing up the Born contribution (without FFs) and all virtual corrections we get the differential cross section in a form:

$$\frac{d\sigma_{B+V}}{d\cos\theta} = \frac{\alpha^2}{4s\beta}\left(2-\beta^2\sin^2\theta\right)\left|\frac{1}{1-\Pi(s)}\right|^2 + \\ + \frac{\alpha^3}{2\pi s\beta}\left\{\left[\left(2-\beta^2\sin^2\theta\right)\mathrm{Re}\left(F_e^{(2)}+F_1^{(2)}\right)+2\,\mathrm{Re}\,F_2^{(2)}\right]+ \\ + \frac{I(t,u,s)}{s}\right\}.$$

Here terms, marked by red color, contain infra-red divergence $\ln(\lambda)$.





Initial state radiation (ISR)　　Final state radiation (FSR)

$$\mathcal{M}_B \sim Ze^2 \qquad |\mathcal{M}_B|^2 \sim Z^2 e^4 \quad \text{charge even}$$

$$\mathcal{M}_{\rm ISR} \sim Z^2 e^3 \qquad |\mathcal{M}_{\rm ISR}|^2 \sim Z^4 e^6 \quad \text{charge even}$$

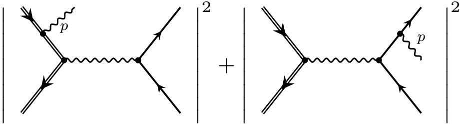
$$\mathcal{M}_{\rm FSR} \sim Ze^3 \qquad |\mathcal{M}_{\rm FSR}|^2 \sim Z^2 e^6 \quad \text{charge even}$$

$$2\,\mathrm{Re}\left(\mathcal{M}_{\rm ISR}\,\mathcal{M}_{\rm FSR}^{+}\right) \sim Z^3 e^6 \quad \text{charge odd}$$

Soft photon emission

The charge even part (sum of ISR and FSR) of the cross section of soft real photon emission was calculated in [A.I. Ahmadov, V.V. Bytev, E.A. Kuraev, and E. Tomasi-Gustafsson. Phys.Rev., D82:094016, 2010]:

$d\sigma^{\text{soft}}_{\text{even}} =$



(6)

in the standard way (the photon energy p_0 in the center-of-mass system is less then some small quantity ω, i.e. $0 < p_0 < \omega \ll E = \sqrt{s}/2$):

$$\frac{d\sigma^{\text{soft}}_{\text{even}}}{d\cos\theta} = \frac{\alpha}{\pi}\frac{d\sigma_B}{d\cos\theta}\left\{-2\left[\ln\frac{2\omega}{\lambda} - \frac{1}{2\beta}L_\beta\right] - 2\ln\frac{\omega m}{\lambda E}\right.$$
$$\left. + 2\frac{1+\beta^2}{2\beta}\left[\ln\frac{2\omega}{\lambda}L_\beta - \frac{1}{4}L_\beta^2 + \Phi(\beta)\right] + 2\left[\ln\frac{2\omega}{\lambda}L_e - \frac{1}{4}L_e^2 - \frac{\pi^2}{6}\right]\right\},$$

where function $\Phi(\beta)$ has the form:

$$\Phi(\beta) = \frac{\pi^2}{12} + L_\beta \ln\frac{1+\beta}{2\beta} + \ln\frac{2}{1+\beta}\ln(1-\beta) + \frac{1}{2}\ln^2(1+\beta) - \frac{1}{2}\ln^2 2 +$$
$$- \mathrm{Li}_2(\beta) + \mathrm{Li}_2(-\beta) - \mathrm{Li}_2\left(\frac{1-\beta}{2}\right), \qquad \Phi(1) = -\frac{\pi^2}{6}. \tag{7}$$

Soft photon emission

We revise charge even part and present the ISR- and FSR-parts separately using more symmetric form. We perform this calculation using formulae presented in [Frits A. Berends, K. J. F. Gaemer, and R. Gastmans. Nucl. Phys., B57:381–400, 1973] and we get for the soft photon emitted from proton line (ISR):

$$\frac{d\sigma_{\rm ISR}^{\rm soft}}{d\sigma_0} = \frac{\alpha}{\pi}\left\{\left(\frac{1+\beta^2}{\beta}L_\beta - 2\right)\ln\frac{2\omega}{\lambda} + \frac{1}{\beta}L_\beta + \frac{1+\beta^2}{2\beta}\left(\mathrm{Li}_2\left(\frac{2\beta}{\beta-1}\right) - \mathrm{Li}_2\left(\frac{2\beta}{\beta+1}\right)\right)\right\}.$$

