Difficulties of N-body cosmological simulations and the physics of dark matter

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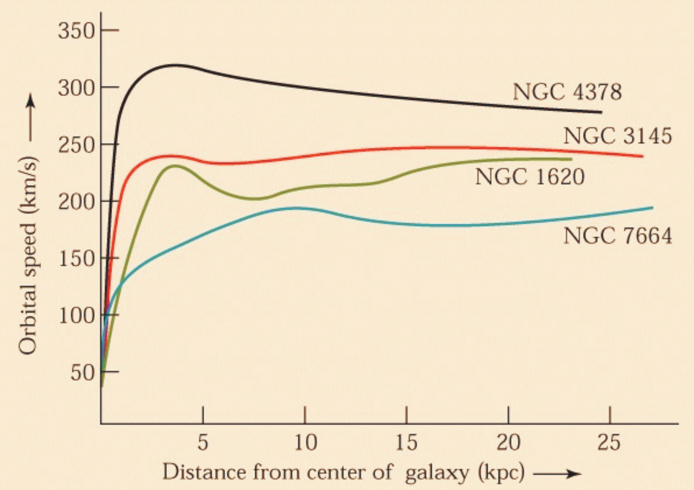
Density profiles. Discovery.



Density profiles.

Isothermal profile

$$\rho \sim r^{-2}$$

Navarro-Frenk-White profile

$$\rho_{NFW} = \frac{\rho_s}{(r/r_s)(1 + r/r_s)^2}$$

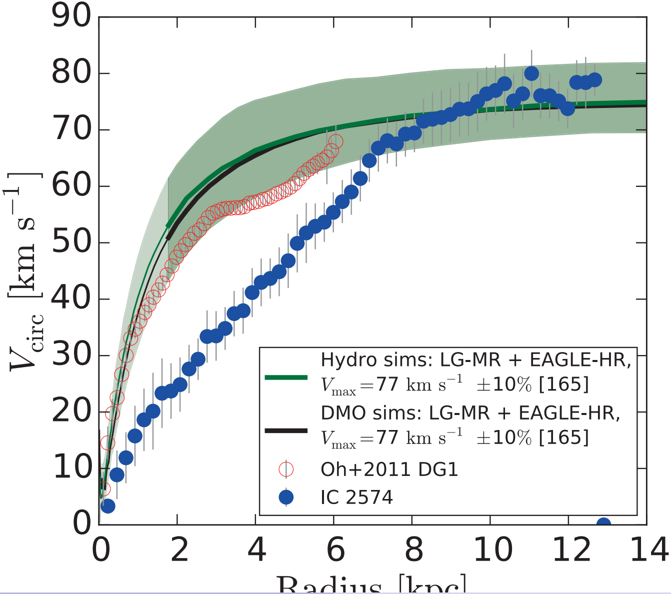
Einasto profile

$$\rho_{Ei} = \rho_s \exp\left\{-2n\left[\left(\frac{r}{r_s}\right)^{\frac{1}{n}} - 1\right]\right\}$$

Hernquist profile

$$\rho_H = \frac{Ma}{2\pi r(r + a)^3}$$

Simulations vs. observations (Oman et al. 2015)



