A legacy of Dima Bardin: ZFITTER and the future

Ein Vermächtnis von Dima Bardin: ZFITTER und die Zukunft

Tord Riemann, DESY **Work done together with: Dima Bardin and many others, see next page**

CALC2018, July 23 – 31, 2018, JINR, Dubna, Russia
The Workshop is dedicated to the memory of Dmitry Yu. Bardin (1945 – 2017).
https://indico.jinr.ru/conferenceOtherViews.py?view=standard&confId=418#2018-07-23
Part of work of T.R. is supported by FNP, Polish Foundation for Science

Introduction

Dima Bardin on a slide

Bardin

Hirsch-index H = 94 (no self-citations)
Hirsch-index H = 126 (all papers, also with collab.'s)

Webpages

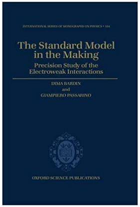
http://theor.jinr.ru/~kuzemsky/bardinbio.html
https://ru.wikipedia.org/wiki/Bardin,_Dmitrij_Yurevitch
http://sanc.jinr.ru/users/zfitter/
http://zfitter.com
http://zfitter.education

Bardin et al.

A. A. Akhundov A. Andonov A. Arbuzov J. Biebel M. Bilenky C. Bondarenko C. Burdik A. Chizhov P. Christova O. Fedorenko Wolfgang Hollik M. Jack S. Jadach Lida Kalinovskaya V. Khovansky M. Klein R. Kleiss V. Kolesnikov D. Lehner A. Leike G. Mitselmakher G. Nanawa A. Olshevsky Giampiero Passarino W. Placzek Sabine Riemann Tord Riemann L. Rumyantsev M. Sachwitz R. Sadykov A. Sapronov A. Sazonov N. M. Shumeiko E. Uglov B. Ward Z. Was and Barbara Badelek Konstantin Chetyrkin Ansgar Denner Stefan Dittmaier Valya Dokuchaeva Fred Jegerlehner Martin Gruenewald Andrei Kataev Johann Kühn Bernd Kniehl Wolfgang Friedrich Lohmann Lew Okun Dorothee Schaile Dmitri Shirkov Alberto Sirlin Vladimir v. Schlippe Hubert Spiesberger Oleg Tarasov

A monstrous book on Standard Model Physics

Born with pain
Incomparable
Unreadable?



©Amazon (14 July 2018)

Foto Passarino
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"The Standard Model in the Making: Precision Study of the Electroweak Interactions"

(The International Series of Monographs on Physics, 1st edition, 1999)

by Dima Bardin and Giampiero Passarino

Bound Edition **EUR 570,20**

ZFITTER main authors, during CALC 2012 at JINR, Dubna

From http://sanc.jinr.ru/users/zfitter/

From left to right: Lida Kalinovskaya, Pena Christova (1943-2016), Dima Bardin (1945-2017), Tord Riemann, Sabine Riemann, Andrej Arbuzov
Middle and right photographs: Sasha Olshevsky and Arif Akhundov.

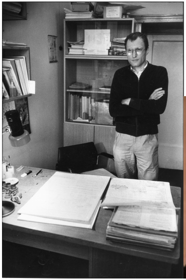
ZFITTER authors, longest list:

(blue: the actual "main" authors)

A. Akhundov, **A. Arbuzov**, M. Awramik, **D. Bardin**, M. Bilenky, A. Chizhov, **P. Christova**, M. Czakon, O. Fedorenko (1951-1994), A. Freitas, M. Grünewald, M. Jack, **L. Kalinovskaya**, **A. Olshevsky**, **S. Riemann**, **T. Riemann**, M. Sachwitz, A. Sazonov, Yu. Sedykh, I. Sheer, L. Vertogradov, H. Vogt

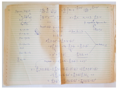
Who is me? – Tord Riemann, Cofounder of ZFITTER, spokesperson since 2005 (2019 $\rightarrow$ A. Arbuzov)

– 1985 in the JINR LTF office
– 2015 in German Wikipedia, https://de.wikipedia.org/wiki/Tord_Robert_Riemann
– 2009 in India, going to Allahabad (+ S. Moch + J. Gluza)
– 2018 in Poland, Univ. of Silesia is spending an affiliation (used for indexed publications)
– 2018 in CERN cafeteria





Dr. Claus-Jochen Biebl, $\approx$ 1980



Michael Peskin
at the 2018 conference “SM50 – 50 years of Standard Model”
→ Authors of precision calculations

https://indico.cern.ch/event/704471/contributions/3012508/subcontributions/255601/attachments/1660617/2660398/Peskin-F
pdf

After this, there was the hard work of supplying precise computations for all of the aspects of Z physics. I do not have time here for a complete accounting of the contributions. Some names that should not be forgotten are

Dima Bardin, Frits Berends, Manfred Bohm, Ansgar Denner, Wolfgang Hollik, Fred Jegerlehner, Ronald Kleiss, Luca Trentadue, and Tord Riemann

Bryan Lynn

at the 2018 conference "SM50 – 50 years of Standard Model"

– 1008 citations on the Standard Model renormalization
– 37 items referring to Dima

https://indico.cern.ch/event/704471/contributions/3012508/subcontributions/255601/attachments/1660617/2660378/LynnStuart-HistoryofPEW.pdf

[815] D.Yu. Bardin, A. Leike, T. Riemann, M. Sachwitz, Energy Dependent Width Effects in e+ e- Annihilation Near the Z Boson Pole, Phys.Lett. B206 (1988) 539-542, PHE-88-03,

[816] Electroweak Radiative Corrections At Hera Energies D.Yu. Bardin, C. Burdik (Dubna, JINR), P.Kh. Khristova (Preslavski U.), T. Riemann (DESY, Zeuthen). 1987. Published in IN *SELLIN 1987, PROCEEDINGS, THEORY OF ELEMENTARY PARTICLES* 324-330.

[817] ZFITTER, A. Akhundov, A. Arbuzov, D. Bardin, P. Khristova, L. Kalinovskaya, A. Olshevsky, S. Riemann, T. Riemann, websites zfitter.com, sanc.jinr.ru/users/zfitter and zfitter.education.

[818] ZFITTER support, D. Bardin et. al., Comput. Phys. Commun. 174(2006)728, hep-ph/0507146, websites sanc.jinr.ru/users/zfitter and zfitter.education (C7)

[819] D. Bardin, M. Bilenky, P. Khristova, M. Jack, L. Kalinovskaya, A. Olshevsky, S. Riemann and T. Riemann, honored as First Scientific Award of JINR, Dubna, 19 January 2001, referee Lew B. Okun.

M. Peskin @ SM50 – ZFITTER + KORALZ

https://indico.cern.ch/event/704471/contributions/3012508/subcontributions/255601/attachments/1660617/2660398/Peskin-PrecisionEW.pdf

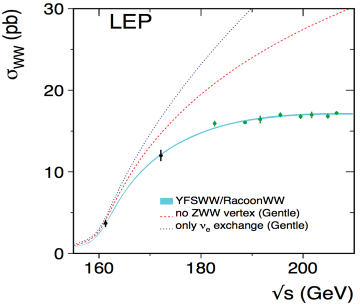
The event generators that implemented this theory for the experimenters played a crucial role. The two most important ones were

ZFITTER : Lida Kalinovskaya, Pena Christova, Dima Bardin, Tord Riemann, Sabine Riemann, Andrej Arbuzov

KORALZ: Stanislaw Jadach, Bennie Ward

M. Peskin @ SM50 – GENTLE
https://indico.cern.ch/event/704471/contributions/3012508/subcontributions/255601/attachments/1660617/2660398/Peskin-PrecisionEW.pdf

The cross section for $e^+e^- \to W^+W^-$ was measured by the LEP experiments, with this result:

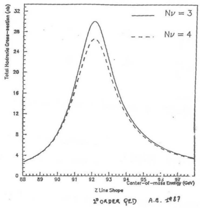


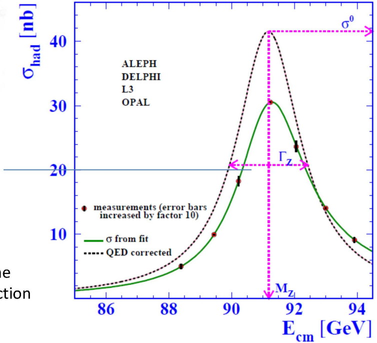
Gentle – Bardin:1996 [1]
GENTLE: A Program for the semianalytic calculation of $e^+e^- \to 4f$
Bardin, Biebel, Lehner, Leike, Olchevski, T. Riemann; 38 pp.
Comput.Phys.Commun. 104 (1997) 161-187

Alain Blondel – The lineshape of the Z boson

at the 2018 conference “SM50 – 50 years of Standard Model”

https://indico.cern.ch/event/704471/contributions/3012507/attachments/1670848/2680273/Blondel.pdf





We had figured out (G. Feldman) that the quantity that is directly sensitive to the number of neutrinos is the peak cross-section (mostly Z→ $q\bar{q}$)

➔ the luminosity measurement had been to object of particular attention with a precision of ±1% (in ALEPH)

By the end of LEP it would be precise to ±0.06%.!)

The key to mass and width measurements is the **beam energy calibration**

$$R_\ell \equiv \Gamma_{had}/\Gamma_\ell$$

$$N_\nu = \frac{\Gamma_\ell}{\Gamma_\nu} \cdot \left(\sqrt{\frac{12\pi R_\ell}{M_Z^2 \sigma_{had}^{peak,0}}} - R_\ell - 3 \right).$$

theory — all measured at the peak

Some basic facts on the ZFITTER project

ZFITTER is a Fortran package

– For the evaluation of quantum field theoretical corrections
– Using the Standard Model of elementary particles and variations of it
– Calculates a variety of observable quantities, notably those related to the Z-boson resonance peak
– Is used for the studies by the LEP collaborations ALEPH, DELPHI, L3, OPAL at CERN and by the prestigeous LEPEWWG – LEP Electroweak Working Group.

ZFITTER is used for an uncountable amount of experimental and phenomenological studies Until today.

The masses of the top quark and of the (assumed) Higgs boson were predicted with ZFITTER prior to their discoveries in 1995 and 2012

This is based on the virtual quantum corrections to lower-energy observables.
And is is visualized in the popular Blue Band Plot of the LEPEWWG.

For the Fortran package ZFITTER applies the
"CPC non-profit use licence agreement of the Computer Physics Communications Program Library"
`http://cpc.cs.qub.ac.uk/licence/licence.html`.