Let us note that this contribution was also presented in Eq. (36) of [Jacques Van de Wiele and Saro Ong. Eur. Phys. J., A49:18, 2013], but unfortunately the term $\frac{1}{\beta}\ln\frac{1+\beta}{1-\beta}$ there has been written in ultrarelativistic form: $\ln\left(s/M_p^2\right)$. This approximation is only valid when $s \gg M_p^2$, which is not the case for PANDA rather small energies $(1.5 \le \sqrt{s} \le 15$ GeV$)$. However numerical difference due to this approximation is rather small.
As for the FSR-part

$$d\sigma_{\rm FSR}^{\rm soft} = d\sigma_{\rm ISR}^{\rm soft}\Big|_{p\to e}, \qquad (8)$$

there is good agreement between [Van de Wiele, Ong:2012] and Eq. (29) in [Berends,Gaemer,Gastmans:1974], because $\frac{1}{\beta_e}\ln\frac{1+\beta_e}{1-\beta_e} \approx \ln\frac{s}{m_e^2}$ holds for PANDA energies with very good accuracy.

Soft photon emission

As for the interference term (ISR-FSR interference, charge odd part of the cross section) of soft photon emission which written in (Eqs. (37) and (38)) of [Van de Wiele, Ong:2012] its form is not symmetric in proton and electron masses:

$$
\begin{aligned}
I^{ep}_{\text{soft}} = 4\pi \Bigg\{ & 2 \ln\left(\frac{M_p^2 - t}{M_p^2 - u}\right) \ln\frac{2\omega}{\lambda} \\
& + \mathrm{Li}_2\left(1 + \frac{(1+\beta)st}{2\,M_p^4}\right) - \mathrm{Li}_2\left(1 + \frac{(1+\beta)su}{2\,M_p^4}\right) \\
& + \mathrm{Li}_2\left(1 + \frac{st}{(M_p^2 - t)^2}\right) - \mathrm{Li}_2\left(1 + \frac{su}{(M_p^2 - u)^2}\right) \\
& + \mathrm{Li}_2\left(1 + \frac{(1-\beta)st}{2M_p^4}\right) - \mathrm{Li}_2\left(1 + \frac{(1-\beta)su}{2M_p^4}\right) \Bigg\}.
\end{aligned}
\tag{9}
$$

Soft photon emission

Again using technics elaborated in [Frits A. Berends, K. J. F. Gaemers, and R. Gastmans. Nucl. Phys., B63: 381–397, 1973] we can rewrite this interference contribution in symmetric (both under the particle masses and under the Mandelstam invariants) form:

$$\frac{d\sigma_{\text{odd}}^{\text{soft}}}{d\sigma_0} = -\frac{\alpha}{2\pi^2}\left(\left(m_e^2 + M_p^2 - t\right) R(s,t) - \left(m_e^2 + M_p^2 - u\right) R(s,u)\right),$$

where function R is presented in (A.11) from [Berends,Gaemer,Gastmans:1973]:

$$R(s,t) = 2\pi\left(2A(s,t)\ln\frac{2\omega}{\lambda} + C(s,t)\right),$$

and

$$A(s,t) = \frac{1}{\sqrt{\lambda(t,m_e^2,M_p^2)}}\ln\left|\frac{t - m_e^2 - M_p^2 - \sqrt{\lambda(t,m_e^2,M_p^2)}}{t - m_e^2 - M_p^2 + \sqrt{\lambda(t,m_e^2,M_p^2)}}\right|,$$

$$C(s,t) = \frac{1}{\sqrt{\lambda(t,m_e^2,M_p^2)}}\sum_{i,j=1}^{4}\epsilon_i\delta_j U_{ij}(\eta_0,\eta_1,y_i,y_j),$$

where $\lambda(x,y,z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$ and

$$\eta_0 = \sqrt{1 - m_e^2/E^2}, \qquad \eta_1 = \sqrt{1 - M_p^2/E^2} + \sqrt{-t}/E,$$