Numerical effects

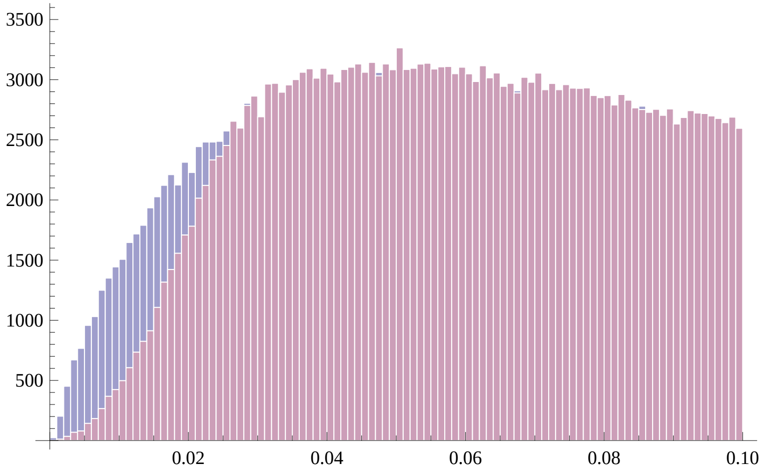
Relaxation time

$$\langle \Delta v \rangle \simeq 0 \qquad \langle \Delta v^2 \rangle \simeq \frac{8v^2 \ln \Lambda}{N(r)}$$

$$\tau_r(r) = \frac{N(r)}{8 \ln \Lambda} \cdot \tau_d(r) \qquad \tau_d(r) \sim \frac{r}{v}$$

(Power et. al. 2003) $t_0 \leq 1.7\tau_r$
(Hayashi et al. 2003; Klypin et al. 2013) $t_0 \leq 30\tau_r$

Core formation



Fokker-Planck equation

$$\frac{\partial n(p)}{\partial t} = \frac{\partial}{\partial p}\left\{\tilde{A}n(p) + \frac{\partial}{\partial p}[Bn(p)]\right\}$$

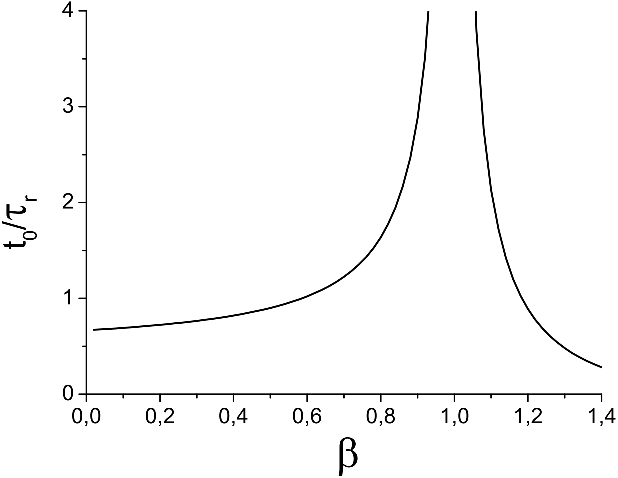
$$\tilde{A} = \frac{\langle \Delta p \rangle}{\delta t} = \mu\frac{\langle \Delta v \rangle}{\delta t} \qquad B = \frac{\langle \Delta p^2 \rangle}{2\delta t} = \frac{\mu^2}{2}\frac{\langle \Delta v^2 \rangle}{\delta t}$$

$$s = -\tilde{A}n(p) - \frac{\partial}{\partial p}[Bn(p)]$$

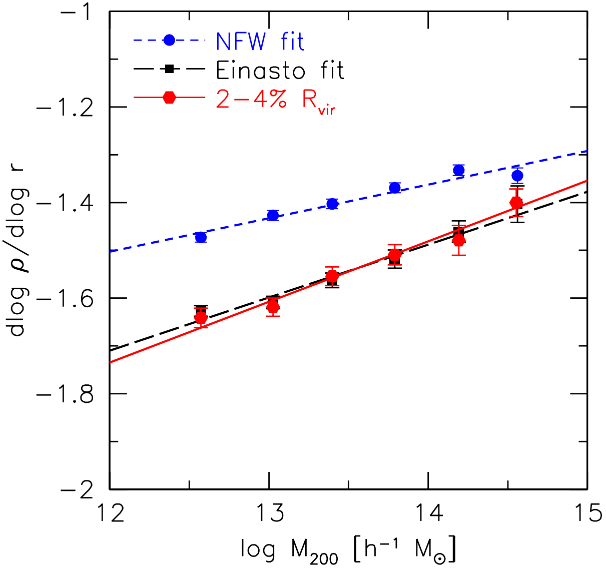
The Fokker-Planck equation has an attractor solution $\rho \propto r^{-\beta}$,

where $\beta \approx (1 - 4/3)$ (Evans & Collett 1997, Baushev 2015)