Logical structure of ZFITTER v6.21, 1999 [2]

14 *Introduction*

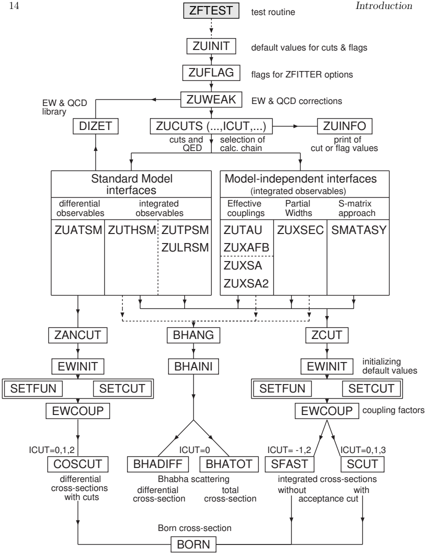


Figure I.2. *Logical structure of the package* ZFITTER

Some Social facts on the ZFITTER project

The ZFITTER project exists since about 1985

Its development was driven mainly at **JINR**, Dubna, Russia, in cooperation with other institutions in **Azerbaidshan, Bulgaria, Germany, Russia**.

The project comprizes about 2.2 millions of Euros

Equivalent to 155 millions of Rubles or 2.4 millions of dollars.
If only counting the investment of about 30 FTE's (Full Time Equivalents) on a regular basis of highly qualified scientists with present-day salary in e.g. Germany.

About half of that is due to project management.
And about half is due to creation of the software.
One half of the second half, in turn, is due to the creation of the Standard Model library of ZFITTER: 7.5 FTE, 550,000 Euro.
The same amount is due to the treatment of QED corrections in ZFITTER.

The estimates are very conservative. They are mentioned here in order to evidence the importance of a proper use of ZFITTER.

Dima and Lida serving the community

Here: At CALC 2006 (photo: T. Riemann, copyright: CC-BY, tord.riemann at hugo-riemann.de, 2006)

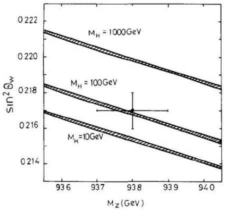
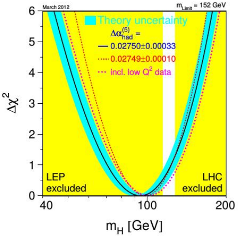


Fig 1 Graph of $\sin^2\theta_W$ versus M_Z, influenced by M_H through radiative corrections The thickness corresponds to the range 30 GeV ≤ m_t ≤ 40 GeV, the error bars indicate the accuracy expected at Z boson factories.



Left: **1985:** A. Akhundov, D. Bardin, TR "Hunting the hidden Higgs" PLB 1985 [3]
Was cited twice: W. Hollik 1987, S. Yost et al. 2011

Right: **2012:** Blue band plot, `http://lepewwg.web.cern.ch/LEPEWWG/`

"The general believe is that this particle is (similar to) the one predicted by Peter Higgs in 1964 [22, 23, 24]. Within less than a year, in October 2013, Peter Higgs and Francois Englert were awarded the Nobel Prize in Physics ... for the theoretical discovery of a mechanism that contributes to our understanding of the origin of mass of subatomic particles, and which recently was confirmed through the discovery of the predicted fundamental particle, by the ATLAS and CMS experiments at CERNs Large Hadron Collider" [25]. The accompanying advanced public information "Scientific Background on the Nobel Prize in Physics 2013: The BEH-Mechanism, Interac- tions with Short Range Forces and Scalar Particles", compiled by the Class for Physics of the Royal Swedish Academy of Sciences [26], ***reproduces the Blue-band plot (March 2012) of the LEPEWWG on page 16. The plot relies on ZFITTER v.6.43.****"*
From: Akhundov, Arbuzov, TR, S. Riemann 2013 [4]
[25] Press release from Royal Swedish Academy of Sciences, 8 October 2013, available from
`http://www.nobelprize.org/nobel_prizes/physics/laureates/2013/press.pdf`.
[26] The Class for Physics of the Royal Swedish Academy of Sciences, Scientific Background on the Nobel Prize in Physics 2013: The BEH-Mechanism, Interactions with Short Range Forces and Scalar Particles. Available from
`http://www.nobelprize.org/nobel_prizes/physics/laureates/2013/advanced-physicsprize2013.pdf`.

First Scientific Award of JINR, Dubna

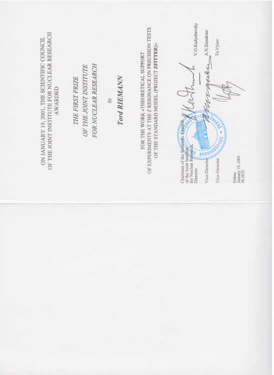
Dima was honored with that price very often, maybe ten times.

The ZFITTER team was honored with the First Scientific Award of JINR, Dubna on 19 January 2001 for

"Theoretical support of experiments at the Z resonance for precision tests of the Standard Model".

The honored ZFITTER authors:

Professor D. Bardin (JINR, Dubna),
Professor M. Bilenky (then JINR, Dubna),
Professor P. Christova (Univ. Shoumen, Bulgaria),
Professor M. Jack (Florida A&M University, Tallahassee, Florida),
Professor L. Kalinovskaya (JINR, Dubna),
Professor A. Olshevsky (JINR, Dubna),
Dr. S. Riemann (DESY, Zeuthen),
Dr. T. Riemann (DESY, Zeuthen).


ON JANUARY 19, 2001, THE SCIENTIFIC COUNCIL
OF THE JOINT INSTITUTE FOR NUCLEAR RESEARCH
AWARDED

THE FIRST PRIZE
OF THE JOINT INSTITUTE
FOR NUCLEAR RESEARCH

to

Tord RIEMANN

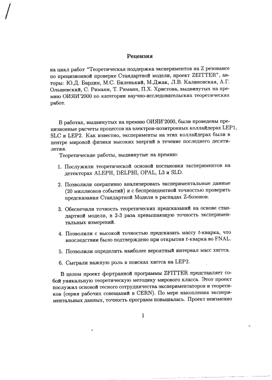
FOR THE WORK «THEORETICAL SUPPORT
OF EXPERIMENTS AT THE Z RESONANCE ON PRECISION TESTS
OF THE STANDARD MODEL (PROJECT ZFITTER)»

Chairman of the Scientific Council of the Joint Institute for Nuclear Research, Director — V.G.Kadyshevsky

Vice-Director — A.N.Sissakian

Vice-Director — Ts.Vylov

Dubna
January 19, 2001
№ 2632


Рецензия

на цикл работ "Теоретическая поддержка экспериментов на Z резонансе по прецизионной проверке Стандартной модели, проект ZEITTER", авторы: Ю.Д. Бардин, М.С. Биленький, М.Джак, Л.В. Калиновская, А.Г. Ольшевский, С. Риманн, Т. Риманн, П.Х. Христова, выдвинутых на премию ОИЯИ'2000 по категории научно-исследовательских теоретических работ.

В работах, выдвинутых на премию ОИЯИ'2000, были проведены прецизионные расчеты процессов на электрон-позитронных коллайдерах LEP1, SLC и LEP2. Как известно, эксперименты на этих коллайдерах были в центре мировой физики высоких энергий в течение последнего десятилетия.

Теоретические работы, выдвинутые на премию:

1. Послужили теоретической основой постановки экспериментов на детекторах ALEPH, DELPHI, OPAL, L3 и SLD.
2. Позволили оперативно анализировать экспериментальные данные (20 миллионов событий) и с беспрецедентной точностью проверить предсказания Стандартной Модели в распадах Z-бозонов.
3. Обеспечили точность теоретических предсказаний на основе стандартной модели, в 2-3 раза превышающую точность экспериментальных измерений.
4. Позволили с высокой точностью предсказать массу t-кварка, что впоследствии было подтверждено при открытии t-кварка во FNAL.
5. Позволили определить наиболее вероятный интервал масс хиггса.
6. Сыграли важную роль в поисках хиггса на LEP2.

В целом проект фортранной программы ZFITTER представляет собой уникальную теоретическую методику мирового класса. Этот проект послужил основой тесного сотрудничества экспериментаторов и теоретиков (серия рабочих совещаний в CERN). По мере накопления экспериментальных данных, точность программ повышалась. Проект неизменно

1

Review

on the work "Theoretical support of experiments on the Z resonance by precision tests of the Standard Model, the project ZFITTER", authors: D. Yu. Bardin, M.S. Bilenky, M. Jack, L. Kalinovskaya, A.G. Olshevski, S. Riemann, T. Riemann, P.C. Christova, nominated for the award "Premia JINR 2000" in the category "Theoretical Research".

In the articles nominated for the award in 2000 JINR, there were conducted precision calculations for processes at the electron-positron colliders LEP1, SLC and LEP2. As is well-known, experiments at these colliders were in the center of the international high energy physics in the last ten years.

The theoretical articles which were proposed for the award:

1. Served as the theoretical basis of the experiments on the detectors ALEPH, DELPHI, OPAL, L3 and SLD.

2. Allowed to analyze the experimental data (20 million events) and to check the predictions of the Standard Model in the decays of Z bosons with unprecedented accuracy.

3. Ensured the accuracy of theoretical predictions based on the standard model at 2-3 times the accuracy of the experimental measurements.

4. Allowed to accurately predict the mass of the t-quark, which subsequently was confirmed by the discovery of the t-quark at FNAL.

5. Allowed to determine the most likely Higgs mass range.

6. Played an important role in the search for the Higgs at LEP2.

Overall, the project "ZFITTER Fortran program" represents a unique theoretical tool of world class. The project formed the basis of a close cooperation of experimetalists and theoreticians (with a series of workshops at CERN). With the accumulation of experimental data, the accuracy of the programs has been increased. The project has always found great interest at conferences. Its importance and the interest to it shows with numerous references in articles, reviews and monographies.

In the long term, with the advent of more precise experiments, ZFITTER will allow to take into account all two-loop electroweak corrections.

The series of theoretical articles on precision tests of the Standard Model at electron-positron colliders certainly deserves the award of the JINR prize 2000.

Academician L.B. Okun

Translated from scanned, OCR interpreted with Terressac and gimagereader by google, with minor improvements by T.R.

Okun: JINR prize 2000

"In the long term, with the advent of more precise experiments, ZFITTER will allow to take into account all two-loop electroweak corrections."

"The series of theoretical articles on precision tests of the Standard Model at electron-positron colliders certainly deserves the award of the JINR prize 2000."

