Joint Institute for Nuclear Research
SCIENCE BRINGING NATIONS TOGETHER

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Few-body problem in exotic cluster nuclei

Three-body problem

Quantum three-body problem

Few-body vs. many-body

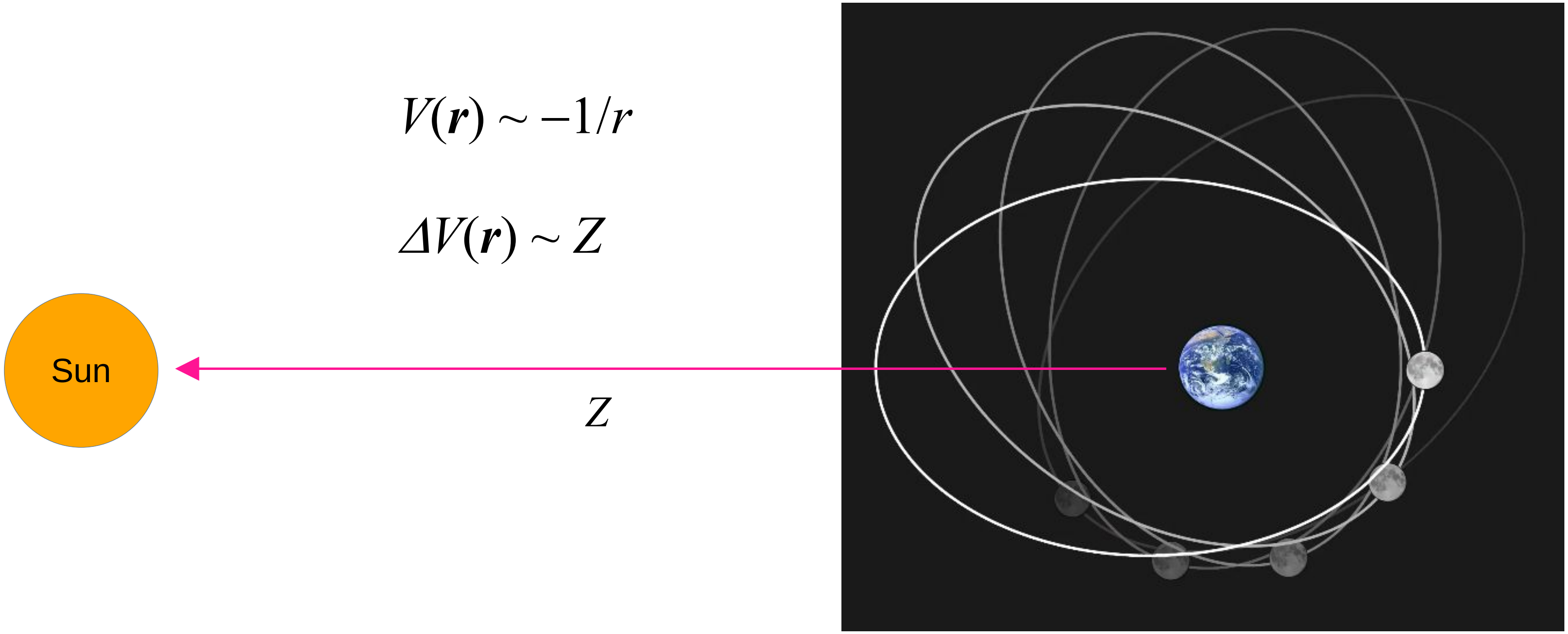
International School “Nuclear Theory and Astrophysical Applications”, Dubna, February 11 – 17, 2024

First three-body effect

Precession of the Moon orbit perihelion

$$V(\boldsymbol{r}) \sim -1/r$$

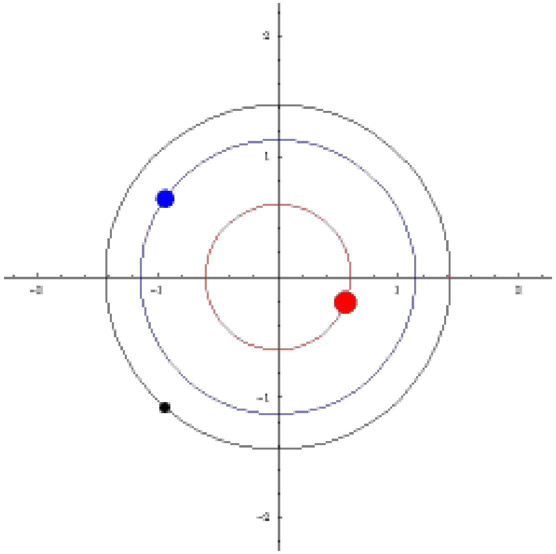
$$\Delta V(\boldsymbol{r}) \sim Z$$

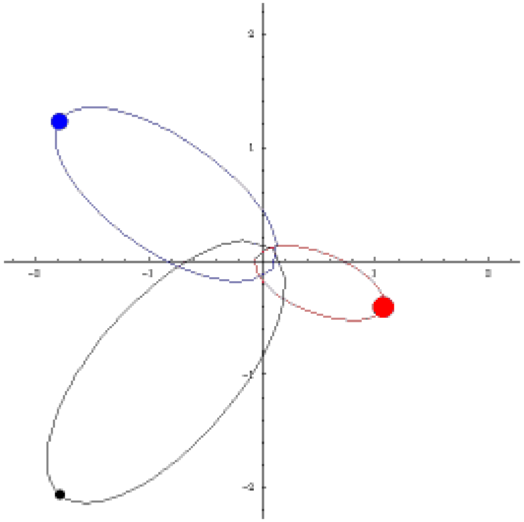


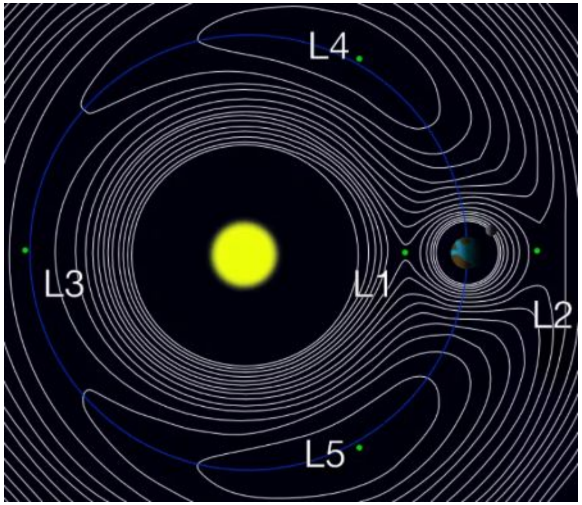
Kepler laws: phenomenology

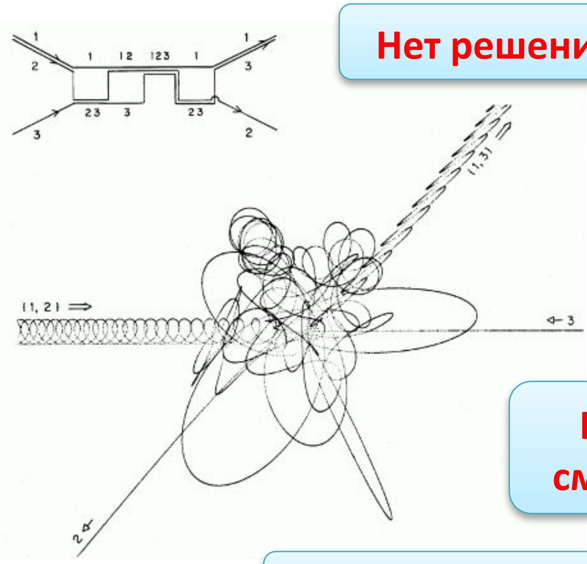
Newton laws: dynamics

Classical three-body problem





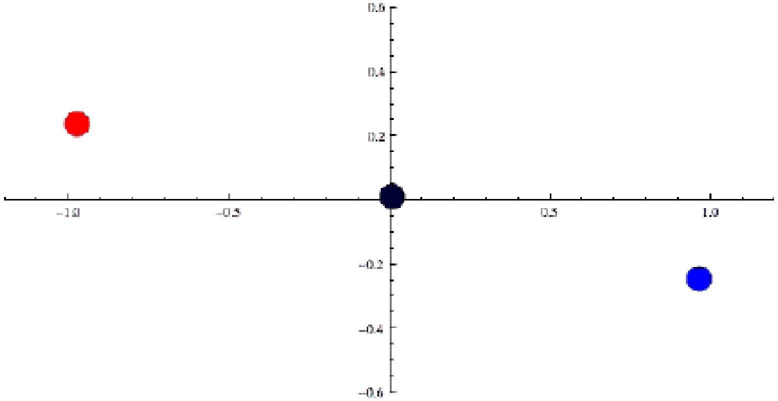




Нет решения в квадратурах

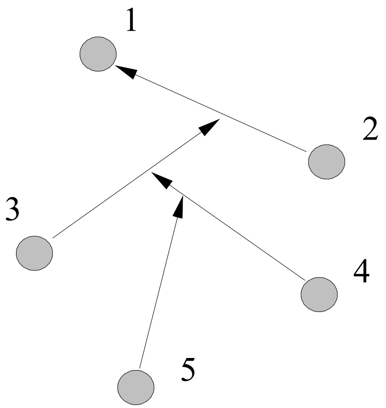
Неустойчива в смысле Ляпунова

Хаотизация в общем случае



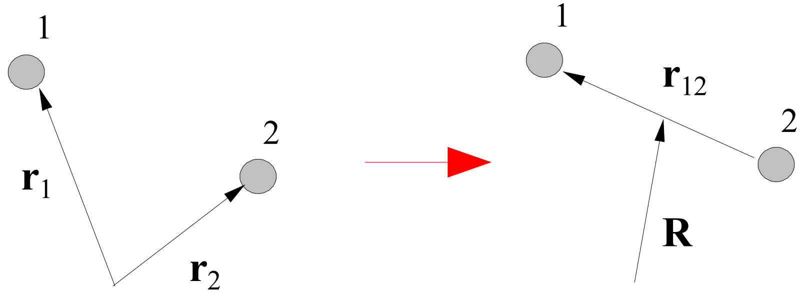
Basics of Few-body techniques

Few-body basics: Jacobi variables



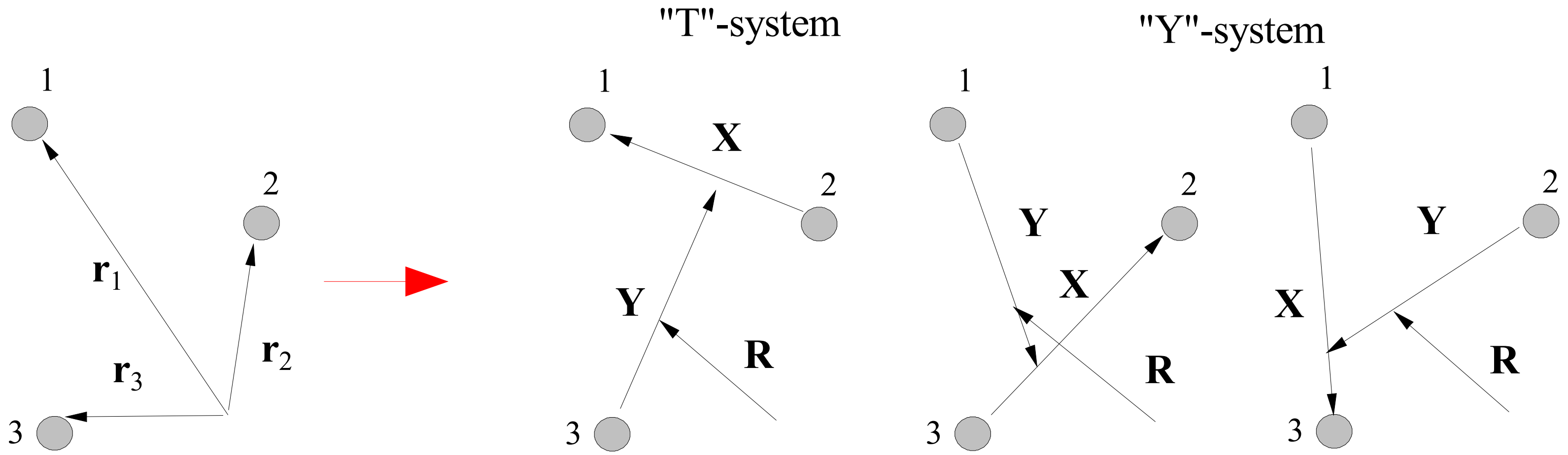
Древовидная схема построения координат Якоби

2-body



Координаты Якоби: задача 2 тел → эффективно задача 1 тела

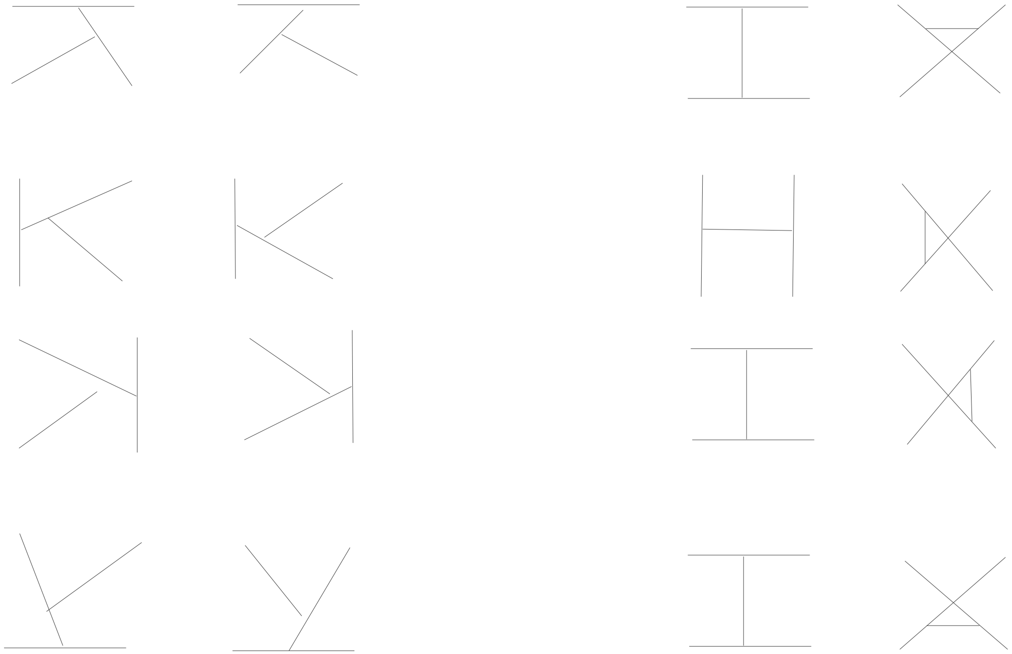
3-body



Few-body basics: Jacobi variables

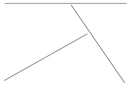
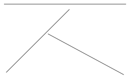
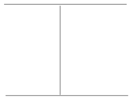
4-body

Древовидные



Н-образные

α-particle

 $= p + T$  $= n + {}^3He$  $= d + d$

Few-body basics: Jacobi variables

- “Non-normalized” and normalized Jacobi variables in coordinate and momentum space
- Meaning of Jacobi vectors in coordinate and momentum space

$$\begin{cases} \mathbf{X} = & \mathbf{r}_1 - \mathbf{r}_2 \\ \mathbf{Y} = & \dfrac{m_1\mathbf{r}_2 + m_2\mathbf{r}_1}{m_1 + m_2} - \mathbf{r}_3 \\ \mathbf{R} = & \dfrac{m_1\mathbf{r}_1 + m_2\mathbf{r}_2 + m_3\mathbf{r}_3}{M} \end{cases}$$

$$\begin{cases} \mathbf{P}_x = & \dfrac{m_1\mathbf{p}_1 - m_2\mathbf{p}_2}{m_1 + m_2} \\ \mathbf{P}_y = & \dfrac{m_3(\mathbf{p}_1 + \mathbf{p}_2) - (m_1 + m_2)\mathbf{p}_3}{M} \\ \mathbf{P}_R = & \mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3 \end{cases}$$

$$\begin{cases} \mathbf{x} = & \sqrt{\dfrac{A_1A_2}{A_1 + A_2}}\,(\mathbf{r}_1 - \mathbf{r}_2) \\ \mathbf{y} = & \sqrt{\dfrac{(A_1 + A_2)A_3}{A}}\left(\dfrac{A_1\mathbf{r}_1 + A_2\mathbf{r}_2}{A_1 + A_2} - \mathbf{r}_3\right) \\ \mathbf{r} = & \dfrac{A_1\mathbf{r}_1 + A_2\mathbf{r}_2 + A_3\mathbf{r}_3}{\sqrt{A}} \end{cases}$$

$$\begin{cases} \mathbf{p}_x = & \sqrt{\dfrac{A_1 + A_2}{A_1A_2}}\,\dfrac{A_1\mathbf{p}_1 - A_2\mathbf{p}_2}{A_1 + A_2} \\ \mathbf{p}_y = & \sqrt{\dfrac{A}{(A_1 + A_2)A_3}}\,\dfrac{A_3(\mathbf{p}_1 + \mathbf{p}_2) - (A_1 + A_2)\mathbf{p}_3}{A} \\ \mathbf{p}_r = & \dfrac{\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3}{\sqrt{A}} \end{cases}$$

$$M = m_1 + m_2 + m_3$$

$$A = A_1 + A_2 + A_3$$

$$\frac{D\left(\mathbf{r}_1\mathbf{r}_2\mathbf{r}_3\right)}{D\left(\mathbf{RYX}\right)} = \frac{D(\mathbf{p}_1\mathbf{p}_2\mathbf{p})}{D(\mathbf{P}_R\mathbf{P}_y\mathbf{P}_x)} = 1$$

$$\frac{D\left(\mathbf{r}_1\mathbf{r}_2\mathbf{r}_3\right)}{D(\,\mathbf{r}\,\mathbf{y}\,\mathbf{x}\,)} = \left(\frac{D(\mathbf{p}_1\mathbf{p}_2\mathbf{p}_3)}{D(\mathbf{p}_r\mathbf{p}_y\mathbf{p}_x)}\right)^{-1} = (A_1A_2A_3)^{-3/2}$$

Three-body correlations in decays and reactions

2-body decay: state is defined by 2 parameters - energy and width

3-body decays:
2-dimensional "internal"
3-body correlations

3-body continuum in reactions: there is a selected direction. 5-dimensional correlations: "internal" + "external"

For direct reactions the selected direction is momentum transfer vector

Which kind of useful information (if any) can be obtained from three-body correlations?