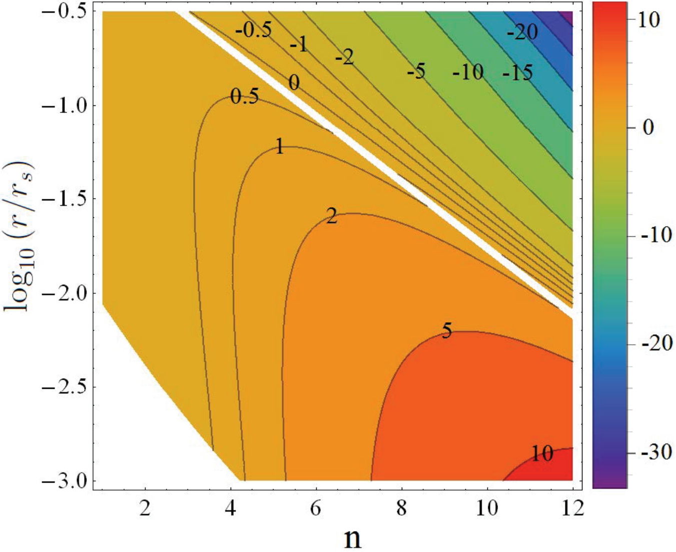
Power-law profiles



Einasto profile (Dutton & Macció 2010)



Einasto profile



Simulation details. Gadget-3.

$$\rho_H = \frac{Ma}{2\pi r(r+a)^3} \qquad \phi(r) = -\frac{GM}{r+a}$$

$M = 10^7 M_\odot$, $a = 100$ pc. We use $N = 10^5$ test bodies.

$$\tau_d = \frac{r+a}{a}\sqrt{r/a}\times 4.72\cdot 10^6\text{years}; \qquad \tau_r = \frac{2r^2\sqrt{r/a}}{a(r+a)}\times 1.36\cdot 10^{10}\text{years}.$$

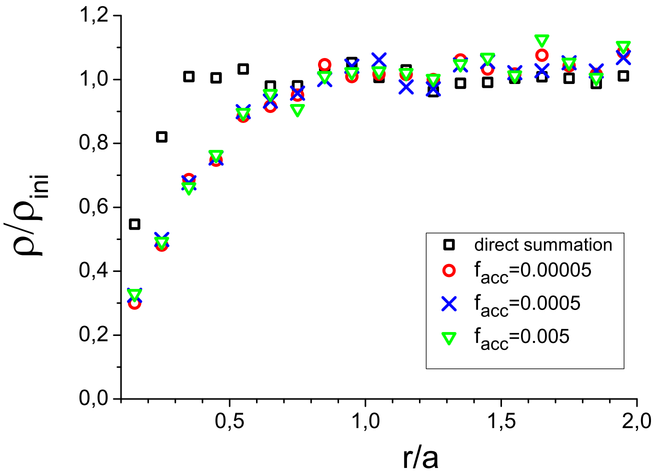
At $r = a$ $\tau_d = 2\cdot\xi = 9.45\cdot 10^6$ years, $\tau_r = 1.36\cdot 10^{10}$ years.

