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From the motion of planets to elementary particles

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Cluster of Excellence
PRISMA
Precision Physics, Fundamental Interactions and Structure of Matter

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Outline

Symmetries in physics

Applications to elementary particle interactions

picture: Quanta Magazine

Symmetries in physics

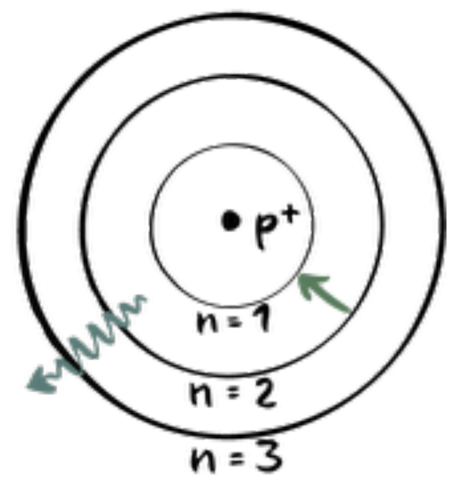
- guiding principle for finding exact description of Nature
- help to exactly solve idealized models
- obvious versus hidden symmetries

Symmetry in important physical systems

Kepler problem

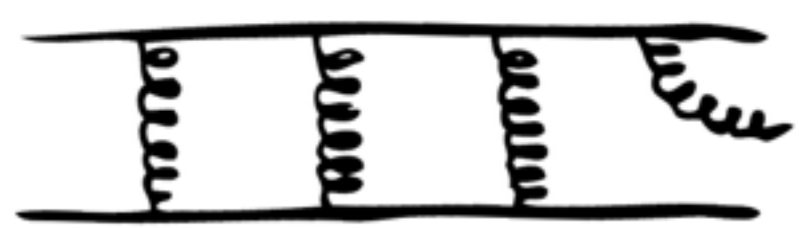
classical mechanics

Hydrogen atom



quantum mechanics

Interactions of elementary particles

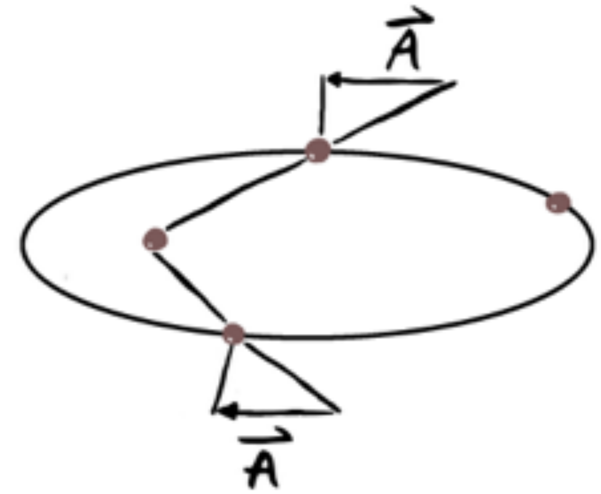


quantum field theory

Governed by the same hidden symmetry!

Regularity of orbits from symmetry

$$V \sim -\frac{\lambda}{r}$$



stable orbits

$$V \sim -\frac{\lambda}{r^{0.9}}$$

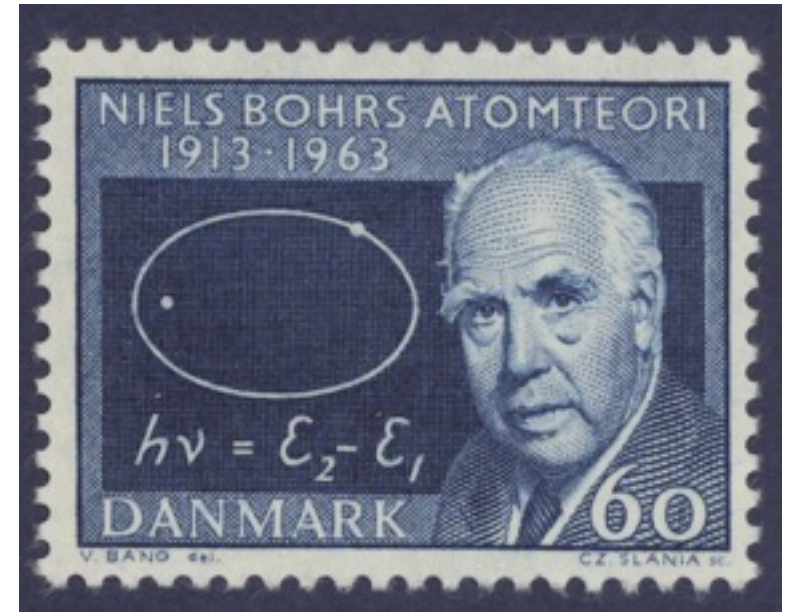
orbits precess

regularity of orbits explained by
conservation of Laplace-Runge-Lenz vector

$$\vec{A} = \frac{1}{2}\left(\vec{p} \times \vec{L} - \vec{L} \times \vec{p}\right) - \mu \frac{\lambda}{4\pi} \frac{\vec{x}}{|x|}$$

Hydrogen atom

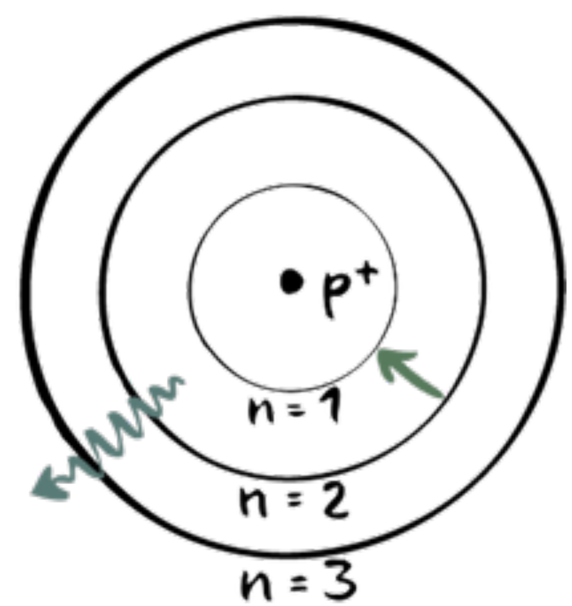
- described by quantum mechanics



- Hamiltonian $$H = \frac{1}{2m}p^2 - \frac{k}{r}$$

- spectrum with degeneracy n^2

$$E_n = -\frac{mk^2}{2\hbar^2}\frac{1}{n^2} \qquad n = 1, 2, \ldots$$



- formula explained by symmetry

Spectrum determined by symmetry

- Hamiltonian $$H = \frac{1}{2m}p^2 - \frac{k}{r}$$

- hidden symmetry:

