Introduction to reactions with heavy ion

1. Introduction (experimental and theoretical aspects)
2. Systematics
3. Scattering and direct reactions
4. Deep-inelastic collisions (properties and description)
5. Pecularities of fusion reactions

1st nuclear reaction with *p* beam: 1931
p, α, *d* beams for study of nuclear structure

50-60th years – ion sourses
heavy ion wave length < 0.1 fm (classical particles)

50th : linear accelerators in USA
60th : cyclotron in Dubna
70-80th : linear accelerator at GSI, cyclotron at GANIL and ...

beam of light nuclei:
nuclear reactions / processes: elastic, inealastic scattering, nuclear transfer reactions, formation and decay of compound nucleus
beam of heavy ions:
the Coulomb fission of heavy nuclei and excitation of high-spin states, population of highly-deformed nuclear states, multinucleon transfer reactions, compound nucleus formation

Problems of synthesis of superheavy nuclei

Production of exotic nuclei, new isotopes

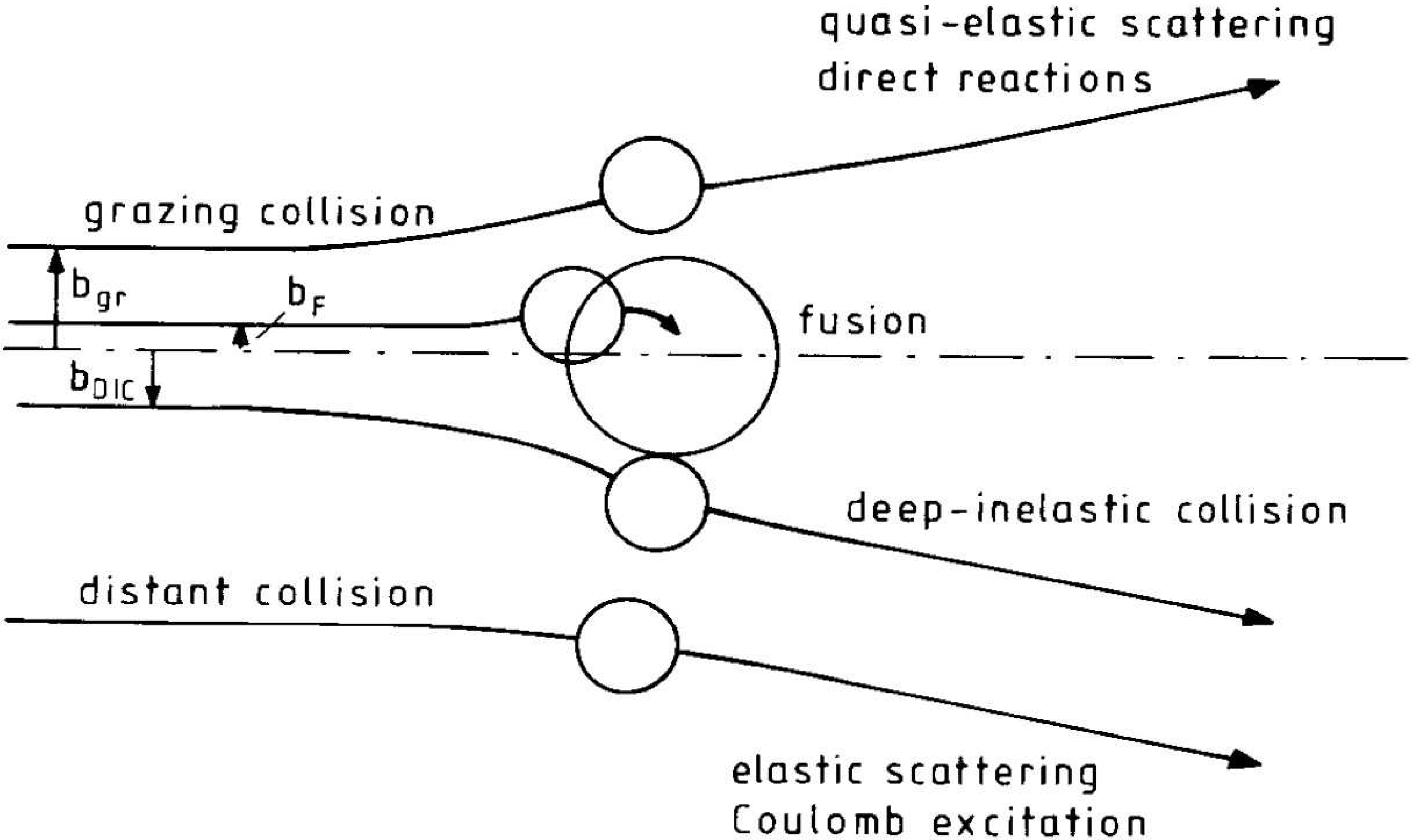
Study of various decay modes including fission, emission of delayed proton, *p* and 2*p* radioactivity

High-spin states

Highly excited compound nuclei

Sub-barrier processes, astrophysical reactions

Cluster or molecule states



Classification of reactions by impact parameter.

impact parameter $b=l/k$

Reaction cross section

$$\sigma_{\mathrm{r}} = \sum_{l} \sigma_{\mathrm{r}}(l),$$

$$\sigma_{\mathrm{r}}(l) = \frac{\pi}{k^2}(2l + 1)T_l$$

For large angular momenta

$$\sigma_{\mathrm{r}} = \int_0^\infty dl\, \sigma_{\mathrm{r}}(l)$$

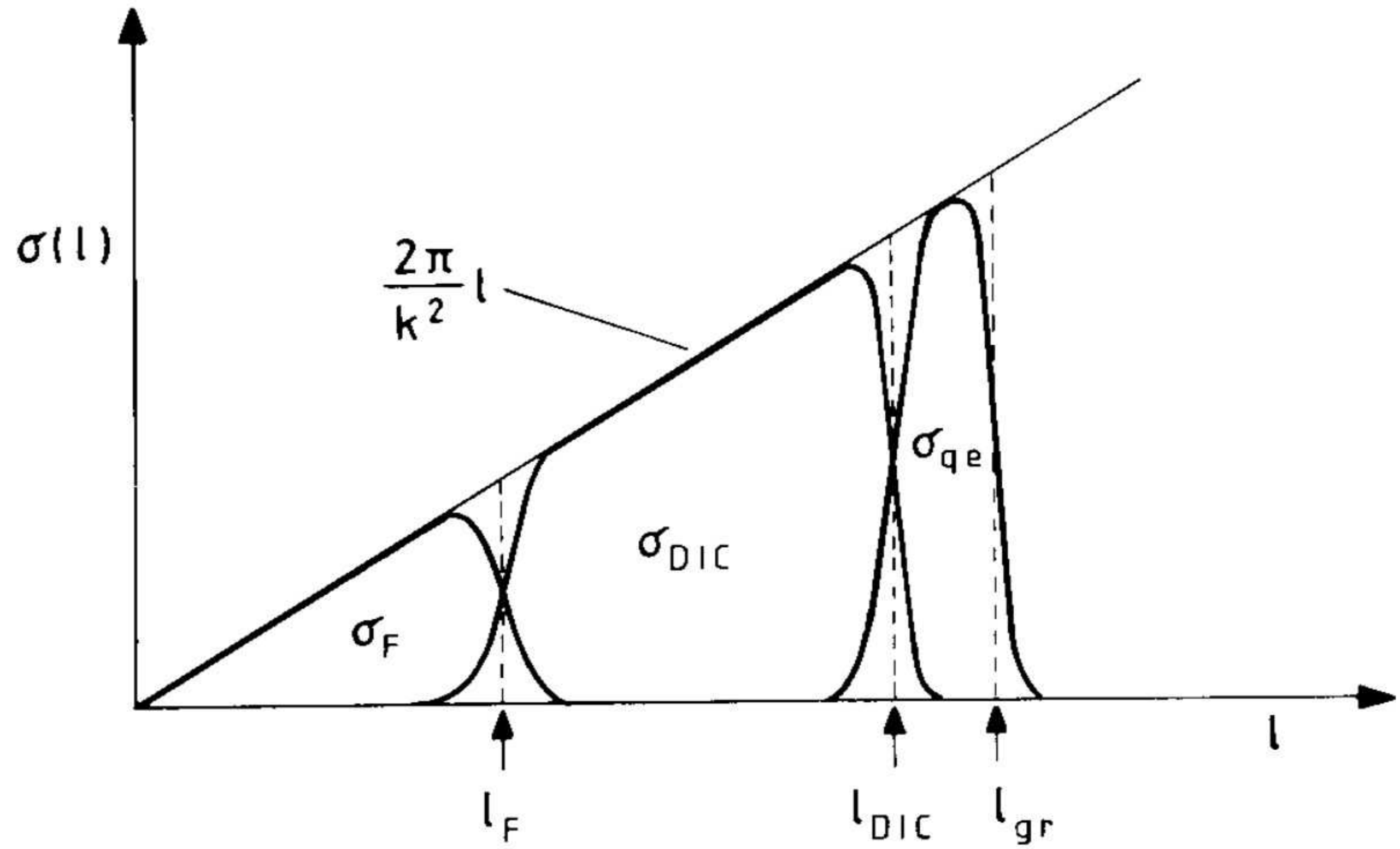
$$\sigma_{\mathrm{r}}(l) = \frac{2\pi}{k^2}\, l\, T(l).$$

$$T(l) = \begin{cases} 1 & \text{for } l < l_{\mathrm{gr}}, \\ 0 & \text{for } l > l_{\mathrm{gr}}, \end{cases} \qquad l_{\mathrm{gr}} = k b_{\mathrm{gr}}$$

$$\sigma_{\mathrm{r}}(l) = \frac{2\pi}{k^2} l$$

between the values $l = 0$ and $l = l_{\mathrm{gr}}$.

$$\begin{array}{lllll} 0 & < & l & < l_{\mathrm{F}} & \text{fusion,} \\ l_{\mathrm{F}} & < & l & < l_{\mathrm{DIC}} & \text{deep-inelastic collisions,} \\ l_{\mathrm{DIC}} & < & l & < l_{\mathrm{gr}} & \text{quasi-elastic collisions.} \end{array}$$



$$\sigma_{\mathrm{F}} = \frac{2\pi}{k^2}\int_0^{l_{\mathrm{F}}} l\,dl = \frac{\pi}{k^2} l_{\mathrm{F}}^2,$$

$$\sigma_{\mathrm{DIC}} = \frac{2\pi}{k^2}\int_{l_{\mathrm{F}}}^{l_{\mathrm{DIC}}} l\,dl = \frac{\pi}{k^2}(l_{\mathrm{DIC}}^2 - l_{\mathrm{F}}^2)$$

Quantum scattering by short-range potential

$$H_{\text{tot}} = \frac{\mathbf{p}_1^2}{2m_1} + \frac{\mathbf{p}_2^2}{2m_2} + V(r) = \frac{\mathbf{P}^2}{2M} + H$$

$$\mathbf{P} = \frac{\hbar}{i}\nabla_R, \quad H = -\frac{\hbar^2}{2\mu}\nabla^2 + V(r),$$

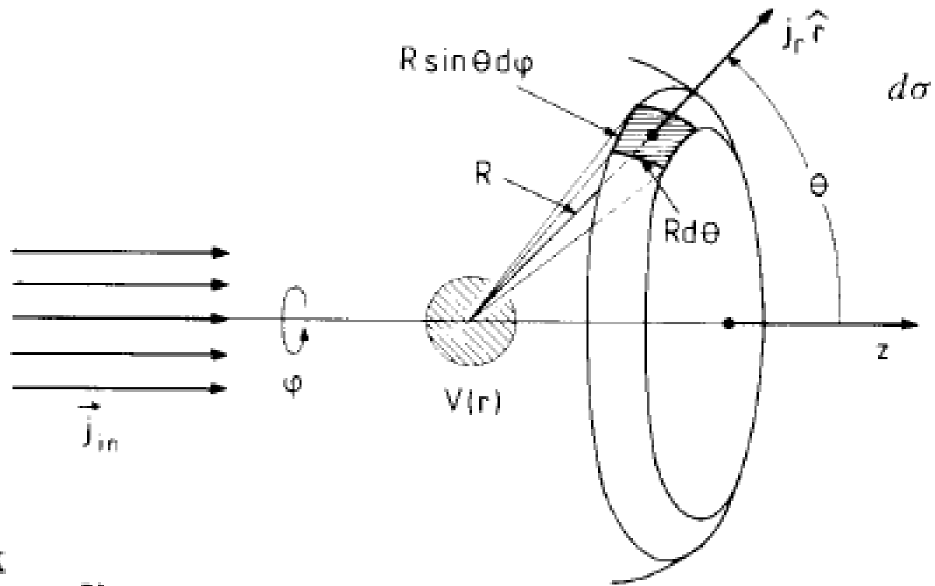
$M = m_1 + m_2$ is the total, and $\mu = m_1 m_2/(m_1 + m_2)$ the reduced mass.

$$\Psi(\mathbf{r}_1, \mathbf{r}_2) = e^{i\mathbf{K}\cdot\mathbf{R}}\,\psi(\mathbf{r}),$$

$$H\psi(\mathbf{r}) = E\psi(\mathbf{r})$$

$$\psi(\mathbf{r}) \;\rightarrow\; e^{i\mathbf{k}\cdot\mathbf{r}} + f(\Omega)\frac{e^{ikr}}{r} \quad \text{for} \quad r \rightarrow \infty;$$

$$\mathbf{j} = \frac{\hbar}{2\mu i}(\psi^*\nabla\psi - \psi\nabla\psi^*),$$



$$d\sigma(\Omega) = \frac{\text{probability current into } d\Omega \text{ in the direction } \Omega}{\text{probability current density of the incident wave}}.$$

$$\mathbf{j} = \frac{\hbar}{2\mu i}(\psi^* \nabla \psi - \psi \nabla \psi^*),$$

$$\mathbf{j}_{\text{in}} = \frac{\hbar \mathbf{k}}{\mu} = \mathbf{v},$$

$$j_r r^2 d\Omega = \frac{\hbar}{2\mu i}\left[f^* \frac{e^{-ikr}}{r} \frac{\partial}{\partial r}\left(f \frac{e^{ikr}}{r}\right) - f \frac{e^{ikr}}{r} \frac{\partial}{\partial r}\left(f^* \frac{e^{-ikr}}{r}\right)\right] r^2 d\Omega$$
$$\rightarrow v|f(\Omega)|^2 d\Omega \quad \text{for } r \rightarrow \infty.$$

$$\frac{d\sigma}{d\Omega} = \frac{j_r R^2}{|\mathbf{j}_{\text{in}}|} = |f(\Omega)|^2$$

$$f(\theta) = \frac{1}{2ik} \sum_{l=0}^{\infty} (2l+1)(S_l - 1) P_l(\cos\theta)$$

$$S_l = e^{2i\delta_l}$$

$$\sigma_{\text{el}} = 2\pi \int_{-1}^{1} d(\cos\theta) \frac{d\sigma}{d\Omega} = \frac{\pi}{k^2} \sum_{l=0}^{\infty} (2l+1)|S_l - 1|^2,$$

reaction cross section

$$\psi(r,\theta) \to \frac{1}{2ik}\sum_{l=0}^{\infty}(2l+1)\left((-)^{l+1}\frac{e^{-ikr}}{r} + S_l\frac{e^{ikr}}{r}\right)P_l(\cos\theta).$$

$$-\int d\mathbf{F}\cdot\mathbf{j} = -\frac{\hbar}{2\mu i}R^2 2\pi\int_{-1}^{1} d(\cos\theta)\left(\psi^*\frac{\partial\psi}{\partial r} - \psi\frac{\partial\psi^*}{\partial r}\right)_{r=R},$$

$$-\frac{\hbar}{2\mu i}R^2 2\pi\frac{1}{4k^2R^2}\sum_{l=0}^{\infty}2(2l+1)\left\{\left[(-)^{l+1}e^{ikR} + S_l^* e^{-ikR}\right]\right.$$

$$\left.\times\left[(-)^{l+1}(-ik)e^{-ikR} + ikS_l e^{ikR}\right] - \text{c.c.}\right\} = \frac{\hbar\pi}{\mu k}\sum_{l=0}^{\infty}(2l+1)(1-|S_l|^2).$$