$$\frac{GM_{cell}}{r^2}\left(\frac{l}{r}\right)^2 \le f_{acc}|\vec{a}|,$$

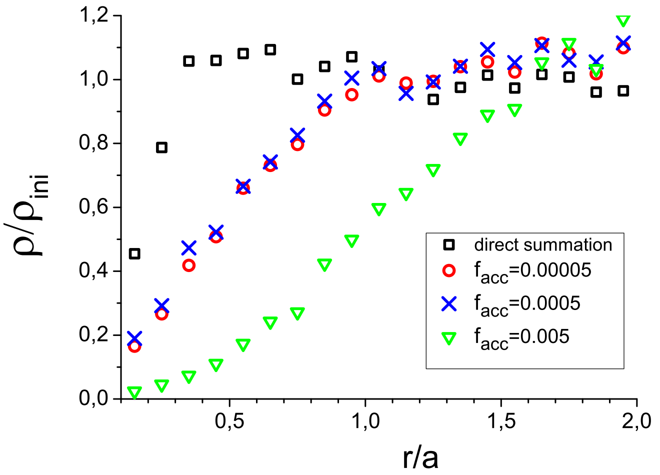
The integrals of motion $\epsilon = \phi(r) + v^2/2$, $\vec{K} = [\vec{v}\times\vec{r}]$, r_0:

$$\epsilon = \phi(r_0) + K^2/2r_0$$

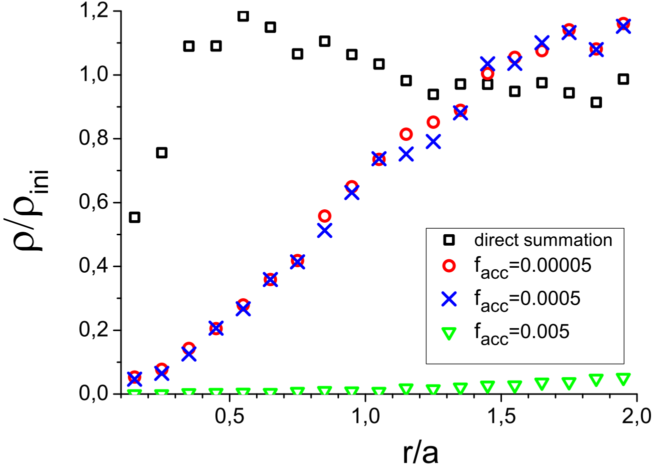
$t = 3.17 \cdot 10^9$years



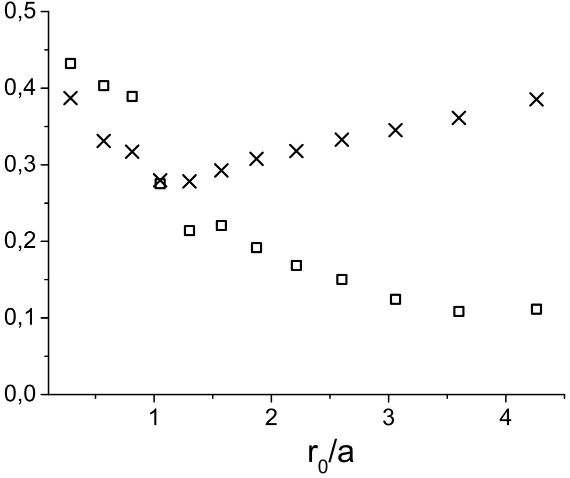
$t = 9.51 \cdot 10^9$years



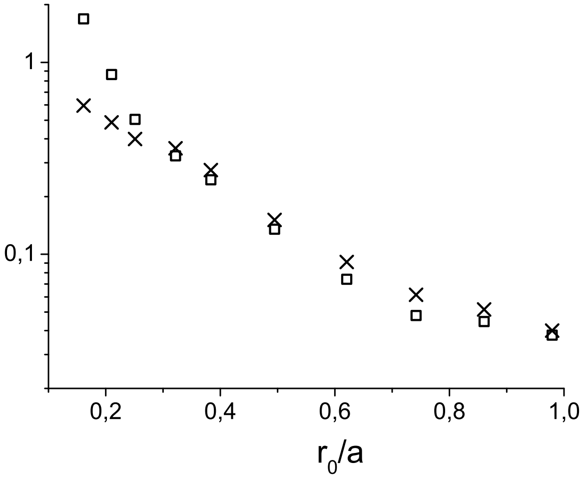
$t = 28.5 \cdot 10^9$years



$\langle \Delta K/K_{circ} \rangle$ (squares) and $\langle \Delta r_0/r_0 \rangle$ (crosses)