Laplace-Runge-Lenz-Pauli operator

$$\vec{A} = \frac{1}{2}(\vec{p} \times \vec{L} - \vec{L} \times \vec{p}) - mk\frac{\vec{r}}{r}$$

- conserved quantity in quantum mechanics

$$[H, L_i] = 0 \qquad [H, A_i] = 0$$

$$[A_i, A_i] = -i\hbar\epsilon_{ijk}L_k\frac{2}{m}H$$

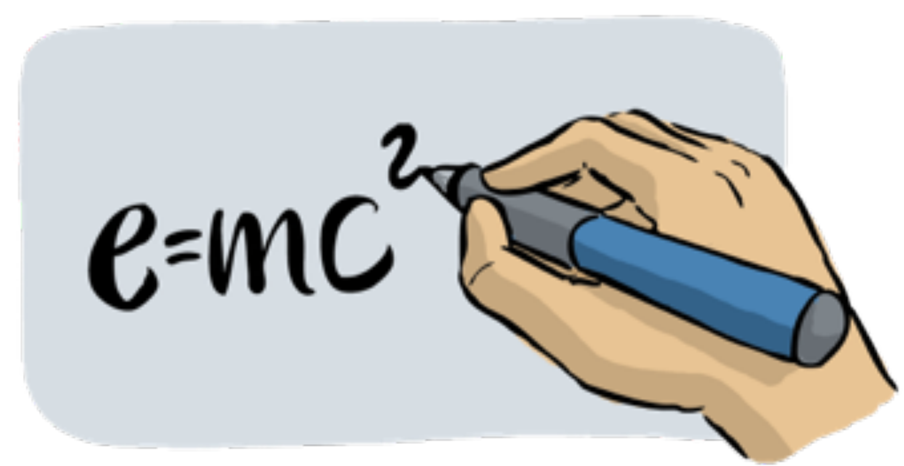
- operator algebra allows to find spectrum

Hidden symmetry in key physical systems

- Kepler problem and hydrogen atom are important classical and quantum mechanics problems that can be exactly solved

- have the same hidden Laplace-Runge-Lenz symmetry

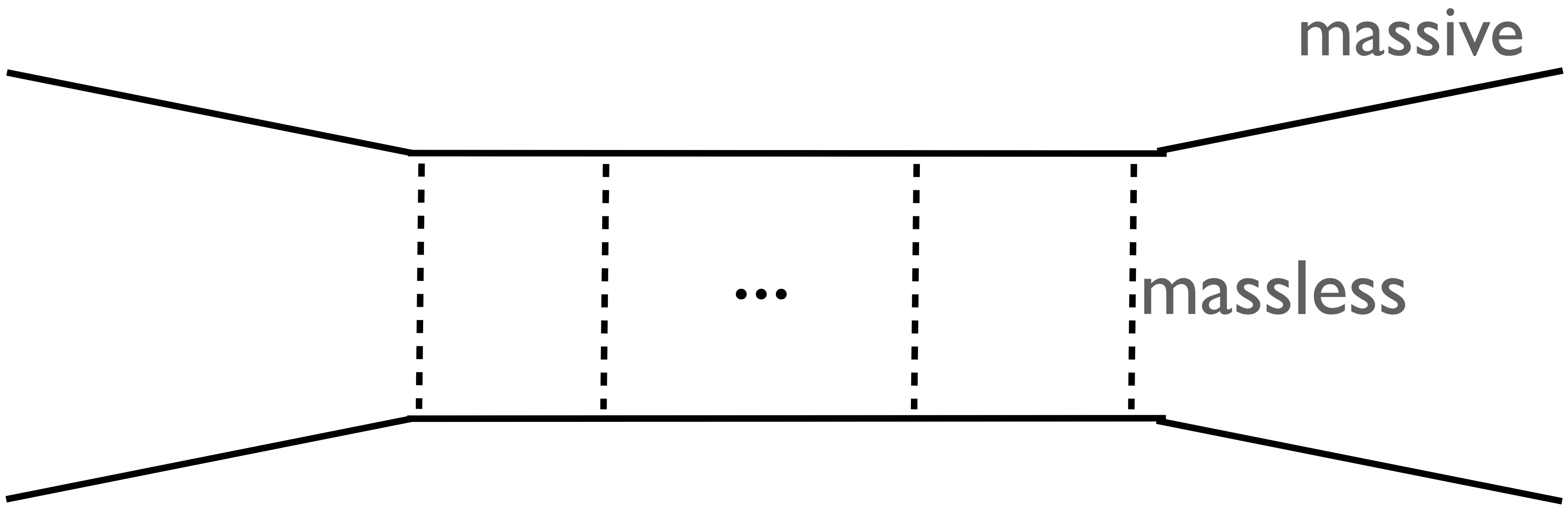
- at higher energies, quantum field theory (QFT) needed



- is there a QFT with the same symmetry?

towards a relativistic QFT

- Wick-Cutkosky model



- ladder approximation to $ep \to ep$, ignoring spin

- In the non-relativistic limit, this reduces to the hydrogen Hamiltonian

symmetry of Wick-Cutkosky model

- model possesses an exact O(4) symmetry, even away from the non-relativistic limit