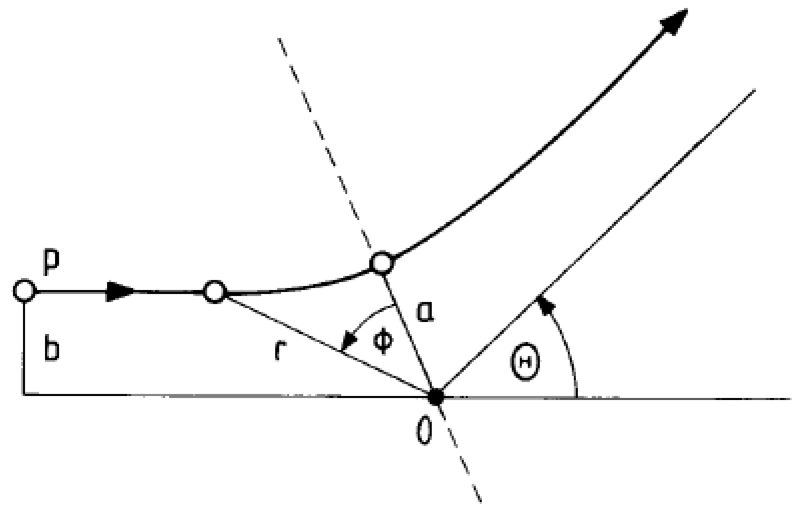
$$\sigma_{\mathrm{r}} = \frac{\pi}{k^2}\sum_{l=0}^{\infty}(2l+1)(1-|S_l|^2). \qquad \sigma_{\mathrm{r}} = \frac{\pi}{k^2}\sum_{l=0}^{\infty}(2l+1)T_l. \qquad T_l = 1-|S_l|^2,$$

total cross section

$$\sigma_{\mathrm{tot}} = \sigma_{\mathrm{el}} + \sigma_{\mathrm{r}}.$$

$$\sigma_{\mathrm{tot}} = \frac{2\pi}{k^2}\sum_{l=0}^{\infty}(2l+1)(1-\mathrm{Re}\, S_l).$$

Classical scattering



$$E = \frac{\mu}{2}\left(\frac{dr}{dt}\right)^2 + \frac{L^2}{2\mu r^2} + V(r)$$

$$L = \mu r^2 \frac{d\phi}{dt}.$$

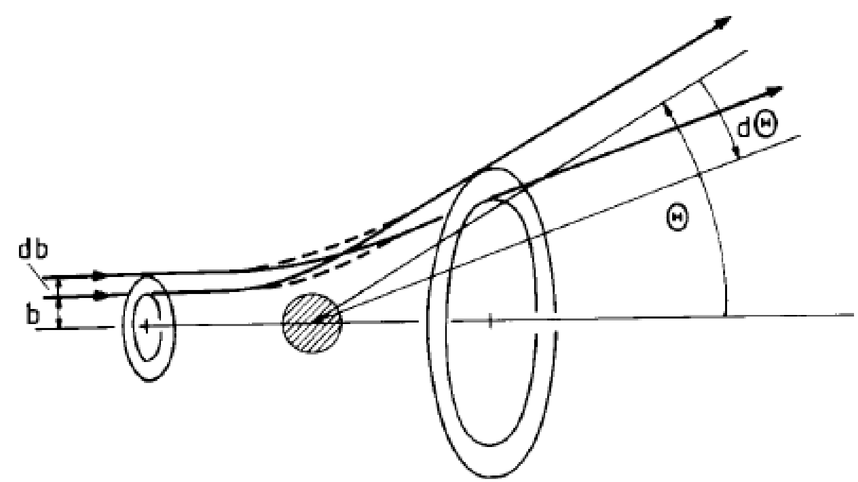
The *deflection angle* Θ

$$\Theta(b) = \pi - 2\int_a^\infty dr \frac{b}{r^2\sqrt{1 - V(r)/E - b^2/r^2}}.$$

$$d\sigma = \frac{\text{particle current into } d\Omega \text{ in the direction } \Omega}{\text{particle current density of the incident particles}}.$$

$$d\sigma = \frac{J\, b\,db\,d\varphi}{J}.$$

$$d\Omega = |\sin\Theta d\Theta| d\varphi$$



$$\frac{d\sigma}{d\Omega} = \frac{b}{\sin\theta |d\Theta/db|};$$

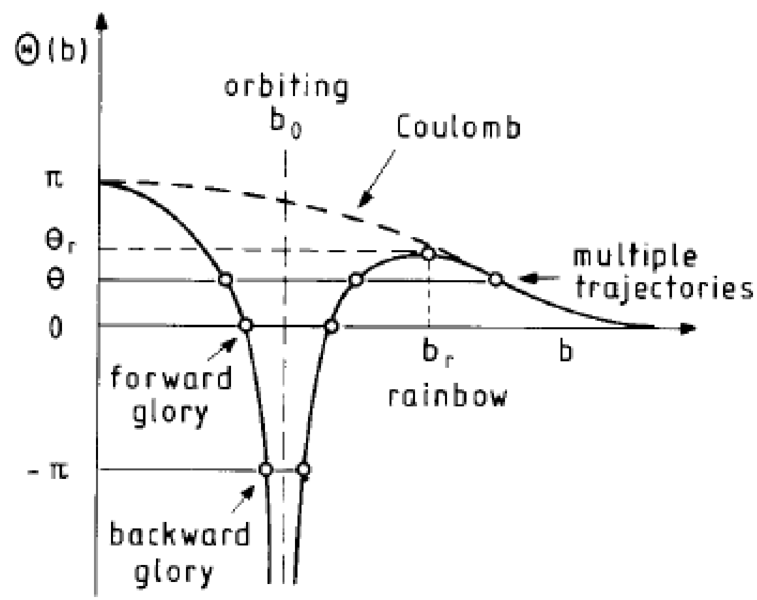
The Coulomb potential has the form

$$V(r) = \frac{Z_1 Z_2 e^2}{r} = \frac{2\eta E}{kr}, \qquad \eta = \frac{Z_1 Z_2 e^2}{\hbar v}$$

$$\frac{d\sigma}{d\Omega} = \frac{\eta^2}{4k^2 \sin^4(\theta/2)}.$$

Rutherford cross section

$$\frac{d\sigma}{d\Omega} = \frac{b}{\sin\theta |d\Theta/db|};$$



Formal theory of reactions

Two-body channels $$x + A \to y + B,$$

$$a = \{\alpha; a_x, a_A; \mathbf{k}_a\},$$

$$\mathbf{k}_a = \frac{m_A \mathbf{k}_x - m_x \mathbf{k}_A}{m_x + m_A}$$

$$\mathcal{E}_a = \frac{\hbar^2 k_a^2}{2\mu_\alpha},$$

$$H_\alpha = T_\alpha + h_\alpha, \qquad T_\alpha = -\frac{\hbar^2}{2\mu_\alpha}\nabla_\alpha^2 \qquad h_\alpha = h_x + h_A.$$

$$(H_\alpha - E_a)|a\rangle = 0$$

$$E_a = \mathcal{E}_a + \epsilon_a,$$

$$h_\alpha |a_x, a_A\rangle = \epsilon_a |a_x, a_A\rangle \text{ with } \epsilon_a = \epsilon_x + \epsilon_A,$$

$$i\hbar \frac{d}{dt}|\Phi_a(t)\rangle = H_\alpha |\Phi_a(t)\rangle$$

The total Hamiltonian H of the system is

$$H = T_\alpha + h_\alpha + V_\alpha = H_\alpha + V_\alpha$$

$$i\hbar \frac{d}{dt} |\Psi_a(t)\rangle = H |\Psi_a(t)\rangle$$

For $t = 0$ we have

$$|\Psi_a^+(0)\rangle = e^{\frac{i}{\hbar} H t_0} |\Phi_a(t_0)\rangle \quad \text{with } t_0 < -t_\infty,$$

the Lippmann-Schwinger equation

$$|a^\pm\rangle = |a\rangle + \frac{1}{E_a - H_\alpha \pm i\eta} V_\alpha |a^\pm\rangle,$$

Cross section

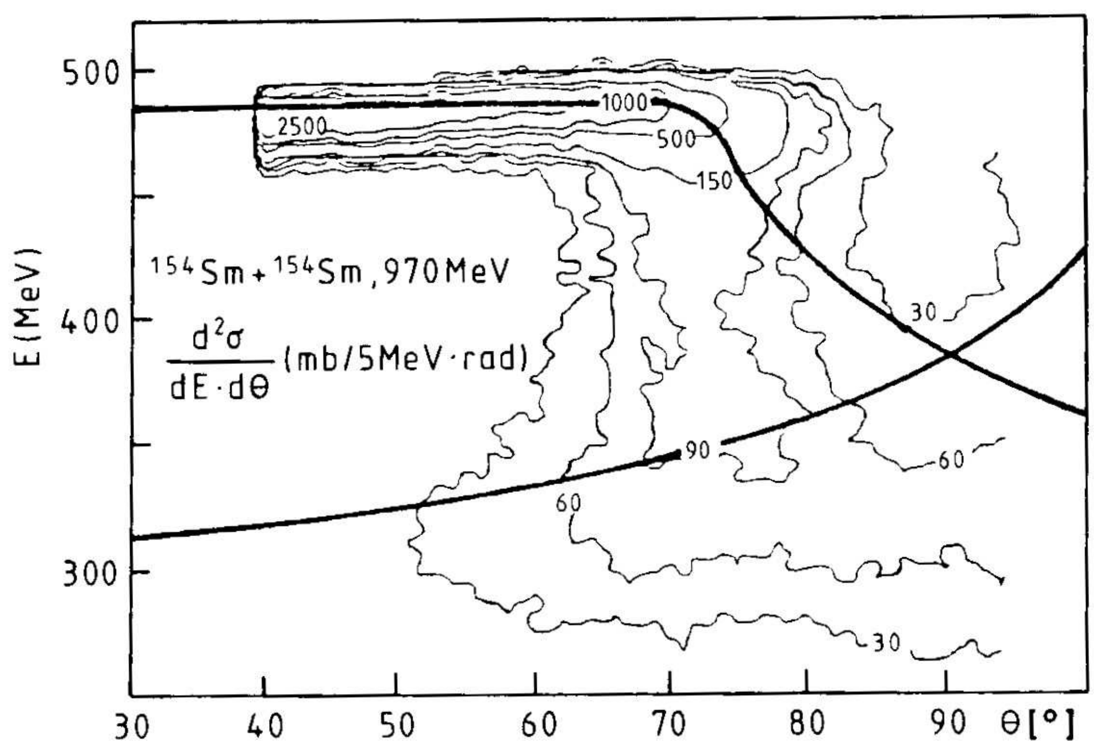
$$\frac{d\sigma_{a\to b}}{d\Omega_b} = \left(\frac{2\pi}{\hbar}\right)^4 \mu_\alpha \mu_\beta \frac{k_b}{k_a} |T_{a\to b}|^2.$$

$$T_{a\to b} = \langle b| \left(V_\alpha + V_\beta \frac{1}{E - H + i\eta} V_\alpha \right) |a\rangle,$$

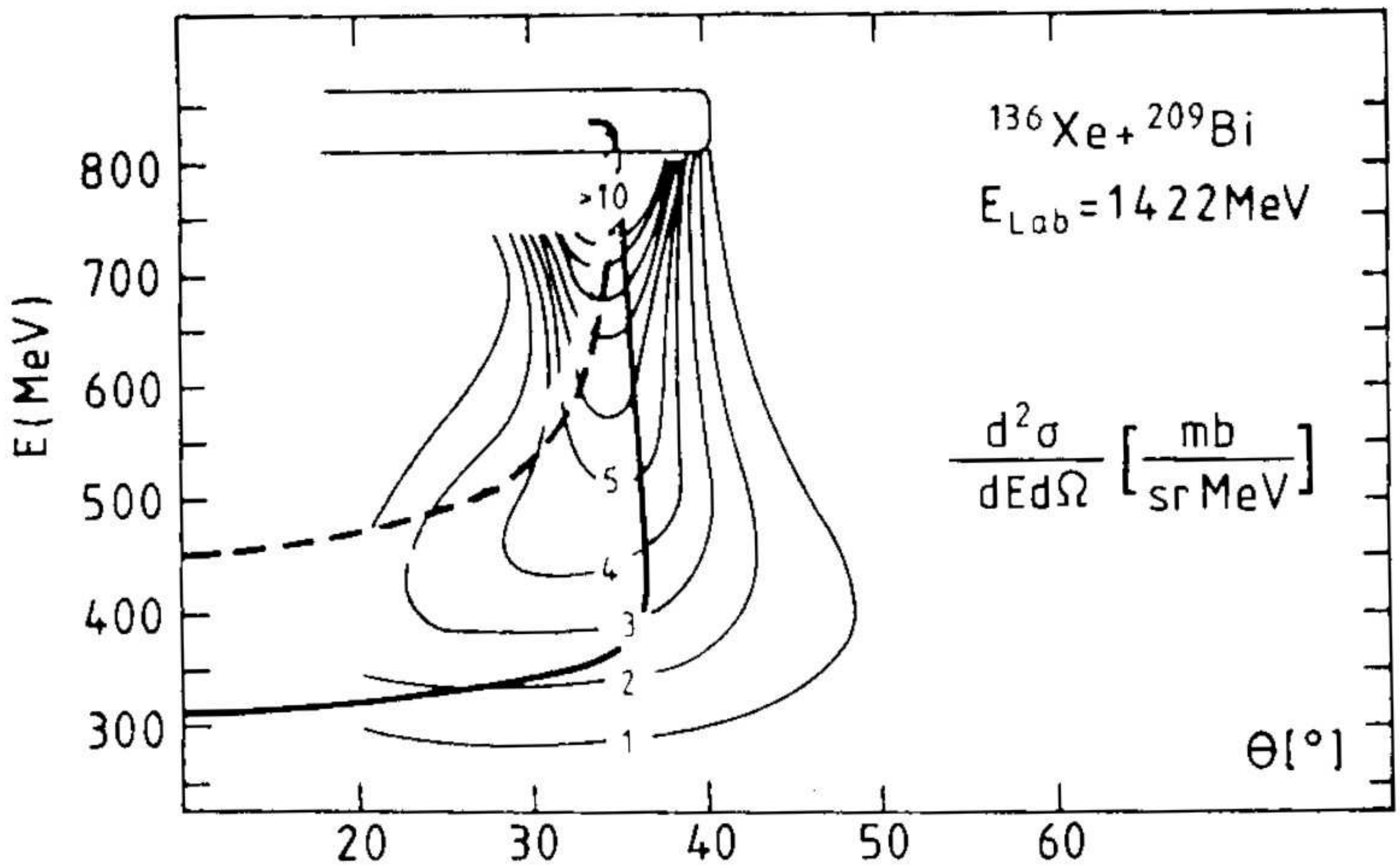
Characteristic features of deep-inelastic collisions (DIC)

Measurments of γ-multiplicities show that the γ-rays emitted after a DIC carry angular momentum, which is taken out of the relative motion of the collision partners. This shows that there is considerable transfer of angular momentum from the relative motion to the internal system.