The ratios $\frac{K_{circ}}{\tau_r}\left\langle\frac{\Delta K}{\Delta t}\right\rangle^{-1}$ (squares) and $\frac{1}{\tau_r}\left\langle\frac{\Delta r_0}{r_0 \Delta t}\right\rangle^{-1}$ (crosses)



Kinetic equations

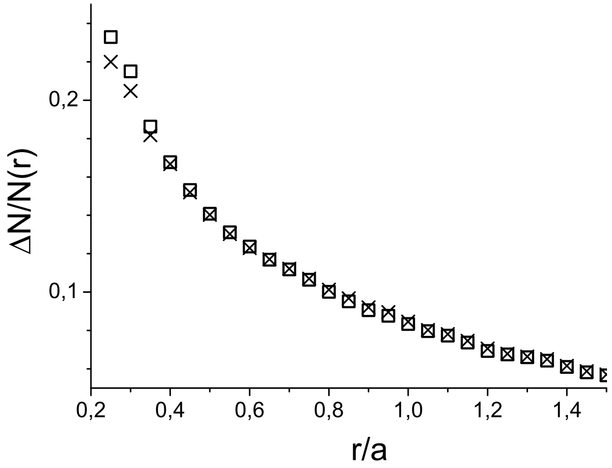
$$\frac{df}{dt} = \frac{\partial}{\partial p_\alpha}\left\{\tilde{A}_\alpha f + \frac{\partial}{\partial p_\beta}[B_{\alpha\beta} f]\right\}$$

where $\vec{q}$ is the momentum changing $\vec{p} \to \vec{p} - \vec{q}$ in a unit time.

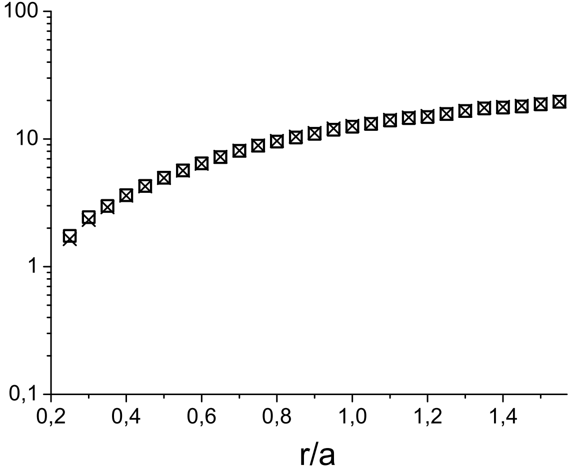
$$\tilde{A}_\alpha = \frac{\sum q_\alpha}{\delta t} \qquad B_{\alpha\beta} = \frac{\sum q_\alpha q_\beta}{2\delta t}$$

$$\frac{df}{dt} = 0 \qquad \text{vs} \qquad \frac{df}{dt} = \frac{\partial^2[B_{\alpha\beta} f]}{\partial p_\alpha \partial p_\beta}$$

The upward $\Delta N_+(r)/\Delta t$ (squares) and downward $\Delta N_-(r)/\Delta t$ (crosses) Fokker-Planck streams



$1.7\tau_r \frac{\Delta N_+(r)}{N(r)\Delta t}$ (squares) and $1.7\tau_r \frac{\Delta N_-(r)}{N(r)\Delta t}$ (crosses)



Conclusions

1) Though the cuspy profile is stable, all integrals of motion characterizing individual particles suffer strong unphysical variations along the whole halo, revealing an effective interaction between the test bodies. This result casts doubts on the reliability of the velocity distribution function obtained in the simulations.

2) The core formation in the halo centers has nothing to do with the collisional relaxation. N-body simulations does not prove that a cusp should be formed in the collision-less CDM halo, and the collisional relaxation tends to transform it into a core.

3) We find unphysical Fokker-Planck streams of particles in the cusp region. The same streams should appear in cosmological N-body simulations, being strong enough to change the shape of the cusp or even to create it.