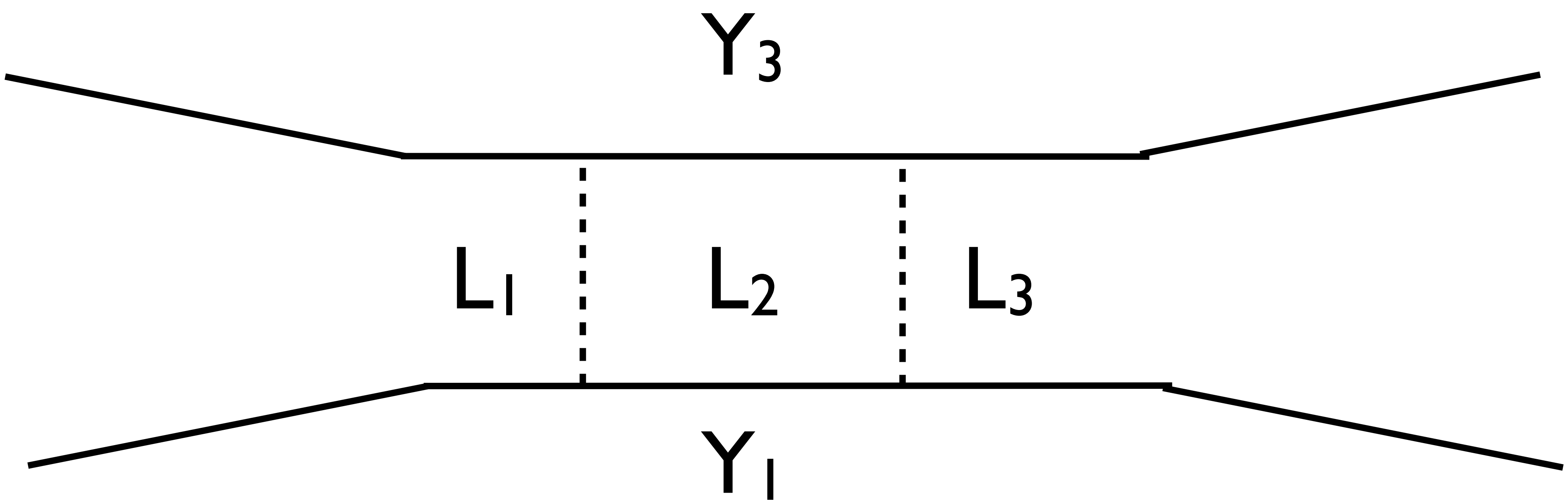
- consider one rung

$$\cdots \int \frac{d^4\ell_2}{(\ell_2-\ell_1)^2\left[(\ell_2-p_1)^2+m^2\right]\left[(\ell_2+p_2)^2+m^2\right](\ell_2-\ell_3)^2} \cdots$$

- symmetry obvious in Dirac's embedding formalism

$$L_i^a \equiv \begin{pmatrix} \ell_i^\mu \\ L_i^+ \\ L_i^- \end{pmatrix} = \begin{pmatrix} \ell_i^\mu \\ \ell_i^2 \\ 1 \end{pmatrix} \qquad L_i \cdot L_j = (\ell_i - \ell_j)^2 \qquad L_i^2 = 0$$

similarly for external momenta



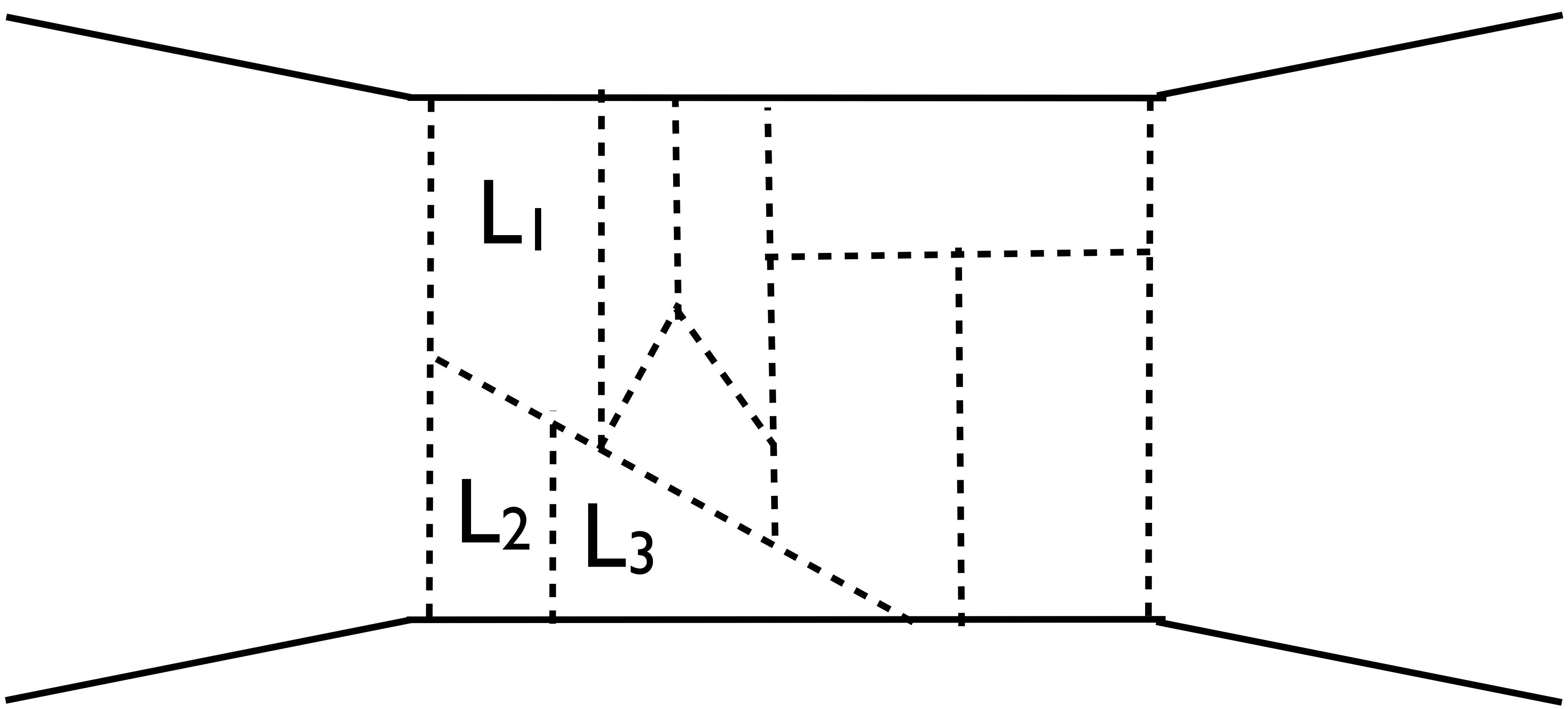
- rung in embedding formalism

$$\cdots \int \text{``}d^4 L_2\text{''} \frac{1}{(L_1 \cdot L_2)(L_2 \cdot Y_1)(L_2 \cdot Y_3)(L_2 \cdot L_3)} \cdots$$

- manifest SO(6) symmetry
- the two vectors Y_1, Y_3 reduce it to SO(4)
- contains the usual SO(3) as a subgroup
- the remaining 3 generators are the Runge-Lenz vector!