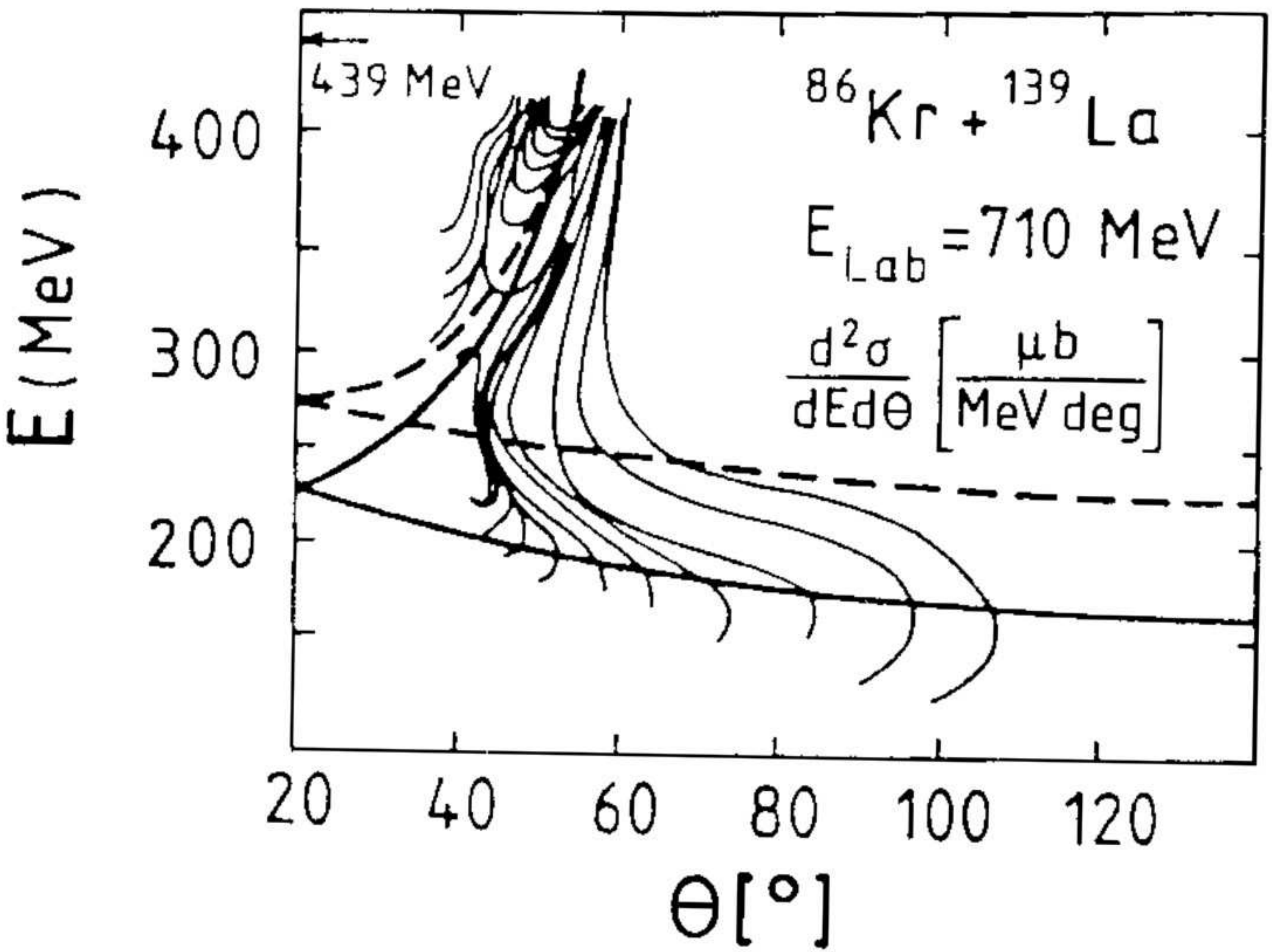
- Coulomb-like collisions. Collision partners are higly charged and the incident energy is relatively low. The Coulomb repulsion dominates and the projectile is strongly reflected to large, backward angles.



- Focussing collisions. Higher energies or lighter nuclei. Scattering into a narrow angular region.



- Orbiting collisions. The attractive nuclear force dominates over the Coulomb force. This pulls the trajectory of the projectile around the target into the region of negative scattering angles.

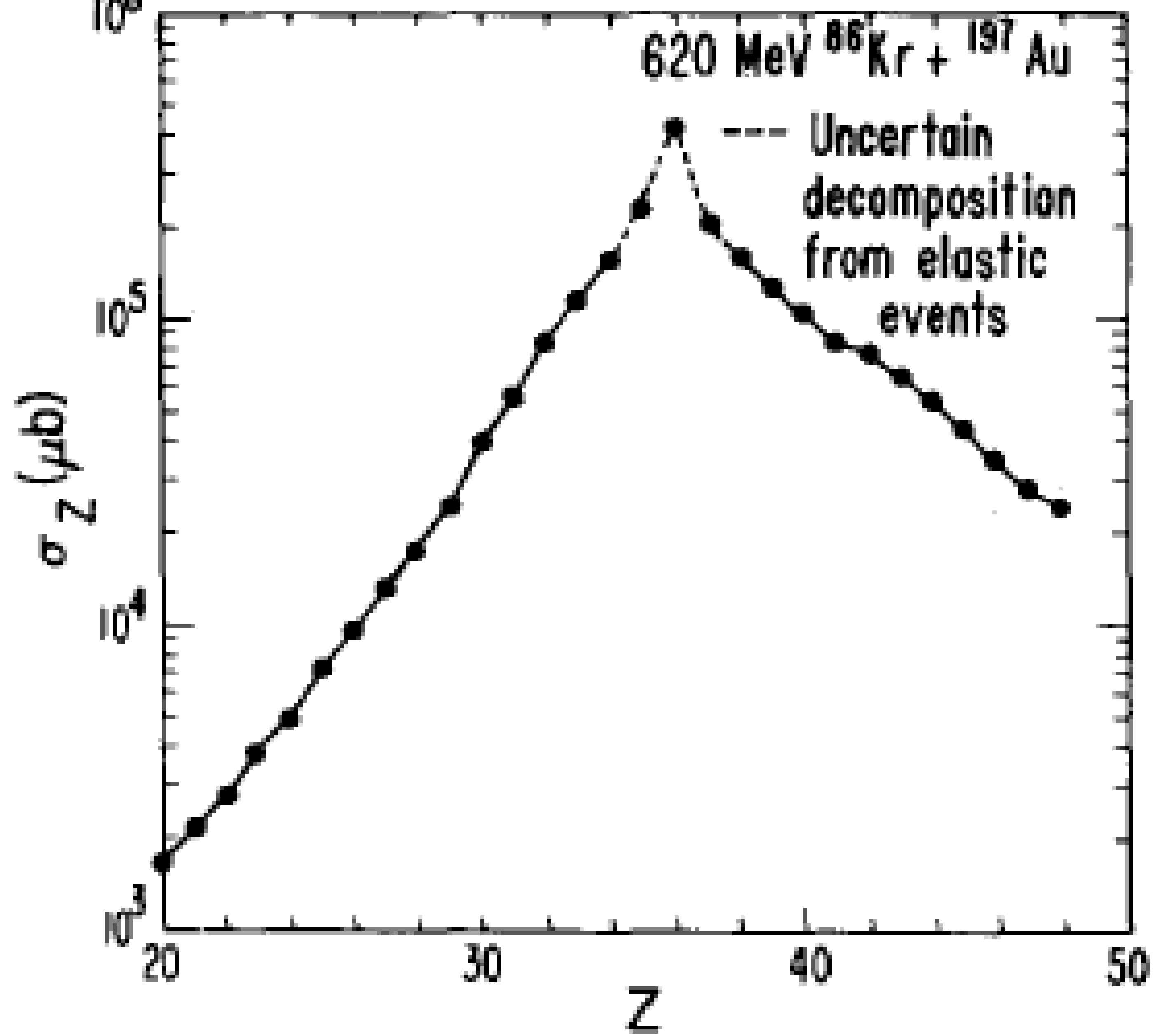


End of 60th – discovery of new type of nuclear reactions – DIC

Mechanism: dynamic & statistic pecularities
formation of DNS – result of nucler viscosity and microscopic effects

nuclear molecule ↔ DNS
quasistationary states dynamics

Study of DIC
identification of the products
scattering chamber
radiochemistry
ΔE-E detectors
time of flight
magnetic spectrometra
two-shoulder detectors
detectors for *n, p*, α, and γ


620 MeV 86Kr + 197Au
Uncertain decomposition from elastic events
σ Z (μb)
Z
10^6
10^5
10^4
10^3
20
30
40
50

Characteristics of DIC

- total dissipation of kinetic energy ⇒ energy distribution has maxima at V_b for the fragments, independent on $E_{c.m.}$
- angular distributions have maxima at forward angles decrease of anisotropy with increasing number of transfered nucleon
- large variation of mass (charge) distributions (max. at A_p (A_t) and Z_p (Z_t))
- N/Z ratio
- sharing of excitation energy and angular momentum

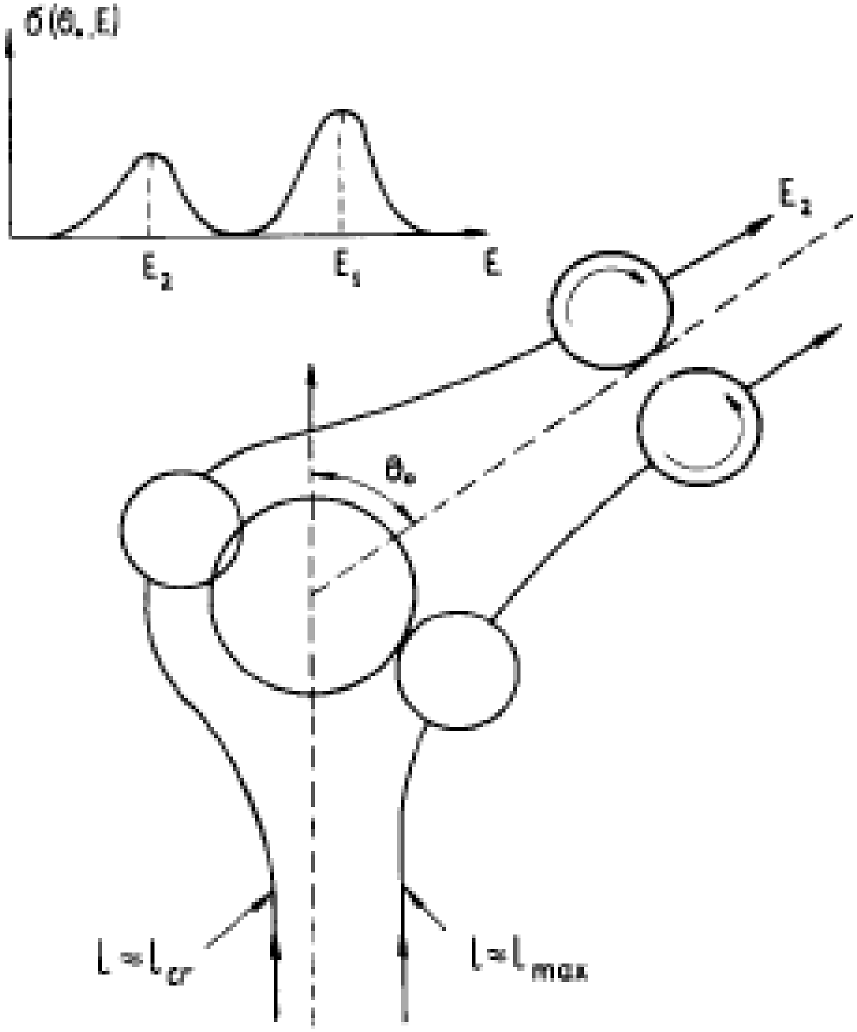
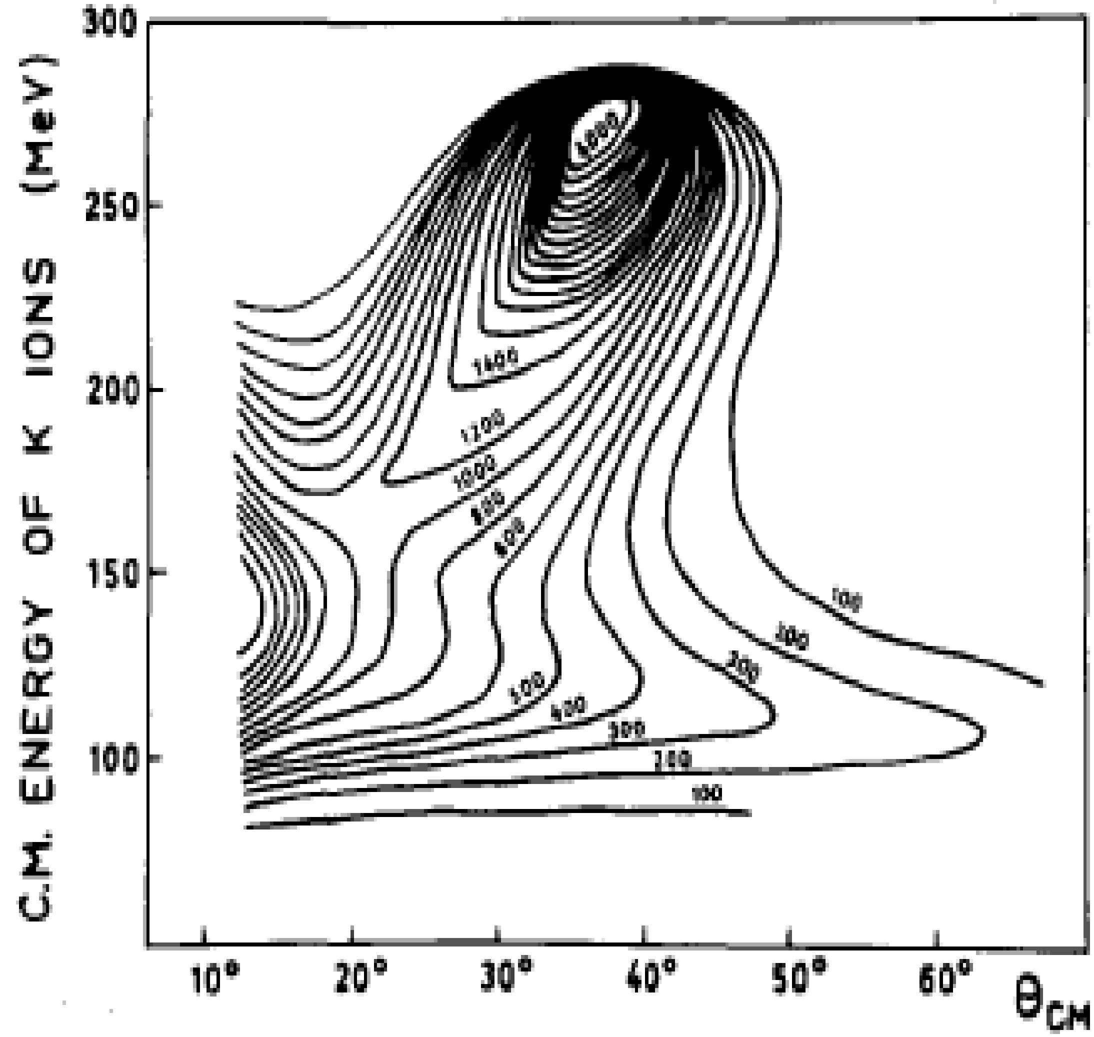