Academician L.B. Okun

EPJC 60 (2009) 543 – wrong Erratum EPJC 71 (2011) 1718

Revisiting the Global Electroweak Fit of the Standard Model and Beyond with Gfitter

The Gfitter Group

H. Flächer[a], M. Goebel[b,c], J. Haller[c], A. Hoecker[a], K. Mönig[b], J. Stelzer[b]

[a]CERN, Geneva, Switzerland
[b]DESY, Hamburg and Zeuthen, Germany
[c]Institut für Experimentalphysik, Universität Hamburg, Hamburg, Germany

The first theoretical framework implemented in Gfitter has been the SM predictions for the electroweak precision observables measured by the LEP, SLC, and the Tevatron experiments. State-of-the-art calculations have been used, and – wherever possible – the results have been cross-checked against the ZFITTER package [5]. For the W mass and the effective weak mixing angle, which

Several theoretical libraries within and beyond the SM have been developed in the past, which, tied to a multi-parameter minimisation program, allowed to constrain the unbound parameters of the SM [5–8]. However, most of these programs are relatively old, were implemented in outdated programming languages, and are difficult to maintain in line with the theoretical and experimental progress. It is unsatisfactory to rely on them during the forthcoming era of the Large Hadron Collider (LHC) and the preparations for future linear collider projects. Improved measurements of important input observables are expected and new observables from discoveries may augment the available constraints. None of the previous programs were modular enough to easily allow the theoretical predictions to be extended to models beyond the SM, and they are usually tied to a particular minimisation package.

These considerations led to the development of the generic fitting package *Gfitter* [9], designed to provide a framework for model testing in high-energy physics. Gfitter is implemented in C++

Several theoretical libraries within and beyond the SM have been developed in the past, which, tied to a multi-parameter minimisation program, allowed to constrain the unbound parameters of the SM [5–8].
However, most of these programs are relatively **old**, were implemented in **outdated** programming languages, and are **difficult to maintain** in ... progress. It is **unsatisfactory** to rely on them during the forthcoming era ...
...
None of the previous programs **were modular** enough to ... allow ... to be extended ... beyond the SM, and they are usually tied to a **particular minimisation package**.
These considerations led to the development of the generic fitting package Gfitter [9], designed to provide a framework for model testing in high-energy physics. Gfitter is implemented in C++ ...

Ref. [5–8] = ZFITTER (Bardin et al.), TOPAZ0 (Passarino et al.), DAPP (Erler)
Ref. [9] = `http://cern.ch/Gfitter`

Comments on the Gfitter-plagiarism:

`http://sanc.jinr.ru/users/zfitter`, `http://zfitter.com`, `http://zfitter.education`
`http://zfitter-gfitter.desy.de/` – with wrong statements of DESY directors, but also with the excellent expertise of the independent expert Dr. Obermöller

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arXiv:hep-ph/9612409, doi:10.1016/S0010-4655(97)00051-9.

[2] D. Bardin, M. Bilenky, P. Christova, M. Jack, L. Kalinovskaya, A. Olchevski, S. Riemann, T. Riemann, ZFITTER 6.21: A semi-analytical program for fermion pair production in e^+e^- annihilation, Comput. Phys. Commun. 133 (2001) 229–395.
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[5] D. Bardin, P. Khristova, O. Fedorenko, On the lowest order electroweak corrections to spin 1/2 fermion scattering. 1. The one loop diagrammar, Nucl. Phys. B175 (1980) 435.
doi:10.1016/0550-3213(80)90021-8.

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[7] G. Passarino, Hard bremsstrahlung corrections for the process $e^+e^- \to \mu^+\mu^-$, Nucl. Phys. B204 (1982) 237–266, doi:10.1016/0550-3213(82)90147-X.
doi:10.1016/0550-3213(82)90147-X.

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[9] P. Christova, M. Jack, T. Riemann, Hard photon emission in $e^+e^- \to \bar{f}f$ with realistic cuts, Phys. Lett. B456 (1999) 264–269.
arXiv:hep-ph/9902408, doi:10.1016/S0370-2693(99)00528-6.

[10] M. A. Jack, Semianalytical calculation of QED radiative corrections to $e^+e^- \to \bar{f}f$ with special emphasis on kinematical cuts to the final state. DESY-THESIS-2000-030.
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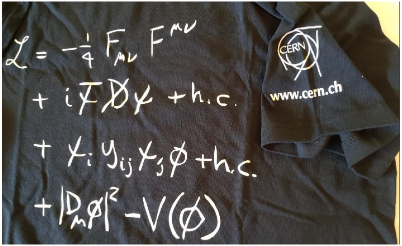
arXiv:1302.1395 The authors of the article got the Prize of the Joint Institute for Nuclear Research (JINR) for the best publication in 2014 in "Physics of Elementary Particles and Nuclei" (PEPAN).

Science



The CERN propaganda Lagrangean (left), compared to the one used by Dima (down; though different gauge)

From: T.D. Gutierrez
at http://nuclear.ucdavis.edu/~tgutierr/files/stmL1.html

Exercise 1.1.1.1.1a:

Given locality, causality, Lorentz invariance, and known physical data since 1860, show that the Lagrangian describing all observed physical processes (sans gravity) can be written:

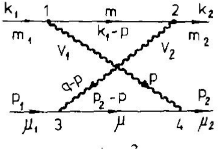
$$\mathcal{L} =$$

$$-\frac{1}{2}\partial_\nu g^a_\mu \partial_\nu g^a_\mu - g_s f^{abc}\partial_\mu g^a_\nu g^b_\mu g^c_\nu - \frac{1}{4}g_s^2 f^{abc} f^{ade} g^b_\mu g^c_\nu g^d_\mu g^e_\nu + \frac{1}{2}ig_s^2(\bar{q}^\sigma_i \gamma^\mu q^\sigma_j)g^a_\mu + \bar{G}^a\partial^2 G^a +$$
$$g_s f^{abc}\partial_\mu \bar{G}^a G^b g^c_\mu - \partial_\nu W^+_\mu \partial_\nu W^-_\mu - M^2 W^+_\mu W^-_\mu - \frac{1}{2}\partial_\nu Z^0_\mu \partial_\nu Z^0_\mu - \frac{1}{2c_w^2}M^2 Z^0_\mu Z^0_\mu -$$
$$\frac{1}{2}\partial_\mu A_\nu \partial_\mu A_\nu - \frac{1}{2}\partial_\mu H \partial_\mu H - \frac{1}{2}m_h^2 H^2 - \partial_\mu \phi^+ \partial_\mu \phi^- - M^2 \phi^+ \phi^- - \frac{1}{2}\partial_\mu \phi^0 \partial_\mu \phi^0 -$$
$$\frac{1}{2c_w^2}M\phi^0\phi^0 - \beta_h[\frac{2M^2}{g^2} + \frac{2M}{g}H + \frac{1}{2}(H^2 + \phi^0\phi^0 + 2\phi^+\phi^-)] + \frac{2M^4}{g^2}\alpha_h - igc_w[\partial_\nu Z^0_\mu(W^+_\mu W^-_\nu -$$
$$W^+_\nu W^-_\mu) - Z^0_\nu(W^+_\mu \partial_\nu W^-_\mu - W^-_\mu \partial_\nu W^+_\mu) + Z^0_\mu(W^+_\nu \partial_\nu W^-_\mu - W^-_\nu \partial_\nu W^+_\mu)] -$$
$$igs_w[\partial_\nu A_\mu(W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - A_\nu(W^+_\mu \partial_\nu W^-_\mu - W^-_\mu \partial_\nu W^+_\mu) + A_\mu(W^+_\nu \partial_\nu W^-_\mu -$$
$$W^-_\nu \partial_\nu W^+_\mu)] - \frac{1}{2}g^2 W^+_\mu W^-_\mu W^+_\nu W^-_\nu + \frac{1}{2}g^2 W^+_\mu W^-_\nu W^+_\mu W^-_\nu + g^2 c_w^2(Z^0_\mu W^+_\mu Z^0_\nu W^-_\nu -$$
$$Z^0_\mu Z^0_\mu W^+_\nu W^-_\nu) + g^2 s_w^2(A_\mu W^+_\mu A_\nu W^-_\nu - A_\mu A_\mu W^+_\nu W^-_\nu) + g^2 s_w c_w[A_\mu Z^0_\nu(W^+_\mu W^-_\nu -$$
$$W^+_\nu W^-_\mu) - 2A_\mu Z^0_\mu W^+_\nu W^-_\nu] - g\alpha[H^3 + H\phi^0\phi^0 + 2H\phi^+\phi^-] - \frac{1}{8}g^2\alpha_h[H^4 + (\phi^0)^4 +$$
$$4(\phi^+\phi^-)^2 + 4(\phi^0)^2\phi^+\phi^- + 4H^2\phi^+\phi^- + 2(\phi^0)^2H^2] - gMW^+_\mu W^-_\mu H - \frac{1}{2}g\frac{M}{c_w^2}Z^0_\mu Z^0_\mu H -$$
$$\frac{1}{2}ig[W^+_\mu(\phi^0\partial_\mu\phi^- - \phi^-\partial_\mu\phi^0) - W^-_\mu(\phi^0\partial_\mu\phi^+ - \phi^+\partial_\mu\phi^0)] + \frac{1}{2}g[W^+_\mu(H\partial_\mu\phi^- - \phi^-\partial_\mu H) -$$
$$W^-_\mu(H\partial_\mu\phi^+ - \phi^+\partial_\mu H)] + \frac{1}{2}g\frac{1}{c_w}(Z^0_\mu(H\partial_\mu\phi^0 - \phi^0\partial_\mu H) - ig\frac{s_w^2}{c_w}MZ^0_\mu(W^+_\mu\phi^- - W^-_\mu\phi^+) +$$