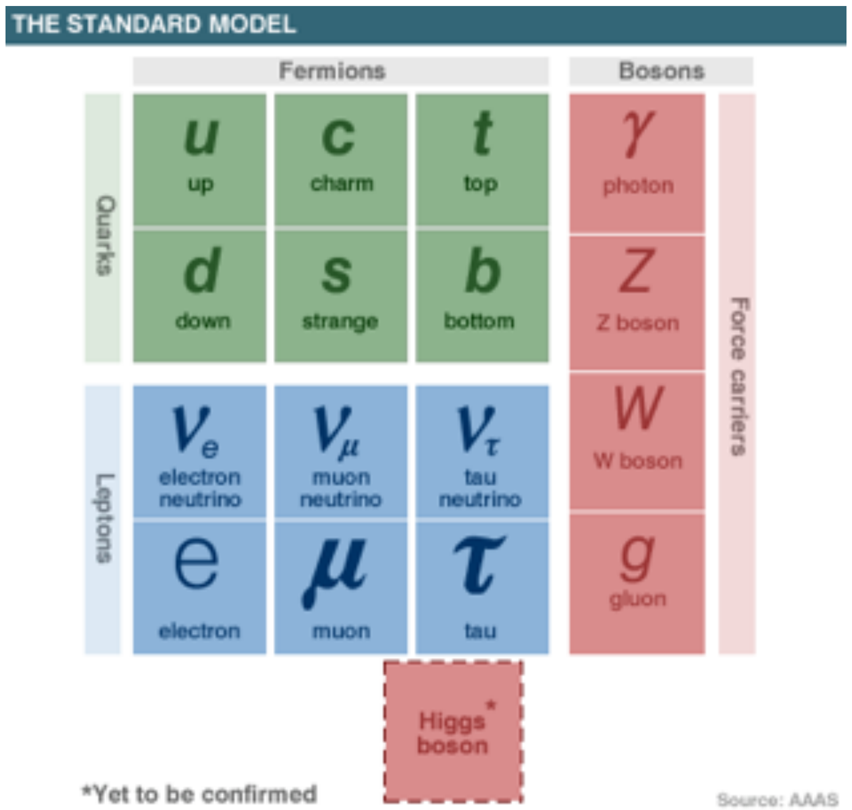
Beyond the ladder approximation

- ladder approximation is arbitrary
- misses multi-particle effects, problems with unitarity
- Is there a consistent QFT with the LRL symmetry?
- the simplest way to imagine this requires a planar limit:



- Feynman rules would have to respect the SO(6) symmetry

Standard model of elementary particles



- Higgs boson: predicted by theorists in the 60's

- as of July 4th, 2012 : discovery by CMS and Atlas experiments

- core part (gluons): non-Abelian gauge theory

$$\mathcal{L} = \frac{1}{4}\mathrm{Tr}\int F_{\mu\nu}F^{\mu\nu}\,, \qquad F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu + ig[A^\mu, A^\nu]$$

$$A^\mu = \sum_{a=1}^{N^2-1} A_a^\mu\, t_{ij}^a$$

gauge group SU(Nc), Nc=3

- large Nc limit selects planar Feynman diagrams

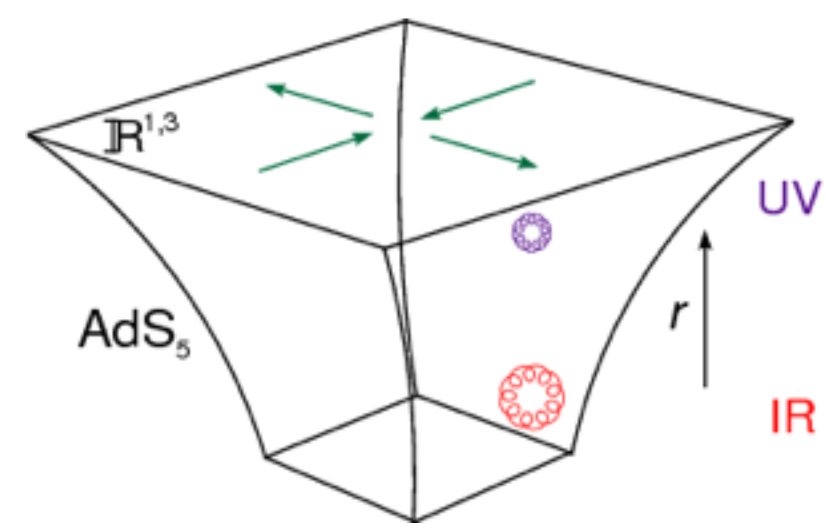
maximally supersymmetric Yang-Mills theory

Particle content similar to QCD:

QCD	N=4 supersymmetric Yang-Mills theory
• SU(3) Yang-Mills theory (gluons) • fermions in fundamental representation	• SU(Nc) Yang-Mills theory • 4 fermions, adjoint repr. • 6 scalars

Bonus features:

- supersymmetry; vanishing beta function
- conjectured holographic AdS description



picture from 0803.2475 [hep-th] (L. Dixon)

Zvi Bern & collaborators studied scattering amplitudes in this theory

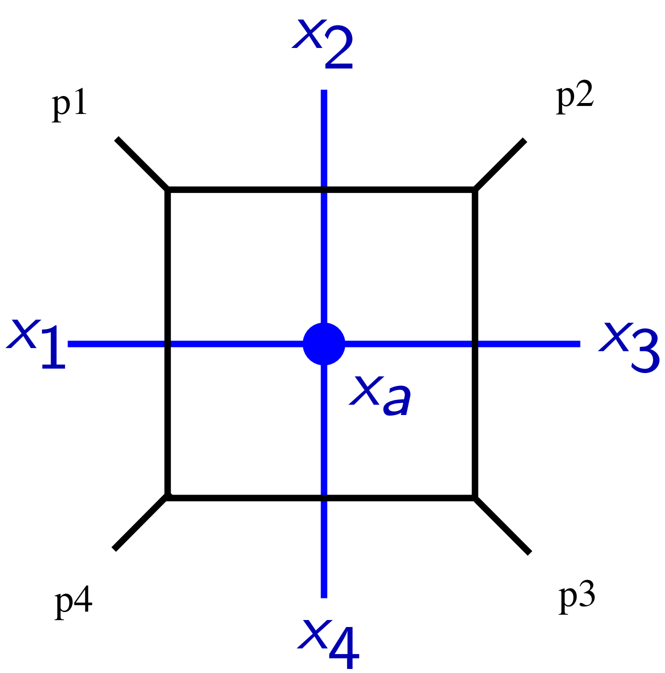
- they used modern (generalized unitarity) methods
- millions of Feynman diagrams sum up to a few ‘effective integrals’
- why is this the case?