"Internal" energy of 3-bodies

$\{k_x, k_y\} \longrightarrow E_T = E_x + E_y$

"Internal" 3-body correlations

$\{\mathbf{k}_x, \mathbf{k}_y\} \longrightarrow$

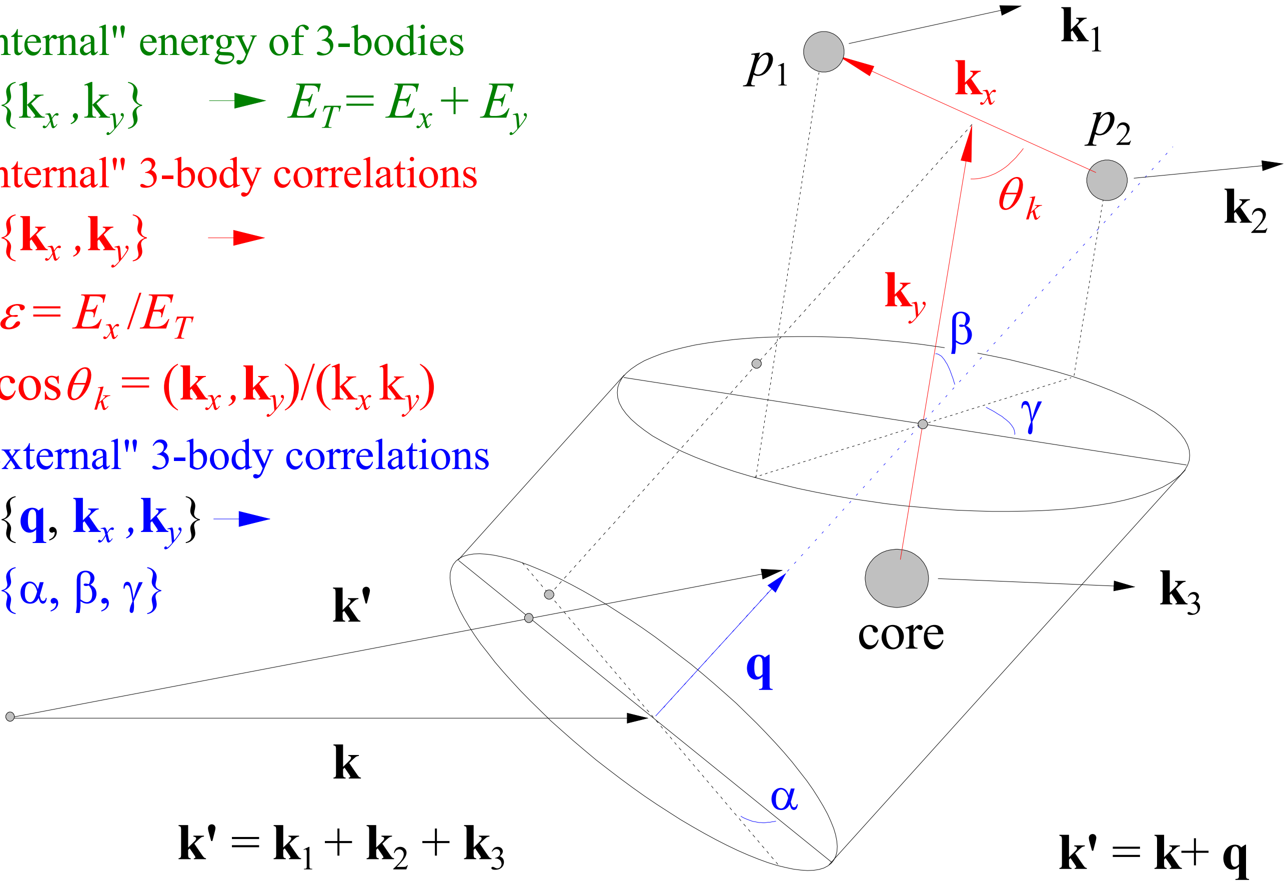
$\varepsilon = E_x/E_T$

$\cos\theta_k = (\mathbf{k}_x, \mathbf{k}_y)/(k_x k_y)$

"External" 3-body correlations

$\{\mathbf{q}, \mathbf{k}_x, \mathbf{k}_y\} \longrightarrow$

$\{\alpha, \beta, \gamma\}$



$$\mathbf{k}' = \mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3$$

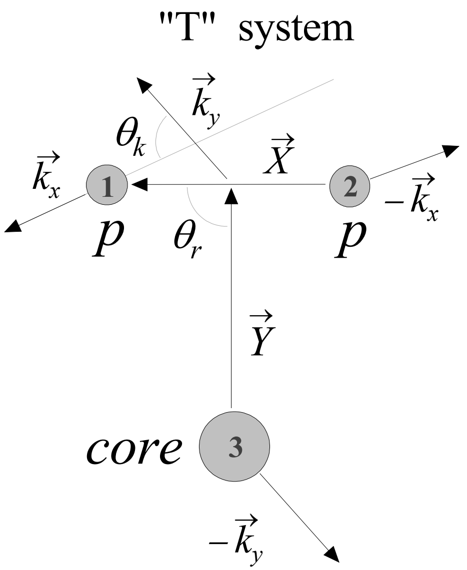
$$\mathbf{k}' = \mathbf{k} + \mathbf{q}$$

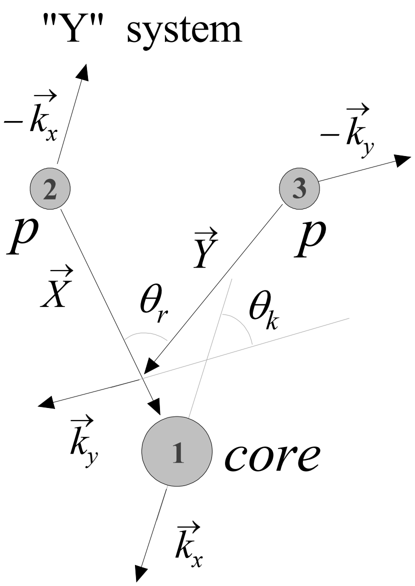
Three-body correlations. “Internal” correlations

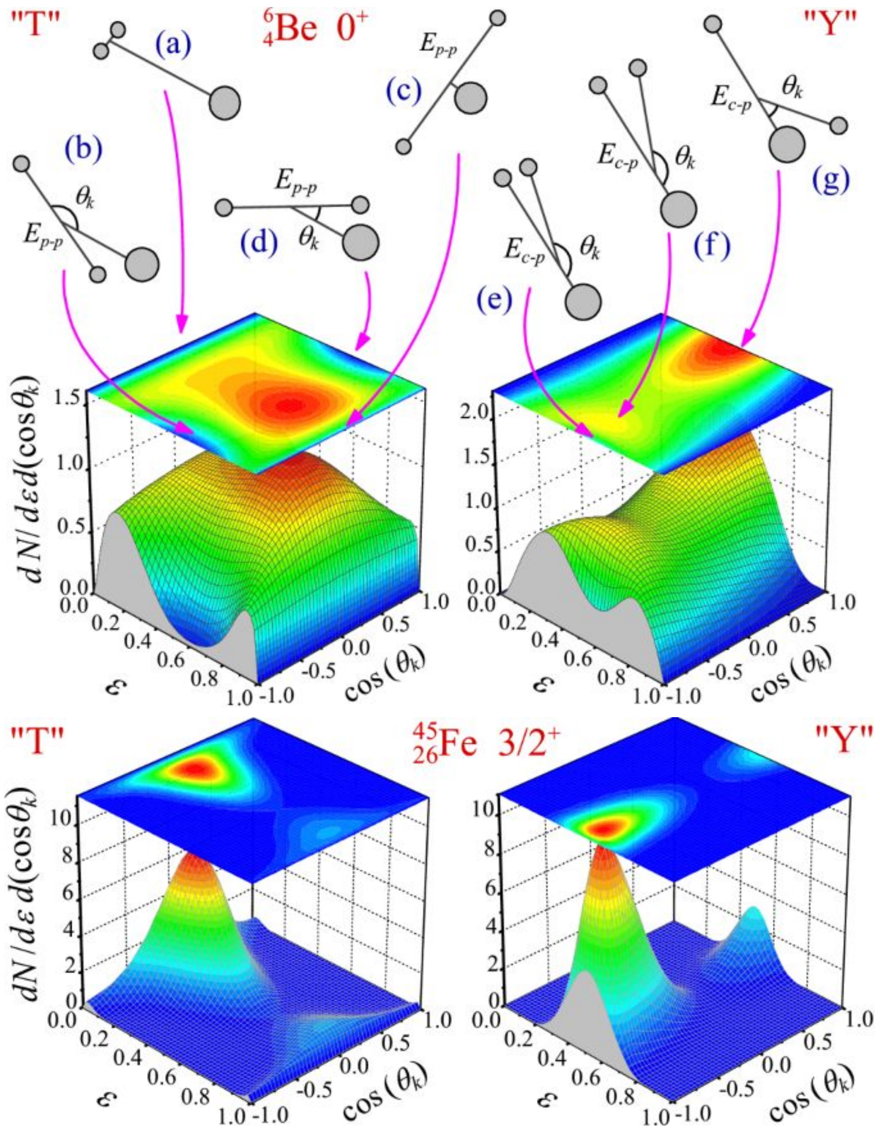
- 2-dimensional “internal three-body correlations” or “energy-angular correlations”

$$\varepsilon = E_x / \mathrm{E_T} \qquad \cos(\theta_k) = (\mathbf{k}_x,\mathbf{k}_y)/\mathrm{k}_x\mathrm{k}_y$$

- “T” and “Y” Jacobi systems reveal different dynamical aspects
- Three-body variables in coordinate and in momentum space.







Simple way to understand three-body correlations: “quasibinary kinematics” ε -> 0 and ε -> 1

Hyperspherical Harmonics method

HH method

$$\rho = \sqrt{x^2 + y^2} \quad ; \qquad \theta_\rho = \arctan(x/y)$$

$$\varkappa = \sqrt{p_x^2 + p_y^2} = \sqrt{2mE} = \sqrt{2m(E_x + E_y)}$$

$$\theta_\varkappa = \arctan\left(\sqrt{E_x/E_y}\right) = \arctan\left(p_x/p_y\right)$$

Hyperspherical variables

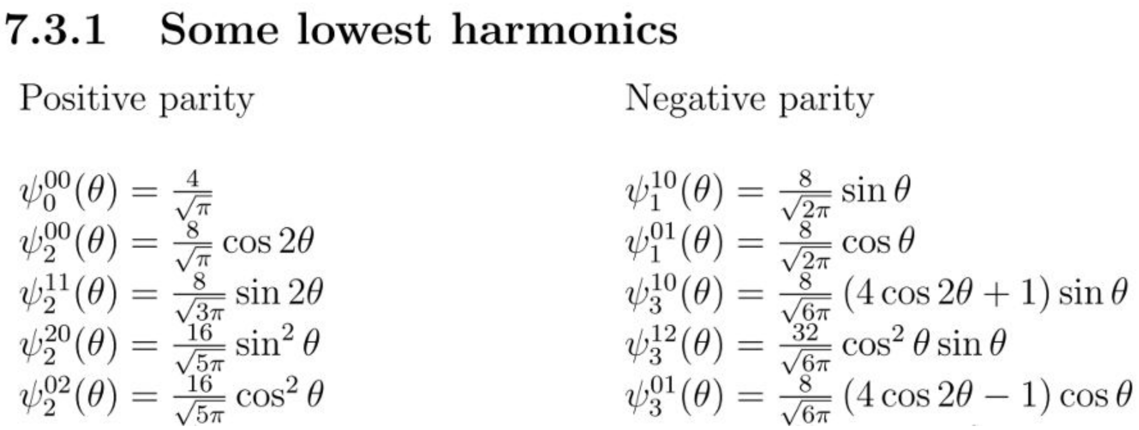
$$\rho^2 = \frac{A_1 A_2 A_3}{A}\left(\frac{\mathbf{r}_{12}^2}{A_3} + \frac{\mathbf{r}_{23}^2}{A_1} + \frac{\mathbf{r}_{31}^2}{A_2}\right)$$

$$\Psi(\mathbf{X},\mathbf{Y}) = \Psi(\rho,\Omega_\rho) = \frac{1}{\rho^{5/2}} \sum_{K\gamma} \chi_{K\gamma}(\rho)\mathcal{J}_{K\gamma}(\Omega_\rho)$$

$$\mathcal{J}_{Kl_xl_y}^{JM}(\Omega) = \psi_K^{l_xl_y}(\theta)\,\left[Y_{l_x} \otimes Y_{l_y}\right]_{JM}$$

$$\psi_K^{l_xl_y}(\theta) = N_K^{l_xl_y}(\sin\theta)^{l_x}(\cos\theta)^{l_y}\,P_{\frac{K-l_x-l_y}{2}}^{l_x+1/2,l_y+1/2}(\cos 2\theta)$$

HH expansion for three-body WF: generalization of the spherical function expansiuon



7.3.1 Some lowest harmonics

Positive parity

$$\psi_0^{00}(\theta) = \frac{4}{\sqrt{\pi}}$$
$$\psi_2^{00}(\theta) = \frac{8}{\sqrt{\pi}}\cos 2\theta$$
$$\psi_2^{11}(\theta) = \frac{8}{\sqrt{3\pi}}\sin 2\theta$$
$$\psi_2^{20}(\theta) = \frac{16}{\sqrt{5\pi}}\sin^2\theta$$
$$\psi_2^{02}(\theta) = \frac{16}{\sqrt{5\pi}}\cos^2\theta$$

Negative parity

$$\psi_1^{10}(\theta) = \frac{8}{\sqrt{2\pi}}\sin\theta$$
$$\psi_1^{01}(\theta) = \frac{8}{\sqrt{2\pi}}\cos\theta$$
$$\psi_3^{10}(\theta) = \frac{8}{\sqrt{6\pi}}(4\cos 2\theta + 1)\sin\theta$$
$$\psi_3^{12}(\theta) = \frac{32}{\sqrt{6\pi}}\cos^2\theta\sin\theta$$
$$\psi_3^{01}(\theta) = \frac{8}{\sqrt{6\pi}}(4\cos 2\theta - 1)\cos\theta$$

HH method

$$\left[\frac{d^2}{d\rho^2}-\frac{\mathcal{L}(\mathcal{L}+1)}{\rho^2}+2M\{E-V_{K\gamma,K\gamma}(\rho)\}\right]\chi_{K\gamma}(\rho)$$

$$=\sum_{K'\gamma'}2MV_{K\gamma,K'\gamma'}(\rho)\chi_{K'\gamma'}(\rho),$$

$$V_{K\gamma,K'\gamma'}(\rho)=\int d\Omega_\rho\ \mathcal{J}^\dagger_{K\gamma}(\Omega_\rho)\left[\sum_{i>j}\hat{V}(\mathbf{r}_{ij})\right]\mathcal{J}_{K'\gamma'}(\Omega_\rho)$$

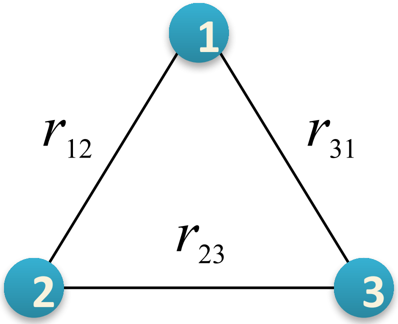
HH partial equations:
effective "single-particle" motion in
effective "strongly deformed" field

$$\mathcal{L}=K+3/2$$

Effective centrifugal barrier is always nonzero !

Quantum mechanics and boundary conditions

	Two-body	Three-body
Discrete	$\sim A\, e^{-kr}/r$	$\sim A_3\, e^{-\kappa\rho}/\rho^{5/2} + \sum_{i>j} A_{ij}\, e^{-k_{ij}r_{ij}}/r_{ij}\; e^{-k_k r_k}/r_k$
Continuum	$\sim A(\theta)\, e^{ikr}/r$	$\sim A_3\, e^{i\kappa\rho}/\rho^{5/2} + \sum_{i>j} A_{ij}\, e^{-k_{ij}r_{ij}}/r_{ij}\; e^{ik_k r_k}/r_k$



- A – **ANC** and **scattering amplitudes**
- Collective variable: **hyperradius**

$$\rho^2 = \frac{A_1 A_2 A_3}{A}\left(\frac{\mathbf{r}_{12}^2}{A_3} + \frac{\mathbf{r}_{23}^2}{A_1} + \frac{\mathbf{r}_{31}^2}{A_2}\right)$$

- A_3 - is always not zero
- A_{ij} - some are typically equal to zero
- **True three-body** system - if all A_{ij} are equal to zero

Dynamics of the processes can not be reduced to the two-body dynamics and studies should be done using methods of the few-body theory

Few-body vs. many-body

Методы теории многих тел строятся отталкиваясь от одночастичных степеней свободы. Одночастичные степени свободы возникают как результат существования в ядре среднего поля. Коллективное движение возникает под действием возмущений над средним полем.

