Computing Nuclear Parton Distributions

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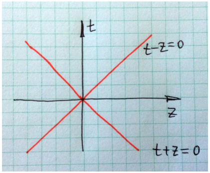
Outline

- Experimental observations on nuclear modification of parton distributions.
- Sketch of basic mechanisms responsible for nuclear effects.
- Results of analysis of nuclear DIS.
- Application to W/Z boson production in $p + \mathrm{Pb}$ collisions at LHC.

Parton distributions in brief

- PDFs are the ligh-cone momentum distributions:

$$q(x,Q^2) = \int \frac{\mathrm{d}z}{2\pi} e^{2iMxz} \langle p|\bar{\psi}_q(t=-z,z,\mathbf{0}_T)\gamma_+\psi_q(0)|p\rangle_{Q^2}$$

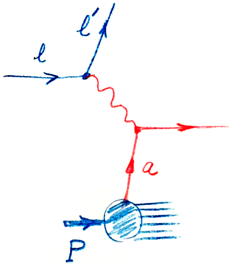


  - PDFs are positively defined and dimensionless functions. The difference of Quark−Antiquark distributions is normalized to the number of valence quarks:

$$\int_0^1 \mathrm{d}x\,\left(u_p(x,Q^2)-\bar{u}_p(x,Q^2)\right)=2, \quad \int_0^1 \mathrm{d}x\,\left(d_p(x,Q^2)-\bar{d}_p(x,Q^2)\right)=1$$

- PDFs are universal, i.e. in the leading order same PDF drive cross sections of different processes

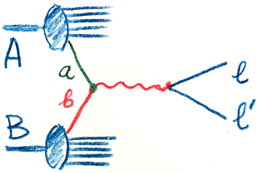
Lepton DIS



$$
\begin{aligned}
F_2^{\gamma} &= (\tfrac{2}{3})^2 x(u+\bar{u}) + (\tfrac{1}{3})^2 x(d+\bar{d}) + \dots \\
F_2^{W^+} &= x(d+\bar{u}+\dots) \\
F_2^{Z} &= (\tfrac{1}{3} - \tfrac{8}{9}\sin^2\theta_W)x(u+\bar{u}) + \\
&\quad (\tfrac{1}{6} - \tfrac{2}{9}\sin^2\theta_W)x(d+\bar{d}) + \dots
\end{aligned}
$$

Hadron hard collisions

$$
\sigma_{AB} = \sum_{a,b} \int \mathrm{d}x_a \mathrm{d}x_b \, q_{a/A}(x_a) q_{b/B}(x_b) \widehat{\sigma}_{ab}
$$

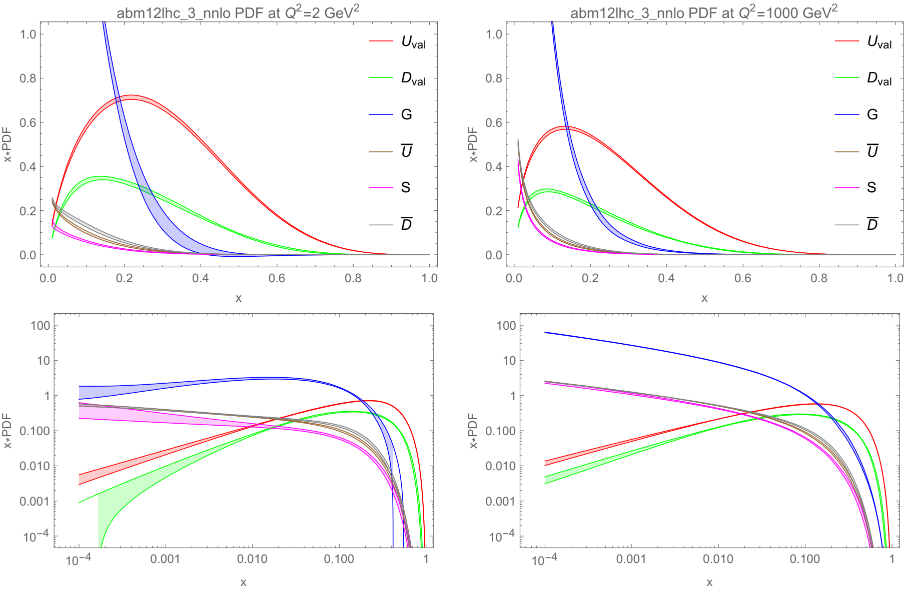


How do we know PDFs?

- PDF are hard to calculate from first principles in QCD since it requires solving QCD in a strong coupling regime. PDF Q^2 dependence is governed by perturbative quark-gluon interaction with the coupling $\alpha_S(Q^2)$ (evolution equation *Dokshitser-Gribov-Lipatov-Altareli-Parisi, 1970s*).
- At practice the proton PDF are obtained from global fits to high-energy data (charged-lepton DIS, Drell-Yan process, W-boson production) assuming some functional form of PDFs at a fixed scale $Q^2 = Q_0^2$

$$p_i(x, Q_0^2) = A_i x^{a_i} (1-x)^{b_i} \left(1 + c_i x + \cdots\right) \quad i = u_V,\ d_V,\ \bar{u},\ \bar{d},\ \bar{s},\ g$$

- A few analyses are available which are regularly updated. To name a few:
  ABM = Alekhin + Blümlein + Moch + ...
  CTEQ = Coordinated Theoretical-Experimental project on QCD
  HERAPDF = H1 and ZEUS Collaborations from HERA
  MRST = Martin + Stirling + Thorn + ...


abm12lhc_3_nnlo PDF at Q^2=2 GeV^2
abm12lhc_3_nnlo PDF at Q^2=1000 GeV^2
U_val
D_val
G
U̅
S
D̅
x*PDF
x

Do we know the *nuclear PDF* if we know those of the proton and the neutron?

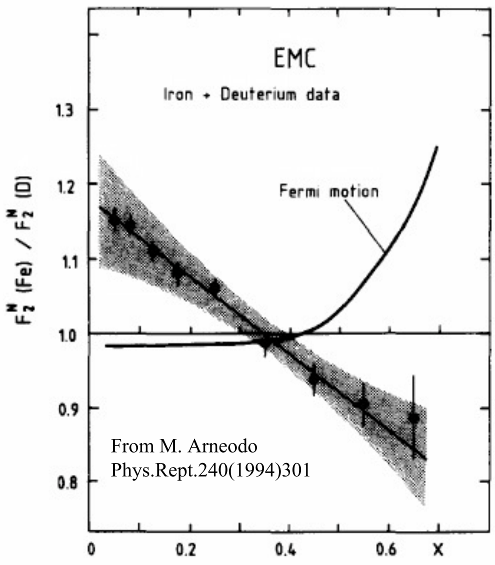
The answer depends on the accuracy we require...

- Nucleus $= Z$ protons $+N$ neutrons; ($A = Z + N$ the total number of bound nucleons).
- Nucleus is a loosely bound system with binding energy $E_B \ll M$ the nucleon mass.
- A naive expectation for nuclear PDF (coarse accuracy):

$$p_{i/A}(x, Q^2) = Zp_{i/p}(x, Q^2) + Np_{i/n}(x, Q^2)$$

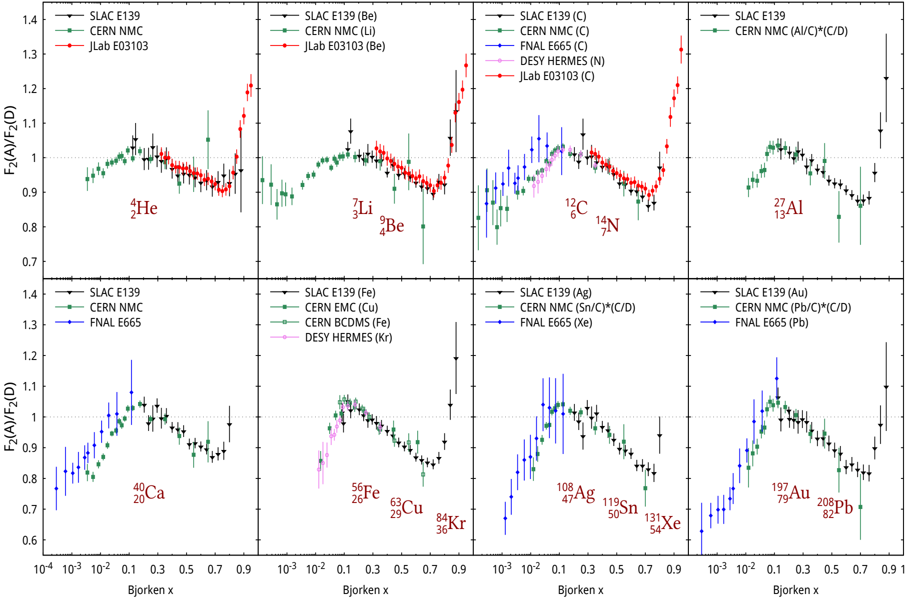
This hypothesis was tested experimentally. The measurements show corrections up to 100% depending on kinematical region.