Hidden symmetry N=4 sYM

planar N=4 sYM has dual conformal symmetry

[Drummond, JMH, Smirnov, Sokatchev 2006; Alday, Maldacena 2007; Drummond, JMH, Korchemsky, Sokatchev 2007]

e.g. 1-loop four-particle amplitude:



$$p_i^\mu = x_i^\mu - x_{i+1}^\mu$$

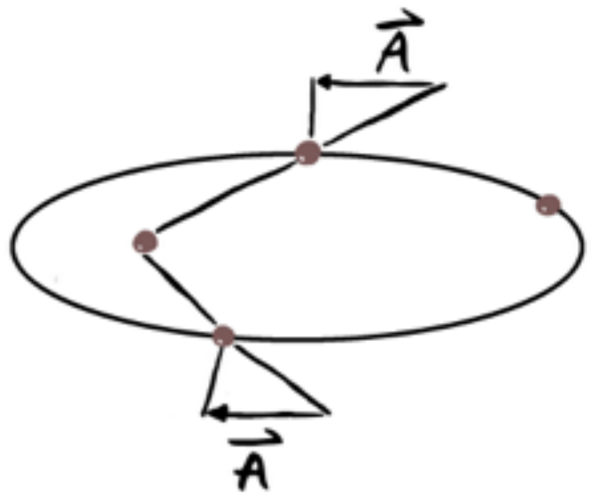
$$= x_{13}^2 x_{24}^2 \int \frac{d^D x_a}{x_{1a}^2 x_{2a}^2 x_{3a}^2 x_{4a}^2}$$

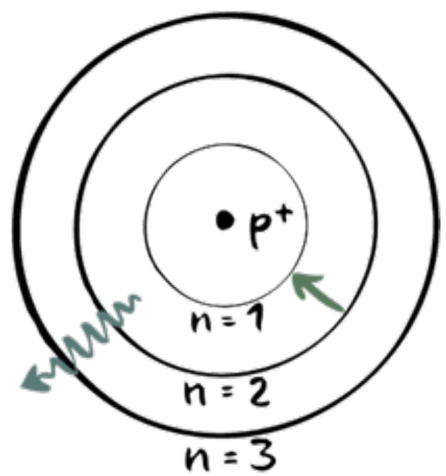
invariant under SO(4,2) in dual space $\quad x^\mu \to x^\mu / x^2$

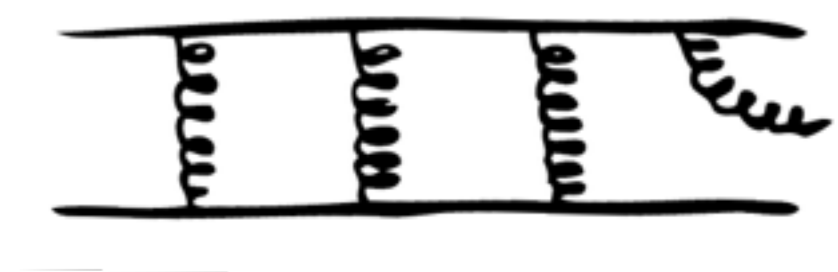
$$= (Y_1 \cdot Y_3)(Y_2 \cdot Y_4) \int \frac{\text{“}d^D L\text{”}}{(Y_1 \cdot L)(Y_2 \cdot L)(Y_3 \cdot L)(Y_4 \cdot L)}$$

summary Laplace-Runge-Lenz symmetry

- LRL symmetry governs several problems







N=4 super Yang-Mills theory is the 'hydrogen atom of the 21st century'

- symmetry explains simplicity
- helpful for finding exact answers

Applications to elementary particle interactions

picture: Quanta Magazine

Multi-particle collisions as the next frontier

picture: Quanta Magazine

- at high energies, many particles produced
- challenge to evaluate the virtual corrections
- long experimenter's wishlist for theorists, e.g.

$pp \to 3$ jets $\quad pp \to H + 2$ jets $\quad pp \to V + 2$ jets

- challenge: 5-particle processes at 2 loops

‘Ideal’ and ‘real’ scattering amplitudes

Is there some simpler version of QCD that allows to understand key properties of scattering amplitudes?

We need to obtain numerical results for cross sections at the LHC.

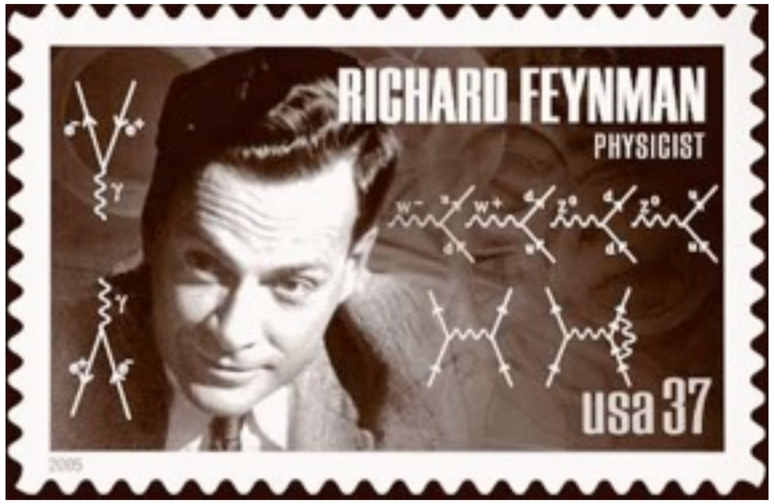
This talk: tools for ‘real’ QCD coming from ‘ideal’ amplitudes