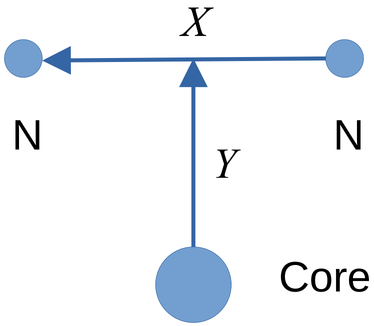
В теории нескольких тел вся динамика коллективизована по построению (пример — эффективная «гиперчастица» в МГГ). Плата — тяжелый формализм, но есть классы задач где нативное рассмотрение корреляций является обязательным.

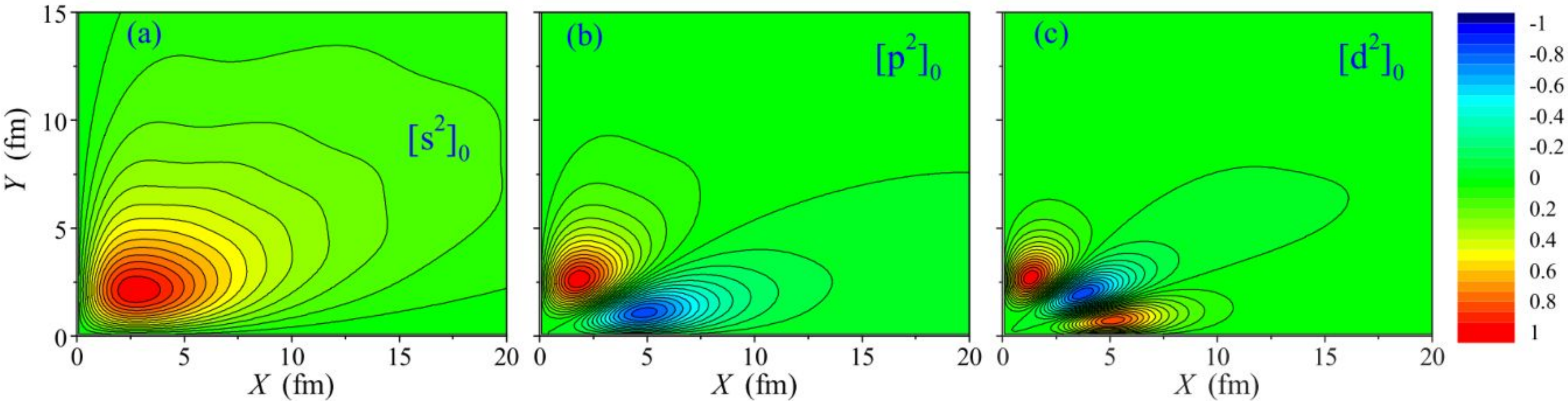
Simple example

Single hyperspherical channel

$$\left[\frac{d^2}{d\rho^2} - \frac{\mathcal{L}(\mathcal{L}+1)}{\rho^2} + 2M\{E - V_{K\gamma,K\gamma}(\rho)\}\right]\chi_{K\gamma}(\rho) = \sum_{K'\gamma'} 2MV_{K\gamma,K'\gamma'}(\rho)\chi_{K'\gamma'}(\rho),$$

$$V_{K\gamma,K'\gamma'}(\rho) = \int d\Omega_\rho \; \mathcal{J}^\dagger_{K\gamma}(\Omega_\rho)\left[\sum_{i>j}\hat{V}(\mathbf{r}_{ij})\right]\mathcal{J}_{K'\gamma'}(\Omega_\rho)$$





$$(\hat{H}_3 - E_T + i\Gamma/2)\Psi^{(+)}_{E_T} = 0, \quad \hat{H}_3 = \hat{T}_3 + \hat{V}_3(\rho)$$

0^+ state in the T-system

K=0, l_x=0, l_y=0, S=0 → $[s^2]_0$
K=2, l_x=0, l_y=0, S=0 → $[p^2]_0$
K=4, l_x=0, l_y=0, S=0 → $[d^2]_0$

«Pauli focusing» type of correlations: Population of the dominating shell model configurations induce strong spatial correlations

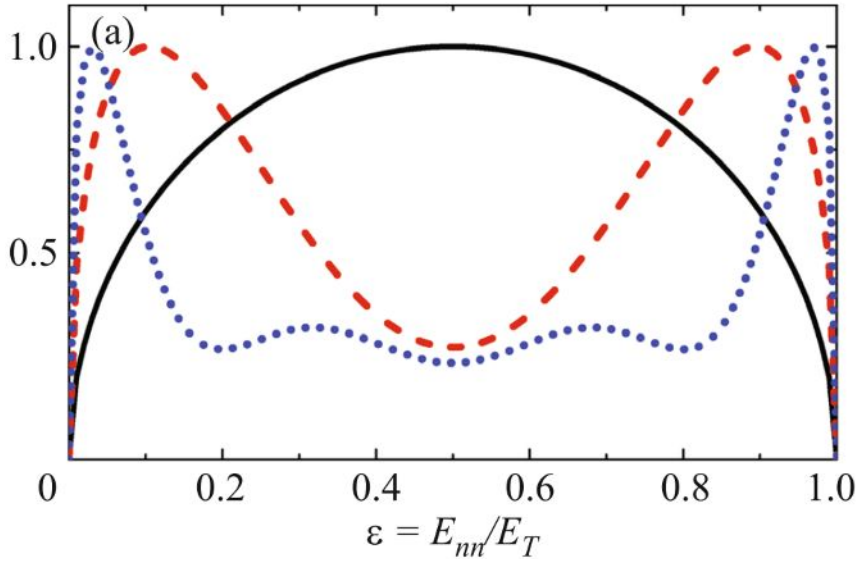
Danilin, Efros, 1988

How to study emission off these configurations?

Just push up the bound state into continuum

0^+ state in the T-system

K=0, l_x=0, l_y=0, S=0 → $[s^2]_0$
K=2, l_x=0, l_y=0, S=0 → $[p^2]_0$
K=6, l_x=0, l_y=0, S=0 → $[f^2]_0$



How to study emission off these configurations?

Lets's consider one (only NN) FSI

$$(\hat{H}_3 - E_T + i\Gamma/2)\Psi^{(+)}_{E_T} = 0, \quad \hat{H}_3 = \hat{T}_3 + \hat{V}_3(\rho) + V_{nn}(X)$$

$$(\hat{H}_3 - E_T)\Psi_{\text{box}} = 0.$$

$$\Psi^{(+)}_{E_T} = -\frac{1}{\hat{T}_3 + V_{nn} - E_T + i\Gamma/2}\hat{V}_3(\rho)\,\Psi^{(+)}_{E_T} \quad \Rightarrow \quad \Psi^{(+)}_{E_T} = -\frac{1}{\hat{T}_3 + V_{nn} - E_T + i\Gamma/2}\hat{V}_3(\rho)\,\Psi_{\text{box}}$$

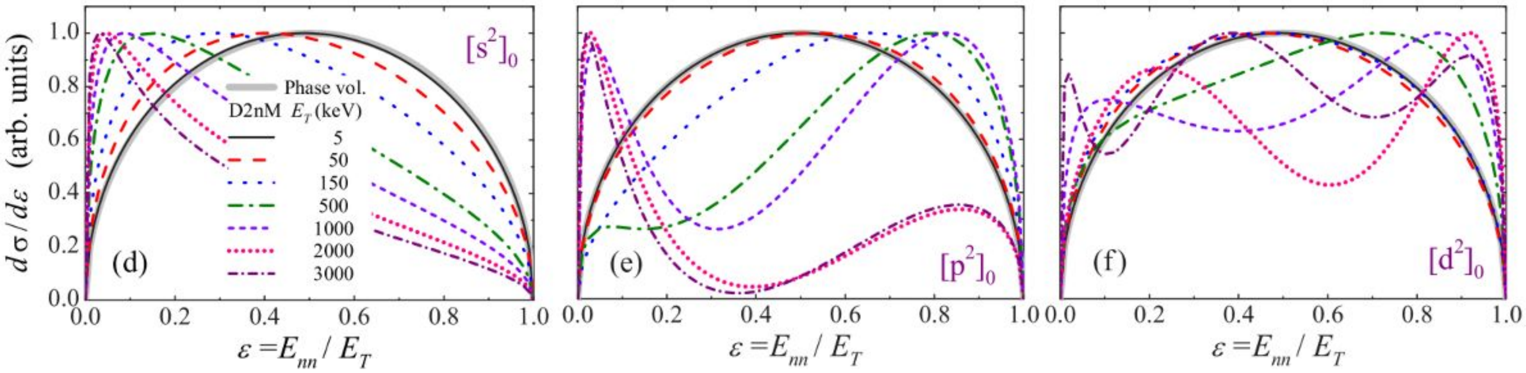
$$\bar{H} = T + V_x^{\text{coul}}(X) + V_x^{\text{nuc}}(X) + V_y^{\text{coul}}(Y) + V_y^{\text{nuc}}(Y)$$

$$\begin{cases} \bar{H}_x - E_x = T_x + V_x^{\text{coul}}(X) + V_x^{\text{nuc}}(X) - E_x \\ \bar{H}_y - E_y = T_y + V_y^{\text{coul}}(Y) + V_y^{\text{nuc}}(Y) - E_y \end{cases}$$

$$G^{(+)}_{E_{3r}}(\mathbf{X}\mathbf{Y}, \mathbf{X}'\mathbf{Y}') = \frac{1}{2\pi i}\int_{-\infty}^{\infty} dE_x\, G^{(+)}_{E_x}(\mathbf{X}, \mathbf{X}')G^{(+)}_{E_y}(\mathbf{Y}, \mathbf{Y}')$$

$$\bar{G}^{(+)}_{k^2/(2m)}(\mathbf{r}, \mathbf{r}') = \frac{2M}{krr'}\sum_l \left\{\begin{matrix} \varphi_l(kr)h_l^{(+)}(kr'), & r \leqslant r' \\ h_l^{(+)}(kr)\varphi_l(kr'), & r > r' \end{matrix}\right\} \times \sum\nolimits_m Y_{lm}(\hat{r})Y^*_{lm}(\hat{r}'),$$

$$\varphi_l(kr) = \exp(i\bar{\delta}_l)[F_l(kr)\cos(\bar{\delta}_l) + G_l(kr)\sin(\bar{\delta}_l)]$$

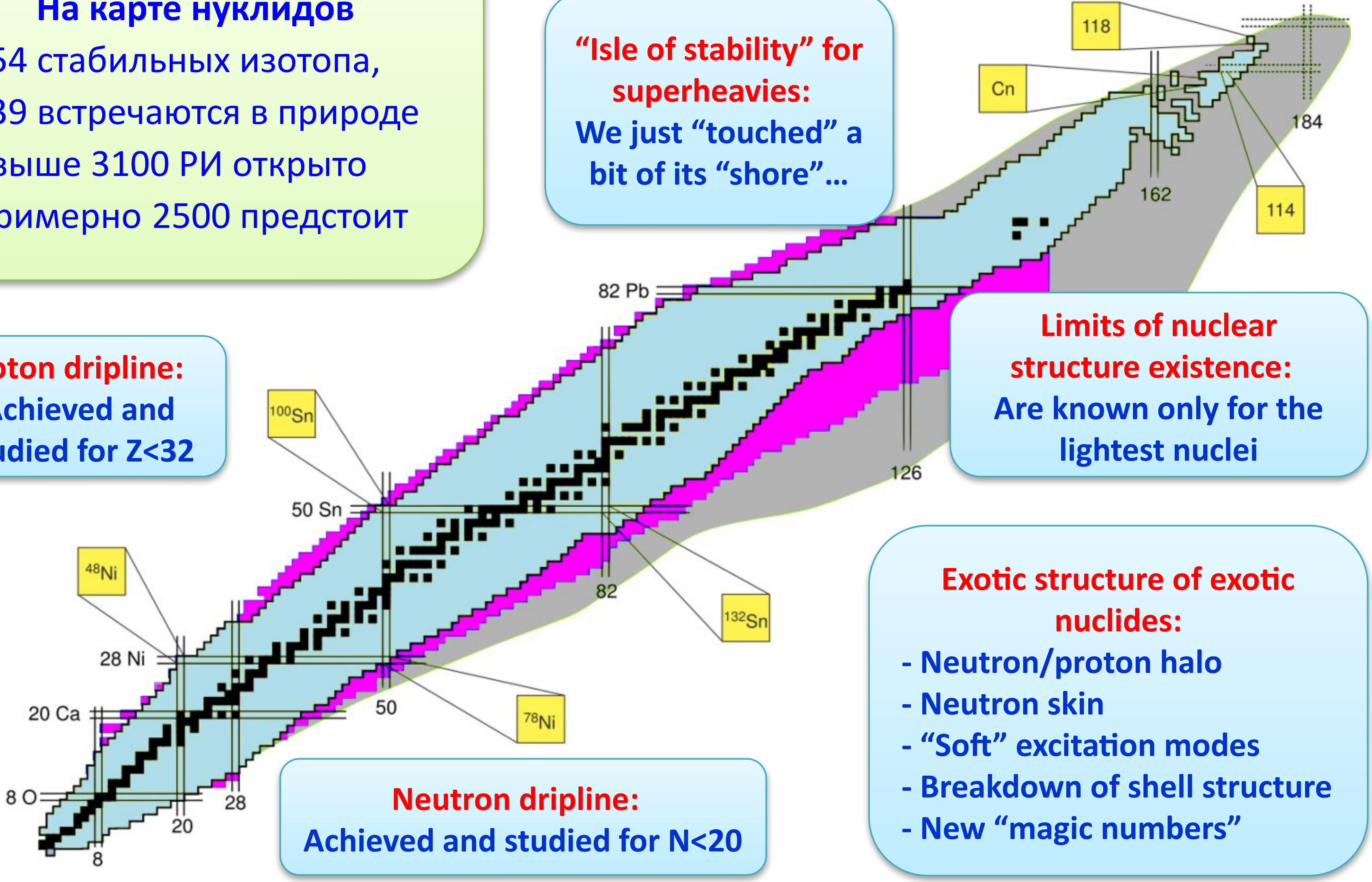


Few-body dynamics in selected problems in the studies of light exotic nuclei

Физика радиоактивных изотопов (Radioactive Ion Beam, RIB) – “магистральный путь” развития современной ядерной физики низких энергий

На карте нуклидов

- 254 стабильных изотопа,
- 339 встречаются в природе
- Свыше 3100 РИ открыто
- Примерно 2500 предстоит



“Isle of stability” for superheavies:
We just “touched” a bit of its “shore”…

Proton dripline:
Achieved and studied for Z<32

Limits of nuclear structure existence:
Are known only for the lightest nuclei

Neutron dripline:
Achieved and studied for N<20

Exotic structure of exotic nuclides:

- **Neutron/proton halo**
- **Neutron skin**
- **“Soft” excitation modes**
- **Breakdown of shell structure**
- **New “magic numbers”**

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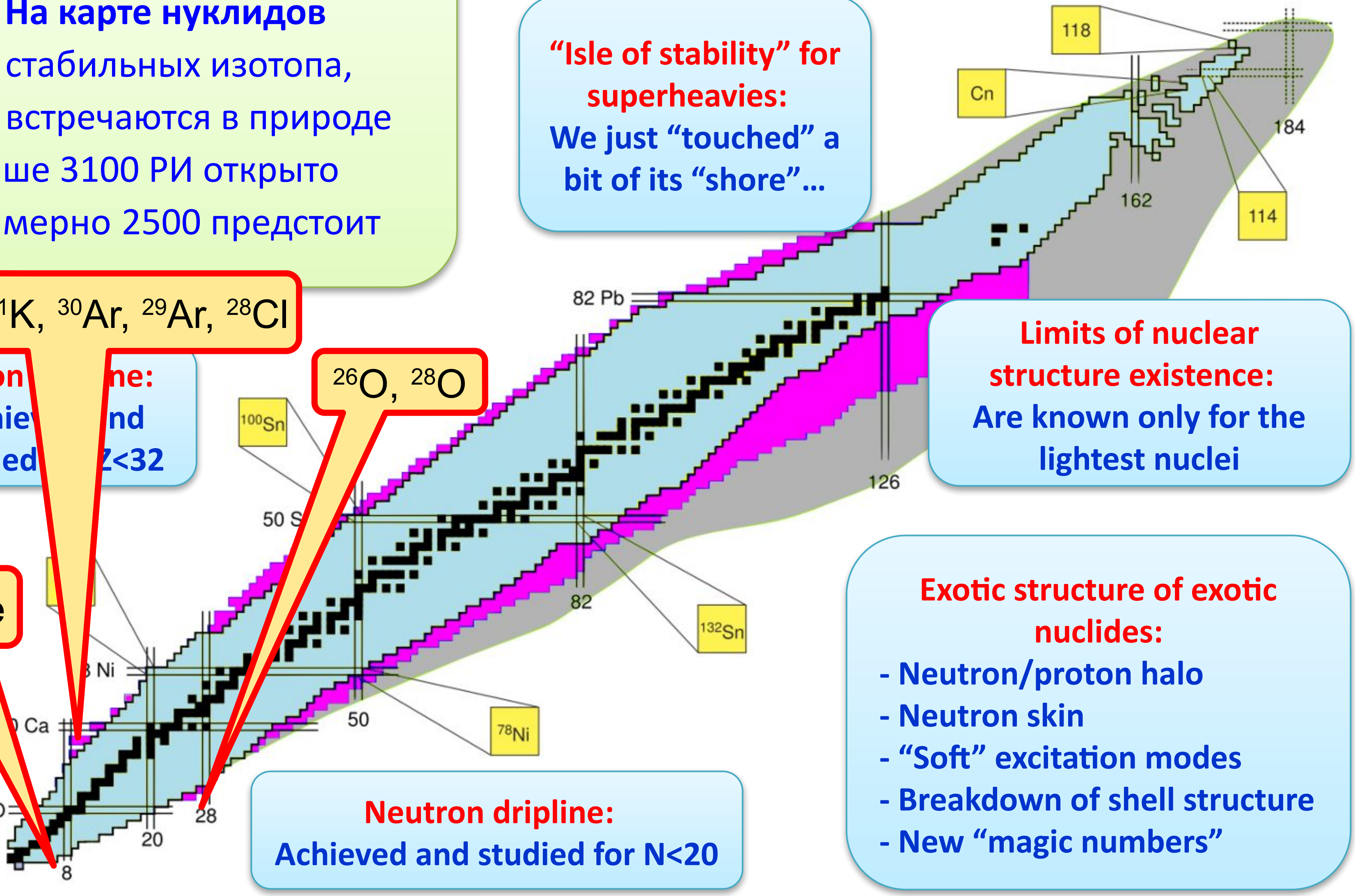