Illustration of the formation of two peaks in the energy spectrum



Contour diagram representing the transfer reaction data for $^{232}Th(^{40}Ar,K)$ at 388 MeV

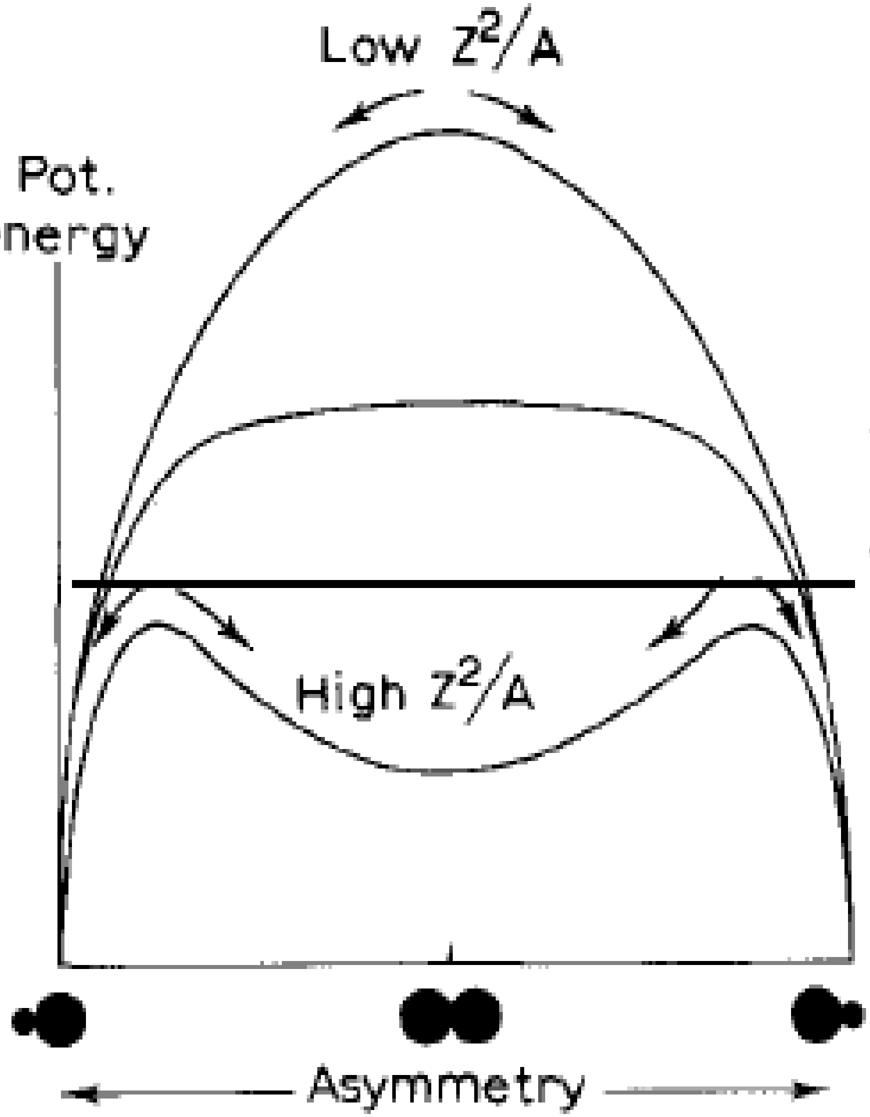


Illustration of the dependence of the potential energy of the system of two touching nuclear drops on mass asymmetry and parameter $(Z_1 + Z_2)^2/(A_1 + A_2)$.

Set of coordinates for the description of DNS evolution:

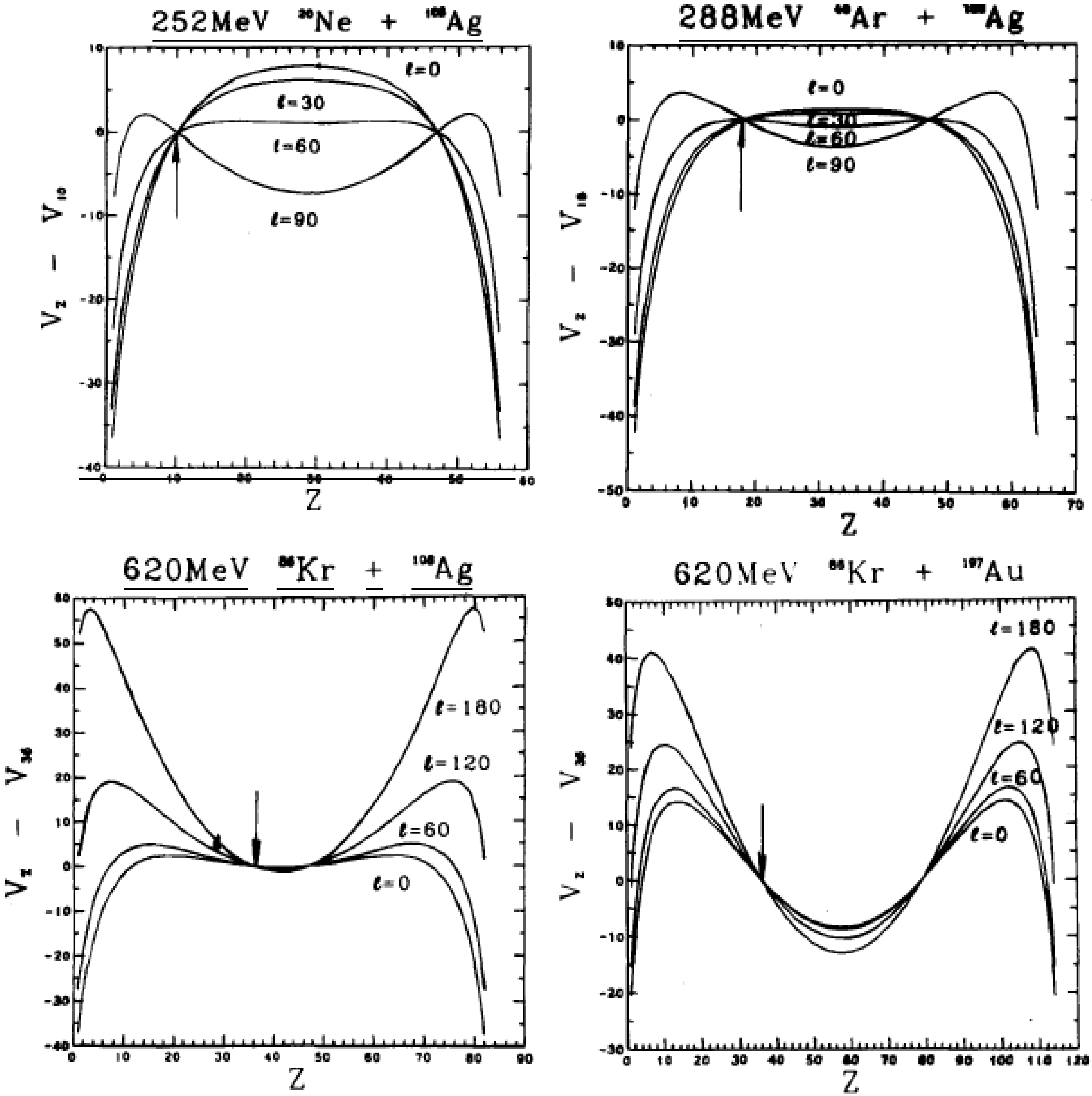
$$\eta_Z=(Z_1-Z_2)/(Z_1+Z_2)\ ,\ \ \eta=(A_1-A_2)/(A_1+A_2)\ ,\ \ R$$

The potential energy of DNS:

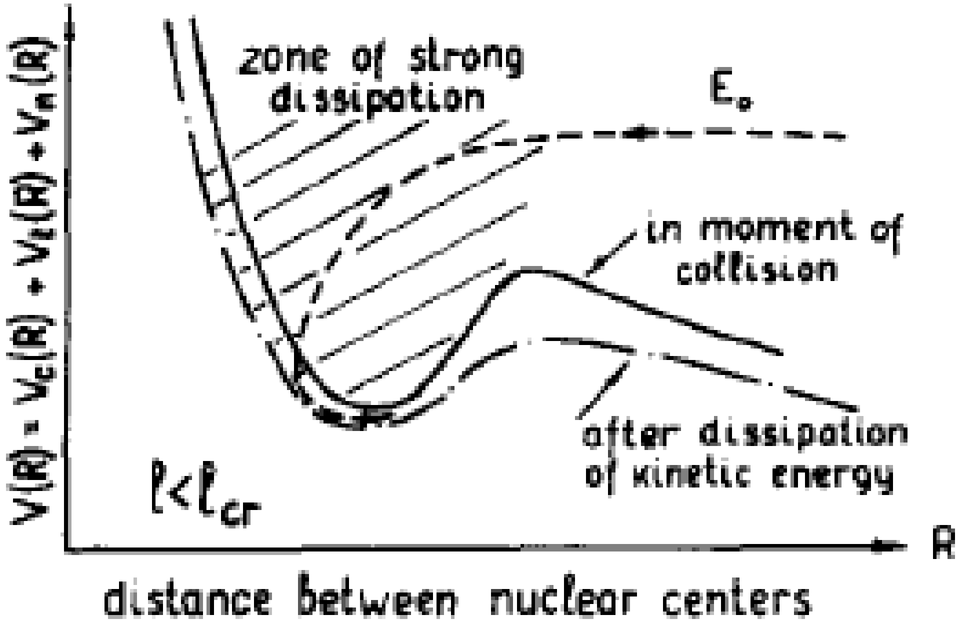
$$U(R,\eta,\eta_Z,\beta_{1,}\beta_{2,}J)=B_1+B_2+V(R,\eta,\eta_Z,\beta_{1,}\beta_{2,}J)$$

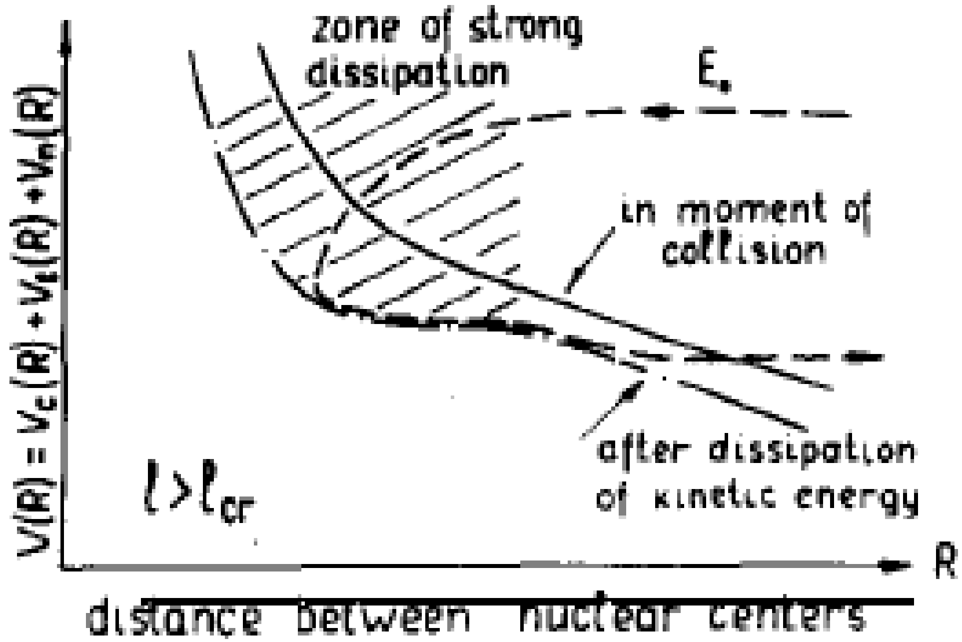
The nucleus-nucleus potential:

$$V(R,\eta,\eta_Z,\beta_{1,}\beta_{2,}J)=V_C(R,\eta_Z,\beta_{1,}\beta_2)+V_N(R,\eta,\beta_{1,}\beta_2)+V_{rot}(\eta,\beta_{1,}\beta_{2,}J)$$