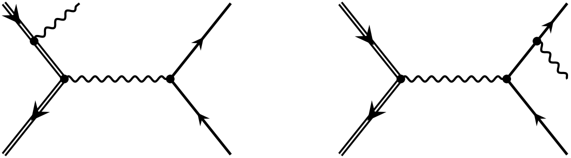
$$y_i = \delta_i - \frac{t + m_e^2 - M_p^2 + \epsilon_i\delta_i\sqrt{\lambda(t,m_e^2,M_p^2)}}{2E\sqrt{-t}}.$$

Infra-red divergence cancellation

Let us demonstrate infra-red divergency cancellation at first order radiative corrections (RC) at $s = 5.4$ GeV^2, for different photon emission angles θ and for $\omega/E = 1\%$. The table shows the stability of the calculations on the fictious photon mass λ. The calculation is very stable for any value of λ on 6 orders of magnitude. However, when it is too large ($\lambda > 10^{-4}\sqrt{s}$) the soft photon contribution can even becomes negative, which is the signature of entering the unphysical region.

θ (degrees)	$\lambda/\sqrt{s}$	σ_0 (pb)	RC(virtual)	RC(soft)	RC(total)
30	10^{-6}	19562.9	-0.780721	0.489790	-0.290931
	10^{-5}	19562.9	-0.581244	0.290310	-0.290931
	10^{-4}	19562.9	-0.381768	0.090837	-0.290931
	10^{-3}	19562.9	-0.182291	-0.108639	-0.290931
60	10^{-6}	17784.1	-0.717256	0.450381	-0.266875
	10^{-5}	17784.1	-0.528899	0.262024	-0.266875
	10^{-4}	17784.1	-0.340542	0.073667	-0.266875
	10^{-3}	17784.1	-0.152185	-0.114690	-0.266875
90	10^{-6}	16894.7	-0.642484	0.403593	-0.23889
	10^{-5}	16894.7	-0.467131	0.228241	-0.23889
	10^{-4}	16894.7	-0.291779	0.052889	-0.23889
	10^{-3}	16894.7	-0.116427	-0.122463	-0.23889
120	10^{-6}	17784.1	-0.567501	0.356806	-0.210695
	10^{-5}	17784.1	-0.405153	0.194458	-0.210695
	10^{-4}	17784.1	-0.242806	0.032111	-0.210695
	10^{-3}	17784.1	-0.080458	-0.130237	-0.210695

Hard photon emission

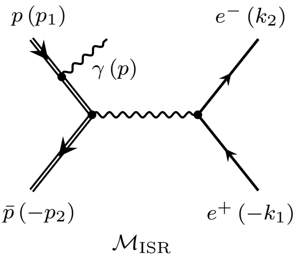


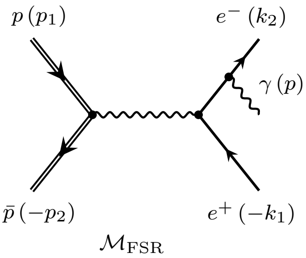
In [A.I. Ahmadov, V.V. Bytev, E.A. Kuraev, and E. Tomasi-Gustafsson. Phys.Rev., D82:094016, 2010] the calculation of hard photon emission was done in in the assumption that proton is point-like particle. This calculation is exact for the reaction $e^+e^- \to \mu^+\mu^-$ [E.A. Kuraev and G.V. Meledin, Nucl. Phys. B 122 (1977) 485]. In [Van de Wiele, Ong:2012] the proton mass is taken into account. The proton structure is taken into account in terms of dipole form factors:

$$F_{1,2}\left(q^2\right) \sim \frac{1}{\left(q^2 - M_0^2\right)^2}, \qquad M_0^2 \approx 0.71\ \text{GeV}^2. \tag{10}$$

It has been checked that different form factor parametrizations do not affect the results.