$$igs_w MA_\mu(W^+_\mu\phi^- - W^-_\mu\phi^+) - ig\frac{1-2c_w^2}{2c_w}Z^0_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) + igs_w A_\mu(\phi^+\partial_\mu\phi^- -$$
$$\phi^-\partial_\mu\phi^+) - \frac{1}{4}g^2 W^+_\mu W^-_\mu[H^2 + (\phi^0)^2 + 2\phi^+\phi^-] - \frac{1}{4}g^2\frac{1}{c_w^2}Z^0_\mu Z^0_\mu[H^2 + (\phi^0)^2 + 2(2s_w^2 -$$
$$1)^2\phi^+\phi^-] - \frac{1}{2}g^2\frac{s_w^2}{c_w}Z^0_\mu\phi^0(W^+_\mu\phi^- + W^-_\mu\phi^+) - \frac{1}{2}ig^2\frac{s_w^2}{c_w}Z^0_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) +$$
$$\frac{1}{2}g^2 s_w A_\mu\phi^0(W^+_\mu\phi^- + W^-_\mu\phi^+) + \frac{1}{2}ig^2 s_w A_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) - g^2\frac{s_w}{c_w}(2c_w^2 -$$
$$1)Z^0_\mu A_\mu\phi^+\phi^- - g^1 s_w^2 A_\mu A_\mu\phi^+\phi^- - \bar{e}^\lambda(\gamma\partial + m_e^\lambda)e^\lambda - \bar{\nu}^\lambda\gamma\partial\nu^\lambda - \bar{u}^\lambda_j(\gamma\partial + m_u^\lambda)u^\lambda_j -$$
$$\bar{d}^\lambda_j(\gamma\partial + m_d^\lambda)d^\lambda_j + igs_w A_\mu[-(\bar{e}^\lambda\gamma^\mu e^\lambda) + \frac{2}{3}(\bar{u}^\lambda_j\gamma^\mu u^\lambda_j) - \frac{1}{3}(\bar{d}^\lambda_j\gamma^\mu d^\lambda_j)] + \frac{ig}{4c_w}Z^0_\mu[(\bar{\nu}^\lambda\gamma^\mu(1 +$$
$$\gamma^5)\nu^\lambda) + (\bar{e}^\lambda\gamma^\mu(4s_w^2 - 1 - \gamma^5)e^\lambda) + (\bar{u}^\lambda_j\gamma^\mu(\frac{4}{3}s_w^2 - 1 - \gamma^5)u^\lambda_j) + (\bar{d}^\lambda_j\gamma^\mu(1 - \frac{8}{3}s_w^2 - \gamma^5)d^\lambda_j)] +$$
$$\frac{ig}{2\sqrt{2}}W^+_\mu[(\bar{\nu}^\lambda\gamma^\mu(1 + \gamma^5)e^\lambda) + (\bar{u}^\lambda_j\gamma^\mu(1 + \gamma^5)C_{\lambda\kappa}d^\kappa_j)] + \frac{ig}{2\sqrt{2}}W^-_\mu[(\bar{e}^\lambda\gamma^\mu(1 + \gamma^5)\nu^\lambda) +$$
$$(\bar{d}^\kappa_j C^\dagger_{\lambda\kappa}\gamma^\mu(1 + \gamma^5)u^\lambda_j)] + \frac{ig}{2\sqrt{2}}\frac{m_e^\lambda}{M}[-\phi^+(\bar{\nu}^\lambda(1 - \gamma^5)e^\lambda) + \phi^-(\bar{e}^\lambda(1 + \gamma^5)\nu^\lambda)] - \frac{g}{2}\frac{m_e^\lambda}{M}[H(\bar{e}^\lambda e^\lambda) +$$
$$i\phi^0(\bar{e}^\lambda\gamma^5 e^\lambda)] + \frac{ig}{2M\sqrt{2}}\phi^+[-m_d^\kappa(\bar{u}^\lambda_j C_{\lambda\kappa}(1 - \gamma^5)d^\kappa_j) + m_u^\lambda(\bar{u}^\lambda_j C_{\lambda\kappa}(1 + \gamma^5)d^\kappa_j] +$$
$$\frac{ig}{2M\sqrt{2}}\phi^-[m_d^\lambda(\bar{d}^\lambda_j C^\dagger_{\lambda\kappa}(1 + \gamma^5)u^\kappa_j) - m_u^\kappa(\bar{d}^\lambda_j C^\dagger_{\lambda\kappa}(1 - \gamma^5)u^\kappa_j] - \frac{g}{2}\frac{m_u^\lambda}{M}H(\bar{u}^\lambda_j u^\lambda_j) - \frac{g}{2}\frac{m_d^\lambda}{M}H(\bar{d}^\lambda_j d^\lambda_j) +$$
$$\frac{ig}{2}\frac{m_u^\lambda}{M}\phi^0(\bar{u}^\lambda_j\gamma^5 u^\lambda_j) - \frac{ig}{2}\frac{m_d^\lambda}{M}\phi^0(\bar{d}^\lambda_j\gamma^5 d^\lambda_j) + \bar{X}^+(\partial^2 - M^2)X^+ + \bar{X}^-(\partial^2 - M^2)X^- +$$
$$\bar{X}^0(\partial^2 - \frac{M^2}{c_w^2})X^0 + \bar{Y}\partial^2 Y + igc_w W^+_\mu(\partial_\mu\bar{X}^0 X^- - \partial_\mu\bar{X}^+ X^0) + igs_w W^+_\mu(\partial_\mu\bar{Y}X^- -$$
$$\partial_\mu\bar{X}^+ Y) + igc_w W^-_\mu(\partial_\mu\bar{X}^- X^0 - \partial_\mu\bar{X}^0 X^+) + igs_w W^-_\mu(\partial_\mu\bar{X}^- Y - \partial_\mu\bar{Y}X^+) +$$
$$igc_w Z^0_\mu(\partial_\mu\bar{X}^+ X^+ - \partial_\mu\bar{X}^- X^-) + igs_w A_\mu(\partial_\mu\bar{X}^+ X^+ - \partial_\mu\bar{X}^- X^-) - \frac{1}{2}gM[\bar{X}^+ X^+ H +$$
$$\bar{X}^- X^- H + \frac{1}{c_w^2}\bar{X}^0 X^0 H] + \frac{1-2c_w^2}{2c_w}igM[\bar{X}^+ X^0\phi^+ - \bar{X}^- X^0\phi^-] + \frac{1}{2c_w}igM[\bar{X}^0 X^-\phi^+ -$$
$$\bar{X}^0 X^+\phi^-] + igMs_w[\bar{X}^0 X^-\phi^+ - \bar{X}^0 X^+\phi^-] + \frac{1}{2}igM[\bar{X}^+ X^+\phi^0 - \bar{X}^- X^-\phi^0]$$

Source: Thomas D. Gutierrez, nuclear.ucdavis.edu/~tgutierr/files/sml2.pdf, cached (14 July 2018)

$$\mathcal{A}(e^+e^- \to Z \to f\bar{f}) = \frac{4ie^2 I_e^{(3)} I_f^{(3)}}{s - M_Z^2 + iM_Z\Gamma_Z}\rho_{ef}$$
$$\times [\gamma_\mu(1+\gamma_5) \otimes \gamma^\mu(1+\gamma_5)$$
$$-4|Q_e|s_W^2\kappa_e\gamma_\mu \otimes \gamma^\mu(1+\gamma_5) - 4|Q_f|s_W^2\kappa_f\gamma_\mu(1+\gamma_5) \otimes \gamma^\mu$$
$$+16|Q_eQ_f|s_W^4\kappa_{ef}\gamma_\mu \otimes \gamma^\mu]$$



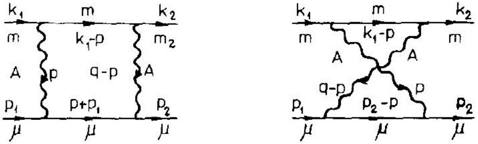


Figure 4.11 from [5], showing a generic massive crossed one-loop box diagram of the ZZ and WW box type and Figure 4.20 with the two photon box diagrams. The latter two diagrams have to be combined with initial-final state interference soft photon bremsstrahlung in order to get a finite result.

$$B(q) = i\frac{f_1^V f_2^V f_3^V f_4^V}{16\pi^2}\Big\{\Big[-u+v-\frac{1}{3}\frac{m_2\mu_2}{M_1^2M_2^2}(A_1+B_1\gamma_5)\otimes(A_3+B_3\gamma_5)$$
$$-\frac{2}{3}\frac{m_1\mu_2}{M_1^2M_2^2}(A_1-B_1\gamma_5)\otimes(A_3+B_3\gamma_5)-\frac{2}{3}\frac{m_2\mu_1}{M_1^2M_2^2}(A_1+B_1\gamma_5)\otimes(A_3-B_3\gamma_5)$$
$$-\frac{1}{3}\frac{m_1\mu_1}{M_1^2M_2^2}(A_1-B_1\gamma_5)\otimes(A_3-B_3\gamma_5)\Big]P+[\omega(q^2,M_1^2,M_2^2)$$
$$+2(S-q^2)B(q^2,q^2-S;M_1^2,M_2^2)]\gamma_\alpha(A_1+B_1\gamma_5)\otimes\gamma_\alpha(A_3+B_3\gamma_5)$$
$$+\frac{1}{S}[A(q^2,q^2-S;M_1^2,M_2^2)+C(q^2,q^2-S;M_1^2,M_2^2)-D(q^2-S,S)]$$
$$\times[\gamma_\alpha(A_1+B_1\gamma_5)\otimes\gamma_\alpha(A_3+B_3\gamma_5)$$
$$+\gamma_\alpha\gamma_5(A_1+B_1\gamma_5)\otimes\gamma_\alpha\gamma_5(A_3+B_3\gamma_5)]\Big\}. \quad (4.12)$$

$$B(q) = i\frac{e^4(f^q)^2(\mathscr{F}^q)^2}{16\pi^2}\Big\{\Big[4\frac{S-m^2-\mu^2}{q^2}\mathscr{I}(-S,m^2,\mu^2)P_{IR}$$
$$+4\frac{S-q^2+m^2+\mu^2}{q^2}\mathscr{I}(S-q^2,m^2,\mu^2)P_{IR}$$
$$+4\frac{1}{q^2}\ln\left|\frac{S-q^2}{S}\right|\ln\frac{|q^2|}{M_W^2}\Big]\gamma_\alpha\otimes\gamma_\alpha$$

$+\frac{1}{S-}$

$+\frac{1}{S}\Big[$

$\times(\gamma_\alpha$

with $\varepsilon = 1$ for q

Eqns. 4.12 and 4.21 from [5] with the contributions of Figs. 4.11 and 4.20 to the general 2→2 matrix element in the unitary gauge.

$$B(q) = i\frac{f_1^V f_2^V f_3^V f_4^V}{16\pi^2}\Big\{\Big[-u + v - \frac{1}{3}\frac{m_2\mu_2}{M_1^2 M_2^2}(A_1 + B_1\gamma_5)\otimes(A_3 + B_3\gamma_5)$$
$$-\frac{2}{3}\frac{m_1\mu_2}{M_1^2 M_2^2}(A_1 - B_1\gamma_5)\otimes(A_3 + B_3\gamma_5) - \frac{2}{3}\frac{m_2\mu_1}{M_1^2 M_2^2}(A_1 + B_1\gamma_5)\otimes(A_3 - B_3\gamma_5)$$
$$-\frac{1}{3}\frac{m_1\mu_1}{M_1^2 M_2^2}(A_1 - B_1\gamma_5)\otimes(A_3 - B_3\gamma_5)\Big]P + [\omega(q^2, M_1^2, M_2^2)$$
$$+2(S - q^2)B(q^2, q^2 - S; M_1^2, M_2^2)]\gamma_\alpha(A_1 + B_1\gamma_5)\otimes\gamma_\alpha(A_3 + B_3\gamma_5)$$
$$+\frac{1}{S}[A(q^2, q^2 - S; M_1^2, M_2^2) + C(q^2, q^2 - S; M_1^2, M_2^2) - D(q^2 - S, S)]$$
$$\times[\gamma_\alpha(A_1 + B_1\gamma_5)\otimes\gamma_\alpha(A_3 + B_3\gamma_5)$$
$$+\gamma_\alpha\gamma_5(A_1 + B_1\gamma_5)\otimes\gamma_\alpha\gamma_5(A_3 + B_3\gamma_5)]\Big\}\,. \qquad (4.12)$$