Scattering amplitudes

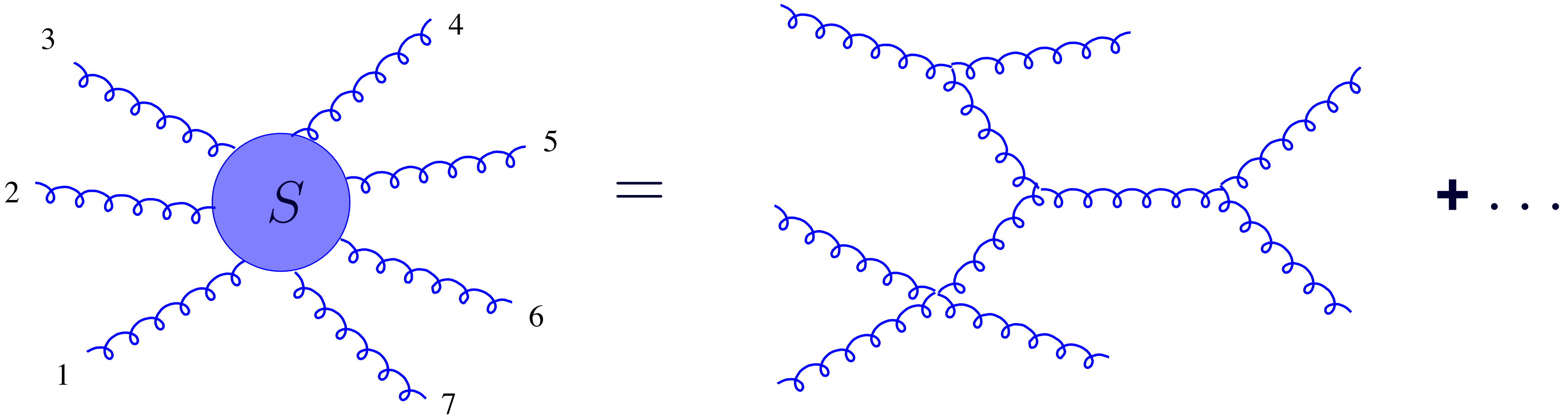
Computational recipe:

(1) draw all Feynman diagrams

(2) compute them!



Often difficult in practice! E.g. tree-level gluon scattering:



number of external gluons	4	5	6	7	8	9	10
number of diagrams	4	25	220	2485	34300	559405	10525900

Final results much simpler than intermediate steps! Why?

Simplicity of amplitudes from symmetry

Tree-level gluon amplitudes are 'secretly' supersymmetric!

They have the full symmetry of N=4 sYM

- conformal supersymmetry $J^a = \sum_{i=1}^{n} J_i^a$
- hidden dual conformal symmetry

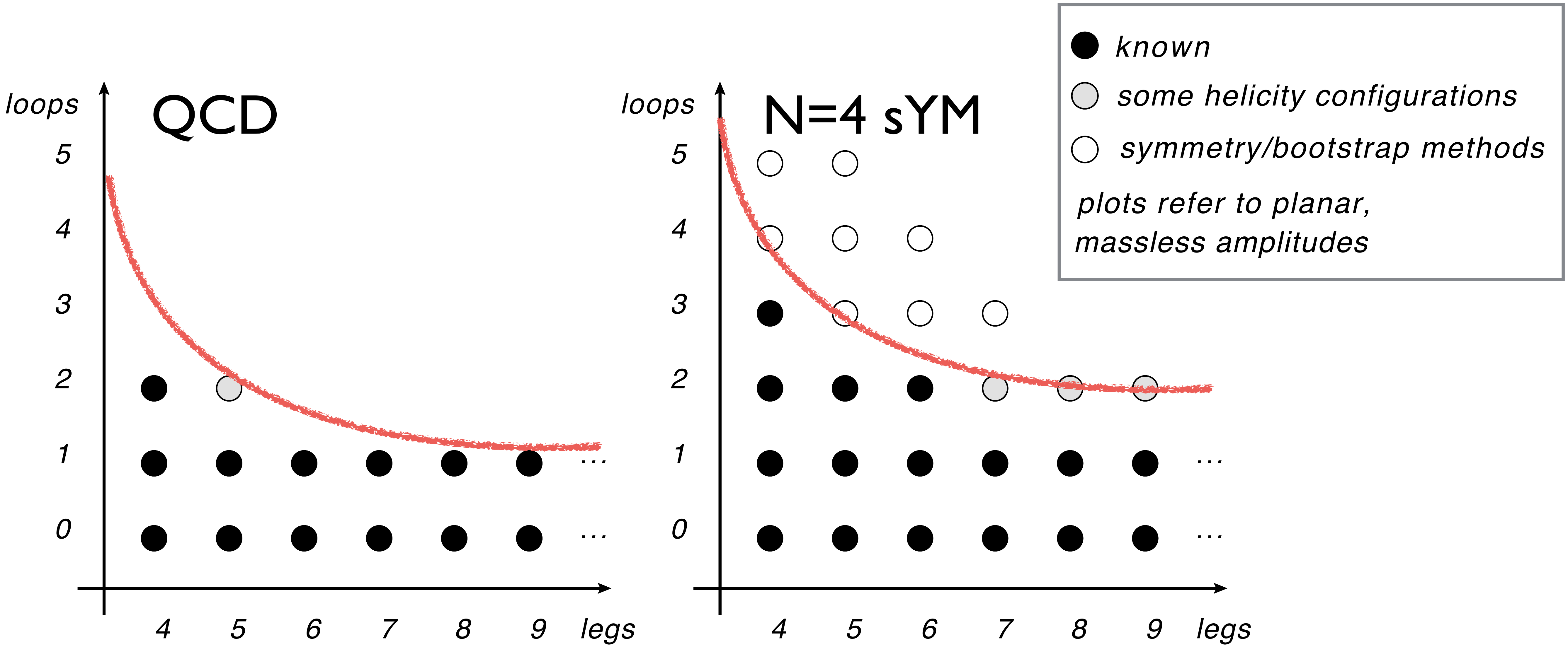
combine to
Yangian symmetry

$$J^a = \sum_{i=1}^{n} J_i^a \qquad Q^c = f^{abc} \sum_{i<j}^{n} J_i^a J_j^b$$

$J^a \mathcal{A}_n = 0$ $\qquad$ $Q^a \mathcal{A}_n = 0$ $\qquad$ explains simplicity!

symmetry & collinear behavior fixes tree-level amplitudes!

State of the art loop amplitudes



- frontier of knowledge pushed forward continuously
- N=4 sYM a good prediction what we can hope to achieve next in QCD

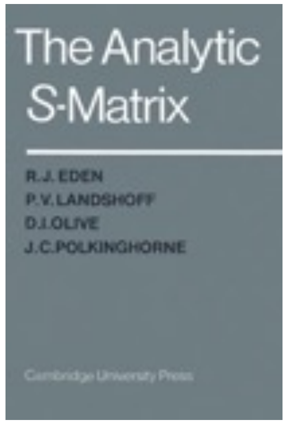
Bootstrapping scattering amplitudes

Can we fix amplitudes from general properties?