Historic EMC measurement of nuclear effects in DIS

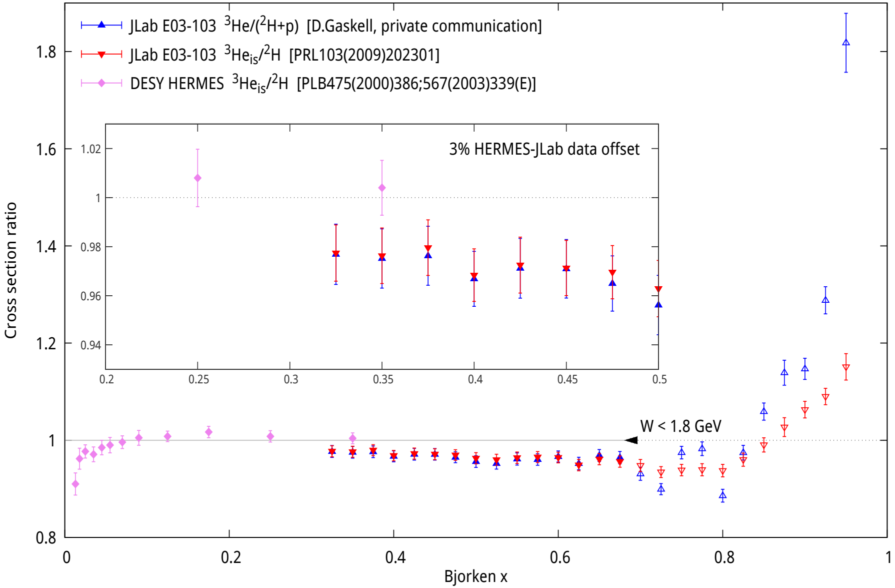


Direct measurement by *EMC Collaboration, 1983* indicated unexpected nuclear effects even in the DIS region. Exciting observation, although the small-x part turned out to be time dependent (the effect changed sign with time).

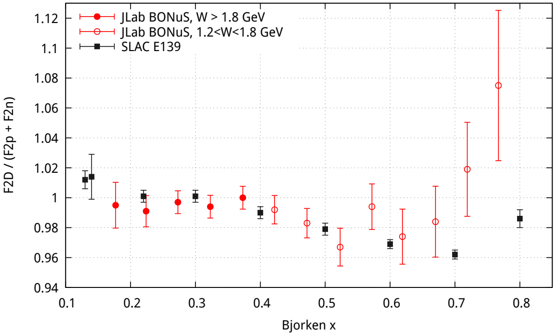
Summary on nuclear ratios data from DIS experiments

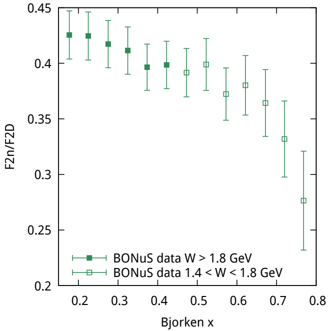


HERMES and JLab data on ^{3}He



SLAC E139 and JLab BONUS data on $^{2}\mathrm{H}$





- SLAC E139 [*PRD49(1994)4348*] obtains $R_D = F_2^D/(F_2^p + F_2^n)$ by extrapolating data on $R_A = F_2^A/F_2^D$ with $A \geq 4$ assuming $R_A - 1$ scales as nuclear density.
- BONuS [*PRC92(2015)015211*] obtains R_D from a direct measurement of F_2^n/F_2^D [*PRC89(2014)045206*] using world data on F_2^D/F_2^p.

Empirical nuclear PDF

Nuclear PDF (NPDF) are phenomenologically extracted from data in a way similar to the proton analyses. Basic steps:

- Assume $p_{i/A}(x,Q^2) = Zp_{i/p}(x,Q^2) + Np_{i/n}(x,Q^2)$ with $p_{i/p}$ and $p_{i/n}$ the *bound* proton and neutron PDF, which are different from the free ones $p^0_{i/p}$ and $p^0_{i/n}$.
- Assume isospin symmetry relations $u_p = d_n$, $d_p = u_n$, $s_p = s_n$, $g_p = g_n$.
- Assume a functional form for $p_{i/A}$ or for the ratio $R^A_i = p_{i/p}/p^0_{i/p}$ where $p^0_{i/p}$ is a PDF of *free* proton.
- Fit R^A_i to nuclear DIS and DY data.

A few analyses are available which differ by functional form (parameterization) of x and A dependencies.

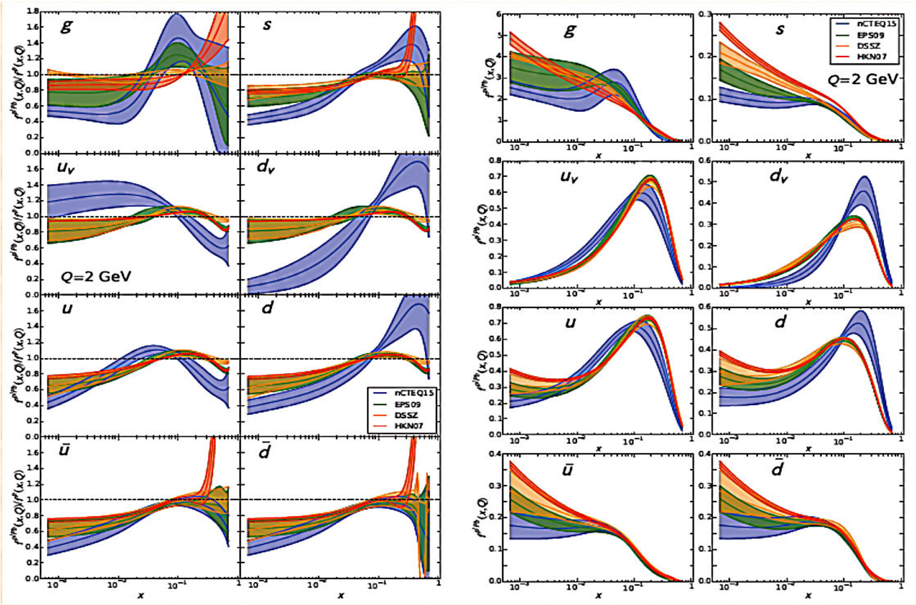
DSZS = de Florian + Sassot + Zurita + Stratmann

EPS = Eskola + Paukkunen + Salgado

HKN = Hirai + Kumano + Nagai

nCTEQ = Kovarik + Kusina + ...

Comparison of different NPDF fits *K.Kovarik et.al. arXiv:1509.00792*



While NPDF fits are useful in constraining general behavior of NPDF they provide a little insight into physics mechanisms responsible for nuclear modification. Furthermore the basic assumptions of the NPDF fits are questionable.

We will follow a different approach to NPDF and perform an analysis of different physics mechanisms of nuclear correction.

Why nuclear corrections survive at DIS?