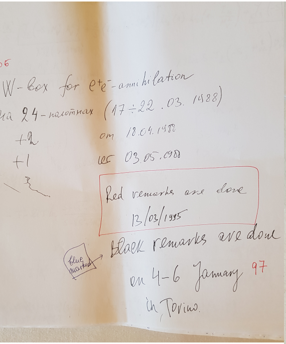
$$\mathcal{A}(e^+e^- \to Z \to f\bar{f}) = \frac{4ie^2 I_e^{(3)} I_f^{(3)}}{s - M_Z^2 + iM_Z\Gamma_Z}\rho_{ef}$$
$$\times\,[\gamma_\mu(1+\gamma_5)\otimes\gamma^\mu(1+\gamma_5)$$
$$-4|Q_e|s_W^2\kappa_e\gamma_\mu\otimes\gamma^\mu(1+\gamma_5) - 4|Q_f|s_W^2\kappa_f\gamma_\mu(1+\gamma_5)\otimes\gamma^\mu$$
$$+16|Q_eQ_f|s_W^4\kappa_{ef}\gamma_\mu\otimes\gamma^\mu]$$

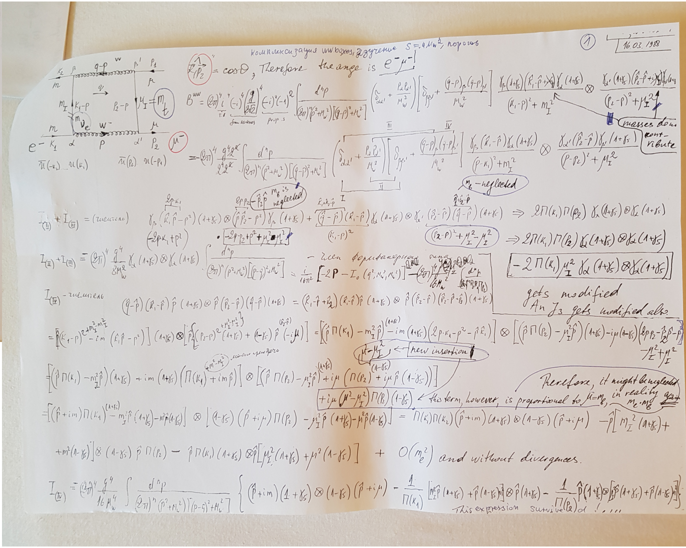
The **four form factors** of the net matrix element are by construction
$\rightarrow$ **gauge-invariant**
and contain
$\rightarrow$ **all** contributions of a given perturbative order.

$$\mathcal{M}^{(0)}(e^-e^+ \to f^-f^+) \sim \rho_Z \Big[v_{ef} \bar{u}(\gamma_\alpha) u \times \bar{v}(\gamma^\alpha) v - a_e v_f \bar{u}(\gamma_\alpha\gamma_5) u \times \bar{v}(\gamma^\alpha) v \\ - v_e a_f \bar{u}(\gamma_\alpha) u \times \bar{v}(\gamma^\alpha\gamma_5) v + a_e a_f \bar{u}(\gamma_\alpha\gamma_5) u \times \bar{v}(\gamma^\alpha\gamma_5) v \Big]. \quad (1)$$

$$\mathcal{A}[e^+e^- \to f\bar{f}] = 4\pi i\,\alpha \frac{Q_e Q_f}{s} \gamma_\mu \otimes \gamma^\mu \\ + i \frac{\sqrt{2} G_F M_Z^2}{1 + i\Gamma_Z/M_Z} I_e^{(3)} I_f^{(3)} \frac{1}{s - \overline{M}_Z^2 + i\overline{M}_Z\overline{\Gamma}_Z} \\ \times \rho_{ef} \Big[\gamma_\mu(1+\gamma_5) \otimes \gamma^\mu(1+\gamma_5) - 4|Q_e| s_W^2 \kappa_e \gamma_\mu \otimes \gamma^\mu(1+\gamma_5) \\ - 4|Q_f| s_W^2 \kappa_f \gamma_\mu(1+\gamma_5) \otimes \gamma^\mu + 16|Q_e Q_f| s_W^4 \kappa_{ef} \gamma_\mu \otimes \gamma^\mu \Big]. \quad (2)$$

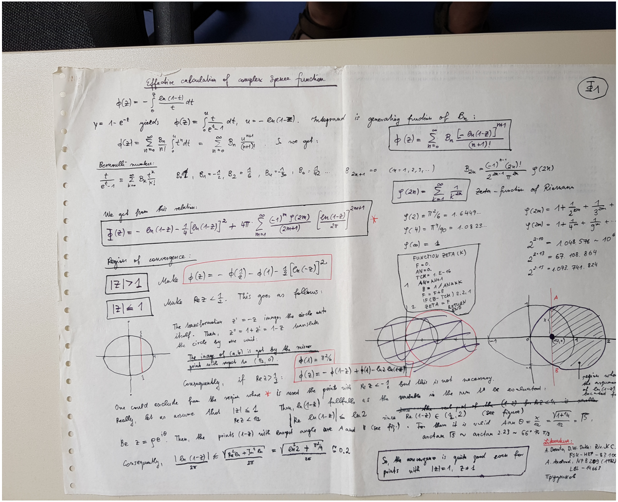
A copy of a sample of Dima's handwritten notes.
Commenting page (left) and first page (right)





How to calculate the Euler dilogarithm with complex argument?

It is the most complicated 1-loop function.
It was needed for LEP physics, and it was not needed before.
Today we have Mathematica and all that.
And we need now much much much more complicated functions for 2-loop calculations.

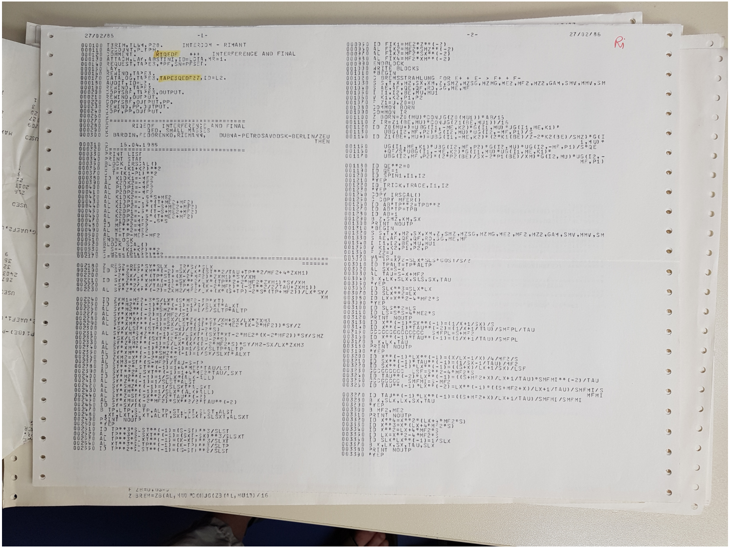


FUNCTION XCDIL(Z)

```
C=====================================================
      IMPLICIT COMPLEX*16(X,Y)
      IMPLICIT REAL*8(A-H,O-W,Z)
C
      COMPLEX*16 Z,Z1,CLZ,CLZP,CADD,LOG
      COMMON/CDZPIF/PI,F1
      DIMENSION ZETA(15)
      EXTERNAL FZETA
      DATA N/0/,TCH/1.D-16/
      SAVE ZETA
C
      JPRINT=0
C
      PI2=PI*PI
      PI4=PI2*PI2
      AAZ=CDABS(Z)
      REZ=DREAL(Z)
      IF(AAZ-1.) 4,2,3
 3    PRINT 1000
 1000 FORMAT(3X,6 (15HERROR MODULUS Z) )
      GOTO 881
 2    IF(REZ-.5) 4,4,3
 4    CONTINUE
      N=N+1
      IF(N-1) 5,5,6
 5    DO 11 I=4,15
      ZETA(I)=0.D0
 11   CONTINUE
      ZETA(1)=F1
      ZETA(2)=PI4/90.
      ZETA(3)=PI4*PI2/945.
 6    CONTINUE
      Z1=DCMPLX(1.D0,0.D0)-Z
      CLZ=LOG(Z1)
      XCDIL=-CLZ-.25*(CLZ)**2
      M=0
      CLZP=CLZ/(2.*PI)
 88   M=M+1
      IF(M-15) 882,882,883
 883  PRINT 1001
 1001 FORMAT(2X,3 (24HERROR-YOU NEED MORE ZETA) )
      GOTO 881
 882  IF(ZETA(M)) 884,884,885
 884  ZETA(M)=FZETA(2*M)
 885  HZETA=ZETA(M)
```

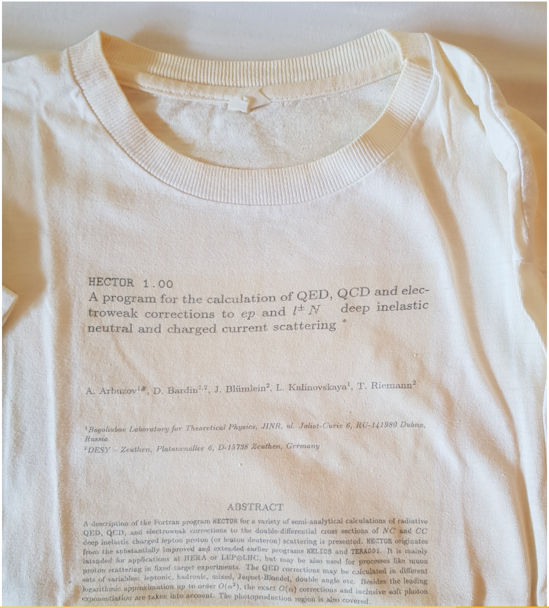
A calculation: Integration of QED corrections, 16. April 1985 – 27. Februar 1986

A typical of our SCHOONSCHIP codes by Tiny Veltman. It was run only at a CDC under US embargo. Later we had FORM by Jos Vermaseren.

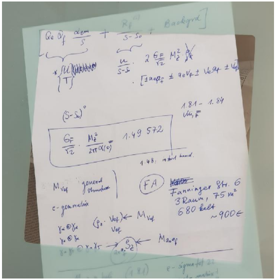


Complete calculations. Exact calculations
Complete calculations. Exact calculations

The title page of the HECTOR article. HECTOR was used for QED calculations at HERA for many years.
Copyright: Prof. Dr. Lida Kalinovskaya



Responsibility and all that

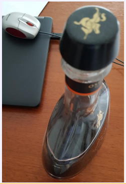


Fictitious desk with a glass plate. The original was in Dima's office. It was used for tea drinking. But also used for retaining hand-signed documents with declarations of correctness of SCHOONSCHIP files. Over many years.