Hard photon emission





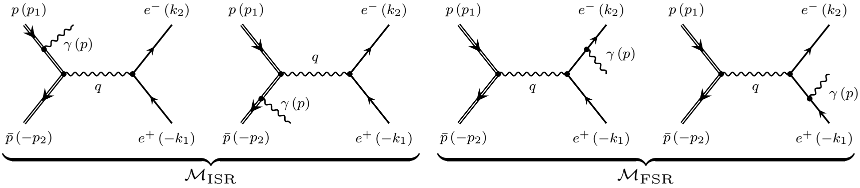
The differential hard photon cross section has the form:

$$d\sigma_\gamma = \frac{\alpha^3}{\pi^2 s\beta}\int\left(\mathcal{M}_{\rm ISR}+\mathcal{M}_{\rm FSR}\right)\left(\mathcal{M}_{\rm ISR}+\mathcal{M}_{\rm FSR}\right)^+\cdot\theta(p_0-\omega)\cdot\theta_P\cdot d\Phi_3,$$

where $\theta(p_0-\omega)$ cuts the hard region ($p_0>\omega$), θ_P is the factor which takes into account specific features of experimental setup (i.e. setup acceptance, efficiency of the detector components etc.) and

$$d\Phi_3 = \delta\left(p_1+p_2-k_1-k_2-p\right)\frac{d^3k_1}{2k_{10}}\frac{d^3k_2}{2k_{20}}\frac{d^3p}{2p_0}, \qquad (11)$$

is the full phase space of the reaction.



Using Feynman rules we write out $\mathcal{M}_{\rm ISR}$ and $\mathcal{M}_{\rm FSR}$ amplitudes in the following way:

$$\mathcal{M}_{\rm ISR} = \frac{ie_\nu(p)Q_p^2Q_e}{(k_1+k_2)^2}\left[\bar{u}(k_2)\gamma_\mu v(k_1)\right]\times$$
$$\times\left[\bar{v}\left(p_2\right)\left(\Gamma^\mu(q^2)\frac{\hat{p}_1-\hat{p}+M_p}{-2\left(p_1p\right)}\Gamma^\nu(p^2)+\Gamma^\nu(p^2)\frac{-\hat{p}_2-\hat{p}+M_p}{2\left(p_2p\right)}\Gamma^\mu(q^2)\right)u(p_1)\right],$$

$$\mathcal{M}_{\rm FSR} = \frac{ie_\nu(p)Q_pQ_e^2}{(p_1+p_2)^2}\left[\bar{u}\left(k_2\right)\left(\gamma^\nu\frac{\hat{k}_2+\hat{p}+m_e}{2\left(k_2p\right)}\gamma^\mu+\gamma^\mu\frac{-\hat{k}_1-\hat{p}+m_e}{2\left(k_1p\right)}\gamma^\nu\right)v(k_1)\right]\times$$
$$\times\left[\bar{v}(p_2)\Gamma_\mu(q^2)u(p_1)\right],$$

where $Q_e=-1$ and $Q_p=+1$ is electric charges of electron and proton in proton charge units $e=\sqrt{4\pi\alpha}$, here q is momentum transfer after emission of the photon $q=p_1+p_2-p$, and $\Gamma^\mu(q^2)$ is the proton vertex:

$$\Gamma^\mu(q^2) = F_1(q^2)\gamma^\mu+\frac{F_2(q^2)}{4M_p}\left(\gamma^\mu\hat{q}-\hat{q}\gamma^\mu\right). \tag{12}$$

Squaring the amplitude we get:

$$(\mathcal{M}_{\rm ISR} + \mathcal{M}_{\rm FSR})\,(\mathcal{M}_{\rm ISR} + \mathcal{M}_{\rm FSR})^{+} = R_{\rm ISR} + R_{\rm FSR} + R_{\rm INT} = \sum_{k} R_k,$$

where

$$R_{\rm ISR} = -\frac{Q_p^4 Q_e^2}{(k_1+k_2)^4}\,\mathsf{Sp}\left[\gamma_\mu(\hat{k}_1 - m_e)\gamma_\nu(\hat{k}_2 + m_e)\right]\times$$
$$\times\,\mathsf{Sp}\left[\left\{\Gamma^\mu(q^2)\frac{\hat{p}_1 - \hat{p} + M_p}{-2\,(p_1 p)}\Gamma^\alpha(p^2) + \Gamma^\alpha(p^2)\frac{-\hat{p}_2 - \hat{p} + M_p}{2\,(p_2 p)}\Gamma^\mu(q^2)\right\}(\hat{p}_1 - M_p)\times\right.$$
$$\left.\times\,\left\{\Gamma_\alpha(p^2)\frac{\hat{p}_1 - \hat{p} + M_p}{-2\,(p_1 p)}\Gamma^\nu(q^2) + \Gamma^\nu(q^2)\frac{-\hat{p}_2 - \hat{p} + M_p}{2\,(p_2 p)}\Gamma_\alpha(p^2)\right\}(\hat{p}_2 + M_p)\right],$$