Space-time scales in DIS

$$W_{\mu\nu} = \int \mathrm{d}^4 x \exp(iq\cdot x)\langle p|[J_\mu(x), J_\nu(0)]|p\rangle$$

$$q\cdot x = q_0 t - |\boldsymbol{q}|z = q_0 t - \sqrt{q_0^2 + Q^2}z \simeq q_0(t-z) - \frac{Q^2}{2q_0}\,z$$

- DIS proceeds near the light cone: $|t-z| \sim 1/q_0$ and $t^2 - z^2 \sim Q^{-2}$.
- In the TARGET REST frame the characteristic time and longitudinal distance are NOT small at all: $t \sim z \sim 2q_0/Q^2 = 1/Mx_{\mathsf{Bj}}$. DIS proceeds at the distance ~ 1 Fm at $x_{\mathsf{Bj}} \sim 0.2$ and at the distance ~ 20 Fm at $x_{\mathsf{Bj}} \sim 0.01$.
- Two different regions in nuclei from comparison of coherence length (Ioffe time) $L = 1/Mx_{\mathsf{Bj}}$ with average distance between bound nucleons r_{NN}:
  - $L < r_{\mathsf{NN}}$ (or $x > 0.2$) $\Rightarrow$ Nuclear DIS $\approx$ incoherent sum of contributions from bound nucleons. Nuclear corrections $\sim EL$ and $\sim |\boldsymbol{p}|^2 L^2$ where $E(p)$ typical energy (momentum) in the nuclear ground state.
  - $L \gg r_{\mathsf{NN}}$ (or $x \ll 0.2$) $\Rightarrow$ Coherent effects of interactions with a few nucleons are important.

Effective theory of nuclear DIS

A good starting point is approximation of incoherent scattering off bound protons and neutrons (*impulse approximation*). Effective T-matrix:

$$\widehat{T}_{\mu\nu} = \int \mathrm{d}^4x\mathrm{d}^4y\, e^{iqy}\, \overline{\Psi}(x)\widehat{\mathcal{T}}_{\mu\nu}(x,y)\Psi(0)$$

- $\widehat{\mathcal{T}}_{\mu\nu}(x,y)$ is effective scattering operator describing nucleon DIS.
- Ψ is the nucleon field operator.
- Matrix element over the proton state $\langle p|\widehat{T}_{\mu\nu}|p\rangle = T^p_{\mu\nu}$ the proton Compton amplitude.
- M.e. over a nuclear state $\langle A|\widehat{T}_{\mu\nu}|A\rangle = T^A_{\mu\nu}$ the nuclear Compton amplitude.

For the hadronic tensor $W_{\mu\nu} = \mathrm{Im}\, T_{\mu\nu}$ we have

$$W^A_{\mu\nu}(P_A,q) = \sum_{\tau=p,n} \int \frac{\mathrm{d}^4 p}{(2\pi)^4}\ \mathrm{Tr}\left[\widehat{\mathcal{W}}^\tau_{\mu\nu}(p,q)\mathcal{A}^\tau(p)\right],$$

$$\mathcal{A}^\tau_{\alpha\beta}(p) = \int \mathrm{d}^4\xi e^{ip\cdot\xi} \langle A|\overline{\Psi}^\tau_\beta(\xi)\,\Psi^\tau_\alpha(0)|A\rangle$$

- The off-shell nucleon tensor $\widehat{\mathcal{W}}_{\mu\nu}(p,q)$ is the matrix in the Dirac space.
- On the mass shell $p^2 = M^2$, averaging $\widehat{\mathcal{W}}_{\mu\nu}(p,q)$ over the nucleon polarizations we obtain the nucleon tensor given in terms of 2 structure functions

$$W^\tau_{\mu\nu}(p,q) = \frac{1}{2}\,\mathrm{Tr}\left[(\not{p}+M)\widehat{\mathcal{W}}^\tau_{\mu\nu}(p,q)\right] = \widetilde{g}_{\mu\nu}\,F_1 + \frac{\widetilde{p}_\mu\widetilde{p}_\nu}{p\cdot q}\,F_2,$$

$$\widetilde{g}_{\mu\nu} = g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}, \qquad \widetilde{p}_\mu = p_\mu - \frac{p\cdot q}{q^2} q_\mu$$

Hadronic tensor in off-shell region

- Expand in the Dirac basis:

$$\widehat{\mathcal{W}}_{\mu\nu} = \sum_n W^n_{\mu\nu}\Gamma_n$$

$$\Gamma_n = I, \gamma^\alpha, \sigma^{\alpha\beta}, \gamma^\alpha\gamma_5, \gamma_5$$

- Require the symmetry under P and T transformations AND keeping ONLY current-conserving terms ($q_\mu W_{\mu\nu} = 0$) we have 7 independent structure functions:

$$2\,\widehat{\mathcal{W}}^{\rm sym}_{\mu\nu}(p,q) = -\,\widetilde{g}_{\mu\nu}\left(\frac{f_1^{(0)}}{M} + \frac{f_1^{(1)}\not{p}}{M^2} + \frac{f_1^{(2)}\not{q}}{p\cdot q}\right) + \frac{\widetilde{p}_\mu\widetilde{p}_\nu}{p\cdot q}\left(\frac{f_2^{(0)}}{M} + \frac{f_2^{(1)}\not{p}}{M^2} + \frac{f_2^{(2)}\not{q}}{p\cdot q}\right) + \frac{f_2^{(3)}}{p\cdot q}\,\widetilde{p}_{\{\mu}\widetilde{g}_{\nu\}\alpha}\gamma^\alpha,$$

the functions $f_i^{(j)} = f_i^{(j)}(x, Q^2, p^2)$ are dimensionless.

- Contribution of each of these structure functions is goverened by corresponding matrix element $\langle\overline{\Psi}\Gamma_n\Psi\rangle$.
- On the mass shell $p^2 = M^2$ we only have 2 independent structure functions

$$F_1 = f_1^{(0)} + f_1^{(1)} + f_1^{(2)}, \qquad F_2 = f_2^{(0)} + f_2^{(1)} + f_2^{(2)} + f_2^{(3)}$$

$1/M$ expansion for a weakly bound nucleus

- Assume a nonrelativistic nuclear ground state:
  - $|\boldsymbol{p}| \ll M, \; |p_0 - M| \ll M$
  - No strong scalar and vector fields in nuclei
- Reduce the four-component relativistic field Ψ to a two-component nonrelativistic operator ψ *Landau & Lifshitz, Field theory*

$$\Psi(\boldsymbol{p}, t) = e^{-iMt} Z \begin{pmatrix} \psi(\boldsymbol{p}, t) \\ \frac{\boldsymbol{\sigma}\cdot\boldsymbol{p}}{2M}\, \psi(\boldsymbol{p}, t) \end{pmatrix}$$

- The renormalization operator $Z = 1 - \boldsymbol{p}^2/8M^2$ provides a correct normalization of the nonrelativistic two-component nucleon field ψ: $\int \mathrm{d}^3 p \Psi^\dagger \Psi = \int \mathrm{d}^3 p \psi^\dagger \psi$ to order $\boldsymbol{p}^2/M^2$.
- Separate the nucleon mass M from the energy p_0, $p = (M + \varepsilon, \boldsymbol{p})$. Examine and reduce all the Lorentz–Dirac structures of $\widehat{W}_{\mu\nu}$. The result to order ε/M and $\boldsymbol{p}^2/M^2$ can be summarized as

$$\frac{1}{M_A} W^A_{\mu\nu}(P_A, q) = \sum_{\tau=p,n} \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \frac{1}{M+\varepsilon} \mathcal{P}^\tau(\varepsilon, \boldsymbol{p}) W^\tau_{\mu\nu}(p,q)$$

Nuclear spectral function describes bound nucleon energy-momentum distribution

$$\mathcal{P}(\varepsilon, \boldsymbol{p}) = \int \mathrm{d}t\, e^{-i\varepsilon t} \langle A|\psi^\dagger(\boldsymbol{p}, t)\psi(\boldsymbol{p}, 0)|A\rangle / \langle A|A\rangle$$

Nucleon off-shell tensor has structure similar to the on-shell one

$$W^\tau_{\mu\nu}(p,q) = \widetilde{g}_{\mu\nu} F_1 + \frac{\widetilde{p}_\mu \widetilde{p}_\nu}{p\cdot q} F_2,$$

$$F_1(x, Q^2, p^2) = f_1^{(0)} \left(1 + \frac{p^2 - M^2}{2M^2}\right) + f_1^{(1)} \frac{p^2}{M^2} + f_1^{(2)},$$

$$F_2(x, Q^2, p^2) = f_2^{(0)} \left(1 + \frac{p^2 - M^2}{2M^2}\right) + f_2^{(1)} \frac{p^2}{M^2} + f_2^{(2)} + f_2^{(3)}$$

Remarks

- While on the mass shell the hadronic tensor has 2 independent tensor structures, off-shell 7 structures are generally required.
- In the vicinity of the mass shell, to order $\boldsymbol{p}^2/M^2 \sim \varepsilon/M$, we still have only 2 independent structure functions which have a correct on-shell limit.