“Isle of stability” for superheavies:
We just “touched” a bit of its “shore”…

^{31}K, ^{30}Ar, ^{29}Ar, ^{28}Cl

^{26}O, ^{28}O

^{4}n,^{7}H,^{10}He

Proton ne:
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Экзотика в ядрах на границе стабильности

Кластеризация

Разделение характерных масштабов в системе

Не работают привычные концепции насыщения ядерной плотности и насыщения ядерного взаимодействия

Ядерное гало

“Нейтронная кожа”

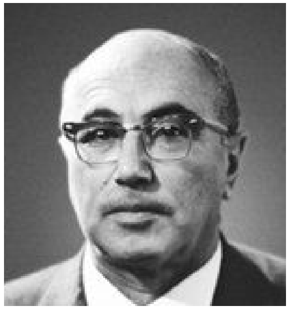
Мягкие моды возбуждения

Экзотические виды радиоактивности

Нарушение стандартных оболочечных закономерностей

Динамических степеней свободы не очень много и они сильно скоррелированы Естественным образом оказались востребованы методы физики нескольких тел

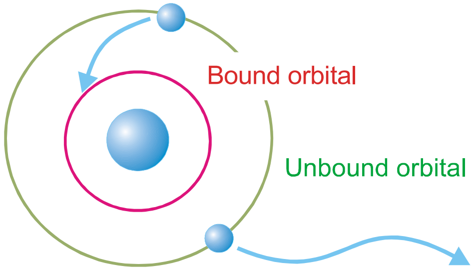
Двухпротонная радиоактивность



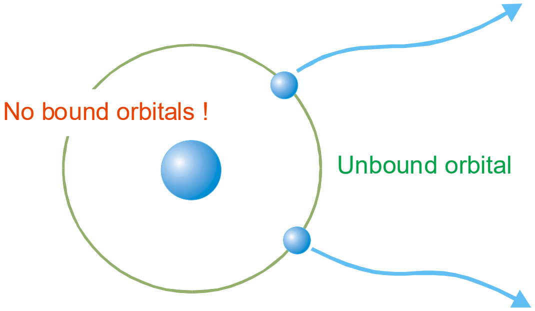
Я.Б. Зельдович и В.И. Гольданский, 1960, предсказание возможности р и 2р радиоактивности

Потребовались 4 десятилетия для реализации предсказаний

Л.В. Григоренко, М.В. Жуков, И. Томпсон, Р. Джонсон, 2000, первая последовательная квантово-механическая теория 2р радиоактивности

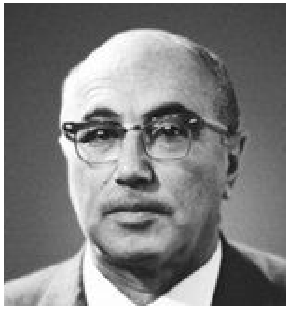


р-радиоактивность – естественное обобщение α-радиоактивности



2р-радиоактивность – необычное и сложное квантовомеханическое явление

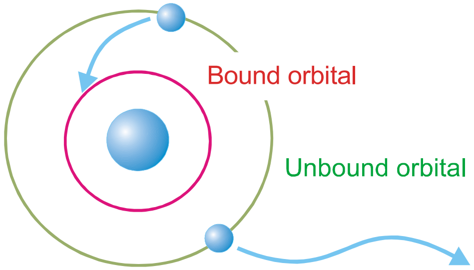
Двухпротонная радиоактивность



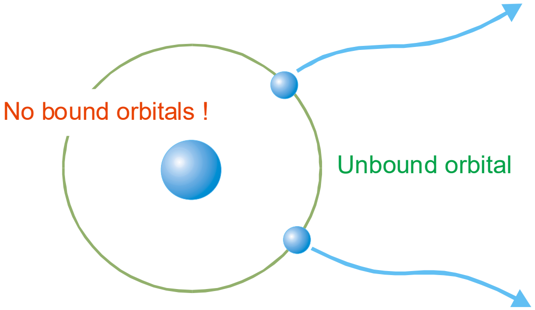
Я.Б. Зельдович и В.И. Гольданский, 1960, предсказание возможности p и 2p радиоактивности

Потребовались 4 десятилетия для реализации предсказаний

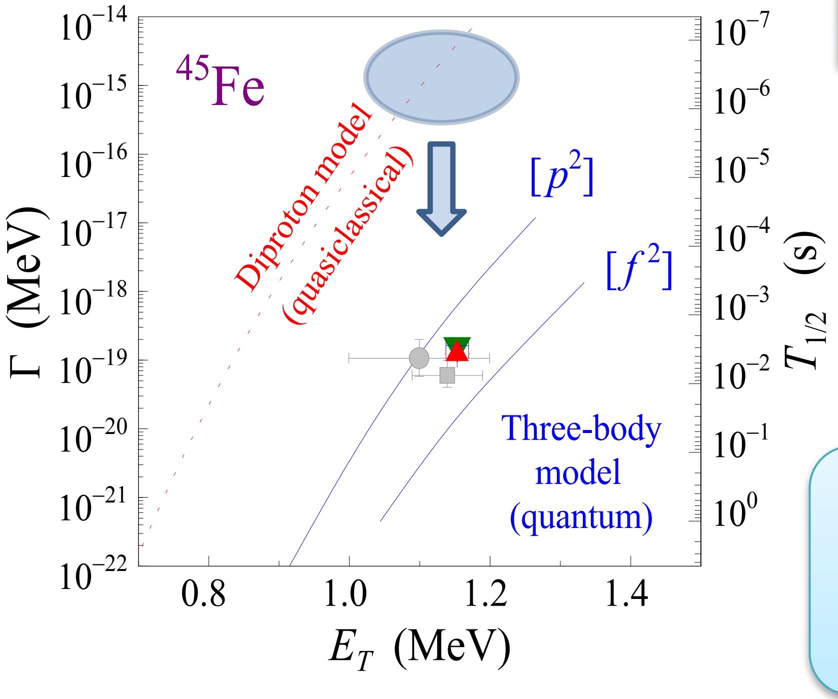
Л.В. Григоренко, М.В. Жуков, И. Томпсон, Р. Джонсон, 2000, первая последовательная квантово-механическая теория 2p радиоактивности



p-радиоактивность – естественное обобщение α-радиоактивности



2p-радиоактивность – необычное и сложное квантовомеханическое явление



M. Pfutzner, 2002, GSI: 2p-радиоактивность ^{45}Fe

Правильные теоретические расчеты сыграли критическую роль в открытии 2p радиоактивности

Ordinary things deep in the nuclear stability valley. Liquid drop model of nucleus. Bethe–Weizsäcker formula.

$$E_B = a_V A - a_S A^{2/3} - a_A \frac{(A-2Z)^2}{A^{1/3}} - a_C \frac{Z(Z-1)}{A^{1/3}} + \delta(A,Z)$$

Volume term, Surface term, Asymmetry term, Coulomb term, Pairing term

For pairing term:

$$\delta(A,Z) = \begin{cases} +\delta_o & \text{A, Z even} \\ 0 & \text{A odd} \\ -\delta_o & \text{A, Z odd} \end{cases}$$

where

$$\delta_o = \frac{a_P}{A^{1/2}}$$

Coefficients:

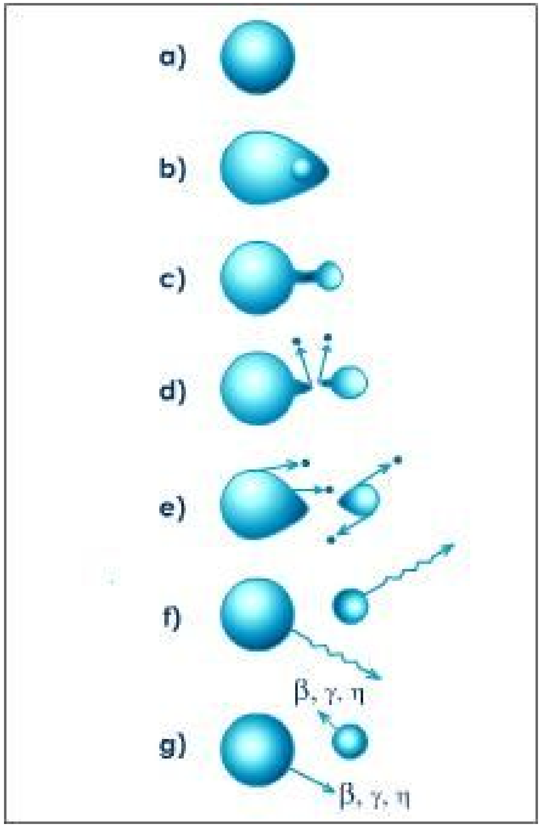
$a_V = 15.85$ MeV

$a_S = 18.34$ MeV

$a_A = 23.21$ MeV

$a_C = 0.714$ MeV

$a_P = 12.00$ MeV



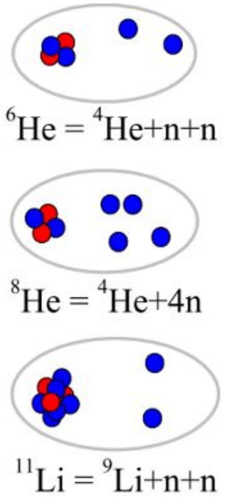
Nucleus is a drop of “nuclear liquid” with $r = r_0 A^{1/3}$

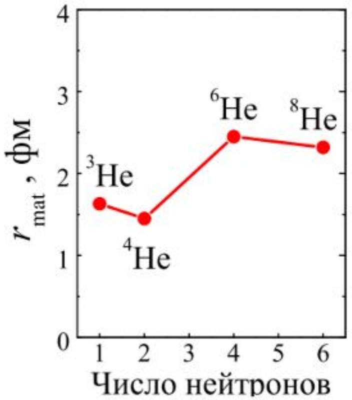
Quantitative explanation of nuclear masses

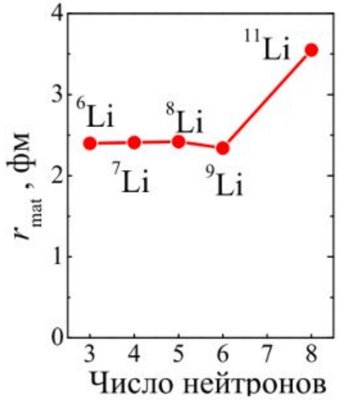
Quantitative explanation of fission in heavy nuclei

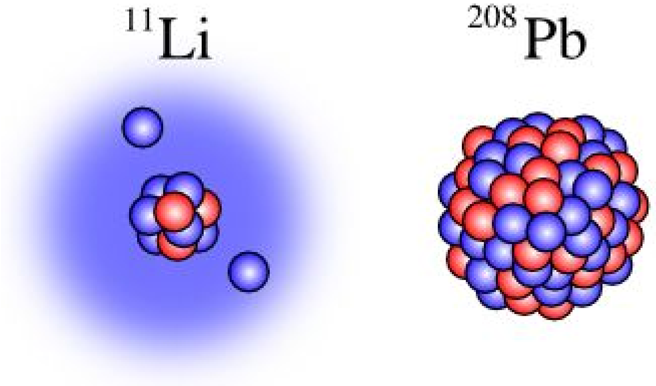
Ядра с гало. Борромиевские ядра

Валентные орбитали аномальных размеров

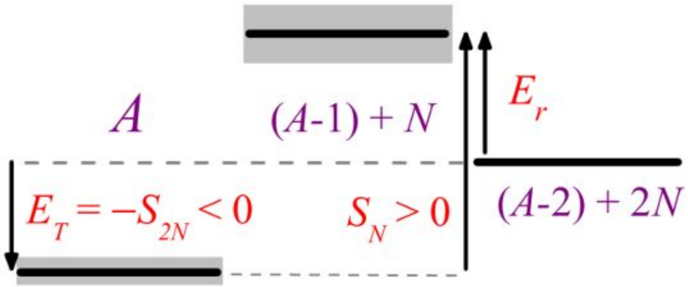




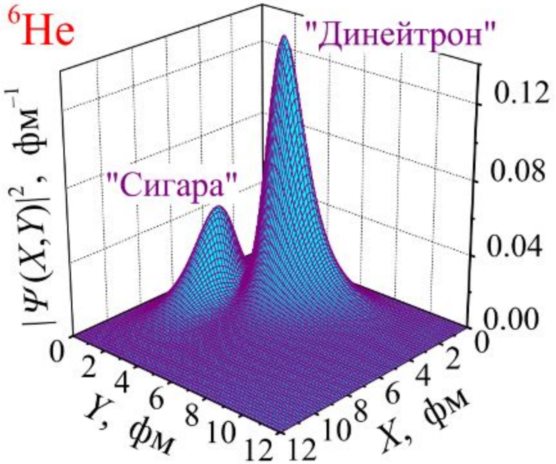




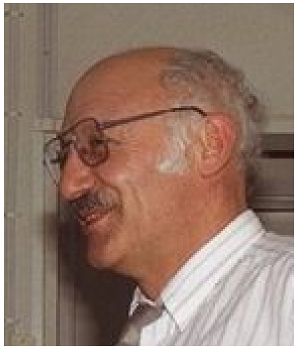
“Борромиевские” системы



Сложные корреляции



Квантовая задача нескольких тел



Научная школа физики нескольких тел в Курчатовском институте

А.И. Базь, М.В. Жуков

СССР - 4 few-body школы: Питер, МГУ, Курчатник, Тбилиси

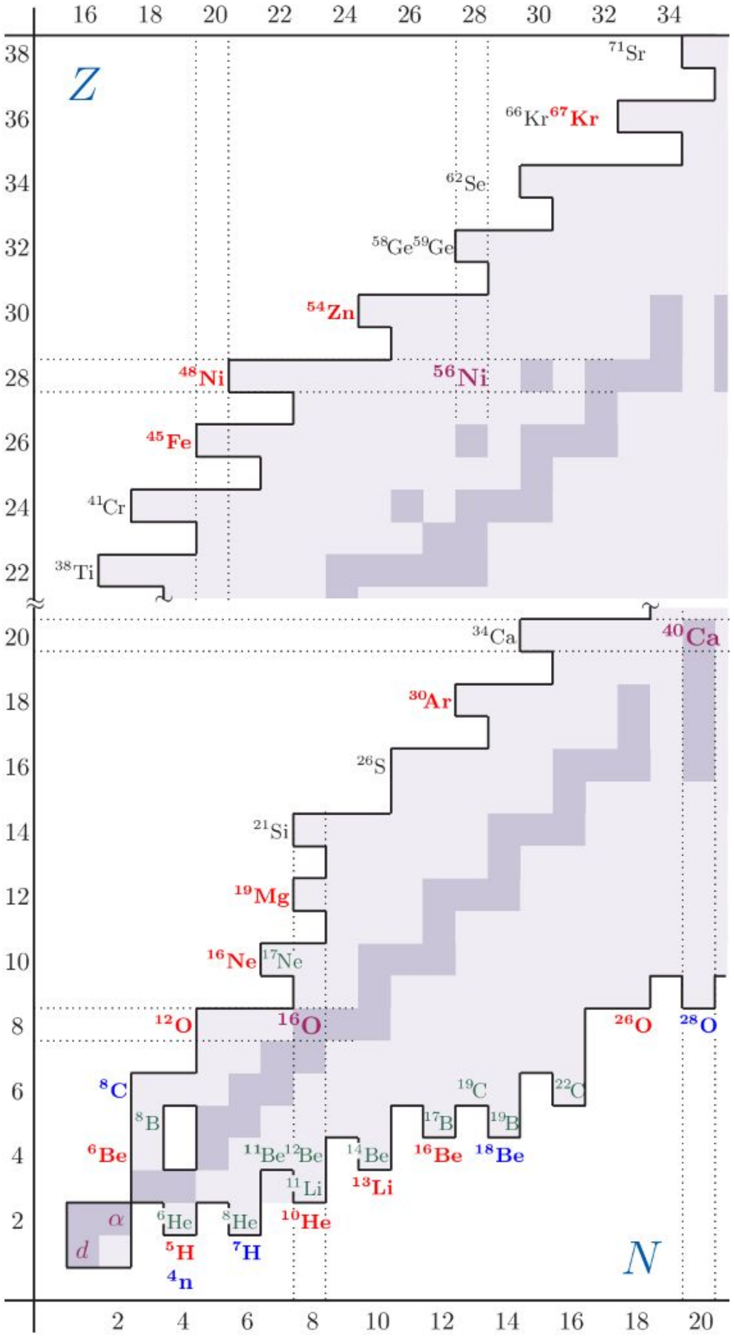
Вблизи границы ядерной стабильности:

i) Кластеризация
ii) Спаривание

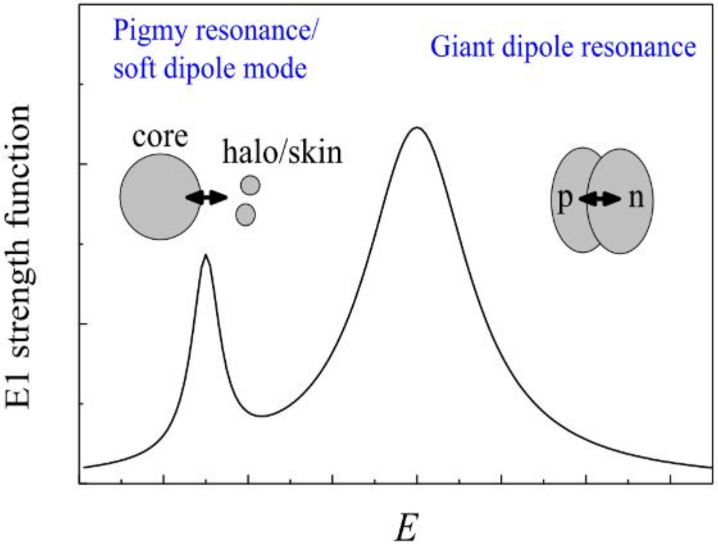
→ **Динамика задачи нескольких тел**

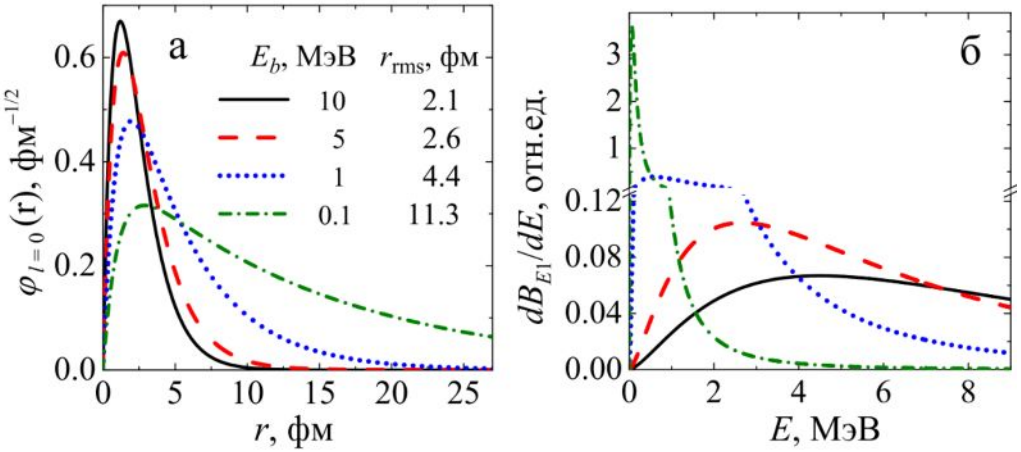
Экзотика вблизи границы ядерной стабильности: Ядра с нуклонным гало, «Истинные» 2p и 2n распады, 4p и 4n распады, Ожидаемые, но не исследованные

(i) Таких систем много (ii) Изученных мало



Мягкие моды возбуждения (мягкая дипольная мода)





$$\phi_{l=0}(r) = N(\exp[-k_1 r] - \exp[-k_2 r])\,, \quad k_1 = \sqrt{2ME_b}\,,$$

$$M_{E1}(E) = \int_0^\infty dr\,(pr) j_{l=1}(pr)\, r\, \phi_{l=0}(r)\,, \quad p = \sqrt{2ME}\,,$$

$$\frac{dB_{E1}}{dE} \sim \frac{|M_{E1}(E)|^2}{\sqrt{E}}$$

Existance of soft dipole mode strongly influence the nonresonant radiative capture rate in astrophysics

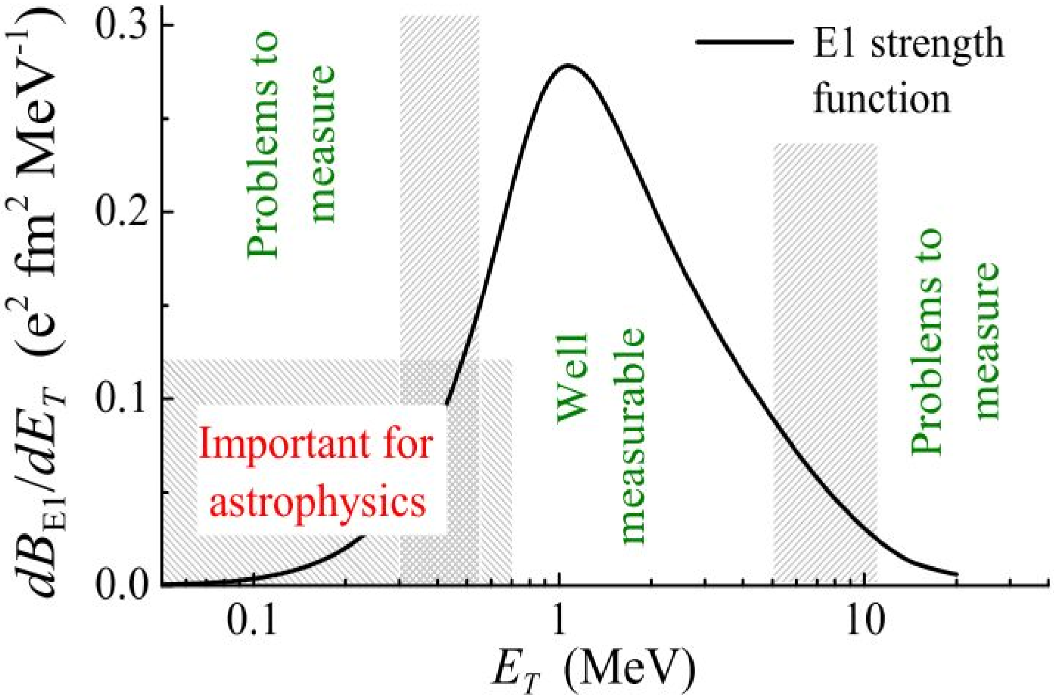
Мягкие моды возбуждения и астрофизика

1. Теоретическая задача

$\sigma \sim B_{E1} \exp(-2\pi\eta)$

$\eta = Z_1 Z_2 \alpha/\nu$

Нужно «экстраполировать» Сечение из области где оно измеримо в область где оно важно для астрофизики.



2. Экспериментальная проблема померять сечение и извлечь силовую функцию E1 как можно ниже по энергии.

Resonance vs nonresonant capture

General case resonant

$$\langle \sigma_{\mathrm{part},\gamma} \rangle (T) \sim \frac{1}{T^{3n/2}} \exp\left(-\frac{E_r}{kT}\right) \frac{\Gamma_\gamma \Gamma_{\mathrm{part}}}{\Gamma_{\mathrm{tot}}}$$

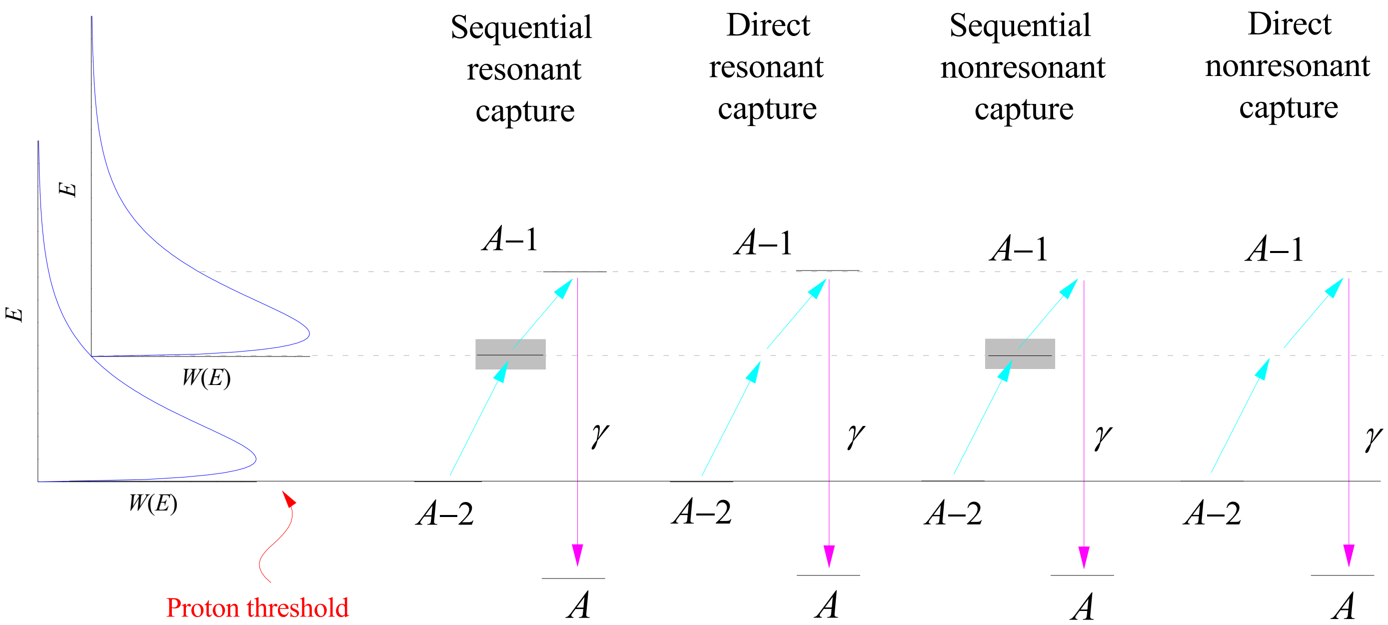
2p resonant

$$\langle \sigma_{2p,\gamma} v \rangle = \left(\frac{A_1 + A_2 + A_3}{A_1 A_2 A_3}\right)^{3/2} \left(\frac{2\pi}{mkT}\right)^3 \sum_n \frac{2J_f(n) + 1}{2(2J_i + 1)} \times \exp\left[-\frac{E_T(n)}{kT}\right] \frac{\Gamma_{2p}(n)\Gamma_\gamma(n)}{\Gamma(n)}, \qquad (16)$$

2p nonresonant

$$\langle \sigma_{2p,\gamma} v \rangle = \left(\frac{A_1 + A_2 + A_3}{A_1 A_2 A_3}\right)^{3/2} \left(\frac{2\pi}{mkT}\right)^3 \frac{2J_f + 1}{2(2J_i + 1)} \times \int dE \, \frac{16\pi}{9} e^2 E_\gamma^3 \frac{dB_{E1}(E)}{dE} \exp\left[-\frac{E}{kT}\right], \qquad (2)$$

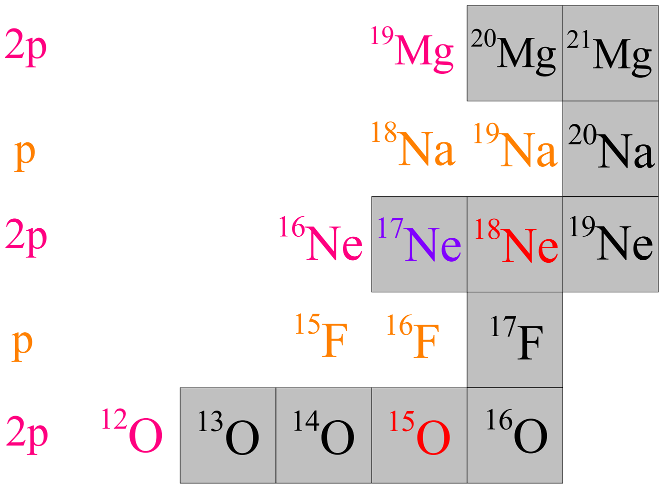
Modes of (2p) radioactive capture



Sequential

Reverse to true 2p decay

Reverse to soft dipole mode



2p				^{19}Mg	^{20}Mg	^{21}Mg
p				^{18}Na	^{19}Na	^{20}Na
2p			^{16}Ne	^{17}Ne	^{18}Ne	^{19}Ne
p			^{15}F	^{16}F	^{17}F	
2p	^{12}O	^{13}O	^{14}O	^{15}O	^{16}O	

- rp-process at high density and temperature.
- ^{15}O, ^{18}Ne, ^{38}Ca : J.Gorres, M.Wiescher, and F.-K.Thielemann, PRC **51** (1995) 392.
- ^{68}Se, ^{72}Kr, … ,^{96}Cd : H.Schatz et al., Phys. Rep. **294** (1998) 167.

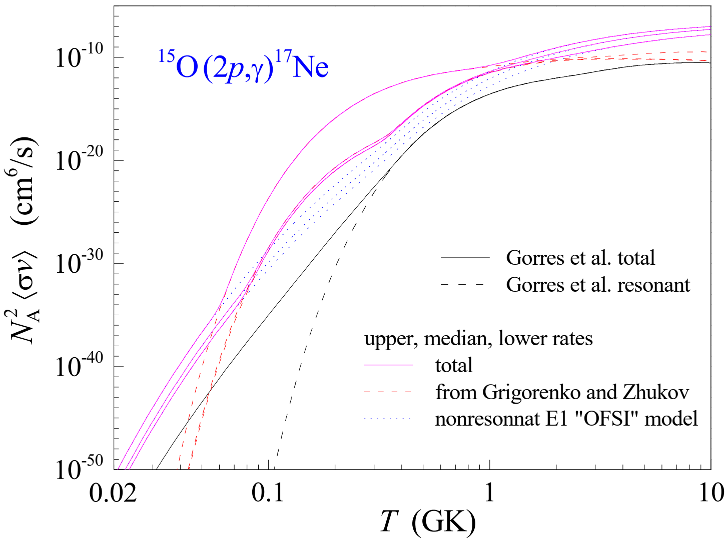
Competition between α and 2p capture

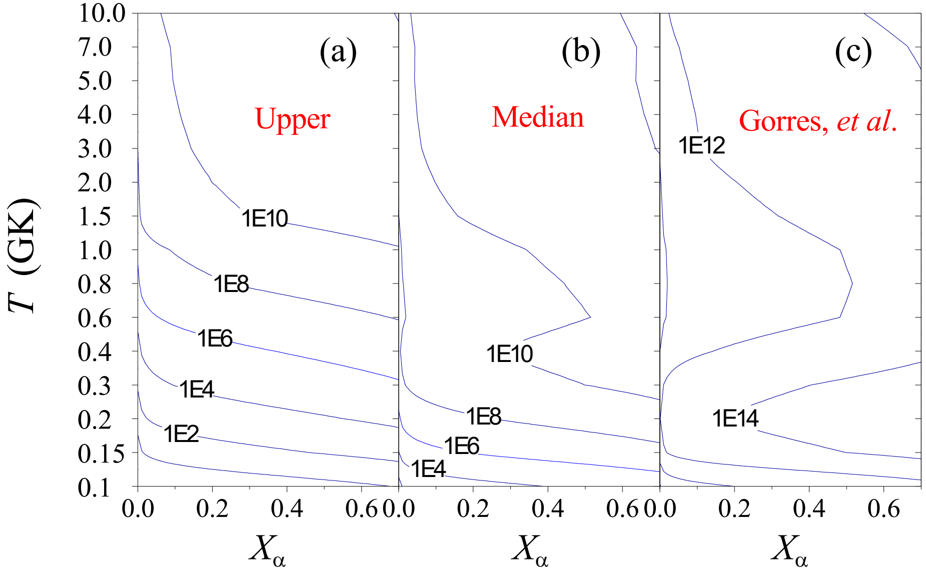
- $^{15}O(2p,\gamma)^{17}Ne$ versus $^{15}O(\alpha,\gamma)^{19}Ne$
- There are two crucial contribution to the radiative 2p capture:
- Nonresonance capture (“reverse soft dipole mode”)
- Resonance capture (defined by the 2p witdth of $3/2^-$ state in ^{17}Ne)

Experiment:
M.J. Chromik et al. PRC66, 024313 (2002)
$\Gamma_{2p} < 2.5*10^{-11}$ MeV

Theory:
L.V. Grigorenko and M.V. Zhukov, PRC 76, 014008 (2007)
$\Gamma_{2p} \sim (5\text{-}8)*10^{-15}$ MeV

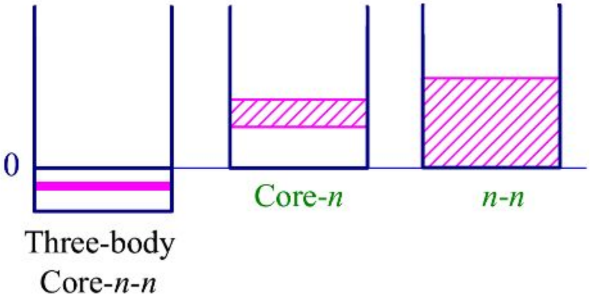
- Densities (in g/ccm) for which ^{17}Ne production in 2p- capture on ^{15}O become dominate over α-capture on ^{19}Ne захватом α (mass concentration of α-s: $X_\alpha = 4\, Y_\alpha$).