252MeV 20Ne + 108Ag
ℓ=0
ℓ=30
ℓ=60
ℓ=90
Vz − V10
Z
288MeV 40Ar + 108Ag
ℓ=0
ℓ=30
ℓ=60
ℓ=90
Vz − V18
Z
620MeV 86Kr + 108Ag
ℓ=180
ℓ=120
ℓ=60
ℓ=0
Vz − V36
Z
620MeV 86Kr + 197Au
ℓ=180
ℓ=120
ℓ=60
ℓ=0
Vz − V36
Z


zone of strong dissipation
E_o
in moment of collision
after dissipation of kinetic energy
$l < l_{cr}$
$V(R) = V_c(R) + V_l(R) + V_n(R)$
R
distance between nuclear centers


zone of strong dissipation
E_o
in moment of collision
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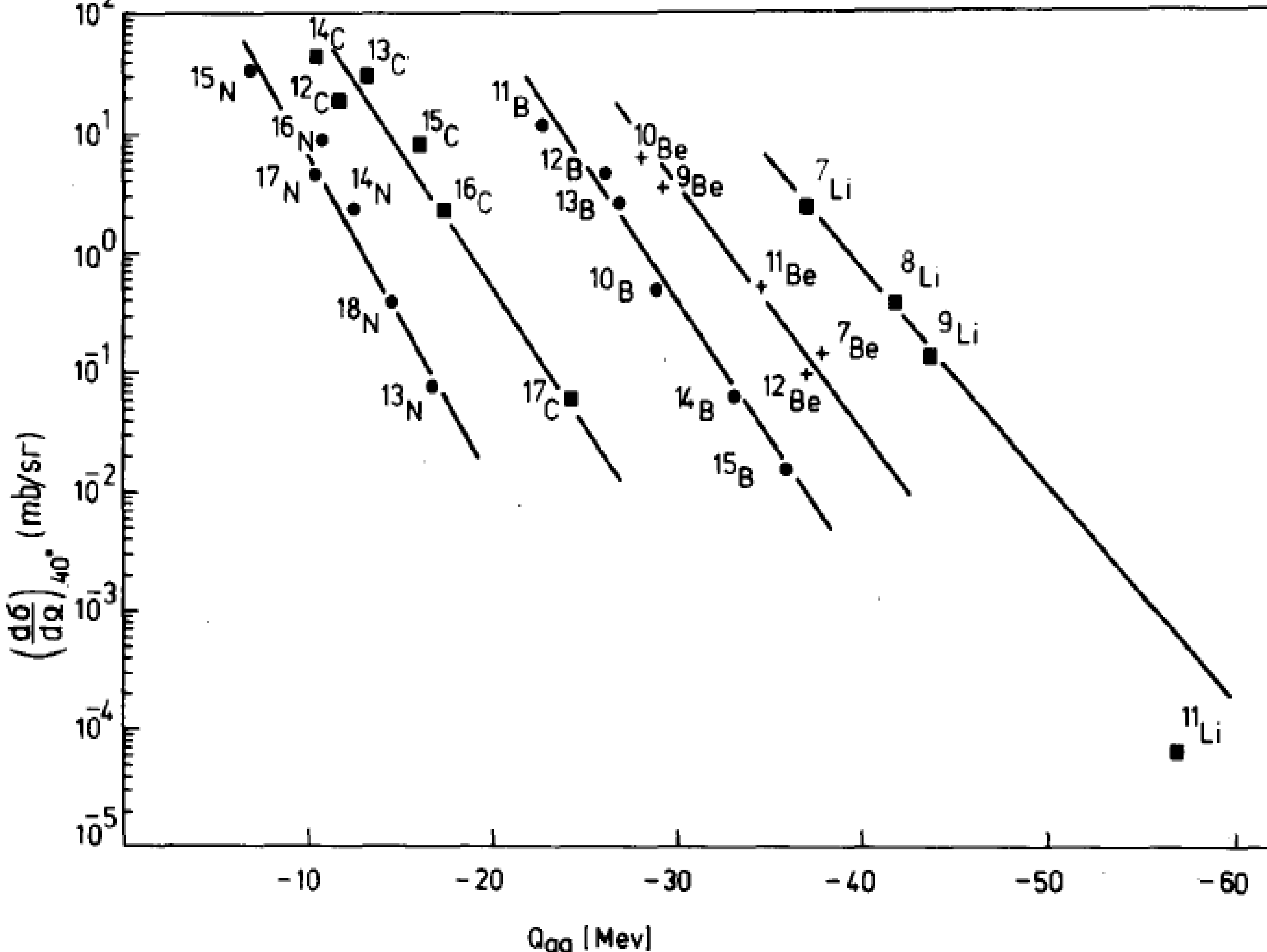
$V(R) = V_c(R) + V_l(R) + V_n(R)$
R
distance between nuclear centers



$Q_{gg} = (M_1 + M_2) - (M_3 + M_4)$.

^{232}Th + ^{16}O 137 MeV

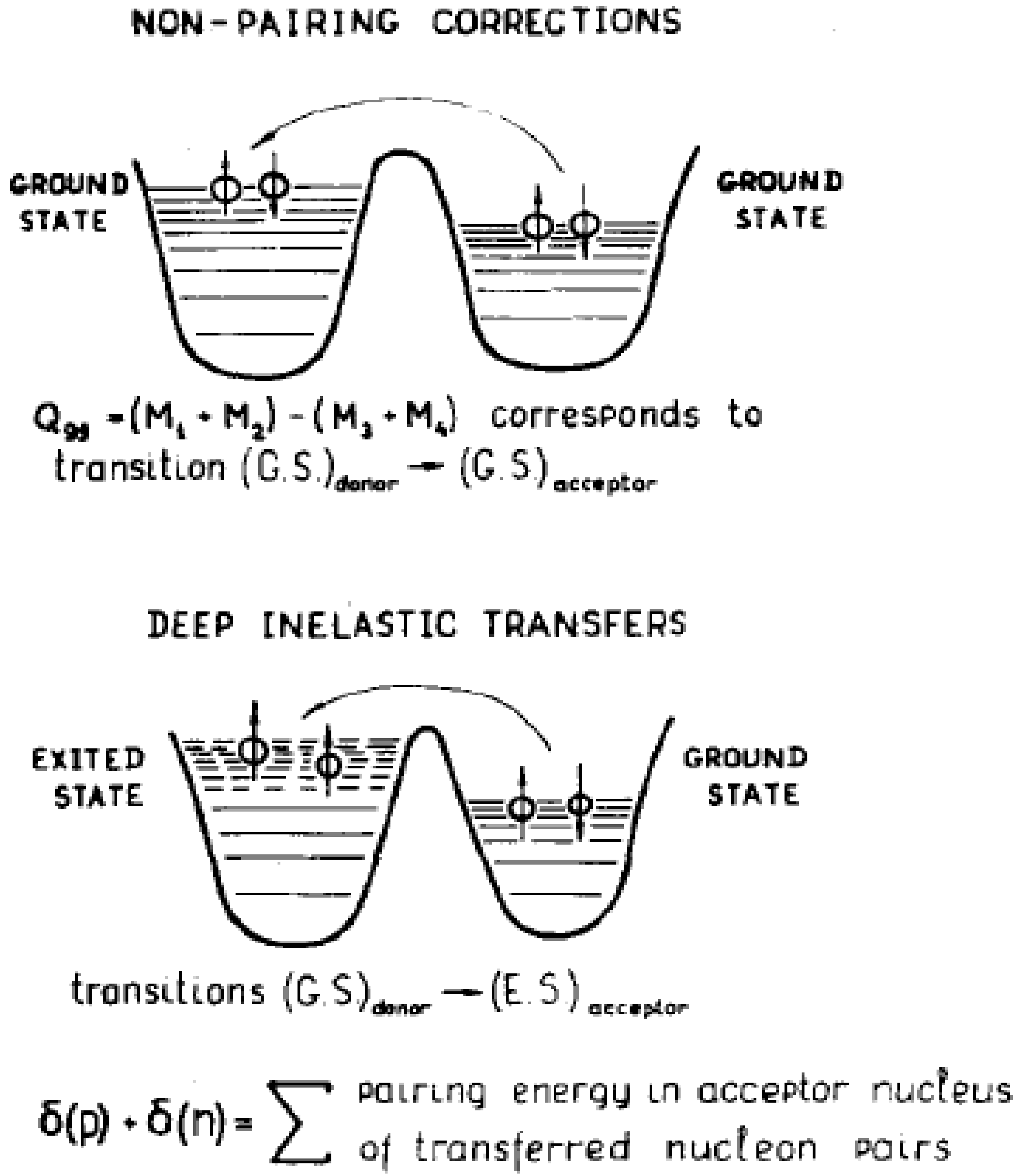
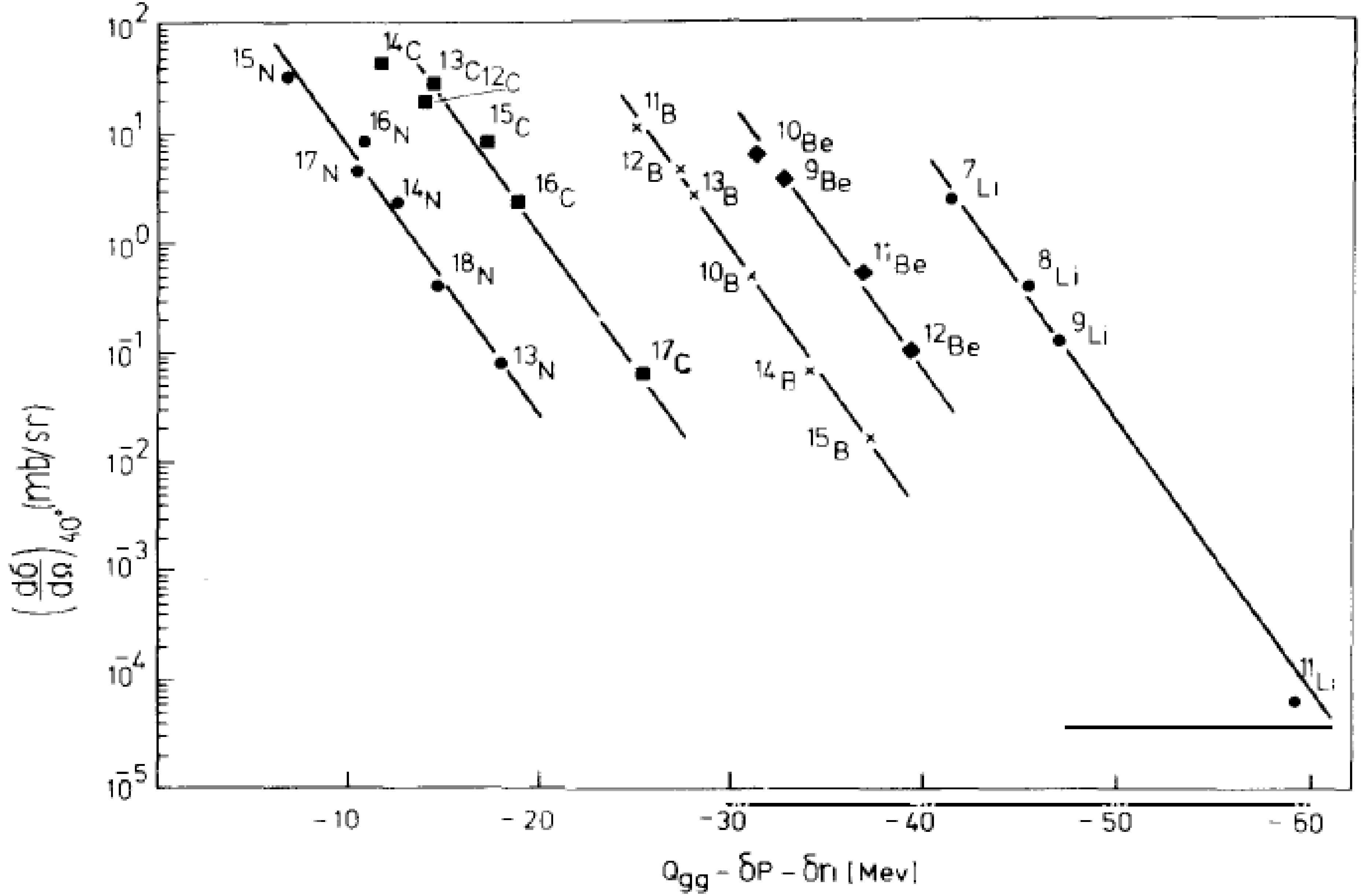
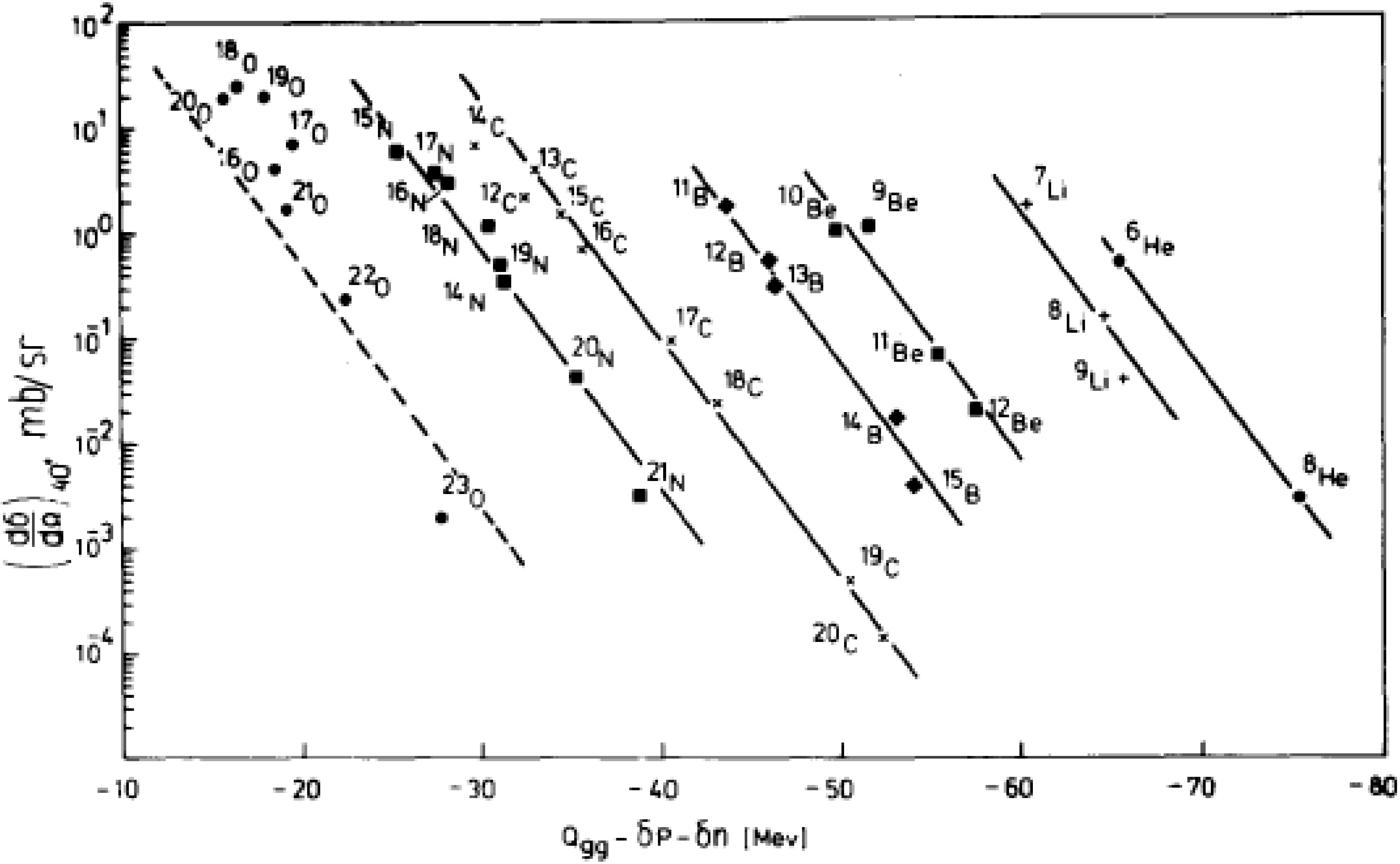


Illustration of the necessity of introducing corrections for non-pairing.