Structure functions

$$F_T^A(x,Q^2) = \sum_{\tau=p,n} \int \frac{\mathrm{d}^4p}{(2\pi)^4} \mathcal{P}^\tau(\varepsilon,\boldsymbol{p}) \left(1+\frac{\gamma p_z}{M}\right) \left(F_T^\tau + \frac{2{x'}^2 \boldsymbol{p}_\perp^2}{Q^2} F_2^\tau\right),$$

$$F_L^A(x,Q^2) = \sum_{\tau=p,n} \int \frac{\mathrm{d}^4p}{(2\pi)^4} \mathcal{P}^\tau(\varepsilon,\boldsymbol{p}) \left(1+\frac{\gamma p_z}{M}\right) \left(F_L^\tau + \frac{4{x'}^2 \boldsymbol{p}_\perp^2}{Q^2} F_2^\tau\right),$$

$$\gamma^2 F_2^A(x,Q^2) = \sum_{\tau=p,n} \int \frac{\mathrm{d}^4p}{(2\pi)^4} \mathcal{P}^\tau(\varepsilon,\boldsymbol{p}) \left(1+\frac{\gamma p_z}{M}\right) \left({\gamma'}^2 + \frac{6{x'}^2 \boldsymbol{p}_\perp^2}{Q^2}\right) F_2^\tau.$$

Notations: $\gamma^2 = \frac{|\boldsymbol{q}|^2}{q_0^2} = 1 + \frac{4M^2x^2}{Q^2}$, $\quad {\gamma'}^2 = 1 + \frac{4p^2{x'}^2}{Q^2}$.

Variables of the off-shell nucleon structure functions $F_i(x', Q^2, p^2)$:

- Bjorken variable $x' = \dfrac{Q^2}{2p\cdot q} = \dfrac{x}{1+\frac{\varepsilon+\gamma p_z}{M}}$
- 4-momentum transfer squared Q^2
- Nucleon virtuality (off-shellness) $p^2 = (M+\varepsilon)^2 - \boldsymbol{p}^2$.

Nuclear parton distributions

From the relations for the structure functions we obtain the relations between the nuclear and the proton/neutron PDFs in the Bjorken limit:

$$p_{i/A}(x,Q^2) = \sum_{\tau=p,n} \int \frac{\mathrm{d}y\mathrm{d}p^2}{y}\, f_{\tau/A}(y,p^2)\, p_{i/\tau}(\frac{x}{y},Q^2,p^2),$$

$$f_{p,n}(y,p^2) = \int \frac{\mathrm{d}^4k}{(2\pi)^4}\mathcal{P}(k,\varepsilon)\left(1+\frac{k_z}{M}\right)\delta\left(y-1-\frac{\varepsilon+k_z}{M}\right)\delta(p^2-k^2)$$

Remarks:

- For any distribution function $f_{p,n}(y,p^2)$ the nuclear PDF $p_{i/A}(x,Q^2)$ automatically obeys to the DGLAP evolution equation provided that the proton/neutron PDF $p_{i/p}(x,Q^2,p^2)$ is a solution to the evolution equation.
- The nucleon distribution function $f_{p,n}(y,p^2)$ does not depend on the PDF type.

- The distribution function is normalized to the number of nucleons:

$$\int \mathrm{d}y\mathrm{d}p^2 f_{p,n}(y,p^2) = Z, N$$

- The distribution is a narrow function peaked about average light-cone momentum $y \sim 1$ (here we average over protons and neutrons)

$$\langle y \rangle = \frac{1}{A}\int \mathrm{d}y\mathrm{d}p^2\, y\, f(y,p^2) = 1 + \frac{\langle \varepsilon \rangle + \frac{2}{3}\langle T \rangle}{M}$$

$$\Delta = \langle y^2 \rangle - \langle y \rangle^2 = \frac{1}{A}\int \mathrm{d}y\mathrm{d}p^2\, (y^2 - \langle y \rangle)^2 f(y,p^2) = \frac{2}{3}\frac{\langle T \rangle}{M}$$

where $\langle \varepsilon \rangle = \langle p_0 - M \rangle$ and $\langle T \rangle = \left\langle \boldsymbol{p}^2 \right\rangle / 2M$. Note that $\varepsilon < 0$ due to binding and $\langle \varepsilon \rangle - \langle T \rangle = \langle V \rangle$ the average potential energy of a bound nucleon.

- Average bound nucleon virtuality $v = (p^2 - M^2)/M^2$

$$\langle v \rangle = \frac{1}{A}\int \mathrm{d}y\mathrm{d}p^2\, v\, f(y,p^2) = 2\frac{\langle \varepsilon \rangle - \langle T \rangle}{M}$$

Parameters of nuclear distribution for the Deuteron and Lead nuclei

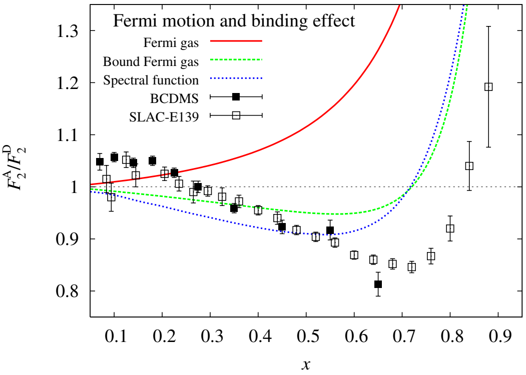
Nucleus	Binding E./A (MeV)	$\langle \varepsilon \rangle$ (MeV)	$\langle T \rangle$ (MeV)	$\langle y \rangle$	Δ	$\langle v \rangle$
^{2}H	1.11	−11.56	9.33	0.994	0.0066	−0.044
^{208}Pb	7.83	−58.51	35.13	0.963	0.025	−0.197

Impulse approximation

First drop off-shell correction: $F_2(x', Q^2, p^2) = F_2(x', Q^2, p^2 = M^2)$

- Momentum distribution leads to a rise at large Bjorken x
- Binding energy correction turns out to be important and results in a “dip” at $x \sim 0.6 - 0.7$

  Akulinichev,Vagradov & S.K., 1984.
- However, even realistic nuclear spectral function fails to accurately explain the slope and the position of the minimum.



Impulse Approximation should be corrected for a number of effects.

Off-shell effect

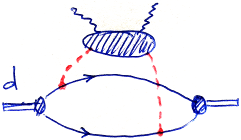
Bound nucleons are off-mass-shell ($p^2 < M^2$).
The treatment of p^2 dependence simplifies in the vicinity of the mass shell by expanding in the relative virtuality $v = (p^2 - M^2)/M^2$ *S.K., Piller & Weise, 1994*

$$q_{i/p}(x, Q^2, p^2) \approx q_{i/p}(x, Q^2) \left(1 + \delta f_i(x, Q^2)\, v\right)$$

- The function $\delta f(x, Q^2)$ describes a relative modification of the nucleon PDFs in the vicinity of the mass shell.
- Off-shell correction is closely related to modification of the nucleon size in nuclear environment *S.K. & R.Petti, 2004*.

Nuclear meson-exchange current effect (MEC)

Leptons can scatter on a meson field which mediate interaction between bound nucleons. This process generate a MEC correction to nuclear sea quark distribution *Llewellyn-Smith, Ericson, Thomas, 1983*

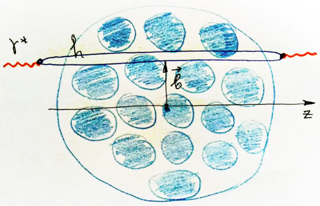


$$\delta q_{\mathsf{MEC}}(x, Q^2) = \int_x \frac{\mathrm{d}y}{y} f_{\pi/A}(y) q^{\pi}(\frac{x}{y}, Q^2)$$

- Contribution from nuclear pions (mesons) is important to balance nuclear light-cone momentum $\langle y \rangle_\pi + \langle y \rangle_N = 1$.
- The nuclear pion distribution function is localized in a region $y < p_F/M \sim 0.3$. For this reason the MEC correction to nuclear (anti)quark distributions is localized at $x < 0.3$.
- The magnitude of the correction is driven by average number of "nuclear pion excess" $n_\pi = \int \mathrm{d}y\, f_{\pi/A}(y)$ and $n_\pi/A \sim 0.1$ for a heavy nucleus like ^{56}Fe.