$$R_{\rm FSR} = -\frac{Q_p^2 Q_e^4}{(p_1+p_2)^4}\,\mathsf{Sp}\left[\Gamma_\mu\left(q^2\right)(\hat{p}_1 - M_p)\Gamma_\nu\left(q^2\right)(\hat{p}_2 + M_p)\right]\times$$
$$\times\,\mathsf{Sp}\left[\left\{\gamma^\mu\frac{\hat{k}_2 + \hat{p} + m_e}{2\,(k_2 p)}\gamma^\alpha + \gamma^\alpha\frac{-\hat{k}_1 - \hat{p} + m_e}{2\,(k_1 p)}\gamma^\mu\right\}\left(\hat{k}_1 - m_e\right)\times\right.$$
$$\left.\times\,\left\{\gamma^\alpha\frac{\hat{k}_2 + \hat{p} + m_e}{2\,(k_2 p)}\gamma^\mu + \gamma^\mu\frac{-\hat{k}_1 - \hat{p} + m_e}{2\,(k_1 p)}\gamma^\alpha\right\}\left(\hat{k}_2 + m_e\right)\right],$$

And the interference of ISR and FSR gives:

$$R_{\rm ISR} = -2\frac{Q_p^3 Q_e^3}{(k_1+k_2)^2(p_1+p_2)^2}\times$$
$$\times\,\mathsf{Sp}\left[\left\{\Gamma^\mu(q^2)\frac{\hat{p}_1-\hat{p}+M_p}{-2\,(p_1 p)}\Gamma^\alpha(p^2)+\Gamma^\alpha(p^2)\frac{-\hat{p}_2-\hat{p}+M_p}{2\,(p_2 p)}\Gamma^\mu(q^2)\right\}\times\right.$$
$$\left.\times\,(\hat{p}_1-M_p)\,\Gamma_\nu\left(q^2\right)(\hat{p}_2+M_p)\right]\times$$
$$\times\,\mathsf{Sp}\left[\gamma_\mu\left(\hat{k}_1-m_e\right)\left\{\gamma^\alpha\frac{\hat{k}_2+\hat{p}+m_e}{2\,(k_2 p)}\gamma^\mu+\gamma^\mu\frac{-\hat{k}_1-\hat{p}+m_e}{2\,(k_1 p)}\gamma^\alpha\right\}\left(\hat{k}_2+m_e\right)\right].$$

Hard photon emission: Phase volume

We will formulate phase volume of the final state of the reaction keeping infinitesimal photon fictitious mass λ (i.e., for example, photon momentum modulus $|\vec{p}| = \sqrt{p_0^2 - \lambda^2}$):

$$\frac{d\sigma_\gamma}{d\cos\theta} = \frac{\alpha^3}{4\pi s}\int\limits_{\lambda,\omega}^{\Omega} dp_0|\vec{p}|\int\limits_0^{\pi} d\theta_p \sin\theta_p \int\limits_0^{2\pi} d\varphi_p \frac{|\vec{k_2}|}{k_{10}g(k_{20})}\sum_k R_k\theta_P, \qquad (13)$$

where Ω is the maximal energy of bremsstrahlung photon and

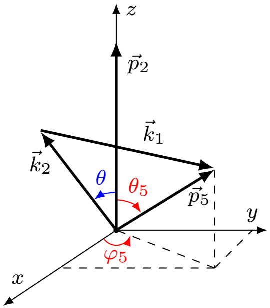
$$g(x) = 1 + \frac{x(1-|\vec{p}|A(x^2-m_e^2)^{-1/2})}{\sqrt{x^2 - 2|\vec{p}|A\sqrt{x^2-m_e^2} + |\vec{p}|^2}},$$

$$A = \cos(\widehat{\vec{p_5},\vec{k_2}}) = \sin\theta\sin\theta_5\cos\varphi_5 + \cos\theta\cos\theta_5,$$

$$k_{20} = \begin{cases} k_{2\,0}^{-}, & \text{if } A > 0, \\ k_{2\,0}^{+}, & \text{if } A < 0, \end{cases} \qquad k_{2\,0}^{\pm} = \frac{BC \pm \sqrt{C^2 + m^2(1-B^2)}}{1-B^2},$$

where $\vec{p_5} \equiv -\vec{p}$, $\theta_p = \pi - \theta_5$, $\varphi_p = \pi + \varphi_5$ and

$$B = \frac{\sqrt{s} - p_0}{A|\vec{p}|}, \qquad C = \frac{|\vec{p}|^2 - (\sqrt{s}-p_0)^2}{2A|\vec{p}|}.$$