$\left(\frac{d\sigma}{d\Omega}\right)_{40°}$ (mb/sr)
$Q_{gg} - \delta P - \delta n$ [Mev]
^{15}N
^{14}C
^{13}C
^{12}C
^{16}N
^{15}C
^{17}N
^{14}N
^{16}C
^{18}N
^{13}N
^{17}C
^{11}B
^{12}B
^{13}B
^{10}B
^{14}B
^{15}B
^{10}Be
^{9}Be
^{11}Be
^{12}Be
^{7}Li
^{8}Li
^{9}Li
^{11}Li

$^{232}Th + ^{16}O$

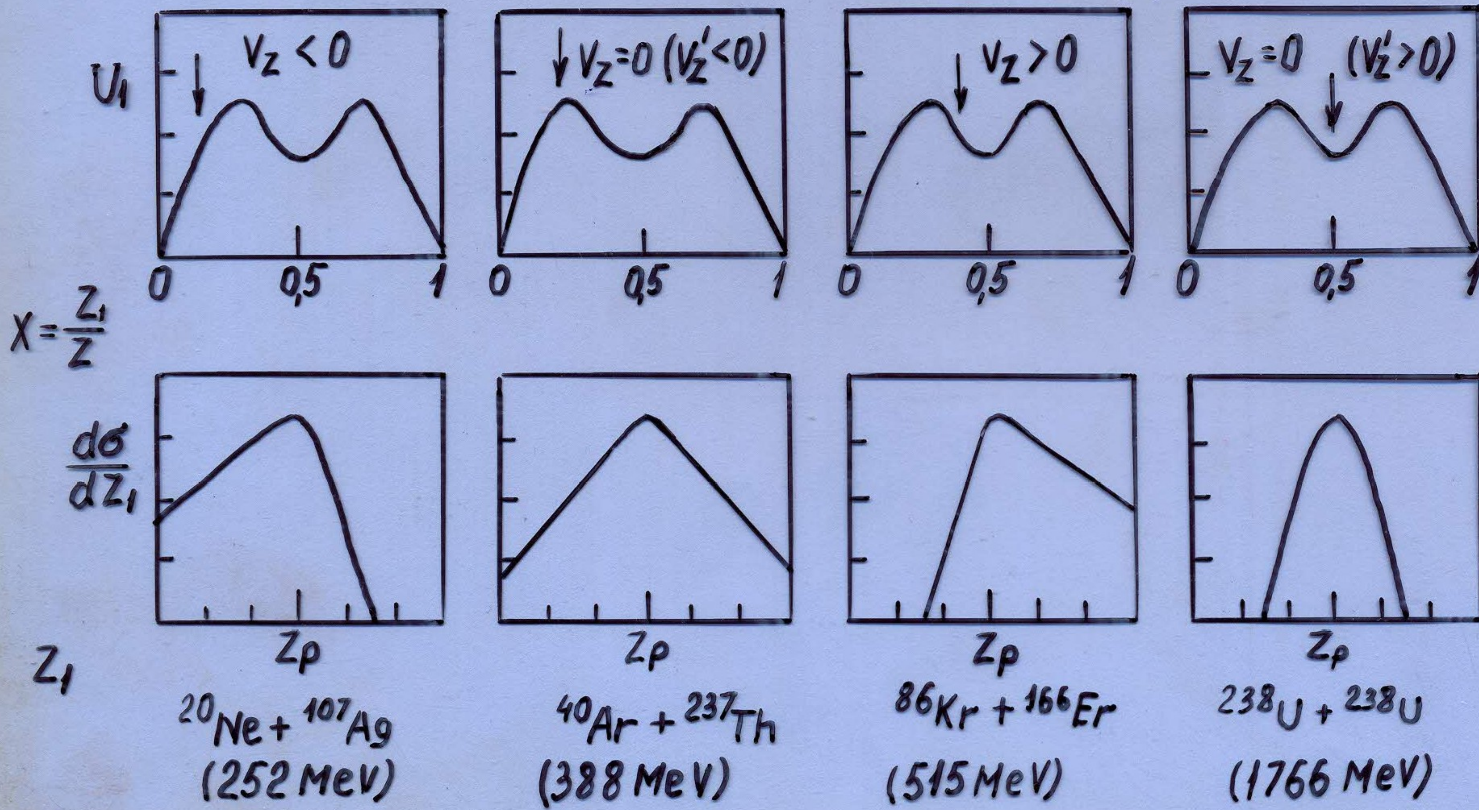


The Q_{gg} systematics for the reaction $^{232}Th + ^{22}Ne$, corrected for non-pairing.

$$\sigma \backsim \exp\{(Q_{gg} + \Delta E_c)/T\}$$

$$\frac{\partial}{\partial t} P(Z_1,t) = -\frac{\partial}{\partial Z_1}(V_Z P) + \frac{\partial^2}{\partial Z_1^2}(D_Z P)$$

$V_Z \sim \frac{\partial U_l}{\partial Z_1}$ $\quad \langle Z_1 \rangle = Z_P + V_Z \cdot t$ $\quad \sigma_Z^2 = 2 D_Z \cdot t$

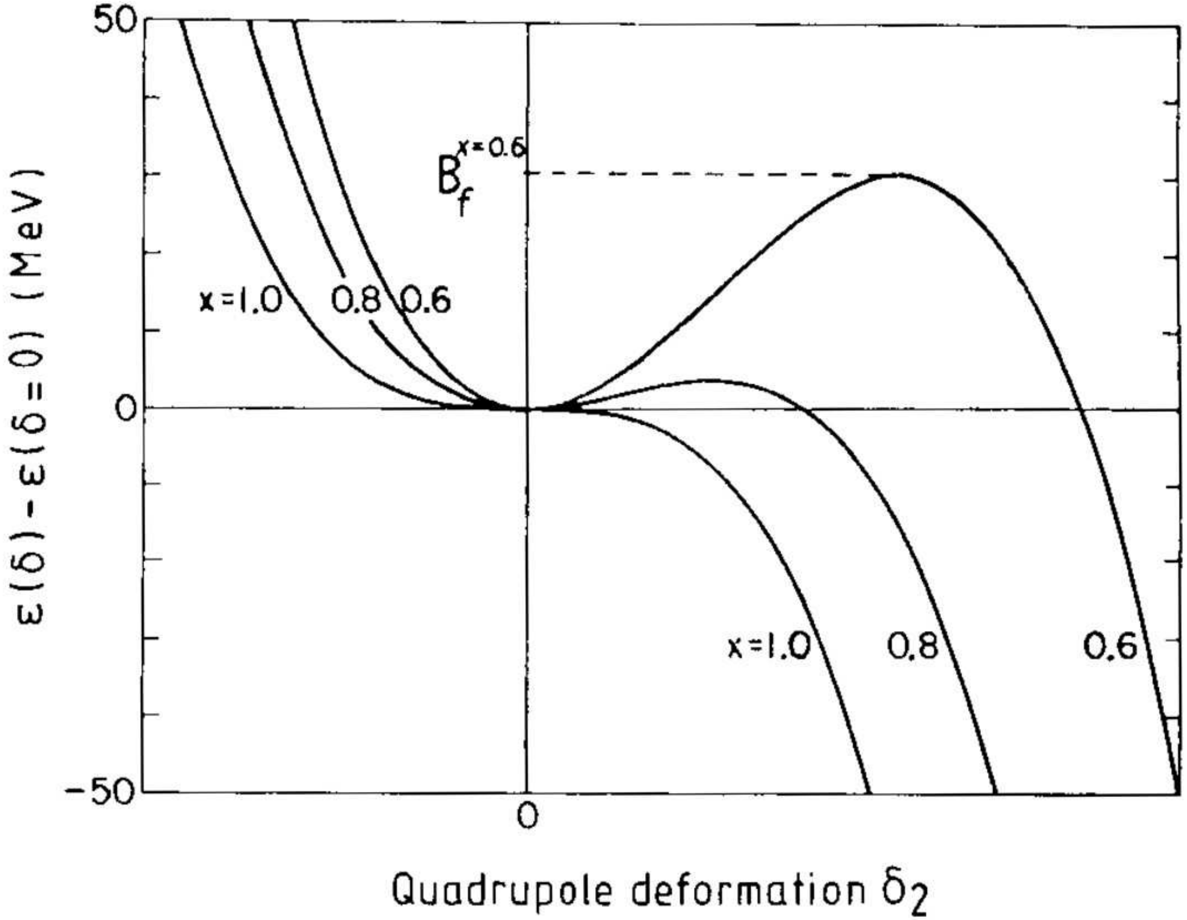


Fusion

stability of the formed compound nuclei
fisility parameter

$$x \approx \frac{1}{50}\frac{Z^2}{A} \approx \frac{Z}{120}.$$

x>1 – unstable
x<1 - stable



The projectile moves in the field of the Coulomb-plus-nuclear potential *V(r)*. For a given impact parameter *b* the radial motion is governed by the potential