Nuclear shadowing

Coherent nuclear correction is due to propagation of intermediate state $\gamma^* \to h$ in nuclear environment, which can be described in the multiple scattering theory *Glauber, Gribov 1970s*.



$$\frac{\delta q_A^{\mathrm{coh}}}{q_N} = \frac{\operatorname{Im}\delta\mathcal{A}}{\operatorname{Im} a}$$

$$\delta\mathcal{A} = \delta\mathcal{A}^{(2)} + \delta\mathcal{A}^{(3)} + \dots$$

$$\delta\mathcal{A}^{(2)} = ia^2 \int_{z_1<z_2} \mathrm{d}^2\boldsymbol{b}\,\mathrm{d}z_1\mathrm{d}z_2\,\rho(\boldsymbol{b},z_1)\rho(\boldsymbol{b},z_2)\,e^{i\frac{z_1-z_2}{L}}$$

- $\rho(\boldsymbol{r})$ is the nuclear number density, $\int \mathrm{d}^3\boldsymbol{r}\rho(\boldsymbol{r}) = A$
- $a = \frac{\sigma}{2}(i+\alpha)$ is the (effective) forward scattering amplitude of intermediate state h off the nucleon
- L is the coherence length of intermediate state which accounts finite life time of intermediate state, $1/L = Mx(1+m_h^2/Q^2)$. Its presence suppresses the coherence effect in the region of large x.

Modelling the nuclear PDFs and analysis of data

Assemble everything together and confront model to data *S.K. & R.Petti, NPA765(2006)126; PRC82(2010)054614; PRC90(2014)045204*

$$q_{i/A} = \langle q_{i/p} \rangle + \langle q_{i/n} \rangle + \delta q_i^{\mathrm{MEC}} + \delta q_i^{\mathrm{coh}}$$

Strategy of the analysis:

- Calculate NPDF using the free proton PDF (*Alekhin et.al.*) with accurate treatment of nuclear momentum distribution and energy spectrum (nuclear spectral function), MEC and coherent correction (nuclear shadowing).
- Consider the off-shell function $\delta f(x)$ and effective amplitude a as unknown and parametrize them. Study the data on the nuclear DIS in terms of the ratios $R_2(A/B) = F_2^A/F_2^B$ and determine $\delta f(x)$ and the amplitude a from data.
- Use the normalization conditions and the DIS sum rules to determine the amplitude a (responsible for nuclear shadowing) in the region of high Q^2, which is not constrained by data.
- Verify the model by comparing the calculations with data not used in analysis (new measurements at JLab, nuclear DY process, W/Z production in $p + \mathrm{Pb}$ collision at LHC).

Targets	χ^2/DOF						
	NMC	EMC	E139	E140	BCDMS	E665	HERMES
^{4}He/^{2}H	10.8/17		6.2/21				
^{7}Li/^{2}H	28.6/17						
^{9}Be/^{2}H			12.3/21				
^{12}C/^{2}H	14.6/17		13.0/17				
^{9}Be/^{12}C	5.3/15						
^{12}C/^{7}Li	41.0/24						
^{14}N/^{2}H							9.8/12
^{27}Al/^{2}H			14.8/21				
^{27}Al/^{12}C	5.7/15						
^{40}Ca/^{2}H	27.2/16		14.3/17				
^{40}Ca/^{7}Li	35.6/24						
^{40}Ca/^{12}C	31.8/24					1.0/5	
^{56}Fe/^{2}H			18.4/23	4.5/8	14.8/10		
^{56}Fe/^{12}C	10.3/15						
^{63}Cu/^{2}H		7.8/10					
^{84}Kr/^{2}H							4.9/12
^{108}Ag/^{2}H			14.9/17				
^{119}Sn/^{12}C	94.9/161						
^{197}Au/^{2}H			18.2/21	2.4/1			
^{207}Pb/^{2}H						5.0/5	
^{207}Pb/^{12}C	6.1/15					0.2/5	

Values of χ^2/DOF between different data sets with $Q^2 \geq 1$ GeV^2 and the model predictions *NPA765(2006)126; PRC82(2010)054614*.

Parameters of the model

- Off-shell structure function $\delta f_2(x) = C_N(x - x_1)(x - x_0)(h - x)$
  - From preliminary studies we observe that h is fully correlated with x_0, i.e. $h = 1 + x_0$.
  - C_N, x_0, x_1 are independent ajustable parameters.
- Effective amplitude

$$\bar{a}_T = \bar{\sigma}_T(i + \alpha)/2, \quad \bar{\sigma}_T = \sigma_1 + \frac{\sigma_0 - \sigma_1}{1 + Q^2/Q_0^2}$$

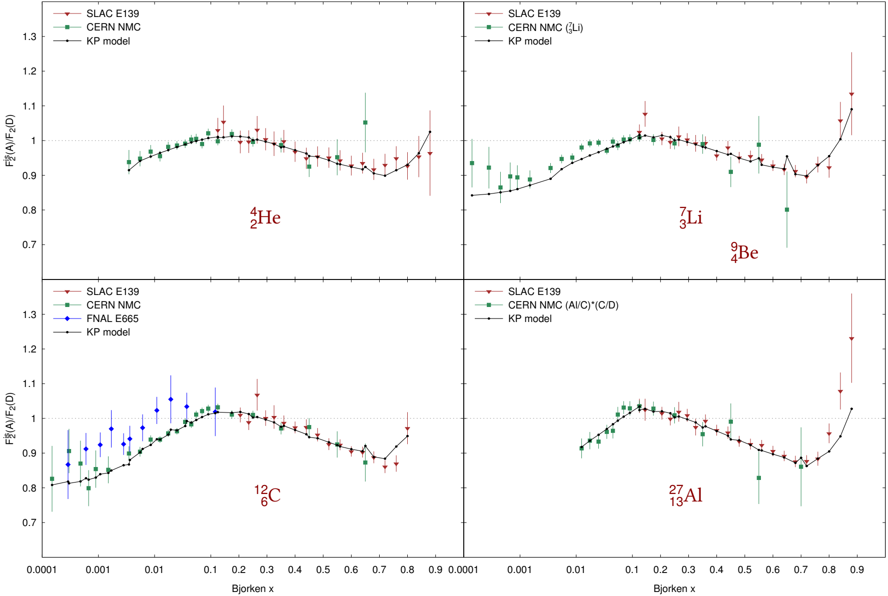
  - Parameters $\sigma_0 = 27\,\mathrm{mb}$ and $\alpha = -0.2$ were fixed in order to match the vector meson dominance model predictions at low Q.
  - Parameter $\sigma_1 = 0$ fixed (preferred by preliminary fits and fixed in the final studies).
  - Q_0^2 is adjustable scale parameter controlling transition between low and high Q regimes.

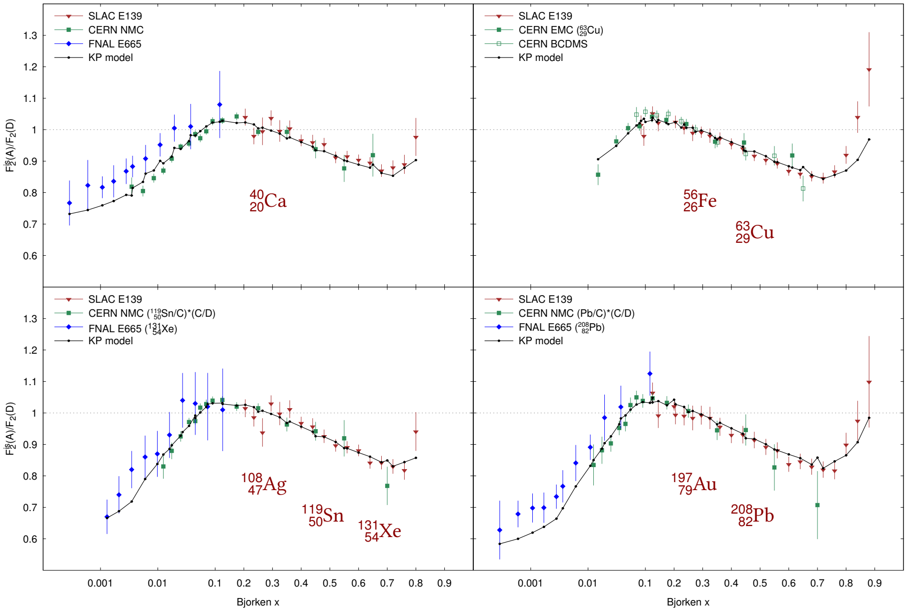
Results

- The x, Q^2 and A dependencies of the nuclear ratios are reproduced for all studied nuclei (^{4}He to ^{208}Pb) in a 4-parameter fit with $\chi^2/\text{d.o.f.} = 459/556$.
- Global fit to all data is consistent with the fits to different subsets of nuclei (light, medium, heavy nuclei).
- Parameters of the off-shell function δf and effective amplitude a_T are determined with a good accuracy.