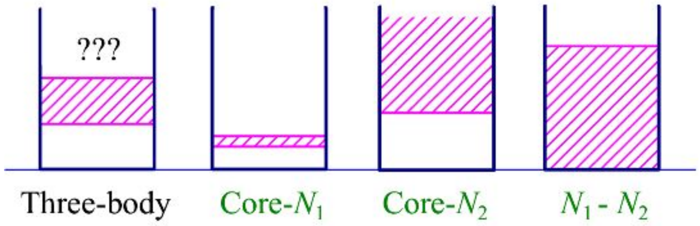


Energy conditions and few-body phenomena

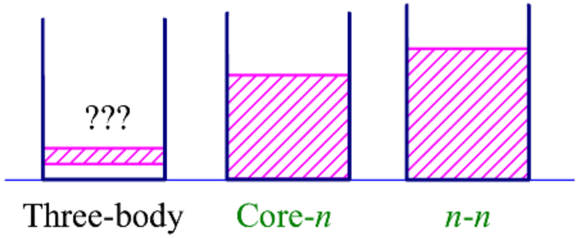
Borromean 2n halo systems



"Soft excitations"

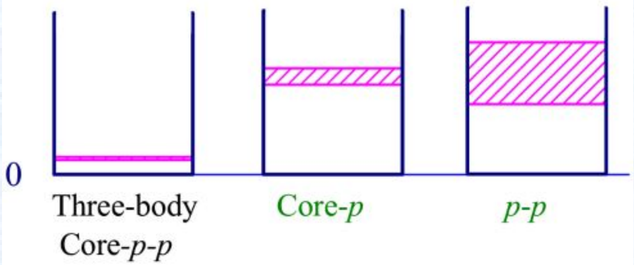


Three-body virtual states

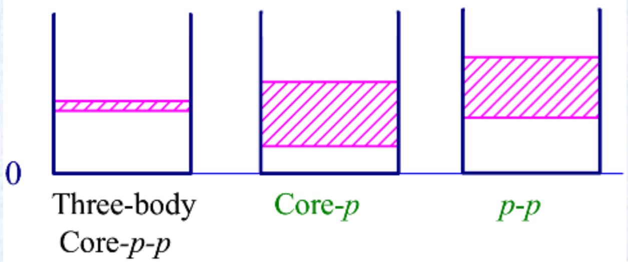


True three-body decay

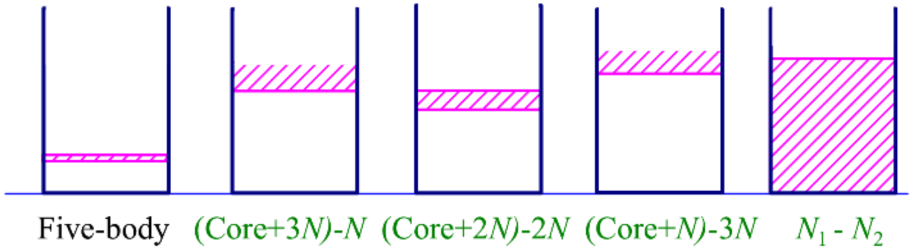
2p radioactivity



Democratic decays



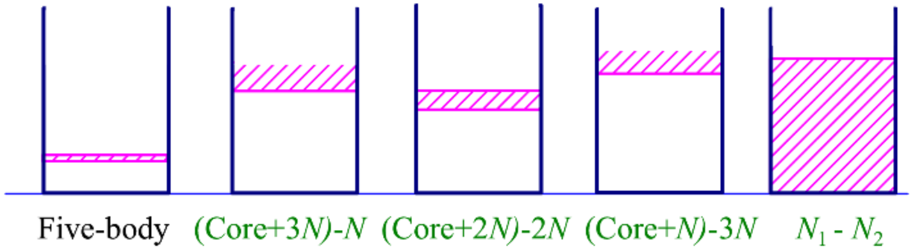
True 5n-body decay (4n radioactivity)

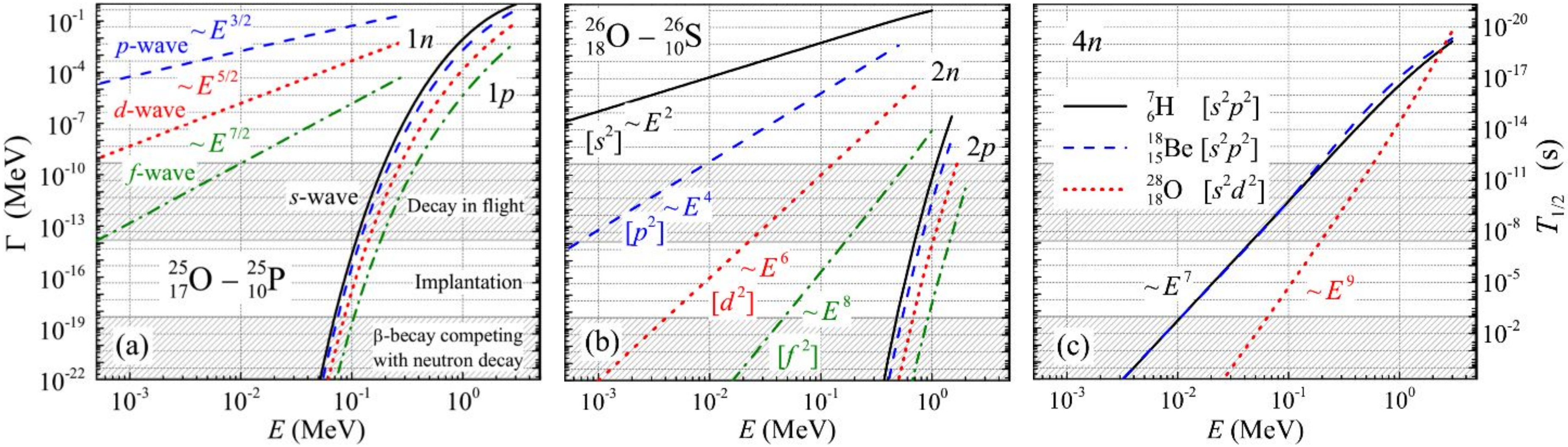


О возможности существования 2 и 4 нейтронной радиоактивностей

L.V. Grigorenko, I.G. Mukha, C. Scheidenberger, and M.V. Zhukov, PRC **84** (2011) 021303(R)

Energy conditions for true 4n decay





Long-living true four-neutron decay states are most probable.

Nearest candidates for 4n radioactive decay: ^{7}H, ^{18}Be, ^{28}O

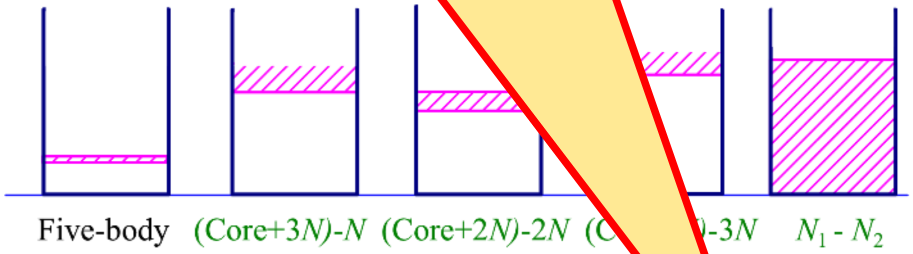
О возможности существова радиоактивностей

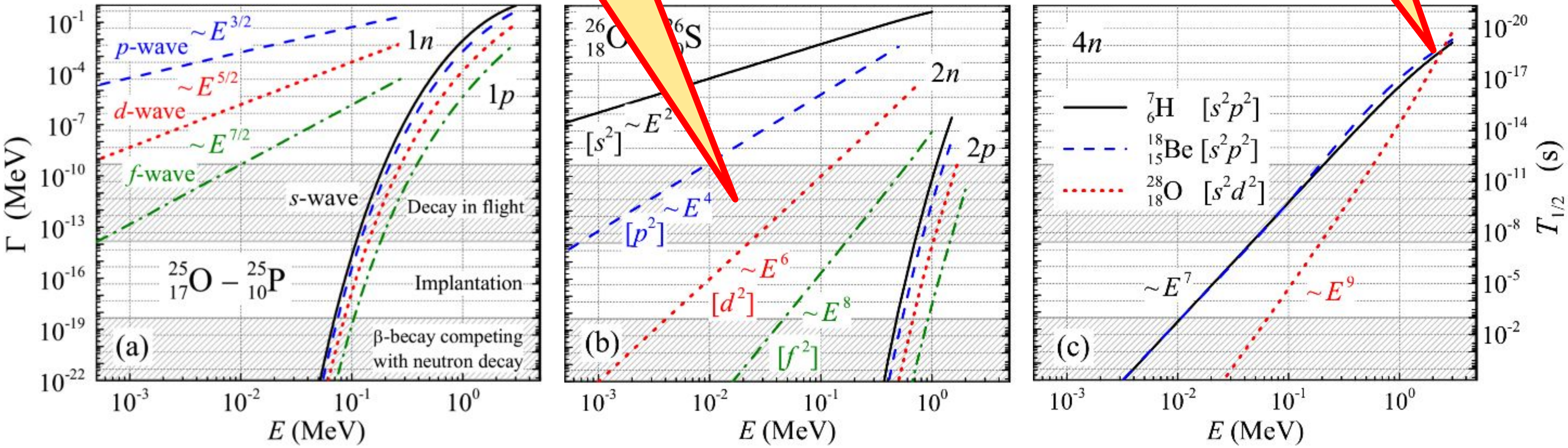
^{7}H — истинно пятичастичный распад, но не радиоактивность

L.V. Grigoren
and M.V. Zhu

^{26}O — противоречие экспериментов

Energy conditions for true 4n y





Long-living true four-neutron decay states are most probable.

Nearest candidates for 4n radioactive decay: ^{7}H, ^{18}Be, ^{28}O

Three-body decay in cluster model

L.V. Grigorenko, R.C. Johnson, I.G. Mukha, I.J. Thompson, M.V. Zhukov, PRL **85** (2000) 22.

- **Decays:**
  Schrodinger equation with complex energy

$$(\hat{H}_3 - E_T + i\Gamma/2)\Psi^{(+)} = 0$$

- "Natural" definition of width

$$\Gamma = \frac{j(\rho_{\max})}{N(\rho_{box})} = \frac{\mathrm{Im}\int d\Omega_\rho \Psi^{(+)\dagger}\rho^{5/2}\frac{d}{d\rho}\rho^{5/2}\Psi^{(+)}\Big|_{\rho_{\max}}}{M\int d\Omega_\rho \int_0^{\rho_{box}} d\rho\, \rho^5 \left|\Psi^{(+)}\right|^2}$$

- Adopted to radioactive decay studies

Typical precision: stable solution for $\Gamma/E_T > 10^{-30}$

L.V. Grigorenko, Phys. of Part. and Nucl. **40** (2009) 674

M. Pfutzner, L.V. Grigorenko, M. Karny, and K. Riisager, Rev. Mod. Phys. **84** (2012) 567 [arXiv:1111.0482].

- Three-body Hyperspherical Harmonic method
- Approximate boundary conditions of the three-body Coulomb problem
- Classical extrapolation to improve momentum distributions

- **Reactions:**
- Inhomogeneous Schrodinger equation with real energy

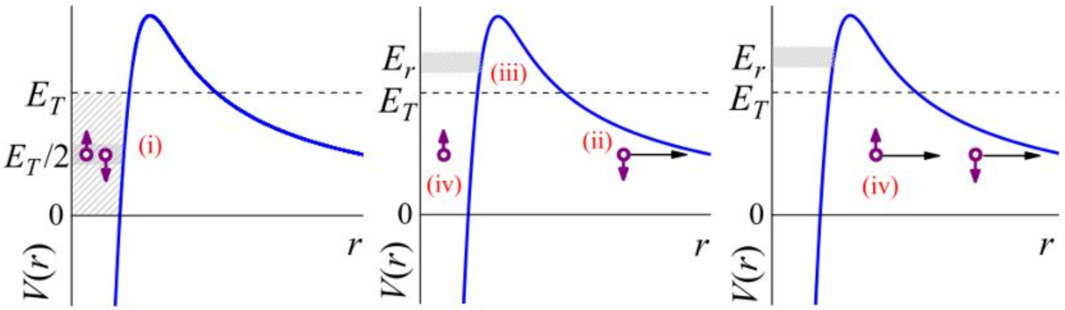
$$(\hat{H}_3 - E_T)\Psi^{(+)} = \Phi_{\mathbf{q}}$$

- Source function formulation is straightforward for several types of direct reactions. E.g. for knockout reactions (^{6}Be populated with ^{7}Be beam):

$$\Phi_{\mathbf{q}} = \int d^3 r_n e^{i\mathbf{q}\mathbf{r}_n}\langle \Psi_{^4\mathrm{He}}|\Psi_{^7\mathrm{Be}}\rangle$$

- Very precise results could be obtained for reactions with well defined "clean" mechanism.

2p decay and "Goldansky-type" correlations



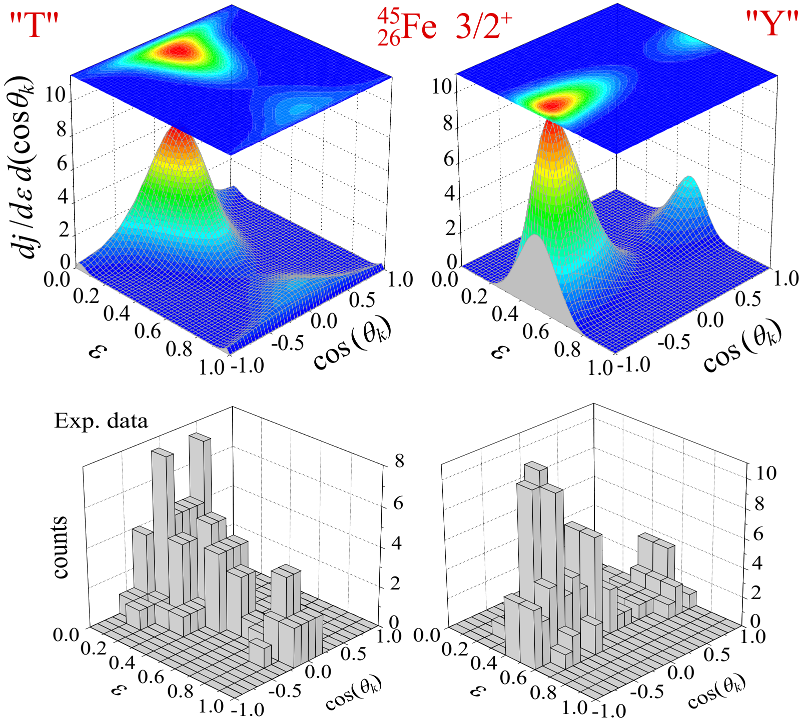
Proton decay

$$\Gamma_2(E_r) \sim \exp\left[-\frac{\pi(Z-1)\alpha\sqrt{M}}{\sqrt{E_r}}\right]$$

Two-proton decay. ONLY sharing of energy. OTHERWISE protons are uncorrelated

$$\begin{aligned} \frac{d\Gamma_3(E_T)}{d\varepsilon} &\sim \exp\left[-\frac{2\pi(Z-2)\alpha\sqrt{M}}{\sqrt{E_T}}\left(\frac{1}{\sqrt{\varepsilon}}+\frac{1}{\sqrt{1-\varepsilon}}\right)\right] , \\ \Gamma_3(E_T) &= \int_0^1 d\varepsilon\ [d\Gamma_3(E_T)/d\varepsilon] . \end{aligned}$$

Original estimate by Goldansky



Correlations within and beyond the vision of Goldansky

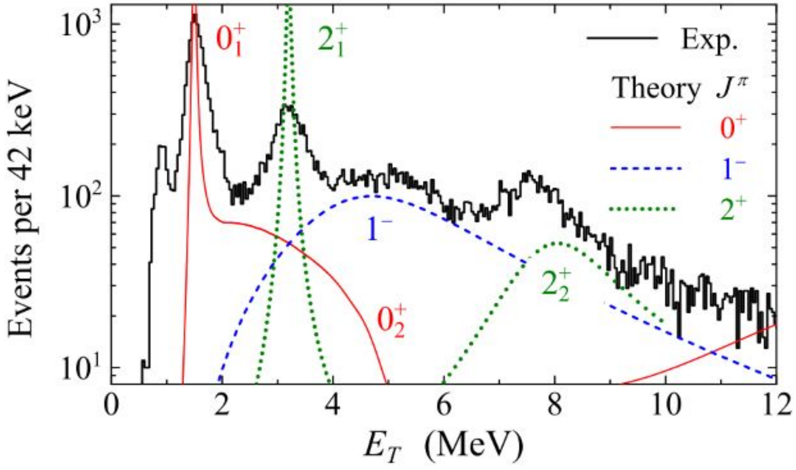
Long-range character of three-body Coulomb by example of ^{16}Ne