- symmetries
- analytic properties
- physical limits

Bootstrap (pre)history

- 1960's: determine S-matrix from analytic properties



- 1994: 'One loop n point gauge theory amplitudes, unitarity and collinear limits' [Bern, Dixon, Dunbar, Kosower]

- 2011: bootstrap in planar maximally supersymmetric Yang-Mills theory [Dixon, Drummond, JMH]

many further developments [Almelid, Bartels, Bargheer, Caron-Huot, Del Duca, Dixon, Druc, Drummond, Duhr, Dulat, Gardi, Harrington, JMH, von Hippel, Marzucca, McLeod, Paulos, Pennington, Parker, Papathanasiou, Scherlis, Schomerus, Sprenger, Spradlin, Trnka, Verbeek, Volovich]

- 2017: first application to multi-loop QCD integrals, non-planar [Chicherin, JMH, Mitev]

Bootstrap approach

$\vec{x}$ kinematic dependence

$D = 4 - 2\epsilon$ dimension

$$\mathcal{A}(\vec{x}, \epsilon) = \sum_{i,j,k} c_{ijk} \frac{1}{\epsilon^i} r_j(\vec{x})\, f_k(\vec{x}) + \mathcal{O}(\epsilon)$$

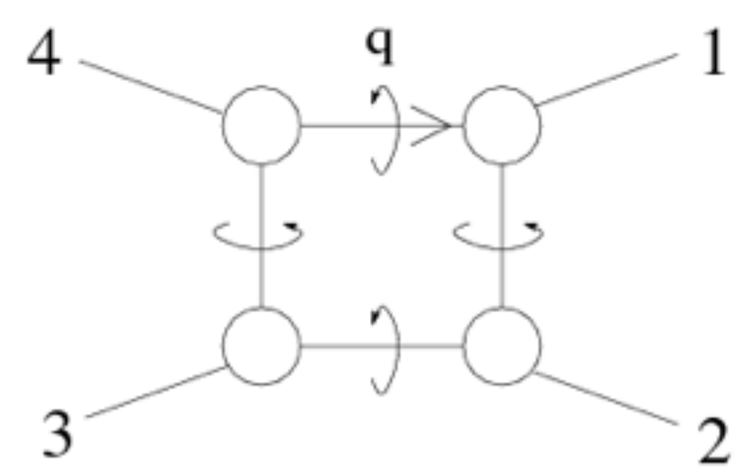
- Laurent expansion in ϵ
- rational/algebraic normalization factors
- special functions
- unknowns: *finite number* of coefficients

Constraints on rational factors

- controlled by leading singularities

[Cachazo `08; Arkani-Hamed, Bourjaily, Cachazo, Trnka `10]

- idea: information contained in loop integrand
- perform integral over closed cycles

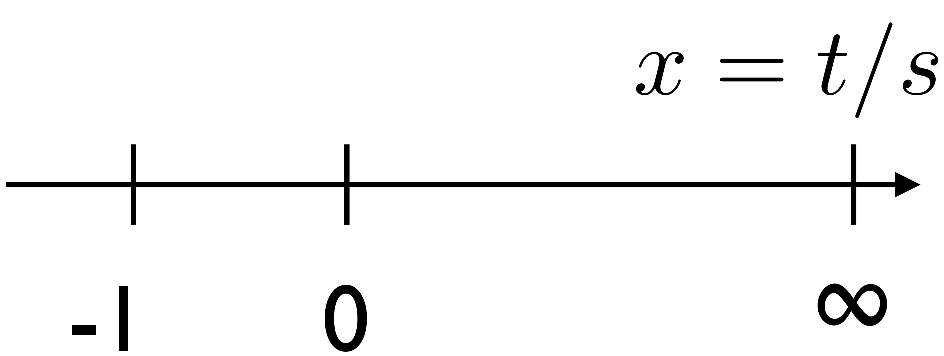


- residue computation much simpler compared to space-time integration

Finding the space of special functions

- improved understanding of iterated integrals
- ‘symbol’ technology [Goncharov, Spradlin, Vergu, Volovich, 2010]
- canonical differential equations defining special functions [JMH, 2013]
- singularities of functions from Landau equations

Example: massless 4-particle scattering

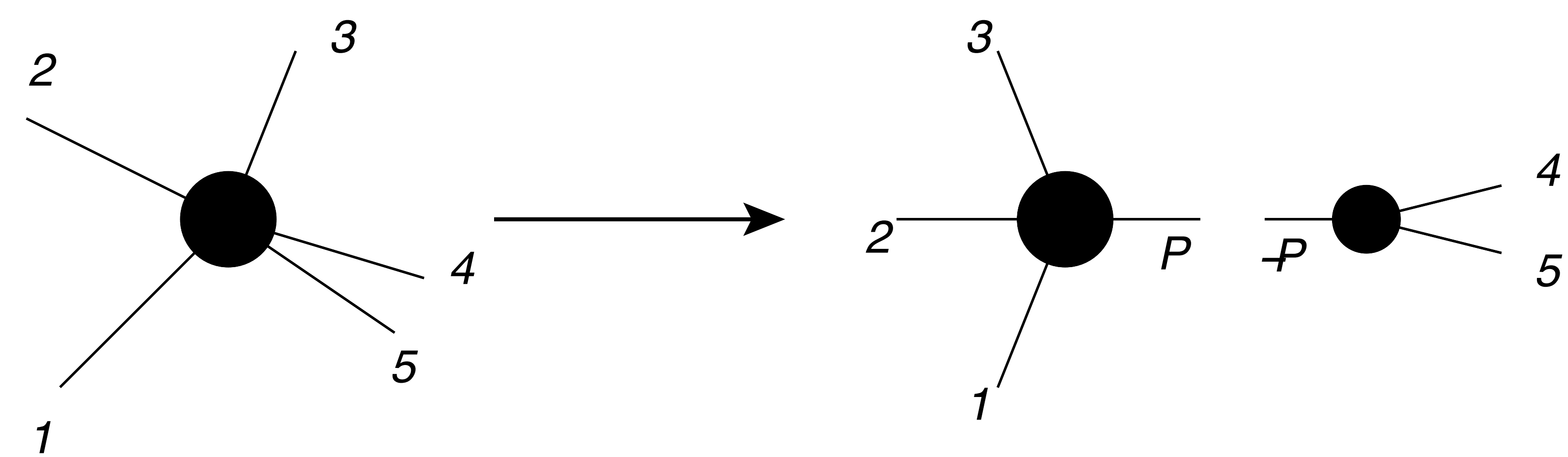


singular points correspond to

$$s = 0\,, \quad t = 0\,, \quad u = -s - t = 0$$

Constraints from symmetries and physical properties

- impose all known symmetries on ansatz
- universal behavior in (singular) limits
- soft, collinear limits