For detailed discussion and comparison with data see *S.K. & R. Petti, Nucl.Phys. A765(2006)126*.

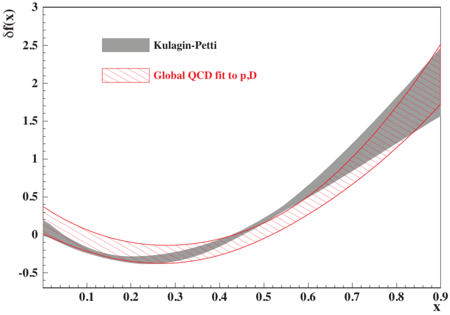
Summary of results on the nuclear ratios F_2^A/F_2^D




SLAC E139
CERN NMC
FNAL E665
KP model
$^{40}_{20}$Ca
SLAC E139
CERN EMC ($^{63}_{29}$Cu)
CERN BCDMS
KP model
$^{56}_{26}$Fe
$^{63}_{29}$Cu
SLAC E139
CERN NMC ($^{119}_{50}$Sn/C)*(C/D)
FNAL E665 ($^{131}_{54}$Xe)
KP model
$^{108}_{47}$Ag
$^{119}_{50}$Sn
$^{131}_{54}$Xe
SLAC E139
CERN NMC (Pb/C)*(C/D)
FNAL E665 ($^{208}_{82}$Pb)
KP model
$^{197}_{79}$Au
$^{208}_{82}$Pb
$F_2^{is}(A)/F_2(D)$
Bjorken x
0.001
0.01
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
0.6
0.7
0.8
0.9
1
1.1
1.2
1.3

Determination of the off-shell function $\delta f(x)$

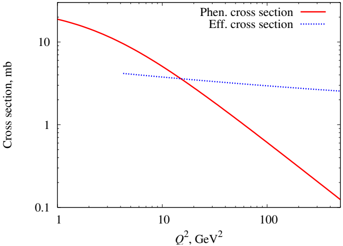
- Analysis of heavy target to deuterium ratios with a model $\delta f(x) = C_N(x-x_1)(x-x_0)(1+x_0-x)$ (gray area)
- Global QCD analysis using deuteron data and a model $\delta f(x) = Ax^2 + Bx + C$ (red) *S.Alekhin, S.K., R.Petti, 2017*. Results are consistent with a heavy-target analysis of *S.K. & R.Petti, 2006*.



- The function $\delta f(x)$ provides a measure of the modification of the quark distributions in a bound nucleon.
- The slope of $\delta f(x)$ in a single-scale nucleon model is related to the change of the radius of the nucleon in the nuclear environment *S.K. & R.Petti, 2006*. The observed slope suggests an increase in the bound nucleon radius in the iron by about 10% and in the deuteron by about 2%.

Determination of effective cross section

- The monopole form $\sigma = \sigma_0/(1 + Q^2/Q_0^2)$ for the effective cross section of C-even $q + \bar{q}$ combination provides a good fit to data on DIS nuclear shadowing for $Q^2 < 15$ GeV^2 with $\sigma_0 = 27$ mb and $Q_0^2 = 1.43 \pm 0.06 \pm 0.195$ GeV^2. Note σ_0 is fixed from $Q^2 \to 0$ limit by the vector meson dominance model. Also we assume $\mathrm{Re}\, a / \mathrm{Im}\, a$ for C-even amplitude to be given by VMD at all energies.



- Nuclear shadowing correction for the C-odd valence distribution $q - \bar{q}$ is also driven by same cross section σ. Note, however, important interference effect between the phases of C-even and C-odd effective amplitude.
- The cross section at high $Q^2 > 15$ GeV^2 is not constrained by data. It is possible to evaluate σ in this region using the the normalization condition. Requiring exact cancellation between the off-shell and the shadowing correction in the normalization we have:

$$\int_0^1 \mathrm{d}x \left(\langle v \rangle \, q_{\mathrm{val}}(x, Q^2) \delta f(x) + \delta q_{\mathrm{val}}^{\mathrm{coh}}(x, Q^2) \right) = 0$$

with $\langle v \rangle = \langle p^2 - M^2 \rangle / M^2$ the average nucleon virtuality. Numeric solution to this equation is shown by dotted curve.

W/Z boson production in $p + Pb$ collisions at LHC

The DY mechanism of W/Z in hadron/nuclear $A + B$ collisions:

$$\frac{\mathrm{d}^2\sigma_{AB}}{\mathrm{d}Q^2\mathrm{d}y} = \sum_{a,b}\int \mathrm{d}x_a\mathrm{d}x_b q_{a/A}(x_a, Q^2)q_{b/B}(x_b, Q^2)\frac{\mathrm{d}^2\widehat{\sigma}_{ab}}{\mathrm{d}Q^2\mathrm{d}y}$$

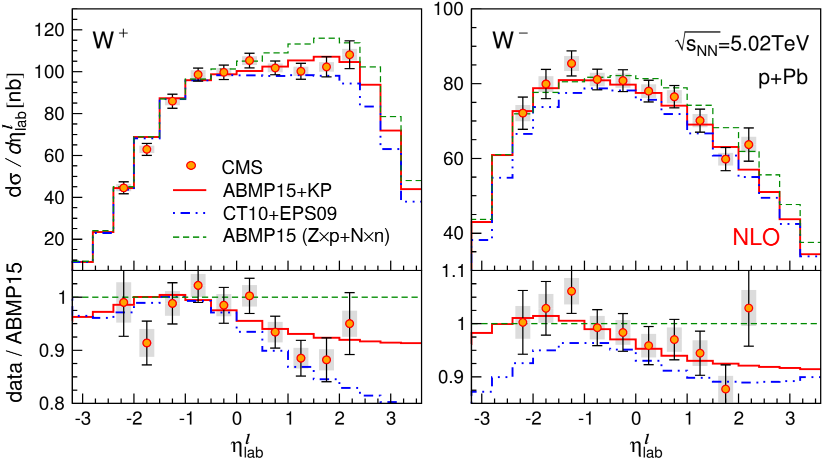
We study rapidity (y) distributions of production of W/Z bosons in $p + Pb$ collisions at LHC with $Q^2 \sim M_Z^2$ and $\sqrt{s} = 5.02\,\mathrm{TeV}$ *P.Ru, S.K., R.Petti, B-W.Zhang, arXiv:1608.06835*.

$$x_p(y) = \frac{M_{W,Z}}{\sqrt{s}}e^y, \quad x_A(y) = \frac{M_{W,Z}}{\sqrt{s}}e^{-y}$$

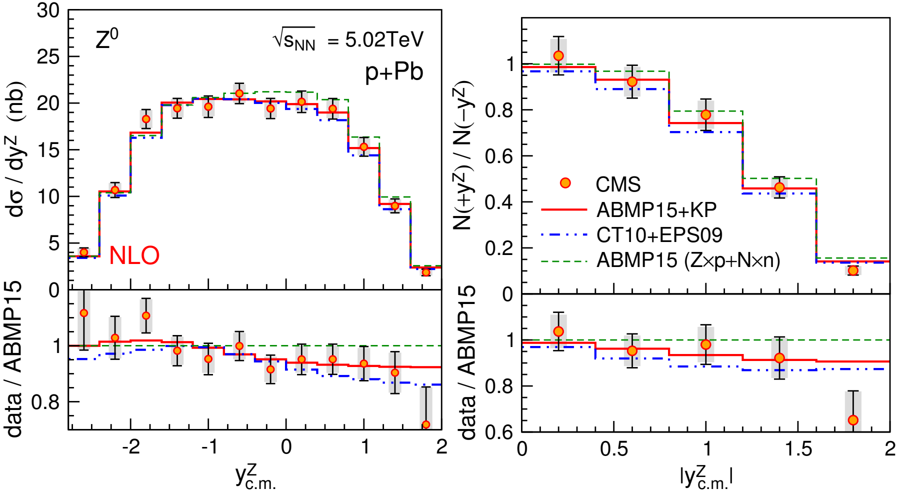
$$x_p(0) = x_A(0) = \frac{M_{W,Z}}{\sqrt{s}} \approx 0.016 \text{ at } \sqrt{s} = 5.02 \text{ TeV}$$

$y > 1 \implies$ small $x_A \implies$ dominated by nuclear antiquarks
$y < -1 \implies$ not so small $x_A \implies$ nuclear valence region.