"Exact Calculations of the Lowest Order Electromagnetic Corrections for the Processes $e^+e^- \to \mu^+\mu^-$"
Akhundov, Bardin, TR: Sov. J. Nucl. Phys. 42 (1985) 762, Yad. Fiz. 42 (1985) 1204-1210 [6]

Checks, checks, checks

Once AAA won a bottle of brandy.
We had a mistake. Needed search! Dima spent a bottle brandy for the lucky guy.
During several weeks, I collected all formulae needed to discover the correct formula for a scattering of massive electrons into massive muons, with some QED corrections. Was later published
I went for lunch.
AAA combined in the meantime the pices – it was correct!
He drank the bottle alone!



From 11 December 1986 till 02 April 1987 (= 4 months) we were searching an error.

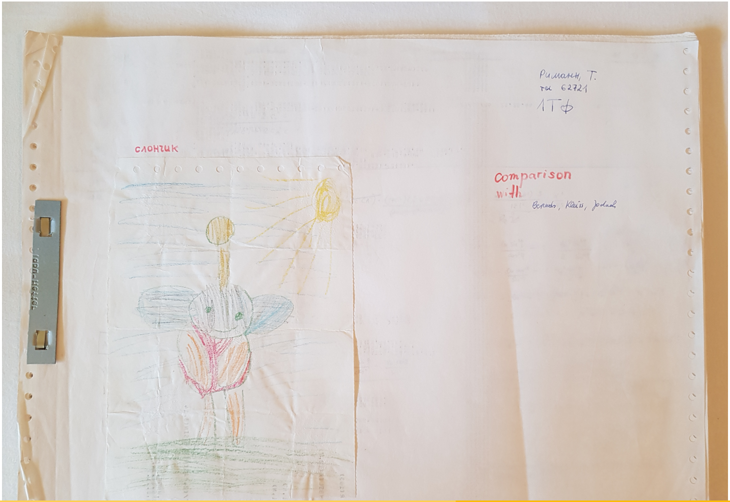
Compared to a Monte Carlo program by Frits Berends, Ronald Kleiss and Stashek Jadach, we were deviating at $s = M_Z^2$.

Dima believed in their numbers.

Finally I found, sleeping at night, the ONE wrong line in a SCHOONSCHIP program.

Of the type $\Re e(A \times B) = \Re e(A) \times \Re e(B) - \Im m(a) \times \Im m(B) \rightarrow \Re e(A) \times \Re e(B)$

Next morning, within 5 minutes (the runtime), our world was OK.



$$
\begin{aligned}
\sigma_T \quad = \quad & \sigma_0 \left\{ Q_f^2 \left[1 + \frac{\alpha}{\pi} (F_0^T + Q_f^2 F_2^T) \right] \right. \\
+ \quad & 2 \cdot |Q_f| v_e v_f Re \left[\chi + \frac{\alpha}{\pi} \chi (G_0^T + Q_f^2 G_2^T) \right] + 2|Q_f| a_e a_f \frac{\alpha}{\pi} Q_f Re(\chi G_4^T) \\
+ \quad & \left. (v_e^2 + a_e^2)(v_f^2 + a_f^2) |\chi|^2 \left[1 + \frac{\alpha}{\pi} Re[(H_0^T + Q_f^2 H_2^T)] \right] + 4 v_e a_e v_f a_f |\chi|^2 \frac{\alpha}{\pi} Q_f Re[H_4^T] \right\}
\end{aligned} \quad (3)
$$

ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ
ДУБНА

E2-88-324

D.Yu.Bardin, M.S.Bilenky, O.M.Fedorenko*,
T.Riemann

THE ELECTROMAGNETIC α^3 CONTRIBUTIONS
TO e^+e^--ANNIHILATION INTO FERMIONS
IN THE ELECTROWEAK THEORY.
TOTAL CROSS SECTION σ_T
AND INTEGRATED ASYMMETRY A_{FB}

Submitted to "Nuclear Physics"

*Dept. of Physics, State University,
of Petrosavodsk, USSR

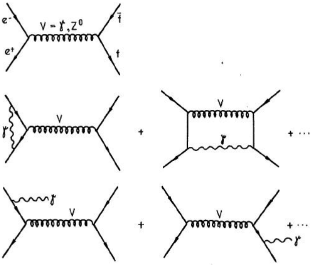


Fig. 1. The QED α^3 radiative contributions to the e^+e^--annihilation into a fermion pair considered in this article.

The trick, using the complex mass function property

integrals /21/. Without going into details, we only ...
to the relation

$$\frac{1}{|s'-M^2|^2} = \frac{i}{2M_Z\Gamma_Z}\left(\frac{1}{s'-M^2} - \frac{1}{s'-M^{*2}}\right)$$

the Z-boson part of the calculation is not considerably mo... cated than the photon-Z-boson interference which is linear... Z-boson propagator.Further, (A.5) shows the mathematical o... tail effects in initial radiation corrections. The s' is t...

The integration is 4-fold

The problem to be solved here is the integration of hard bremsstrahlung. The Lorentz-invariant integration phase space is parametrized as follows:

$$\int d\Gamma = \frac{\pi^2}{4s}\int_{-1}^{+1} d\cos\theta \int_0^1 x\,dx\,\frac{1-x}{1-x+m_f^2/s}\,\frac{1}{4\pi}\int_{-1}^{+1} d\cos\theta_R \int_0^{2\pi} d\varphi_R\,. \qquad (A.2)$$

The θ_R and φ_R are angles of the photon in the rest system of fermion f^- plus photon:

$$\vec{p}_1 + \vec{p} = 0, \qquad (A.3)$$

The twofold integrati...

We need the definition of L_R and of L_s.
Tail

The initial state radiation produces the "radiative tail", if $s > M_Z^2$

$$H_0^T = d + t\left[2R + \frac{1}{2} - |R|^2 + \frac{2R}{R-R^*}(1-R^*)(1+R^2)L_R\right.$$

with

$$d = \frac{\pi^3}{3} - \frac{1}{2}\,, \qquad t = L_e - 1\,.$$

The pure QED function F_0^T is known from /11/. The ot... tial-state corrections depend on one additional, compl... R,

$$R = \frac{M^2}{s}$$

with M^2 as defined in (2.9). The R^* is its complex... Further,

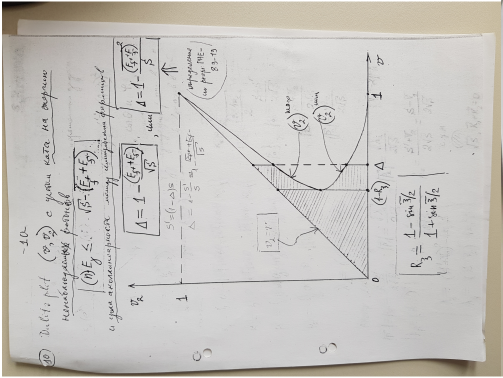
$$L_a = \frac{s}{m_a^2}\,, \quad a = e, f\,,$$

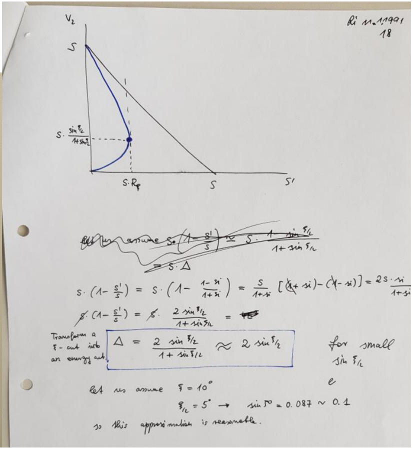
$$L_R = \ln(1 - 1/R)\,.$$

The initial-final state radiation (plus γZ boxes do not create a "radiative tail"

$$H_4^T = -3L_Z\,. \qquad (3.17)$$

The integration region for QED with acollinearity cut.
See Passarino 1982, M. Bilenky 1989, Christova+Jack+TR 1999, Jack PhD 2000 [7, 8, 9, 10]
Original sample: Akhundov for S.Riemann, second: page 17 of my notes, I understood how experimentalists argue (Prof. Min Chen, Chicago + L3 at CERN)





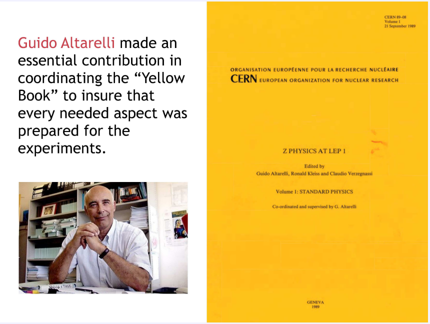
Formulas and Software are not enough · · ·

- ZFITTER version numbers
- ZFITTER descriptions
- ZFITTER webpages and anonymous downloads
- Copyright questions → see e.g. `http://zfitter.com`, anonymous author

M. Peskin at the conference "50 Years of the Standard Model – SM50"

– Altarelli + Yellow Report "Physics at LEP", 1989
https://indico.cern.ch/event/704471/contributions/3012508/subcontributions/255601/attachments/1660617/2660398/Peskin-PrecisionEW.pdf

Peskin's statement



The 1989 LEP workshop and the Yellow Report

F. Berends ... Bardin et al. 1989: [11]: "Z LINE SHAPE"
M. Böhm ... Bardin et al. 1989: [12]: "Forward-backward asymmetries"

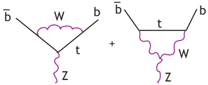
M. Peskin @ SM50 – Zbb 1-loop

https://indico.cern.ch/event/704471/contributions/3012508/subcontributions/255601/attachments/1660617/2660398/Peskin-PrecisionEW.pdf

Peskin's statement

[Michael Peskin forgot Arif Akhundov, but he added Misha B. and Oleg F.]

A special process is the decay $Z \to b\bar{b}$. This obtains a special (finite) correction due to the top quark, enhanced by m_t^2/m_W^2 , as pointed out by Bardin, Bilenky, Fedorenko, and Riemann.



This gives a -2% correction, most visible in the ratio

$$R_b = \frac{\Gamma(Z \to b\bar{b})}{\Gamma(Z \to \text{ hadrons})}$$

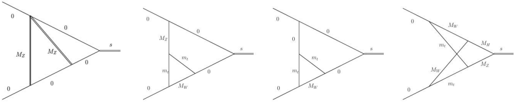
The LEP measurements require this correction.