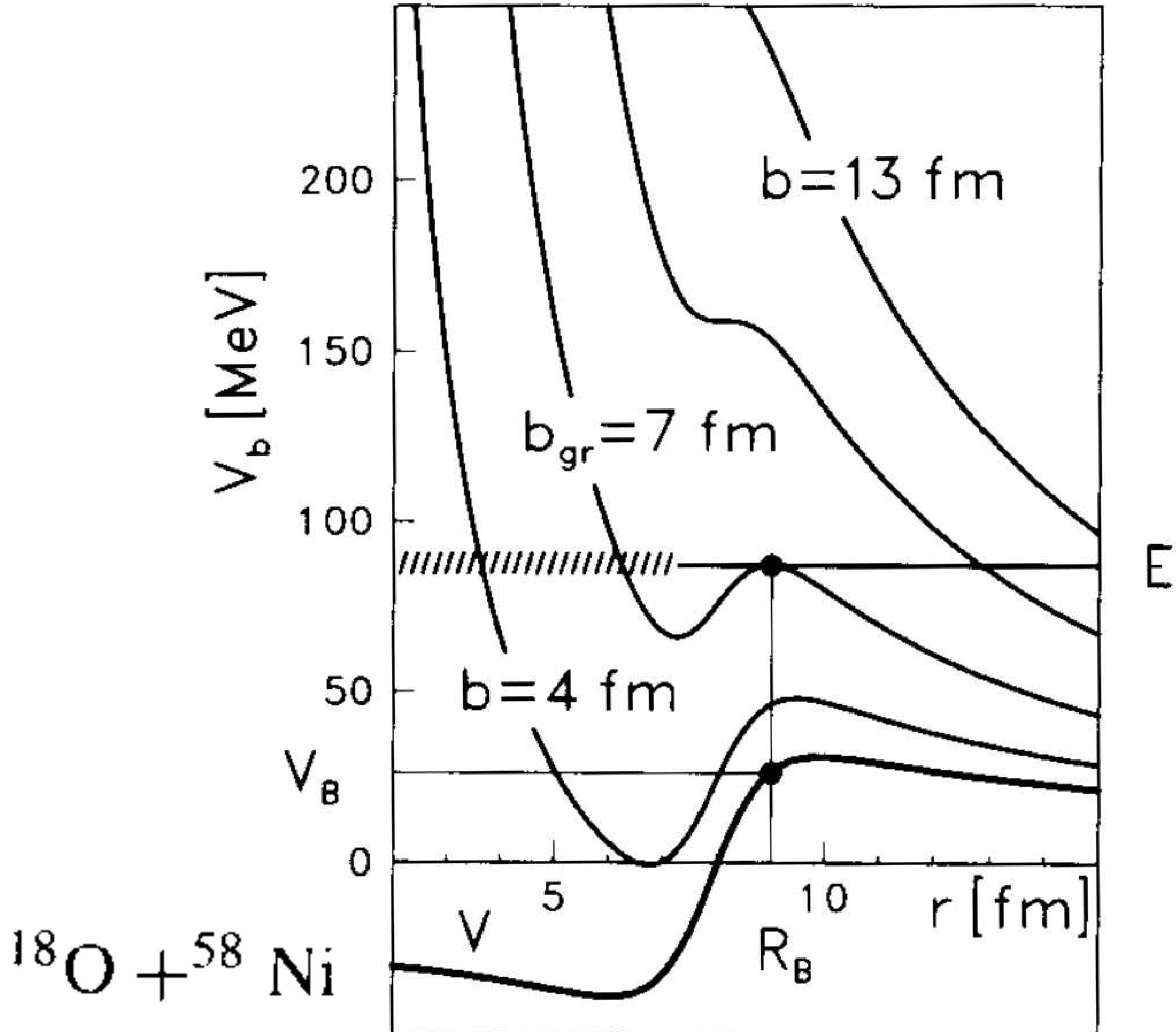
$$V_b(r) = V(r) + E\frac{b^2}{r^2},$$

$$b = L/\sqrt{2\mu E}$$

$$V_{\mathrm{B}} = V(R_{\mathrm{B}}) = V_{b=0}(R_{\mathrm{B}}),$$

$$V_{\mathrm{B}} + E\frac{b_{\mathrm{gr}}^2}{R_{\mathrm{B}}^2} = E,$$

$$b_{\mathrm{gr}} = R_{\mathrm{B}}\sqrt{1 - \frac{V_{\mathrm{B}}}{E}}.$$

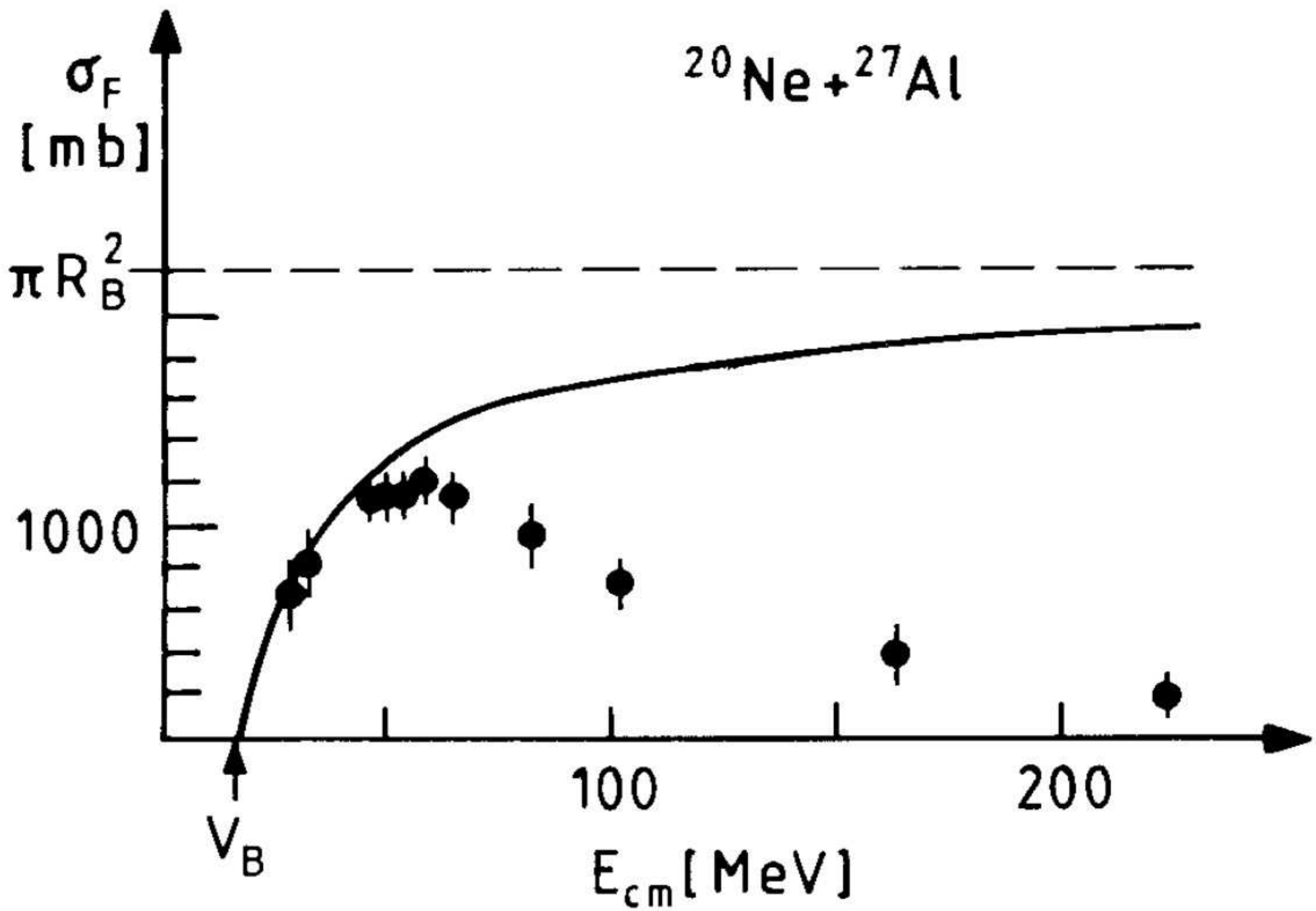


V_b [MeV]
200
150
100
50
V_B
0
5
10
r [fm]
R_B
b=13 fm
b_{gr}=7 fm
b=4 fm
E
V
$^{18}O + ^{58}Ni$

Total fusion cross section

$$\sigma_{\mathrm{F}} = \pi b_{\mathrm{gr}}^2 .$$

$$\sigma_{\mathrm{F}}(E) = \pi R_{\mathrm{B}}^2 \left(1 - \frac{V_{\mathrm{B}}}{E} \right) .$$



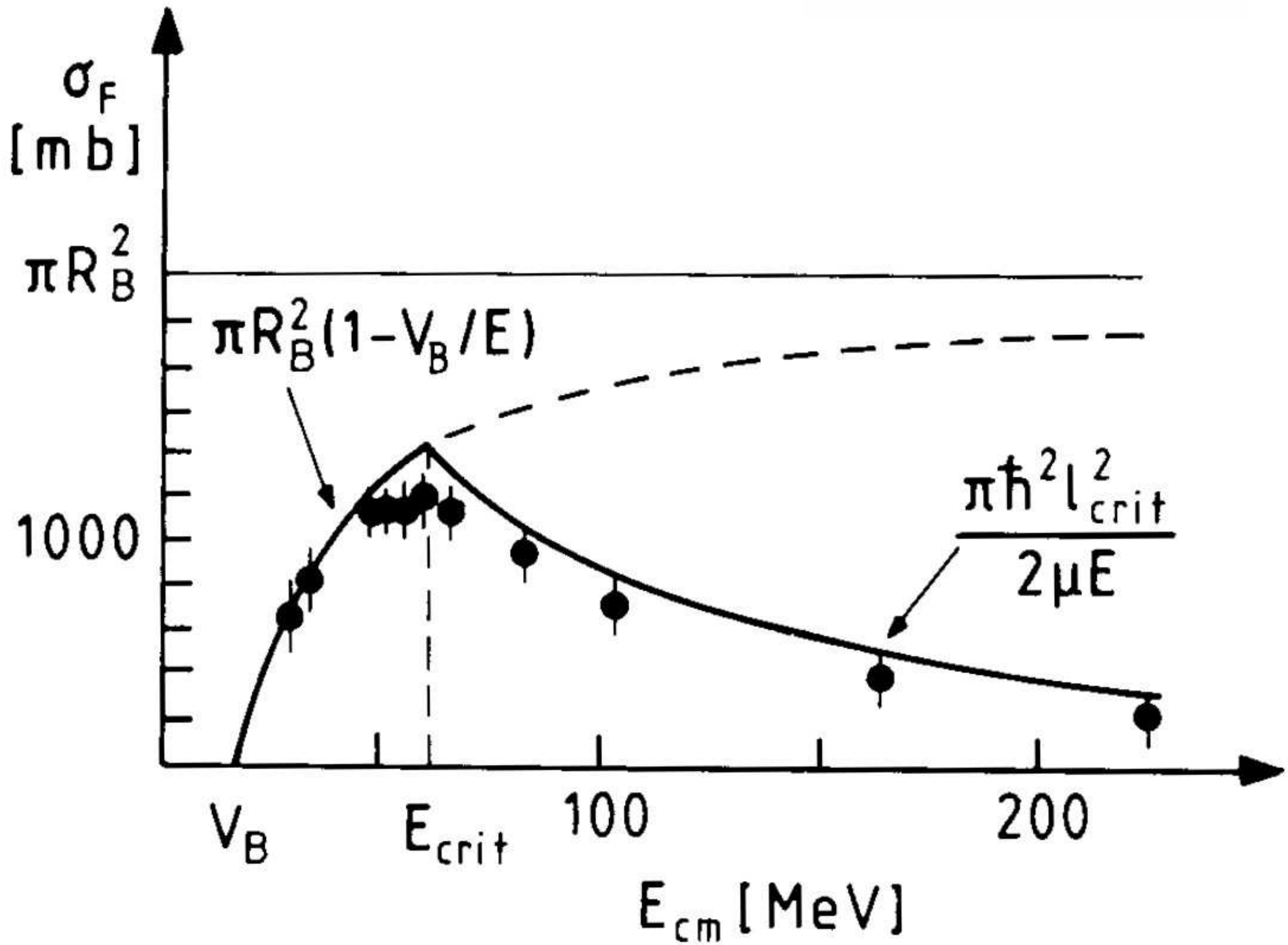
Limitation by angular momentum

The compound nucleus becomes unstable against fission above the certain value of angular momentum $l_{crit}=l_{crit}^{f}$, $b_{\mathrm{crit}} = l_{\mathrm{crit}}/k$.

$$\sigma_{\mathrm{F}} = \begin{cases} \pi b_{\mathrm{gr}}^2 & \text{for } b_{\mathrm{gr}} < b_{\mathrm{crit}}, \\ \pi b_{\mathrm{crit}}^2 & \text{for } b_{\mathrm{gr}} > b_{\mathrm{crit}}. \end{cases}$$

$$\sigma_{\mathrm{F}} = \begin{cases} \pi R_{\mathrm{B}}^2 (1 - V_{\mathrm{B}}/E) & \text{for } E < E_{\mathrm{crit}}, \\ \pi \hbar^2 l_{\mathrm{crit}}^2 / 2\mu E & \text{for } E > E_{\mathrm{crit}}. \end{cases}$$

$^{20}Ne + ^{27}Al$



σF
[mb]
πR_B^2
πR_B^2(1-V_B/E)
1000
πħ^2l_crit^2 / 2μE
V_B
E_crit
100
200
E_cm [MeV]

<u>Sub-barrier fusion</u>

transmission coefficient in the WKB approximation

$$T = \exp\left(-\frac{2}{\hbar}\int_b^a |p(x')|dx'\right),$$

$$p(x) = \sqrt{2\mu[E - V(x)]},$$

For the parabolic barrier, Hill-Wheeler formula

$$T = T(E) = \frac{1}{1 + \exp[2\pi(V_B - E)/\hbar\omega]}.$$

$$T_l(E) = \frac{1}{1 + \exp\{2\pi[V_B + \hbar^2 l(l+1)/2\mu R_B^2 - E]/\hbar\omega_B\}},$$

$$\omega_B^2 = \left| \frac{1}{\mu} \frac{d^2}{dr^2} \left(V(r) + \frac{\hbar^2 l(l+1)}{2\mu r^2} \right) \right|_{R_B}$$

$$\sigma_F(E) = \frac{\pi}{k^2} \sum_{l=0}^{\infty} (2l+1) T_l(E)$$
$$\approx \frac{2\pi}{k^2} \int_0^{\infty} \frac{l\,dl}{1 + \exp\{2\pi[V_B + \hbar^2 l^2/2\mu R_B^2 - E]/\hbar\omega_B\}}.$$

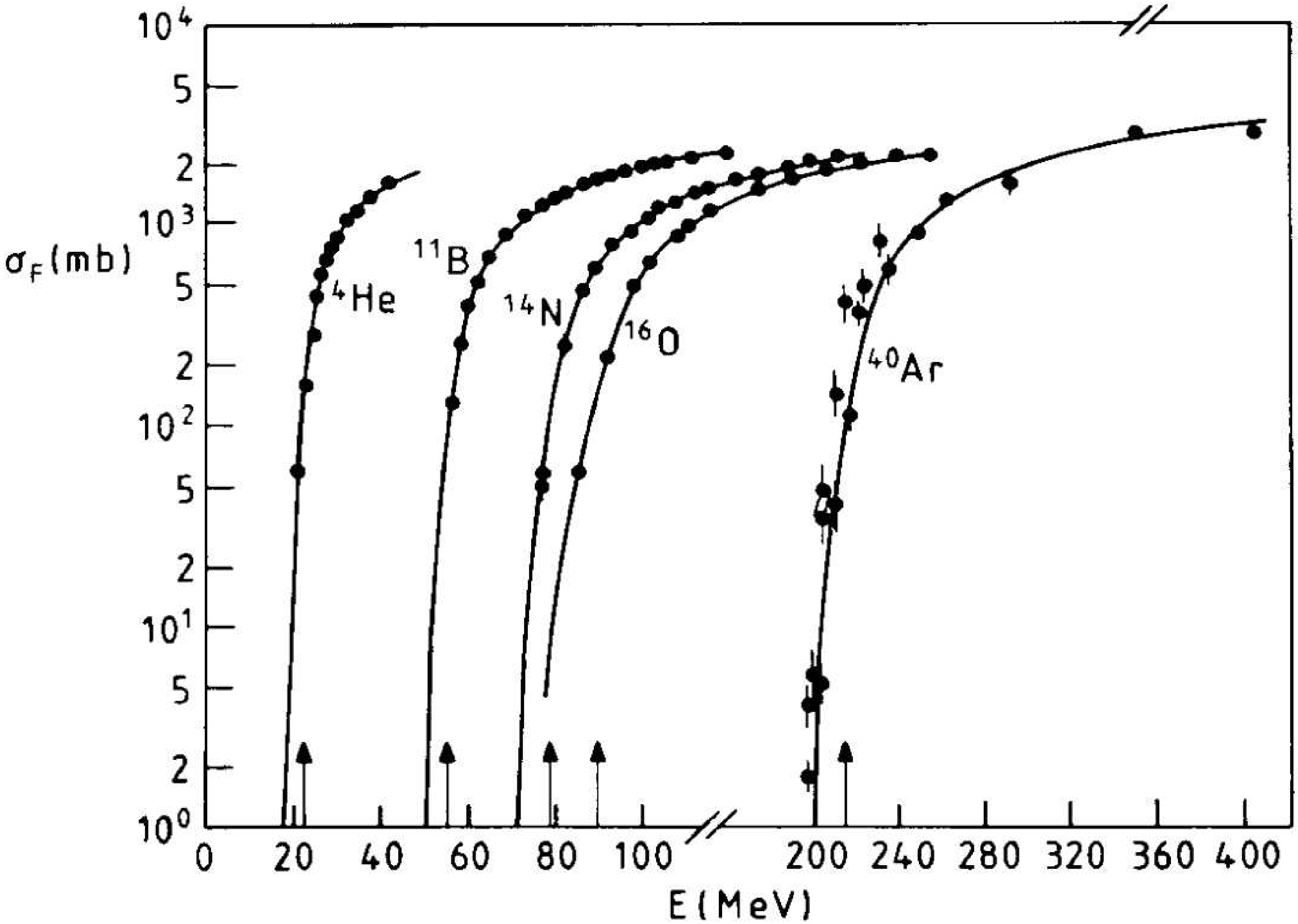
With the substitutions $y = l^2$, $a = \exp[2\pi(V_B - E)/\hbar\omega_B]$ and $b = \pi\hbar/\mu R_B^2\omega_B$ we obtain

$$\sigma_F(E) = \frac{\pi}{k^2}\int_0^\infty \frac{dy}{1 + a\exp(by)} = \frac{\pi}{k^2}\frac{1}{b}\ln\left(1 + \frac{1}{a}\right).$$

Going back to the original parameters, we arrive at the *Wong formula* for the fusion cross section