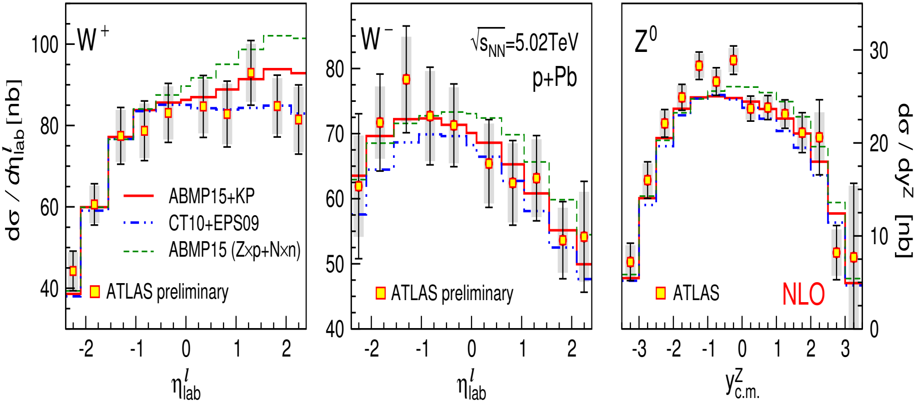
Predictions for $W^{\pm}$ in comparison with CMS data



Predictions for Z^0 in comparison with CMS data

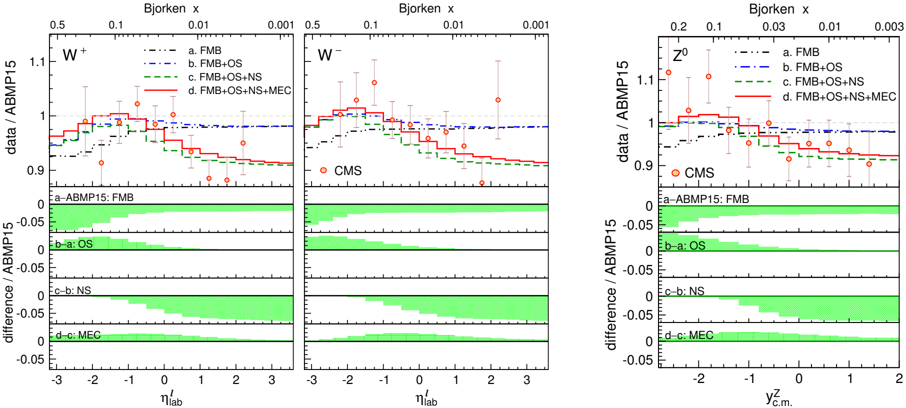


Comparison with ATLAS data on W/Z production



Separating the nuclear effects in W/Z boson production

Different nuclear effects on the production cross section of W (left) and Z boson (right) in $p + \mathrm{Pb}$ collisions at $\sqrt{s} = 5.02\,\mathrm{TeV}$ *P.Ru, S.K., R.Petti, B-W.Zhang arXiv:1608.06835*.



Upper axis is Bjorken x of Pb while the lower axis is (pseudo)rapidity $(\eta)y$.

Performance of the model in terms of χ^2

Observable	N_{Data}	ABMP15 + KP	CT10 + EPS09	ABMP15 (Zp + Nn)
			CMS experiment:	
$d\sigma^+/d\eta^l$	10	1.052	1.532	3.057
$d\sigma^-/d\eta^l$	10	0.617	1.928	1.393
$N^+(+\eta^l)/N^+(-\eta^l)$	5	0.528	1.243	2.231
$N^-(+\eta^l)/N^-(-\eta^l)$	5	0.813	0.953	2.595
$(N^+ - N^-)/(N^+ + N^-)$	10	0.956	1.370	1.064
$d\sigma/dy^Z$	12	0.596	0.930	1.357
$N(+y^Z)/N(-y^Z)$	5	0.936	1.096	1.785
CMS combined	57	0.786	1.332	1.833
			ATLAS experiment:	
$d\sigma^+/d\eta^l$	10	0.586	0.348	1.631
$d\sigma^-/d\eta^l$	10	0.151	0.394	0.459
$d\sigma/dy^Z$	14	1.449	1.933	1.674
CMS+ATLAS combined	91	0.796	1.213	1.635

Summary

- A detailed microscopic model of nuclear PDF was presented.
  - A QCD treatment of the proton and neutron PDF and structure functions.
  - A number of nuclear effects have been addressed: Fermi motion and nuclear binding together with off-shell correction; meson-exchange currents in nuclei; coherent nuclear effects (nuclear shadowing).
  - The nuclear effects are not universal and differ for the valence and the sea-quark distributions.
- A detailed study of nuclear DIS data shows a good performance of the approach:
  - An accurate description of the ratios of nuclear structure functions F_2^A/F_2^B (nuclear EMC effect) both in the valence and the sea region.
  - A good description of the cross section of W and Z boson production in $p + \mathrm{Pb}$ collisions at LHC.
  - Not discussed today:
    - An accurate description of data on the ratio of cross sections of nuclear DY process (nuclear sea at relatively large Bjorken x).
    - A good performance in the description of (anti)neutrino differential and total cross sections from the measurements by CCFR, NuTeV, NOMAD, CHORUS *S.K.&R.Petti,PRD76(2007)094023; S.K. arXiv:1606.07016*

Backup slides

Structure functions are not only PDF

Operator product (twist) expansion in QCD:

$$F_2(x,Q^2) = F_2^{LT,TMC}(x,Q^2) + \frac{H_2(x,Q)}{Q^2} + \cdots$$

The leading term is given in terms of PDFs convoluted with coefficient functions:

$$F_2^{LT} = C_q(\alpha_S) \otimes x \sum_q e_q^2 (q + \bar{q}) + C_g(\alpha_S) \otimes xg$$

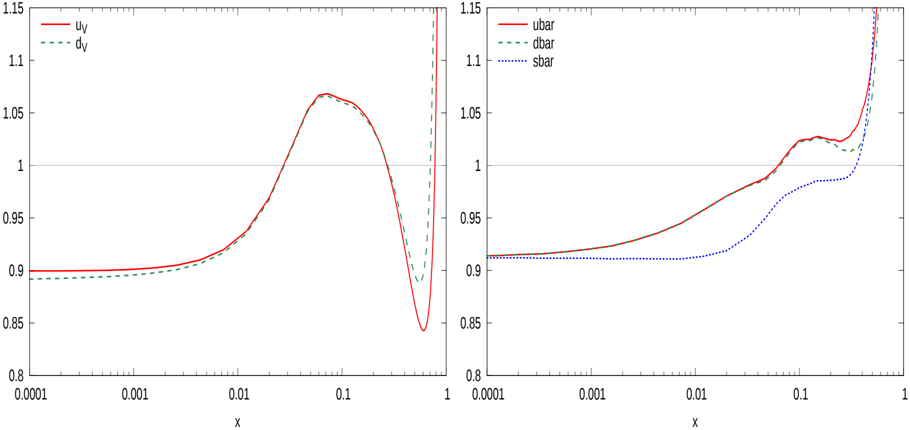
The HT terms involve interaction between quarks and gluons and lack simple probabilistic interpretation.
In the region of high Bjorken x and/or low Q^2 one has to worry about target mass correction *Georgi & Politzer, 1976*

$$F_2^{LT,TMC}(x,Q^2) = \frac{x^2}{\xi^2\gamma^2} F_2^{LT}(\xi,Q^2) + \frac{6x^3M^2}{Q^2\gamma^4} \int_\xi^1 \frac{\mathrm{d}z}{z^2} F_2^{LT}(z,Q^2) + \mathcal{O}(Q^{-4})$$

$\xi = 2x/(1+\gamma)$ is Nachtmann variable and $\gamma^2 = 1 + 4x^2M^2/Q^2$

Nuclear effects on valence quarks vs. antiquarks

The ratios $R_a = q_{a/A}/(Zq_{a/p} + Nq_{n/A})$ computed for the valence u and d (left) and the corresponding antiquarks (right) *S.K. & R.Petti, PRC90(2014)045204*.