The $Z\bar{b}b$-vertex is the most complicated of the Zf^+f^--vertices – has one scale more than the others, $\frac{m_t^2}{M_Z^2}$

The complete 1-loop Z boson vertex:
Akhundov, Bardin, TR 1985 [13]
Later also: Jegerlehner 1988 [14], Beenakker+Hollik 1988 [15], Bernabeu 1988 [16].

About 1000 2-loop integrals. See few samples of the complete 2-loop calculation:

Samples of Feynman integrals for the $Z\bar{b}b$ vertex



2016 – Dubovyk, Freitas, Gluza, TR
from [17].

Order	Value $[10^{-4}]$	Order	Value $[10^{-4}]$
α	468.945	$\alpha_t^2\alpha_s$	1.362
$\alpha\alpha_s$	−42.655	α_t^3	0.123
$\alpha_t\alpha_s^2$	−7.074	$\alpha_{\rm ferm}^2$	3.866
$\alpha_t\alpha_s^3$	−1.196	$\alpha_{\rm bos}^2$	−0.986

different orders of radiative corrections to $\Delta\kappa_b$, u

2018 – Dubovyk, Freitas, Gluza, TR, Usovitsch, table from [18].

Γ_i [MeV]	Γ_e	Γ_ν	Γ_d	Γ_u	Γ_b
Born	81.142	160.096	371.141	292.445	369.562
$\mathcal{O}(\alpha)$	2.273	6.174	9.717	5.799	3.857
$\mathcal{O}(\alpha\alpha_s)$	0.288	0.458	1.276	1.156	2.006
$\mathcal{O}(\alpha_t\alpha_s^2,\ \alpha_t\alpha_s^3,\ \alpha_t^2\alpha_s,\ \alpha_t^3)$	0.038	0.059	0.191	0.170	0.190
$\mathcal{O}(N_f^2\alpha^2)$	0.244	0.416	0.698	0.528	0.694
$\mathcal{O}(N_f\alpha^2)$	0.120	0.185	0.493	0.494	0.144
$\mathcal{O}(\alpha_{\rm bos}^2)$	0.017	0.019	0.059	0.058	0.167

Table 2: *Contributions of different orders in perturbation theory to the partial and total Z widths. A fixed value of M_W has be*

A. Blondel @ miniWSJan18, also @S M50 – wishlist for the FCC-ee-Z

observable	Physics	Present precision		FCC-ee stat Syst Precision	FCC-ee key	Challenge
M_Z MeV/c2	Input	91187.5 ±2.1	Z Line shape scan	0.005 MeV <±0.1 MeV	E_cal	QED corrections
Γ_Z MeV/c2	$\Delta\rho$ (T) (no $\Delta\alpha$!)	2495.2 ±2.3	Z Line shape scan	0.008 MeV <±0.1 MeV	E_cal	QED corrections
$R_l = \frac{\Gamma_h}{\Gamma_l}$	α_s, δ_b	20.767 (25)	Z Peak	0.0001 (2-20)	Statistics	QED corrections
N_ν	Unitarity of PMNS, sterile ν's	2.984 ±0.008	Z Peak Z+γ(161 GeV)	0.00008 (40) 0.001	->lumi meast Statistics	QED correction Bhabha scat.
R_b	δ_b	0.21629 (66)	Z Peak	0.000003 (20-60)	Statistics, small IP	Hem. corr, gluon spli
A_{LR}	$\Delta\rho$, ε_3, $\Delta\alpha$ (T, S)	$\sin^2\theta_W^{eff}$ 0.23098(26)	Z peak, Long. polarized	$\sin^2\theta_W^{eff}$ ±0.000006	4 bunch scheme	Design experim
A_{FB}^{lept}	$\Delta\rho$, ε_3, $\Delta\alpha$ (T, S)	$\sin^2\theta_W^{eff}$ 0.23099(53)		$\sin^2\theta_W^{eff}$ ±0.000006	E_cal & Statistics	
M_W MeV/c2	$\Delta\rho$, ε_3, ε_2, $\Delta\alpha$ (T, S, U)	80385 ± 15	Threshold (161 GeV)	0.3 MeV <0.5 MeV	E_cal & Statistics	QED corections
m_{top} MeV/c2 13/01/2018	Input	173200 ± 900	Threshold scan	~10 MeV	E_cal & Statistics	Theory limit at MeV? 10

Dinner at the "Mini-Workshop on precision calculations for the FCC-ee-Z", January 2018

From ZFITTER with 1 1/2 loops to ZFITTER/SANC or so with 2 1/2 loops

Where are our JINR friends ??

From matrix elements to cross-sections and EWPOs (Pseudo-Observables)

Weak amplitudes $\mathcal{M}_{i,Z}$ with $s, U = 1/2\,(1+\cos\theta), T = 1/2\,(1-\cos\theta)$:

$$\mathcal{M}_{1,Z} = \mathcal{M}_Z(e_L^- e_R^+ \to f_L^- f_R^+) = 2\frac{G_F}{\sqrt{2}}M_Z^2\rho_Z\,[1+v_f+v_e+v_{ef}]\,\frac{U}{s-s_0}, \quad (4)$$

$$\mathcal{M}_{2,Z} = \mathcal{M}_Z(e_L^- e_R^+ \to f_R^- f_L^+) = 2\frac{G_F}{\sqrt{2}}M_Z^2\rho_Z\,[-1+v_f-v_e+v_{ef}]\,\frac{T}{s-s_0}, \quad (5)$$

$$\mathcal{M}_{3,Z} = \mathcal{M}_Z(e_R^- e_L^+ \to f_L^- f_R^+) = 2\frac{G_F}{\sqrt{2}}M_Z^2\rho_Z\,[-1-v_f+v_e+v_{ef}]\,\frac{T}{s-s_0}, \quad (6)$$

$$\mathcal{M}_{4,Z} = \mathcal{M}_Z(e_R^- e_L^+ \to f_R^- f_L^+) = 2\frac{G_F}{\sqrt{2}}M_Z^2\rho_Z\,[1-v_f-v_e+v_{ef}]\,\frac{U}{s-s_0}. \quad (7)$$

We mention here that an inclusion of the photon exchange amplitude into the Z amplitude may be performed by the following replacements:

$$v_{ef} \to v_{ef} + \frac{s-s_0}{s}\,Q_eQ_f\,\frac{4\pi\alpha}{\frac{G_F}{\sqrt{2}}M_Z^2\rho_z}. \quad (8)$$

This is a consequence of the above definitions, where both v_{ef} and the γ exchange go together with the matrix element structure $\gamma\otimes\gamma$.

$$\begin{aligned} A_{FB} &= \frac{\sigma_{FB}}{\sigma_{tot}} \\ &= \frac{\left[\int_0^{+1} d\cos\theta - \int_{-1}^{0} d\cos\theta\right] (+|\mathcal{M}_1|^2 + |\mathcal{M}_2|^2 + |\mathcal{M}_3|^2 + |\mathcal{M}_4|^2)}{\int_{-1}^{+1} d\cos\theta (+|\mathcal{M}_1|^2 + |\mathcal{M}_2|^2 + |\mathcal{M}_3|^2 + |\mathcal{M}_4|^2)} \\ &= \frac{3}{4} \frac{2\Re e(v_e v_f^* + v_{ef})}{1 + |v_e|^2 + |v_f|^2 + |v_{ef}|^2} \\ &\approx \frac{3}{4} \frac{2a_e \Re e(v_e)}{a_e^2 + |v_e|^2} \frac{2a_f \Re e(v_f)}{a_f^2 + |v_f|^2} \\ &= \frac{3}{4} \, A_e \, A_f \end{aligned} \tag{9}$$

In the last line of (10) we identify the asymmetry parameters A_e, A_f:

$$\begin{aligned} A_f &= \frac{2a_f \Re e(v_f)}{a_f^2 + |v_f|^2} \equiv \frac{2\frac{\Re e(v_f)}{a_f}}{1 + \frac{|v_f|^2}{a_f^2}} \\ &= \frac{1 - 4|Q_f| \sin^2\theta_f^{\mathrm{eff}}}{1 - 4|Q_f| \sin^2\theta_f^{\mathrm{eff}} + 8|Q_f|^2 \sin^4\theta_f^{\mathrm{eff}}}. \end{aligned} \tag{10}$$

The effective weak mixing angle is defined as

$$\sin^2\theta_f^{\mathrm{eff}} = [1 + \Re e(\kappa_f)] \sin^2\theta_W. \tag{11}$$

The Laurent series

We come now to a next universal point. As was pointed out at LEP times, the MI or SM matrix elements have no arbitrary structure, when considered at the Z peak; let us mention [19] for the MI approach and [20, 21] for a discussion in the SM. Near a resonance the perturbative series has an additional small parameter, $s - M_Z^2$, and when this parameter becomes small, the Z boson width gets important, which itself is a higher order term in perturbative QFT. The width appears in the Breit-Wigner resonance term $1/(s - M_Z^2 + iM_Z\Gamma_Z)$. Further, one has to respect gauge invariance, unitarity, analyticity. These conditions may be fulfilled by the "pole scheme", where the matrix element is set to be a Laurent expansion in the s-plane with a single, simple pole and a Taylor series called "background" [22, 23, 20, 24, 21, 25, 26, 27, 28]:

$$M = \frac{R}{s - s_0} + \sum_{n=0}^{\infty} (s - s_0)^n \, B^{(n)}, \tag{12}$$

where

$$s_0 = \bar{M}_Z^2 + i\bar{M}_Z\bar{\Gamma}_Z. \tag{13}$$

Deriving from this ansatz a scheme for realistic analyses is called the S-matrix approach [24, 29, 30, 31]. A mini review with a rather complete collection of experimental applications is [32].

The residue R and the coefficients $B^{(n)}$ are complex numbers characteristic of the process, and $\bar{M}_Z$ and $\bar{\Gamma}_Z$ are the universal mass and width of the Z particle. The matrix element (**??**) is a scalar function. The equivalent of this can be constructed here by introducing the four independent helicity matrix elements which describe the $2\rightarrow2$ scattering of massless fermions. For the Born case, we follow [33]. The general case may be most easily derived from the Born case by the replacement $v_e v_f \rightarrow v_{ef}$, wherever this combination appears. We will use the notion

$$i \; = \; [1, 2, 3, 4] \; = \; [(LR)(LR), (LR)(RL), (RL)(LR), (RL)(RL)]. \tag{14}$$

As a result of the foregoing considerations, the four helicity matrix elements

$$\mathcal{M}_i(e^+e^- \rightarrow f\bar{f}) \quad = \quad \mathcal{M}_{i,\gamma}^f + \mathcal{M}_{i,Z}^f \tag{15}$$

have the generic form

$$\mathcal{M}_i(e^+e^- \to f\bar{f}) \quad \sim \quad s\,(1 \pm \cos\theta)\left(Q_e Q_f \frac{\alpha_{em}(s)}{s} + \frac{R_f^{(i)}}{s - s_0} + \sum_{n=0}^{\infty} B_{f,n}^{(i)} (s - s_0)^n\right). \tag{16}$$

The last two equations are the central elements of a phenomenological analysis. Their forms result from the demand of a correct determination of the loop corrections in accordance with unitarity, analyticity, gauge invariance. A phenomenological ansatz which is expected not to contradict pertubation theory should respect their forms as well.