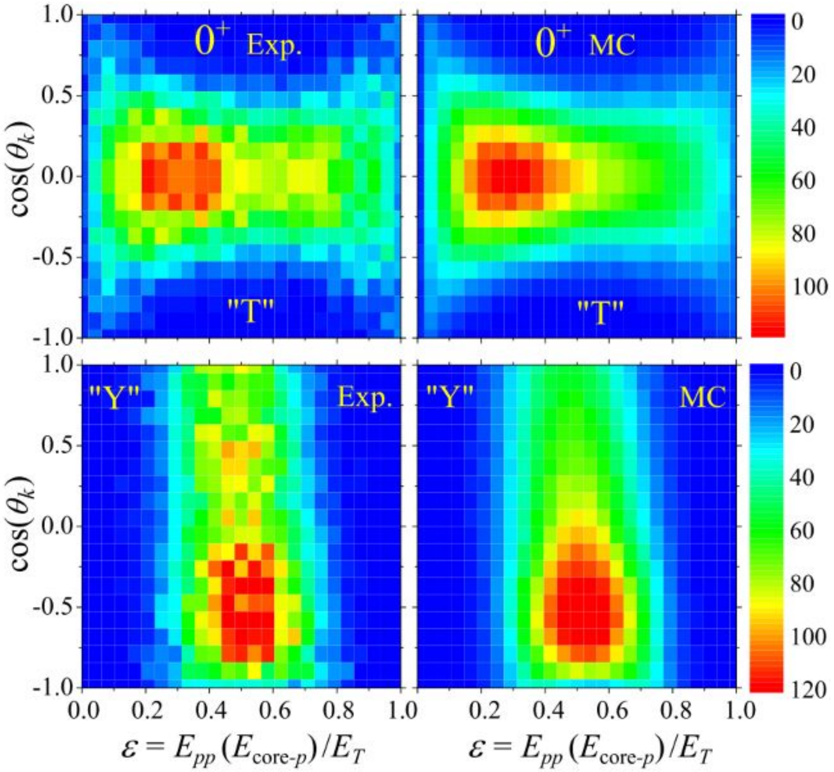
^{16}Ne g.s., E_T= 1.466 MeV

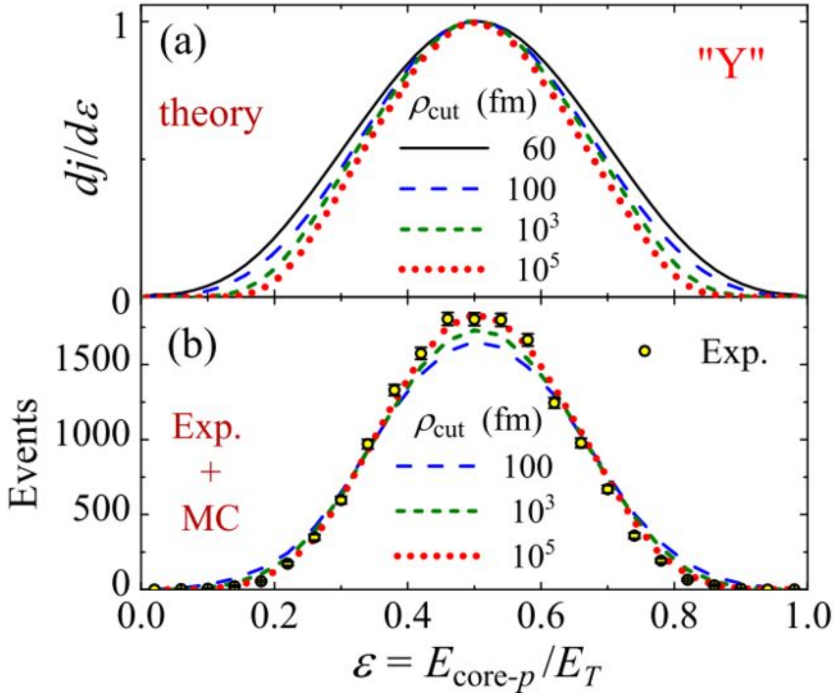
- New level of experimental precision. MSU 2013: ^{16}Ne populated in n knockout from ^{17}Ne

K. Brown et al., PRL **113** (2014) 232501

- The energy distribution in “Y” Jacobi system only reproduced for extreme range of calculation





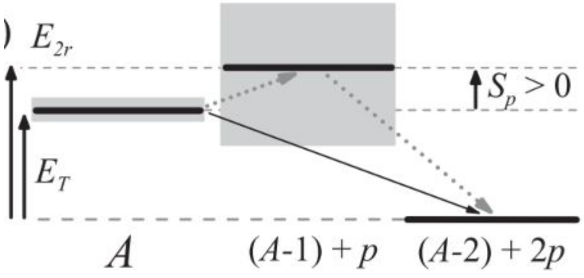


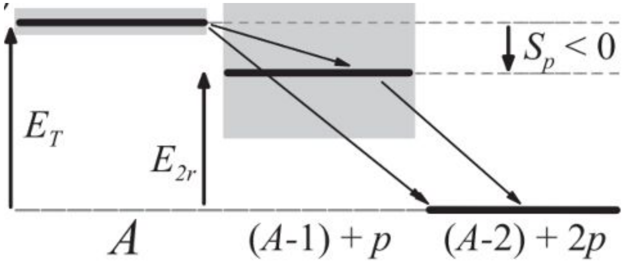
Dominance of core-nucleon dynamics.

Transition dynamics and “phase transition” diagram for the three-body decay

I. Mukha *et al.,* PRL **115** (2015) 202501.
T.A. Golubkova *et al.*, PLB **762** (2016) 263.
X. Xu *et al.*, PRC **97** (2018) 034305.

Mechanisms of 2p decay defined by separation energies

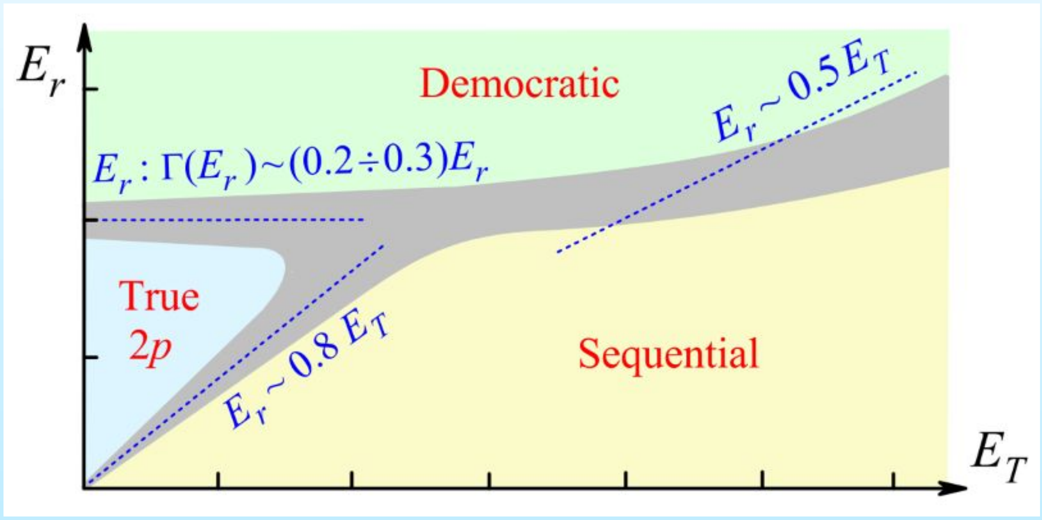




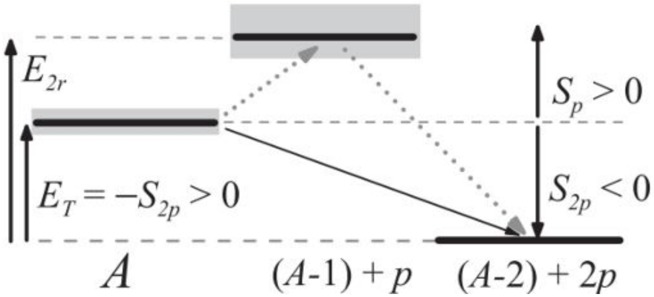
Three major decay mechanisms: True 2p, Democratic 2p Sequential 2p

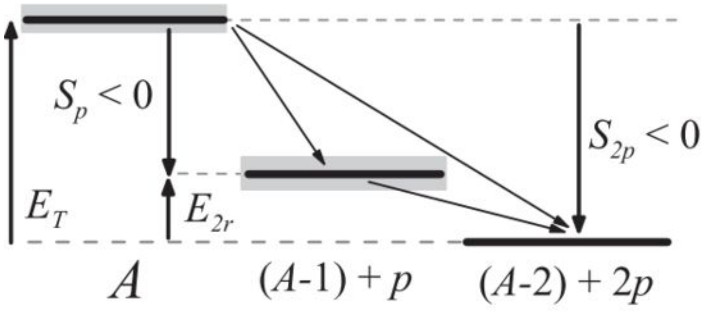
Three principal parameters:

$E_T \quad E_r \quad \Gamma_r$



There SHOULD EXIST transition region between them





Transition decay dynamics in simplified semianalytical models

$$\Gamma_{j_1 j_2}(E_T) = \frac{E_T \langle V_3 \rangle^2}{2\pi} \int_0^1 d\varepsilon \, \frac{\Gamma_{j_1}(\varepsilon E_T)}{(\varepsilon E_T - E_{j_1})^2 + \Gamma_{j_1}(\varepsilon E_T)^2/4}$$

$$\times \frac{\Gamma_{j_2}((1-\varepsilon)E_T)}{((1-\varepsilon)E_T - E_{j_2})^2 + \Gamma_{j_2}((1-\varepsilon)E_T)^2/4}$$

Stems from simplified three-body Hamiltonian

Basic feature strong dependence on two resonances in the subsystems

Recent upgrade to “improved direct decay model”

T.A. Golubkova *et al.*, PLB **762** (2016) 263

Correct angular momentum coupling, amplitude symmetries, NN FSI correction, etc.

$$\frac{d\Gamma(E_{3r})}{d\Omega_\varkappa} = \sum_{LS} \frac{E_{3r}}{2\pi(2L+1)} \sum_{M_L} \left| \sum\nolimits_\gamma A^{LM_L}_{S\gamma}(\Omega_\varkappa) \right|^2 ,$$

$$A^{LM_L}_{S\gamma}(\Omega_\varkappa) = C^{JLS}_\gamma \, V^J_\gamma \, [l_1 \otimes l_2]_{LM_L} \, A_{j_1 l_1}(E_1) A_{j_2 l_2}(E_2) \, ,$$

$$[l_1 \otimes l_2]_{LM_L} = \sum\nolimits_{m_1 m_2} C^{LM_L}_{l_1 m_1 l_2 m_2} Y_{l_1 m_1}(\hat{r}_1) Y_{l_1 m_1}(\hat{r}_1) \, .$$

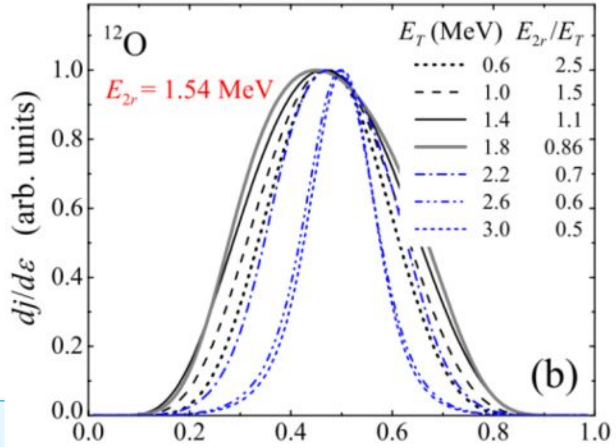
$$A^{LM_L}_{S\gamma}(\Omega_\varkappa) \rightarrow \frac{C^{JLS}_\gamma \, V^J_\gamma \, A^{(pp)}_S(E^T_x)}{E_{r1} + E_{r2} - E_T - i \left[\Gamma_1(E_{r1}) + \Gamma_2(E_{r2}) \right]/2}$$

$$\times \hat{\mathcal{O}}_S \left([l^{Y_1}_x \otimes l^{Y_1}_y]_{LM_L} \, A_{j^{Y_1}_x l^{Y_1}_x}(E^{Y_1}_x) \sqrt{\Gamma_1(E^{Y_1}_y)} \right.$$

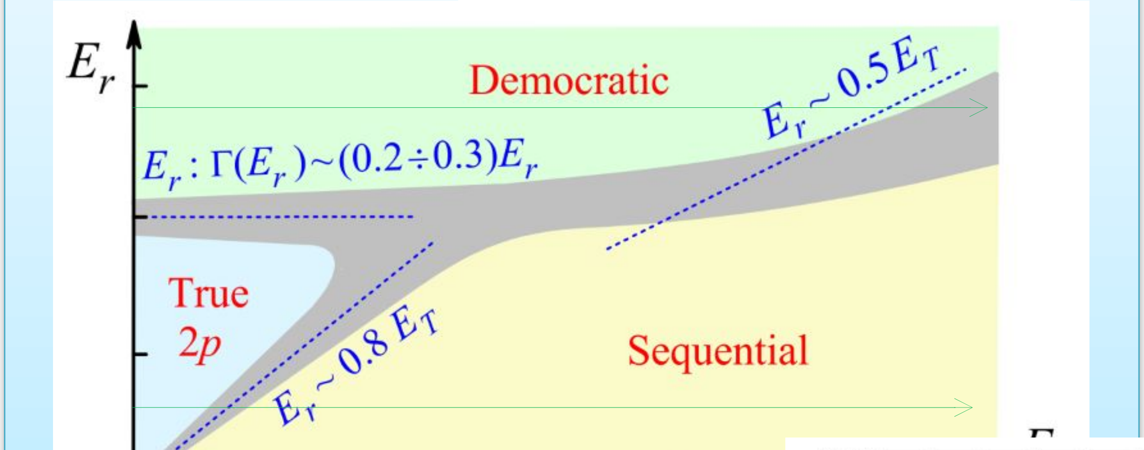
$$\left. + \, [l^{Y_2}_x \otimes l^{Y_2}_y]_{LM_L} \, A_{j^{Y_2}_y l^{Y_2}_y}(E^{Y_2}_x) \sqrt{\Gamma_2(E^{Y_2}_y)} \right) \, .$$

$$A_{jl}(E) = \frac{\sqrt{\Gamma_r(E)}}{E_r - E - i\Gamma_r(E)/2} + A^{(p)}_{jl}(E)$$

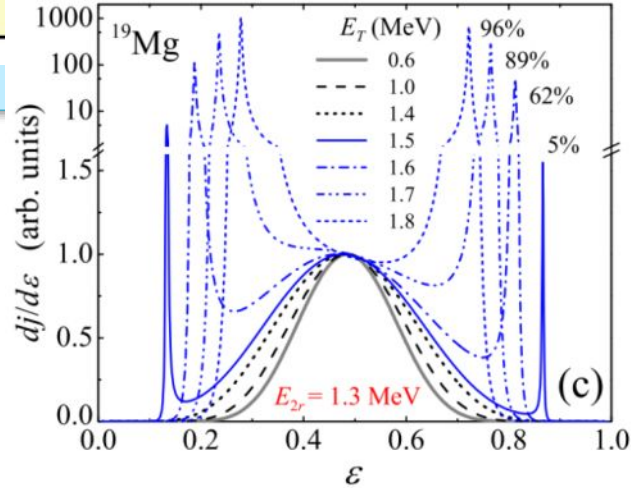
Mechanisms of 2p decay



Democratic 2p <-> Sequential 2p



Energy correlations between core and one proton

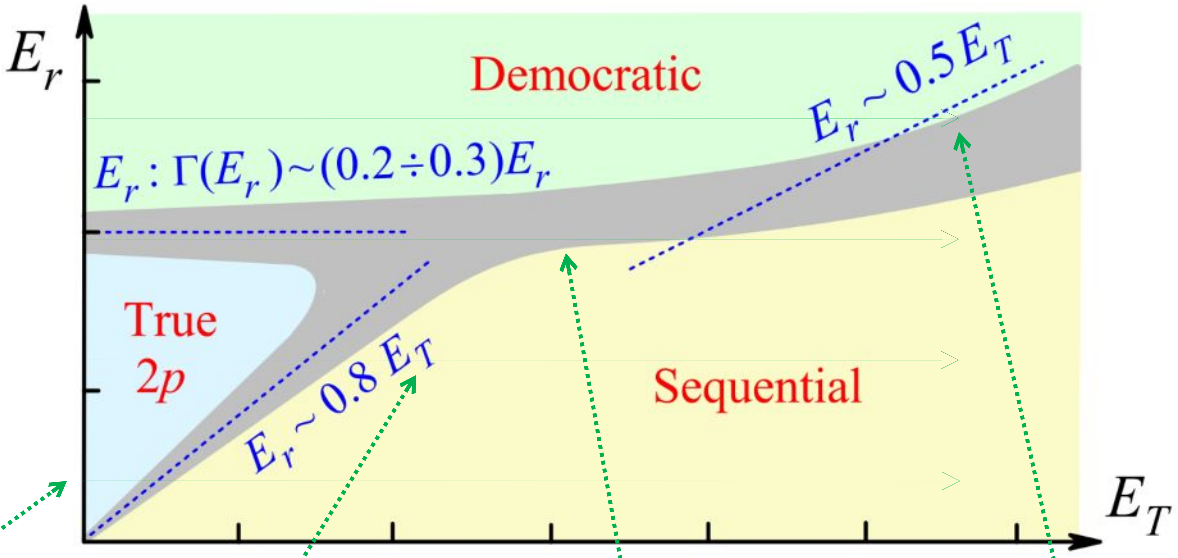


True 2p <-> Sequential 2p

General view of transition dynamics

“^{30}Ar”

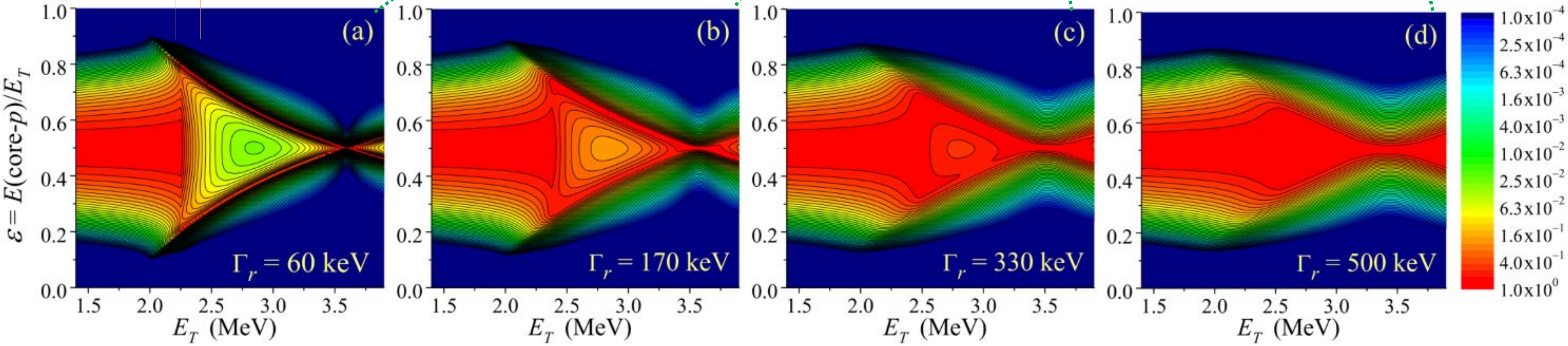
Energy correlations between core and one proton