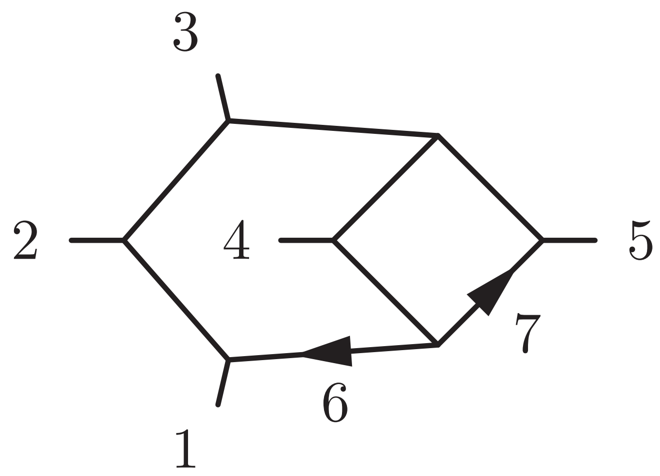
- high-energy, Regge limit
- constraints on discontinuities (e.g. Steinmann relations)

Sample applications

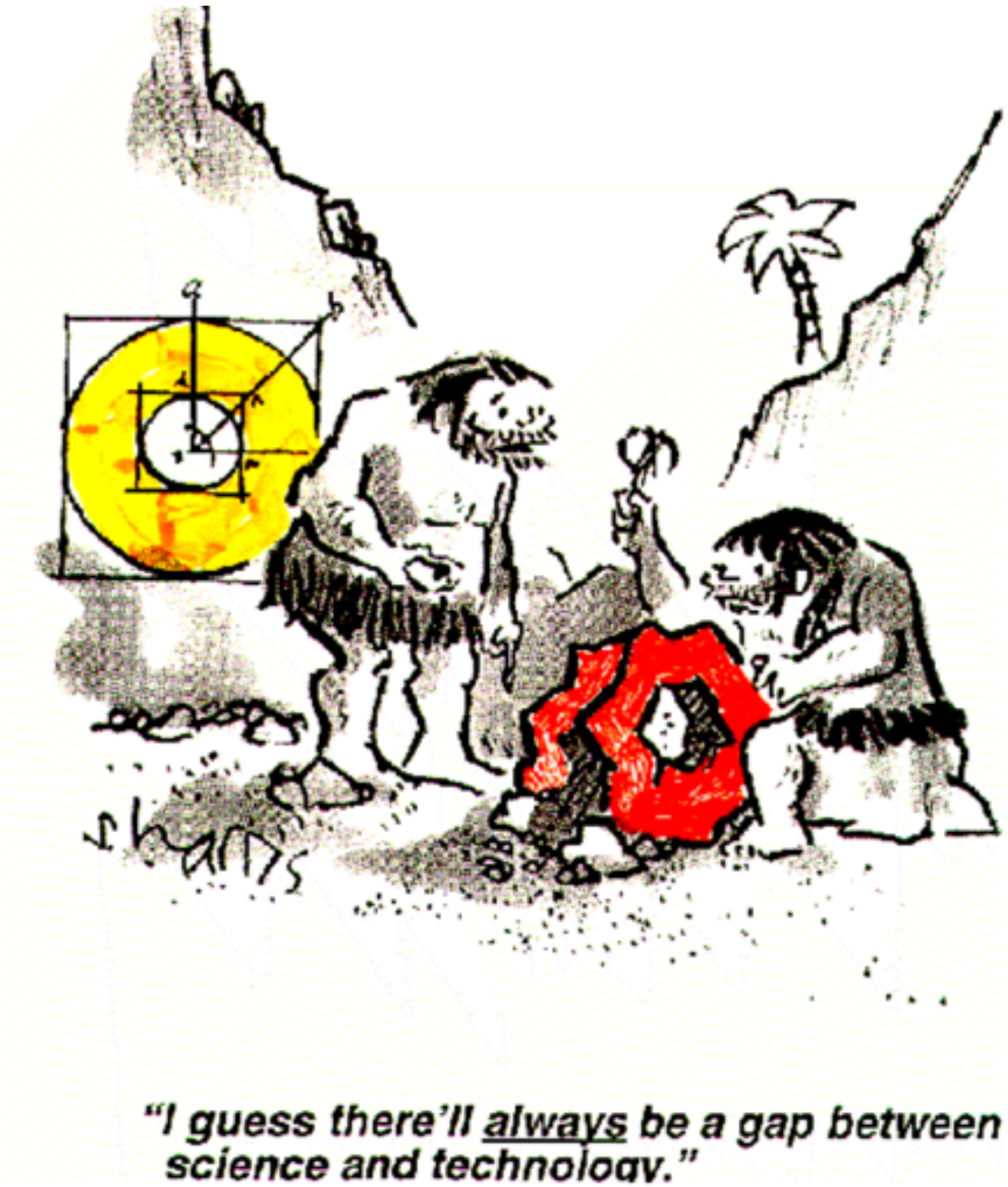
- Six-particle amplitudes in N=4 sYM known to high loop orders [Caron-Huot, Dixon, McLeod, von Hippel, 2016]

- Applications to quantities in effective field theory [Li, Zhu, 2017]

- First application to non-planar five-particle integrals in QCD [Chicherin, JMH, Mitev, 2017]



Comment on novel methods



- N=4 sYM exciting laboratory for developing ideas
- with refinements, applications to QCD possible
- e.g. open door to 2-loop QCD amplitudes, needed for LHC physics

Conclusion

- the same hidden symmetry governs several important problems:
    - motion of planets
    - hydrogen atom
    - elementary particle interactions

- amplitude bootstrap

scattering amplitudes determined from symmetries, analytic properties, and physical limits

Outlook lecture 2 &3

- symmetries of scattering amplitudes
- special functions in scattering amplitudes

Thank you!

(most) illustrations by Joy Katzmarzik,
www.leap4joy.de