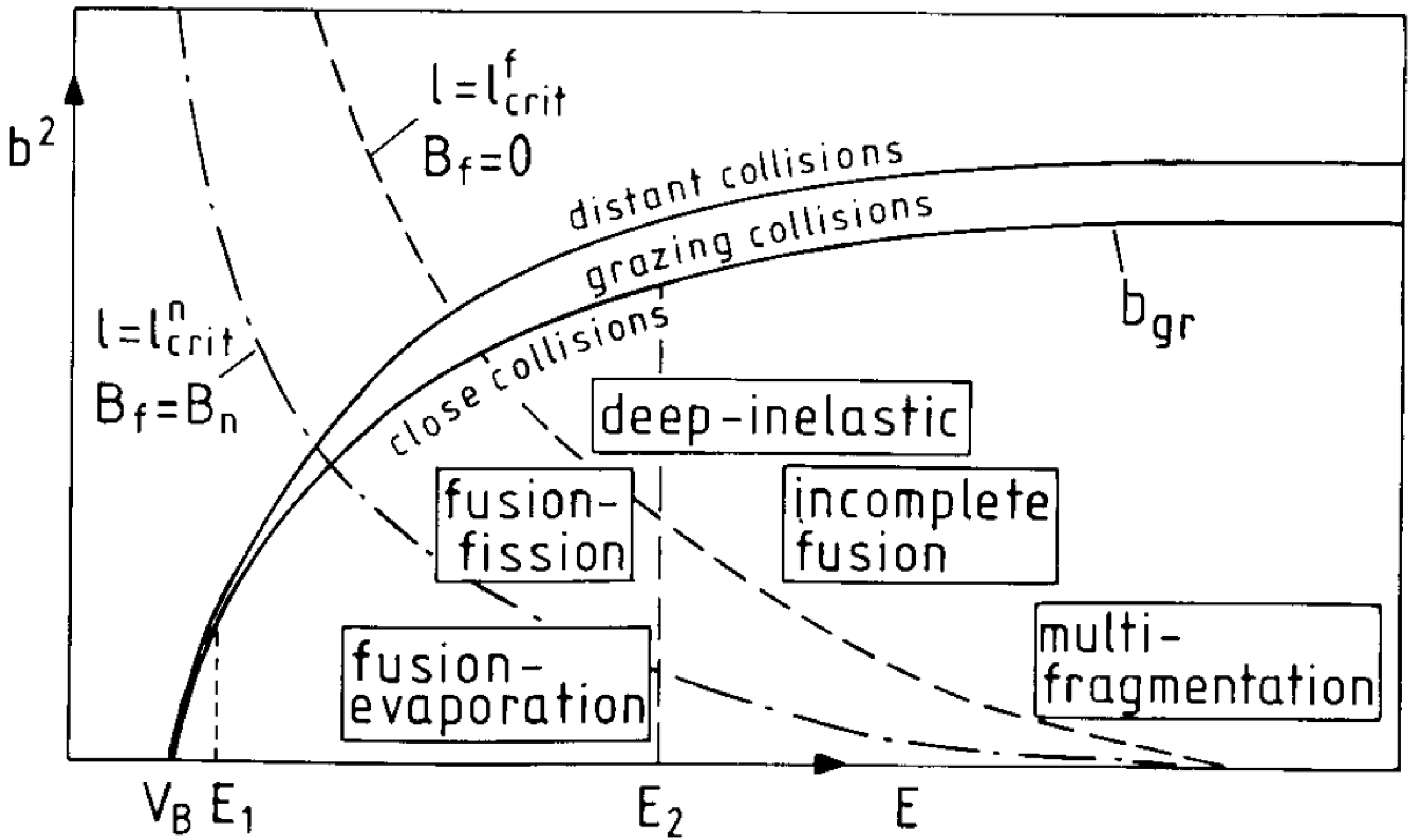
$$\sigma_F(E) = \frac{\hbar\omega_B R_B^2}{2E}\ln\{1 + \exp[2\pi(E - V_B)/\hbar\omega_B]\}.$$

$$\sigma_F(E) = \begin{cases} \pi R_B^2[1 - (V_B/E)] & \text{for } E > V_B, \\ \left(\hbar\omega_B R_B^2/2E\right)\exp[-2\pi(V_B - E)/\hbar\omega_B] & \text{for } E < V_B. \end{cases}$$

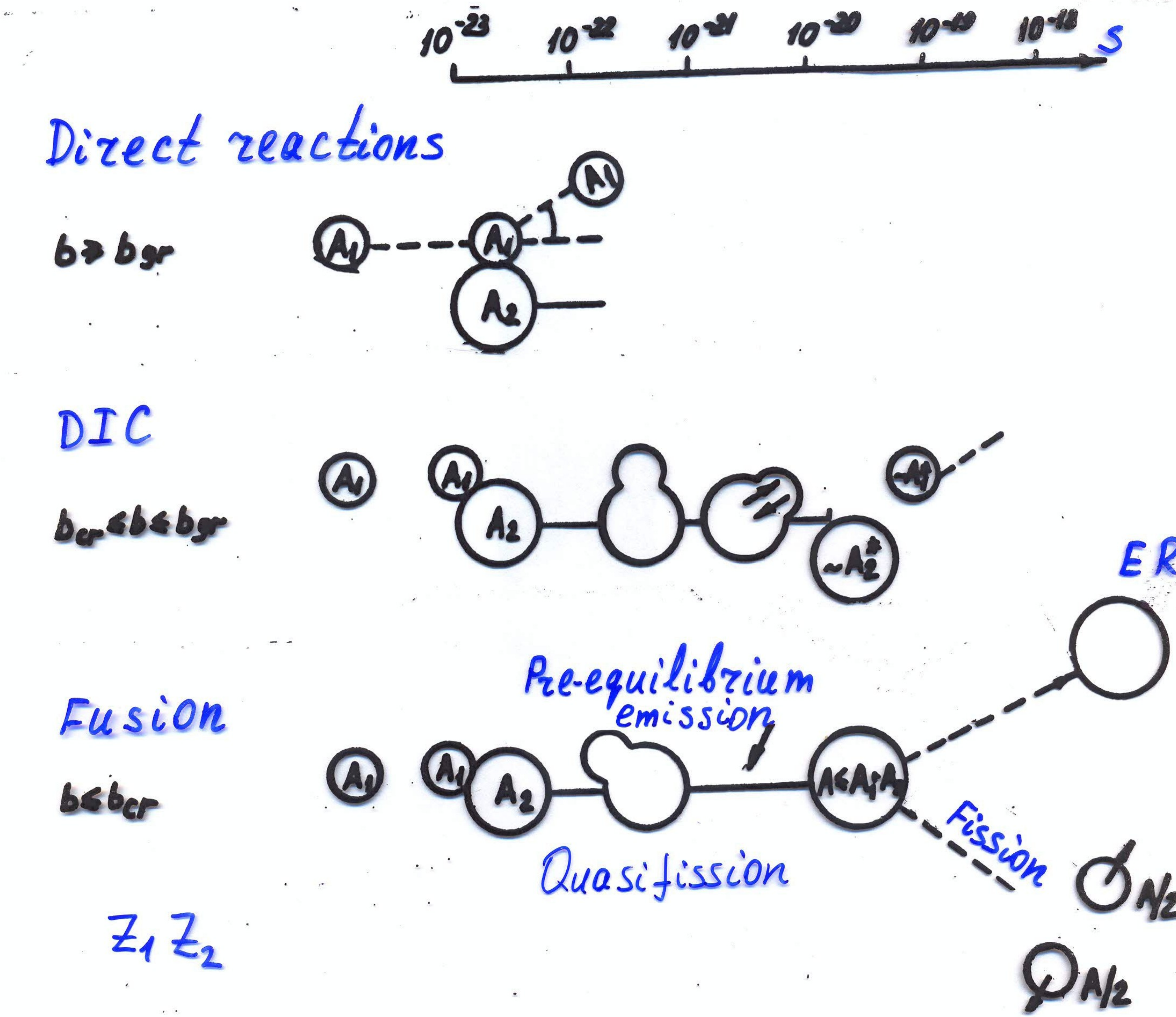


10⁴
5
2
10³
σ_F (mb)
5
2
10²
5
2
10¹
5
2
10⁰
0 20 40 60 80 100 200 240 280 320 360 400
E (MeV)
⁴He
¹¹B
¹⁴N
¹⁶O
⁴⁰Ar

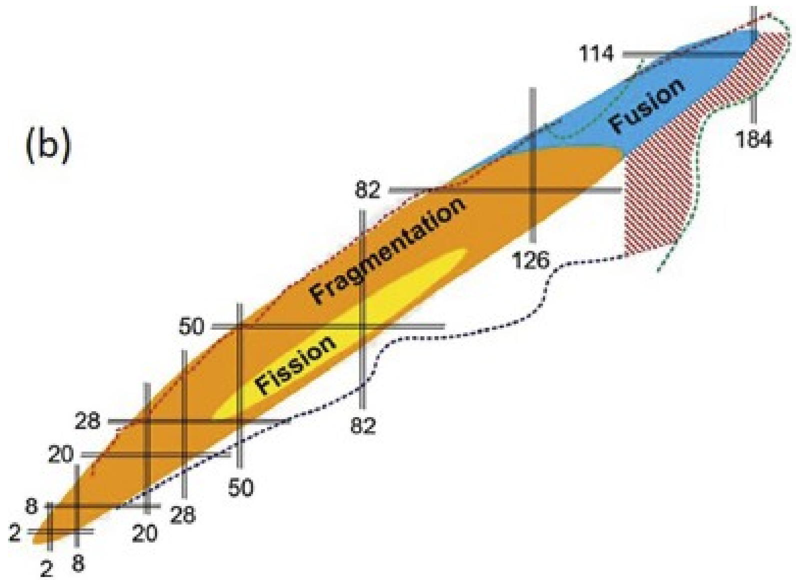
$+^{238}U$

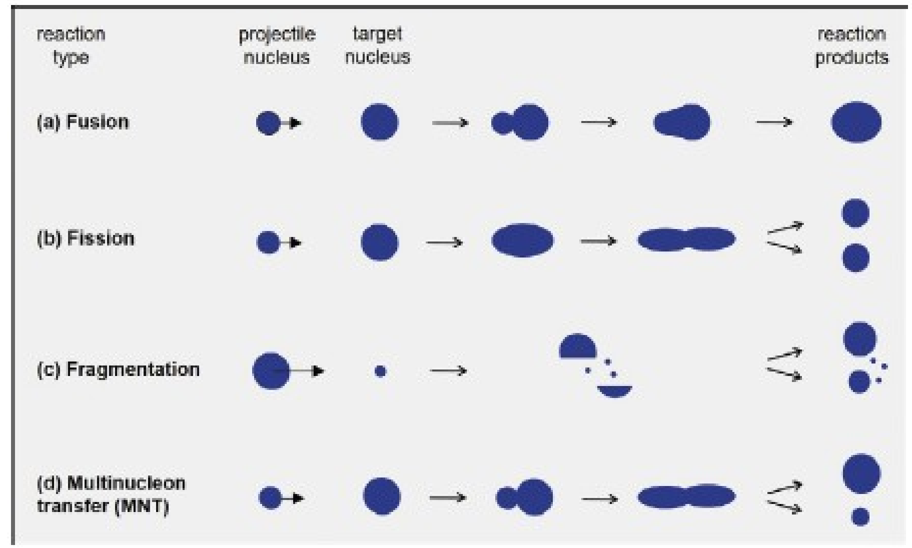


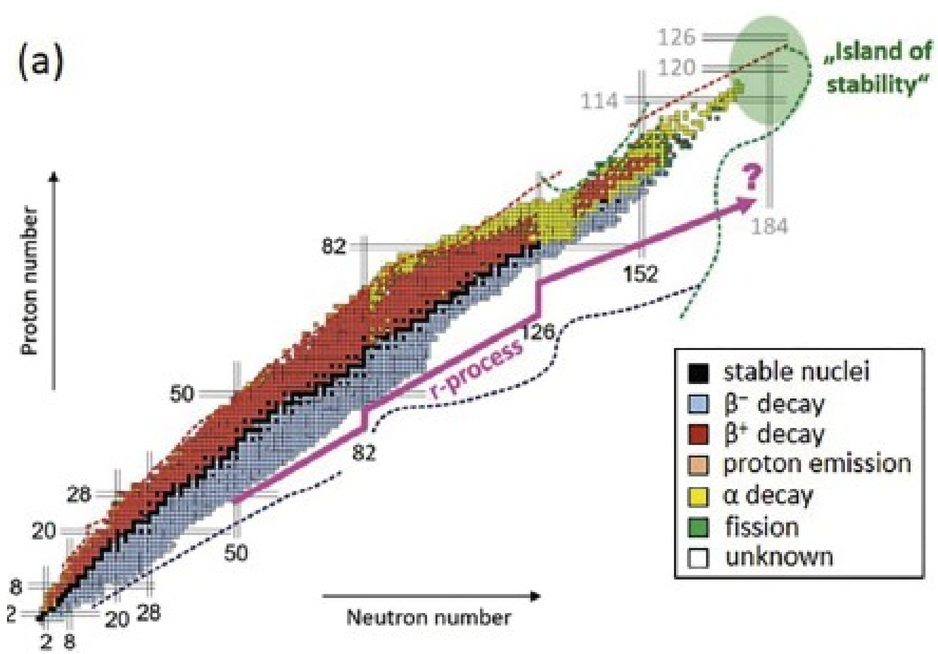
$l = l^f_{crit}$
$B_f = 0$
b^2
distant collisions
grazing collisions
close collisions
$l = l^n_{crit}$
$B_f = B_n$
b_{gr}
deep-inelastic
fusion-fission
incomplete fusion
fusion-evaporation
multi-fragmentation
V_B
E_1
E_2
E

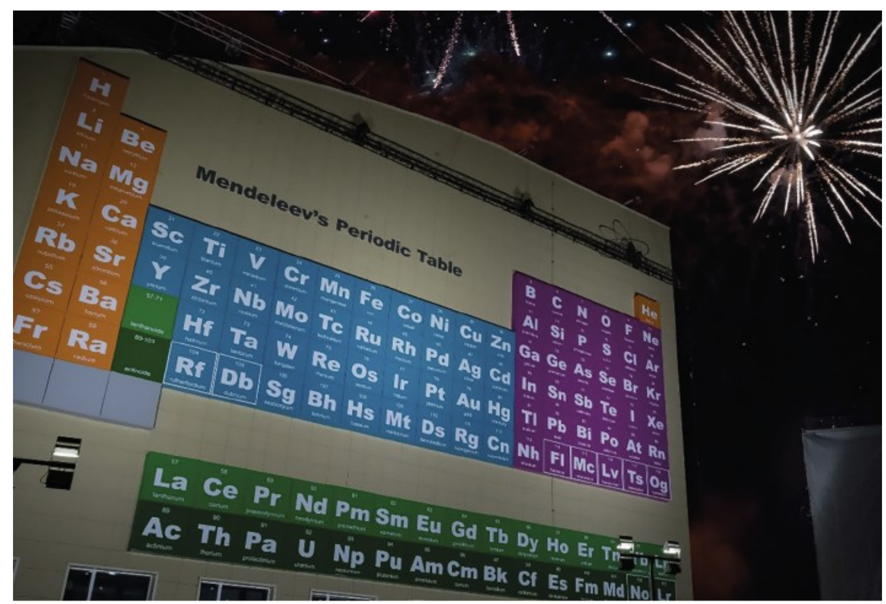


10^-23
10^-22
10^-21
10^-20
10^-19
10^-18
s
Direct reactions
b > b_gr
A_1
A_2
DIC
b_cr < b < b_gr
~A_1
~A_2*
ER
Pre-equilibrium emission
Fusion
b < b_cr
A < A_1+A_2
Fission
Quasifission
A/2
A/2
Z_1 Z_2


(b)
Fusion
Fragmentation
Fission
114
184
82
126
50
82
28
20
50
8
2
20
28
2
8


reaction type
projectile nucleus
target nucleus
reaction products
(a) Fusion
(b) Fission
(c) Fragmentation
(d) Multinucleon transfer (MNT)


(a)
„Island of stability"
126
120
114
?
184
82
152
126
r-process
50
82
28
20
50
8
2
20 28
2 8
Proton number
Neutron number
stable nuclei
β⁻ decay
β⁺ decay
proton emission
α decay
fission
unknown


Mendeleev's Periodic Table