Hard photon emission: Infra-red stability

Here we present numerical estimation of radiative correction to the cross section:

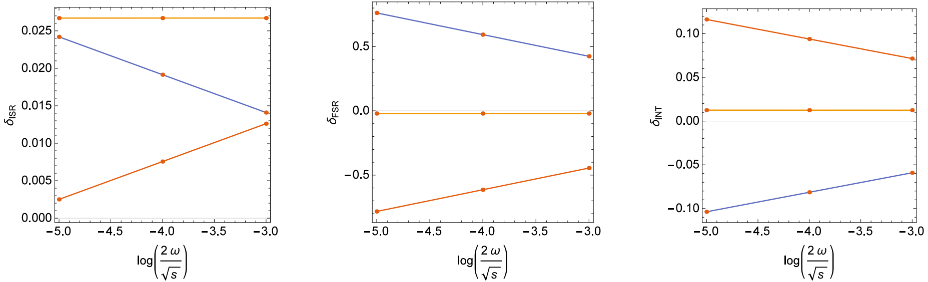
$$\delta = \frac{d\sigma_{RC}}{d\sigma_B}, \tag{14}$$

at fixed scattering angle θ and energy (defined via maximum photon energy Ω). The main interest is to show the independence of final result of fictitious photon mass λ:

$$\lambda = 10^n \sqrt{s}. \tag{15}$$

$\theta,^\circ$	n	$\Omega = 0.1 \cdot \sqrt{s}/2$			$\Omega = 0.3 \cdot \sqrt{s}/2$		
		V	R	$V+R$	V	R	$V+R$
30	−4	−0.27015	0.20910	−0.06106	−0.27015	0.26661	−0.00354
	−5	−0.42168	0.35708	−0.06460	−0.42168	0.41607	−0.00560
	−6	−0.57320	0.50840	−0.06480	−0.57320	0.56744	−0.00576
	−7	−0.72473	0.65995	−0.06478	−0.72473	0.71898	−0.00575
	−8	−0.87626	0.81145	−0.06480	−0.87626	0.87051	−0.00575
90	−4	−0.36121	0.27262	−0.08859	−0.36121	0.34036	−0.02085
	−5	−0.53510	0.44298	−0.09212	−0.53510	0.51224	−0.02286
	−6	−0.70899	0.61665	−0.09234	−0.70899	0.68591	−0.02308
	−7	−0.88288	0.79052	−0.09237	−0.88288	0.85978	−0.02310
	−8	−1.05677	0.96440	−0.09238	−1.05677	1.03366	−0.02312
150	−4	−0.45175	0.33595	−0.11580	−0.45175	0.41343	−0.03832
	−5	−0.64801	0.52859	−0.11941	−0.64801	0.60767	−0.04033
	−6	−0.84426	0.72458	−0.11968	−0.84426	0.80367	−0.04059
	−7	−1.04052	0.92082	−0.11970	−1.04052	0.99986	−0.04066
	−8	−1.23678	1.11703	−0.11975	−1.23678	1.19614	−0.04064

Hard photon emission: Dependence of auxiliary parameter ω



Radiative corrections as a function of hard regime separation parameter ω:

- Virtual correction with soft photon emission regime (orange line): $\lambda < p_0 < \omega$
- Hard photon regime (blue line): $\omega < p_0 < \Omega = 0.5\frac{\sqrt{s}}{2}$

And their sum is presented as yellow line.

Conclusion

In this talk radiative corrections for the process $\bar{p}p \to e^+e^-$ in the condition of the PANDA experiment were calculated:

1. We showed different contributions: virtual and real photon emission.
2. Infra-red stability of the result was demonstrated.
3. Formulae ready to use in Monte-Carlo generator are derived.