Drell-Yan process with nuclear targets

DY process $p + A \to \gamma^* \to \mu^+\mu^- + X$

Cross section is driven by

$\sum e_q^2 \left[q^B(x_B)\bar{q}^T(x_T) + \bar{q}^B(x_B)q^T(x_T)\right]$

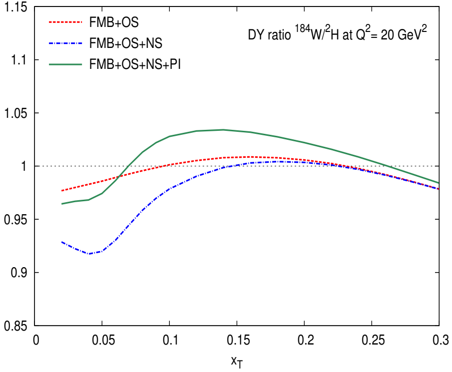
In the context of Fermilab E772 & E866 experiments:

Energy $E_p = 800$ GeV, $s \sim 1600$ GeV^2

Muon pair masses: $4 < Q < 9$ GeV and $Q > 11$ GeV (exclude quarkonium)

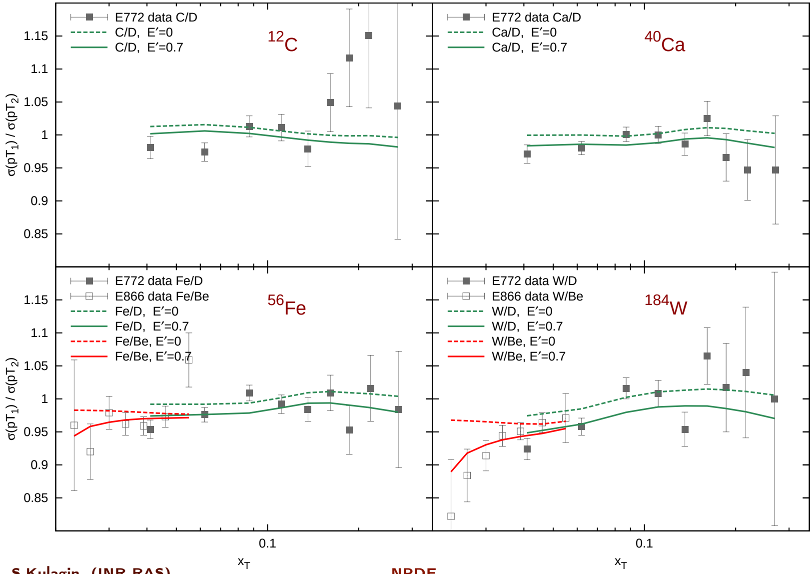
Probed region of target's Bjorken variable $0.04 < x_T < 0.27$

$\sigma(p^{184}\mathrm{W})/\sigma(pD)$ at $Q^2 = 20$ GeV^2

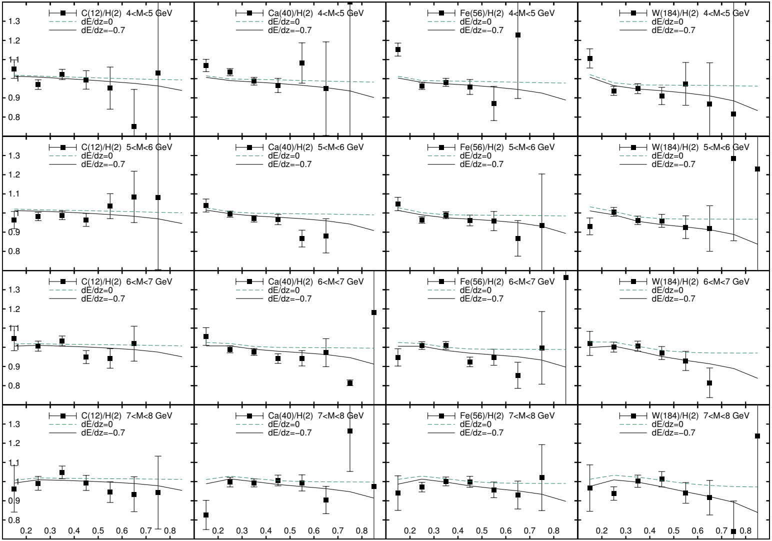


Comparison with E772 & E866 measurements

S.K. & R.Petti, PRC90(2014)045204



Detailed comparison with E772 by dimuon mass bin

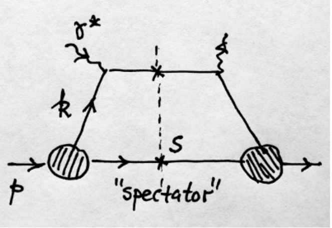


Off-shell effect and the bound nucleon radius

The valence quark distribution in (off-shell) nucleon (see, e.g., *Kulagin, Piller & Weise, PRC* **50**, *1154 (1994)*)

$$q_{\mathsf{val}}(x,p^2) = \int^{k^2_{\mathsf{max}}} \mathrm{d}k^2 \Phi(k^2,p^2)$$

$$k^2_{\mathsf{max}} = x\left(p^2 - s/(1-x)\right)$$



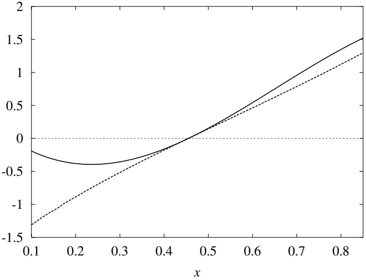
- A one-scale model of quark k^2 distribution: $\Phi(k^2) = C\phi(k^2/\Lambda^2)/\Lambda^2$, where C and ϕ are dimensionless and Λ is the scale.
- Off-shell: $C \to C(p^2),\ \Lambda \to \Lambda(p^2)$
- The derivatives $\partial_x q_{\mathsf{val}}$ and $\partial_{p^2} q_{\mathsf{val}}$ are related

$$\delta f(x) = \frac{\partial \ln q_{\mathsf{val}}}{\partial \ln p^2} = c + \frac{\mathrm{d}q_{\mathsf{val}}(x)}{\mathrm{d}x} x(1-x)h(x)$$

$$h(x) = \frac{(1-\lambda)(1-x) + \lambda s/M^2}{(1-x)^2 - s/M^2}$$

$$c = \frac{\partial \ln C}{\partial \ln p^2},\ \ \lambda = \frac{\partial \ln \Lambda^2}{\partial \ln p^2}$$

- A simple pole model $\phi(y) = (1-y)^{-n}$ (note that $y < 0$ so we do not run into singularity) provides a resonable description of the nucleon valence distribution for $x > 0.2$ and large Q^2 ($s = 2.1$ GeV^2, $\Lambda^2 = 1.2$ GeV^2, $n = 4.4$ at $Q^2 = 15 \div 30$ GeV^2).
- The size of the valence quark confinement region $R_c \sim \Lambda^{-1}$ (nucleon core radius).
- Off-shell corection is independent of specific choice of profile $\phi(y)$ and is given by $(\ln q_{\text{val}}(x))'$.
- Fix c and λ to reproduce $\delta f_2(x_0) = 0$ and the slope $\delta f_2'(x_0)$.
  We obtain $\lambda \approx 1$ and $c \approx -2.3$. The positive parameter λ suggests decreasing the scale Λ in nuclear environment (swelling of a bound nucleon)



$$\frac{\delta R_c}{R_c} \sim -\frac{1}{2}\frac{\delta\Lambda^2}{\Lambda^2} = -\frac{1}{2}\lambda\frac{\langle p^2 - M^2\rangle}{M^2}$$

^{56}Fe : $\delta R_c/R_c \sim 9\%$

^{2}H : $\delta R_c/R_c \sim 2\%$