The residues $R_f^{(i)}$, the universal pole location s_0 and the coefficients $B_{f,n}^{(i)}$ depend on the chosen model-independent ansatz or on the higher order loop calculations in the beloved theory, maybe in the Standard Model. They are the basic building blocks for any experimental analysis:

$$R_f^{(i)} \quad = \quad 2\frac{G_F}{\sqrt{2}} M_Z^2\, \rho_Z\, [\pm a_e a_f \pm a_e v_f \pm v_e a_f \pm v_{ef}] \tag{17}$$

In Born approximation, the background terms vanish. They arise only from radiative corrections or from New Physics, if not the photon is formally made part of background.

All the above considerations were independent of the underlying picture, be it model independent or be it due to the Standard Model. They are introduced at this length in order to show that the hard scattering process, in which form ever it might be used in some simulation of the real cross sections and asymmetries, there are, around the Z peak, exactly four form factors per final state which suffice to describe any observable:

$$\rho_Z, v_e, v_f, v_{ef}, \tag{18}$$

or

$$\rho_z, \kappa_e, \kappa_f, \kappa_{ef}, \tag{19}$$

or

$$\rho_z, \sin^2\theta_W^{e,\mathrm{eff}}, \sin^2\theta_W^{f,\mathrm{eff}}, \sin^2\theta_W^{ef,\mathrm{eff}}. \tag{20}$$

While, quantities like A_f are tools whose usefulness relies on the correctness of certain approximations. The art of a Z line shape analysis relies on the ability to reduce the many degrees of freedom from experiment to relate sufficiently correct and sufficiently simple to a sufficiently small set of intermediate variables, which are easily described by the theory beloved by the phenomenologist. With one loop accuracy (and a bit beyond) this was prepared in the ZFITTER package [34, 2, 35, 4] and studied at many occasions, notably in [36, 37, 38] and references therein.

For enthusiasts: ZFITTER and correct 2-loop programming

One finds the difference of $\sin^2\theta^f_{\rm eff}$ between ZFITTER and the pole scheme:

$$\sin^2\theta^f_{\text{eff, ZFITTER}} = s^2_W\,\Re e\left\{\kappa^f_Z(M^2_Z)\right\} \tag{24}$$

$$\begin{aligned}\sin^2\theta^f_{\text{eff},\text{pole}} &= \bar{s}^2_W\,\Re e\left\{\bar{\kappa}^f_Z(M^2_Z)\right\} \qquad (25)\\ &= \sin^2\theta^f_{\text{eff, ,ZFITTER}} - \frac{\Gamma_Z}{M_Z}\,\frac{q_f^{(0)}}{a_e^{(0)}(a_f^{(0)}-v_f^{(0)})}\,\Im m\left\{p_e^{(1)}\right\}\end{aligned}$$

with

$$\bar{s}^2_W = \left(1-\frac{\overline{M}^2_W}{\overline{M}^2_Z}\right) = s^2_W\left[1+\frac{c^2_W}{s^2_W}\left(\frac{{\Gamma_W}^2}{{M_W}^2}-\frac{\Gamma^2_Z}{M^2_Z}\right)\right]^{-1}. \tag{26}$$

With equation (8) in [39], it is concluded (we quote from that article): *"In summary, it was found that the treatment of non-resonant contributions in ZFITTER is not consistent with the pole scheme at next-to-next-to-leading order. As a result, the value of* $\sin^2\theta^f_{\rm eff}$ *needs to be corrected by a shift*

$$s^2_W\,\delta\kappa_f = -\frac{\Gamma_Z}{M_Z}\,\frac{q_f^{(0)}}{a_e^{(0)}(a_f^{(0)}-v_f^{(0)})}\,\Im m\left\{p_e^{(1)}\right\}. \tag{21}$$

Numerically this shift amounts to $s^2_W\,\delta\kappa_f \approx 1.5\times10^{-6}$*, well below the current experimental error of* 1.7×10^{-4} *[37]. It was checked that a similar shift* $\delta\kappa_{ef}$ *in the form factor* κ_{ef} *also leads to a negligible numerical effect on* $\sin^2\theta^f_{eff}$*."*

We use the definitions

$$\chi_Z(s) = \frac{G_F M^2_Z}{\sqrt{2}\,2\pi\alpha_{em}}\,\rho_Z(s) \approx 1.49572\,\rho_Z(s), \tag{22}$$

In which features is ZFITTER – and other projects like e.g. HECTOR – original and unique?

- **Complete calculations** in some order of perturbation theory – no need to estimate the role of approximations
- **QED and weak** combined – with all kinds of terms (ini, int, fin, soft, hard)
- **Several independent calculations** often were advocated
- **QED subtraction method** – tricky (and complicated) for huge singularities; see Bardin+Shumeiko: 1976 [40] ... Arbuz.+Bardin+Christ.+Kalin.+TR: 1995 [41]
- **User support**: extensive, with monstrous program descriptions, easy-running of codes, **anonymous download of programs (finished 2012)**
- **Modularity** – allows easily modifications by users and integrations of new elements; examples: ZEFIT (heavy Z' boson), SMATASY (S-matrix approach with Laurent series)
- **FLAGS and BRANCHES and INTERFACES** – allow adaptions to specific purposes
- **4-form-factor notions** – tree $\sin^2\theta_W$ invented for ALL loops at all orders of perturbation theory
- **Bremsstrahlung as complex functions** in ZFITTER and HECTOR:

$$\frac{M_Z^2}{|s-M_Z^2+iM_Z\Gamma_Z|^2} \equiv \frac{i}{2}\frac{M_Z}{\Gamma_Z}\left[\frac{1}{s-M_Z^2+iM_Z\Gamma_Z}-\frac{1}{s-M_Z^2-iM_Z\Gamma_Z}\right] \sim \frac{i}{2}\frac{M_Z}{\Gamma_Z}\times 2\Re e\frac{1}{s-M_Z^2+iM_Z\Gamma_Z} \quad (25)$$

- **Unitary gauge** and many higher order terms – **ZFITTER at 1+1/2 loops**

Dima was a genuine "1-loop person."

See the original 1-loop library "THE ONE LOOP DIAGRAMMAR" BFK (Bardin, Khristova, Fedorenko) of 1980 via ZWRATE (1986) until ZFITTER/DIZET 1989/2005 [5, 42, 13, 43, 44, 45]

Nevertheless, a man of his class looked beyond 1 loop wherever it was possible and/or necessary.

So we can ask:

Dima and 2 loops and higher orders – what about?

Let me make only short remarks.

- In 1989 [46], it became clear even to us what was known since 1975 (Greco et al.) [47]:
  Many photons of low energy are emitted, and this can be treated with Soft Photon Exponentiation
  $1+\epsilon\left[+\frac{1}{2}\epsilon^2+\frac{1}{6}\epsilon^3+\ldots\right]=e^{\epsilon}$
  But this needs a specific phase space parameterization: $s'=\frac{E_\gamma}{E_{beam}}=M_{1l}(f^+f^-)$ must be one of the 4 integration variables!
  We had to change from $\int d\cos\theta\, d\phi_\gamma, d\frac{E}{2E_{beam}}\, d\cos\theta_\gamma\, d\Phi_\gamma$ to some other variables, among them as last integral $\int ds'$
- We (Bardin, Leike, TR, Sachwitz 1988) [48] discovered that the change
  $\frac{1}{s-M_Z^2+iG_Z/M_Z s}\rightarrow\frac{1}{s-M_Z^2+iG_Z M_Z^2}$
  leads to a redefinition
  $M_Z\rightarrow\bar{M}_Z=M_Z-34\text{ MeV},$
  at an experimental LEP accuracy of only 2 MeV.
  This is a 2-loop effect from defining the location of the Z pole in the complex energy plane.
- With Chizhov 1988 [49]: $\mathcal{O}(\alpha\alpha_s)$ a la Djouadi [50], later better: Kniehl [51]
- Starting in about 1990, Dima inserted into ZFITTER all know 2-loop effects, in contact with the authors: Tarasov, Chetyrkin, Kataev, Steinhauser and several others. 1-year workshop [52].
- Many years later, Bardin, Kalinovskaya, Tkachov 2000 [53]:
  "New algebraic numeric methods for loop integrals: Some one loop experience" – How not to do!!

Number of diagrams in 1-loop and 2-loop and 3-loop accuracy

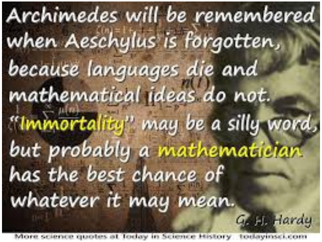
$Z \to b\bar{b}$			
Number of topologies	1 loop	2 loops	3 loops
	1	14 $\overset{(A)}{\to}$ 7 $\overset{(B)}{\to}$ 5	211 $\overset{(A)}{\to}$ 84 $\overset{(B)}{\to}$ 51
Number of diagrams	15	2383 $\overset{(A,B)}{\to}$ 1074	490387 $\overset{(A,B)}{\to}$ 120472
Fermionic loops	0	150	17580
Bosonic loops	15	924	102892
Planar / Non-planar	15 / 0	981/133	84059/36413
QCD / EW	1 / 14	98 / 1016	10386/110086

$Z \to e^+e^-, \ldots$			
Number of topologies	1 loop	2 loops	3 loops
	1	14 $\overset{(A)}{\to}$ 7 $\overset{(B)}{\to}$ 5	211 $\overset{(A)}{\to}$ 84 $\overset{(B)}{\to}$ 51
Number of diagrams	14	2012 $\overset{(A,B)}{\to}$ 880	397690 $\overset{(A,B)}{\to}$ 91472
Fermionic loops	0	114	13104
Bosonic loops	14	766	78368
Planar / Non-planar	14 / 0	782/98	65487/25985
QCD / EW	0 / 14	0 / 880	144/91328

Table 1: Number of topologies and diagrams for $Z \to f\bar{f}$ decays in the Feynman gauge. Statistics for planarity, QCD and EW type diagrams is also given. Label (A) denotes statistics after elimination of tadpoles and wavefunction corrections, and label (B) denotes statistics after elimination of topological symmetries of diagrams.

Besides straightforward improvements in numerical calculations based on Sector decomposition and Mellin-Barnes methods, work on new innovative numerical and analytical techniques (and combinations thereof) should continue and may lead to accelerated progress

What will remain?



Source: ©jpg (14 July 2018)