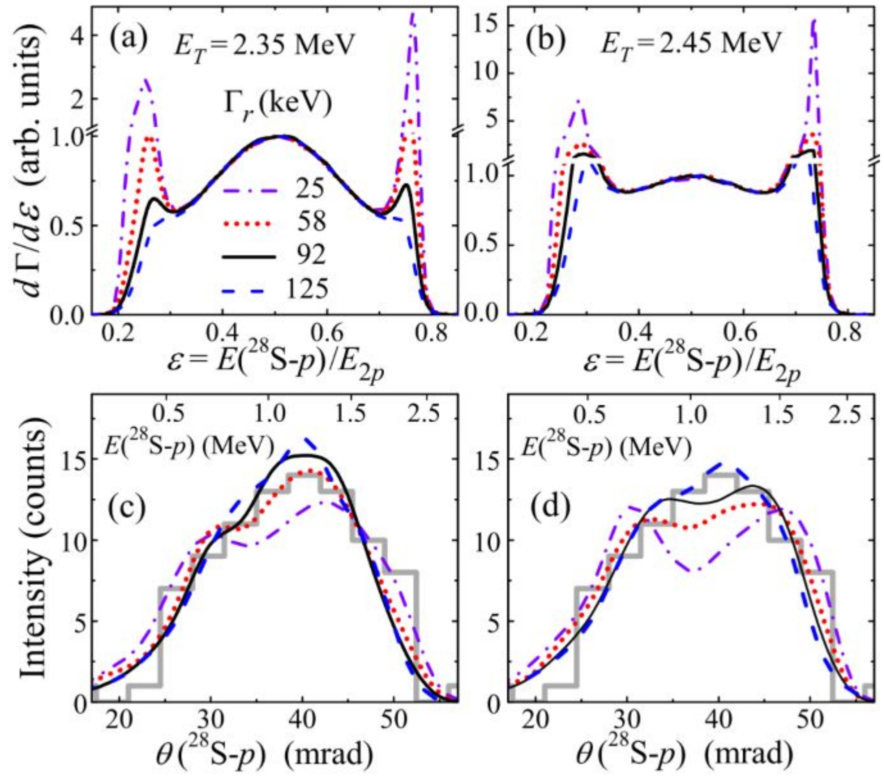
Transition

True 2p

Sequential 2p



^{29}Cl g.s. width from ^{30}Ar data



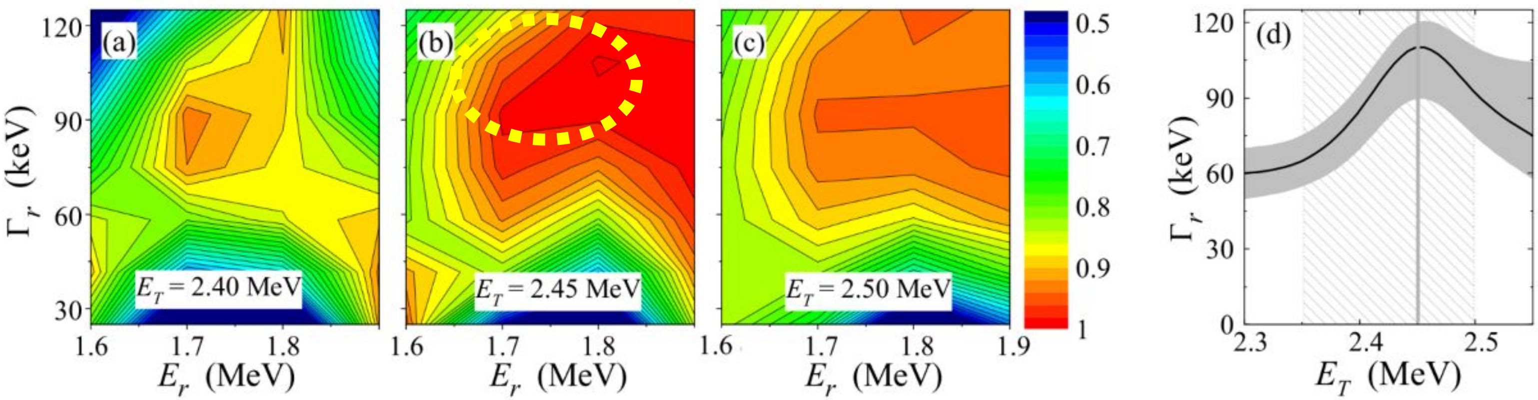
Energy is "easy" to measure, width could be very complicated.
From $T_{1/2}$~1 ps to Γ~100-200 keV there is a "blind spot"

^{30}Ar-> ^{28}S+2p decay was found to have transition decay dynamics

Strong dependence of the experimental signal on the g.s. properties of core+p subsystem – ^{29}Cl

Stringent limits for ^{29}Cl g.s. width

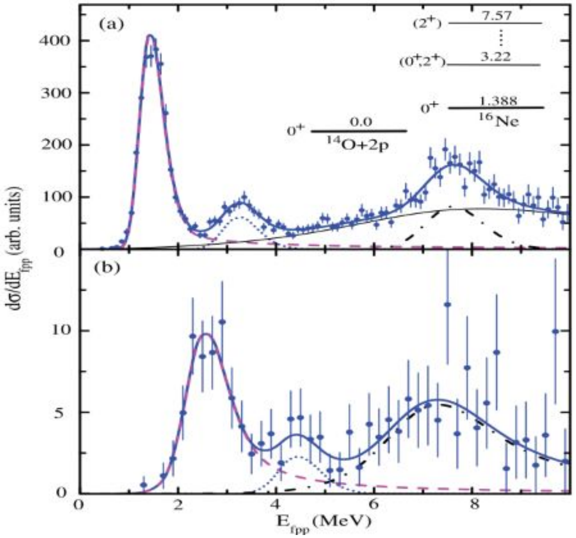
I. Mukha et al., PRL **115** (2015) 202501.
T.A. Golubkova *et al.*, PLB **762** (2016) 263.
X. Xu *et al.*, Phys. Rev. C **97** (2018) 034305.



Prospects to observe transition dynamics in ^{15}Ne

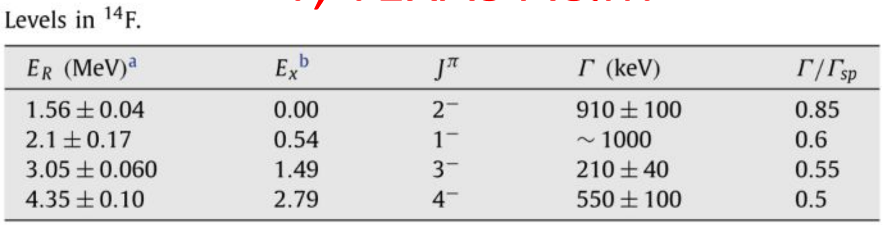
F. Wamers *et al.*, PRL **112** (2014) 132502

^{16}Ne, ^{15}Ne, GSI



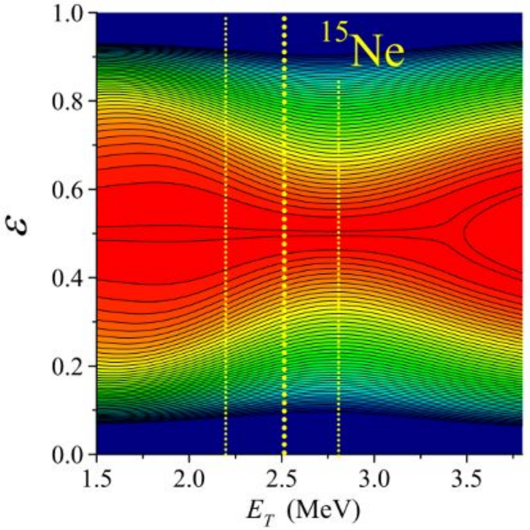
V. Goldberg *et al.*, PLB **692** (2010) 307

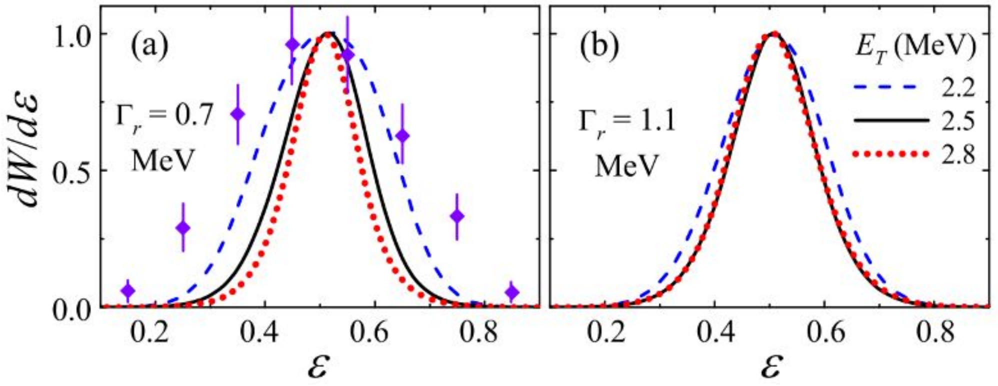
^{14}F, TEXAS A&M



Levels in ^{14}F.

E_R (MeV)[a]	E_x[b]	J^π	Γ (keV)	Γ/Γ_{sp}
1.56 ± 0.04	0.00	2$^-$	910 ± 100	0.85
2.1 ± 0.17	0.54	1$^-$	~ 1000	0.6
3.05 ± 0.060	1.49	3$^-$	210 ± 40	0.55
4.35 ± 0.10	2.79	4$^-$	550 ± 100	0.5





Proposal: to study energy evolution of correlations across broad g.s. of ^{15}Ne to extract ^{14}F width

Mini-conclusion

There is expectation of transition dynamics to be widespread in the poorly explored proton-rich s-d shell systems beyond the dripline

Three principal parameters:

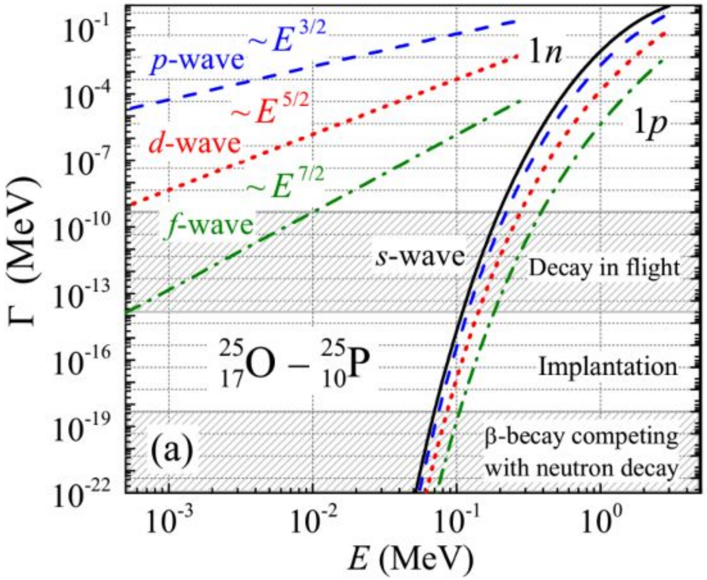
$E_T \;\; E_r \;\; \Gamma_r$

It is expected for “phase transition” situation that there is sharp change in observables depending on exact values of the principal parameters

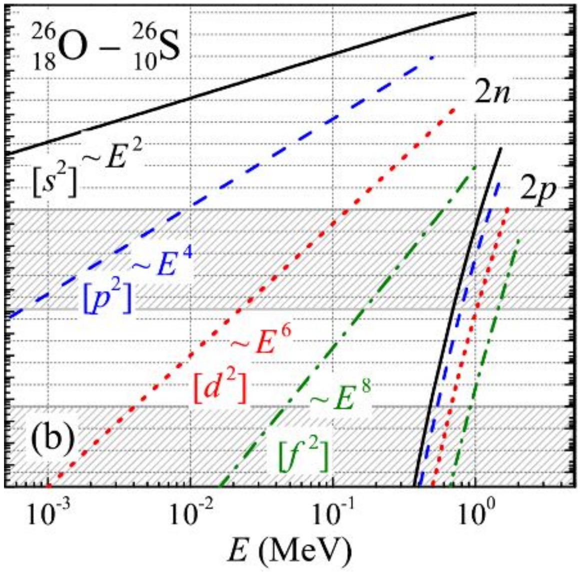
Proposal: new method for two-body width determination from three-body correlation data

What about neutron radioactivity?

L.V. Grigorenko, I.G. Mukha, C. Scheidenberger, and M.V. Zhukov, PRC **84** (2011) 021303(R)



Since we realized that three-body "virtual state" is likely to form narrow peak…



- Two-proton radioactivity is the long awaited and the most recently found mode of the radioactive decay. Can neutron radioactivity exist?
- Estimates: **one-neutron** radioactivity is highly unlikely.
- There are additional effective few-body "centrifugal" barriers making few-body emission relatively slower.
- Long-living **Two-neutron** decay states are reasonably probable.

Collective tunneling in the hyperspherical harmonics method

- Tunneling via collective (hyperspherical) barrier for A particles

$$\left[\frac{d^2}{d\rho^2} - \frac{\mathcal{L}(\mathcal{L}+1)}{\rho^2} + 2ME\right]\chi_K^{(+)}(\rho) = f_A(\rho)$$

$$\mathcal{L} = K + (3A-6)/2$$

For 2n emission the minimal effective HH momentum is 3/2 for minimal K=0

Lowest K=0 may be dynamically supressed due to Pauli principle

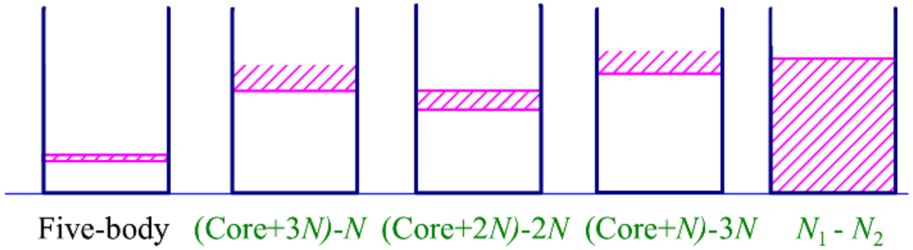
For 4n emission the minimal effective HH momentum is 15/2 for minimal K=2

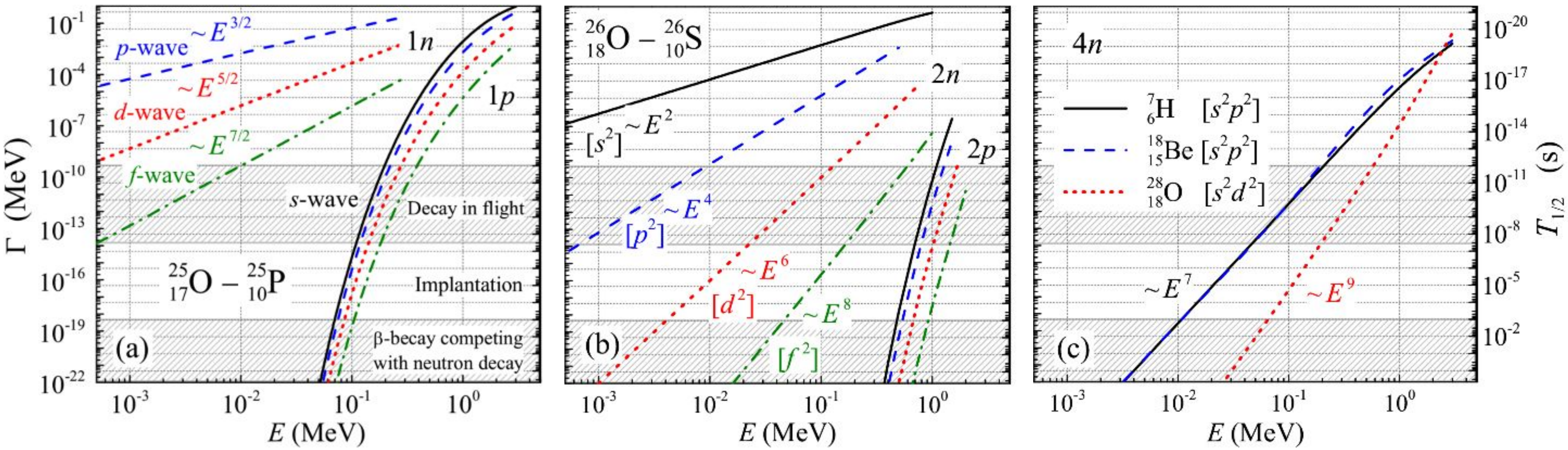
K=0 is strictly prohibited due to Pauli principle

Two- (and more)-neutron radioactivity search prospects

L.V. Grigorenko, I.G. Mukha, C. Scheidenberger, and M.V. Zhukov, PRC **84** (2011) 021303(R)

Energy conditions for true 4n decay





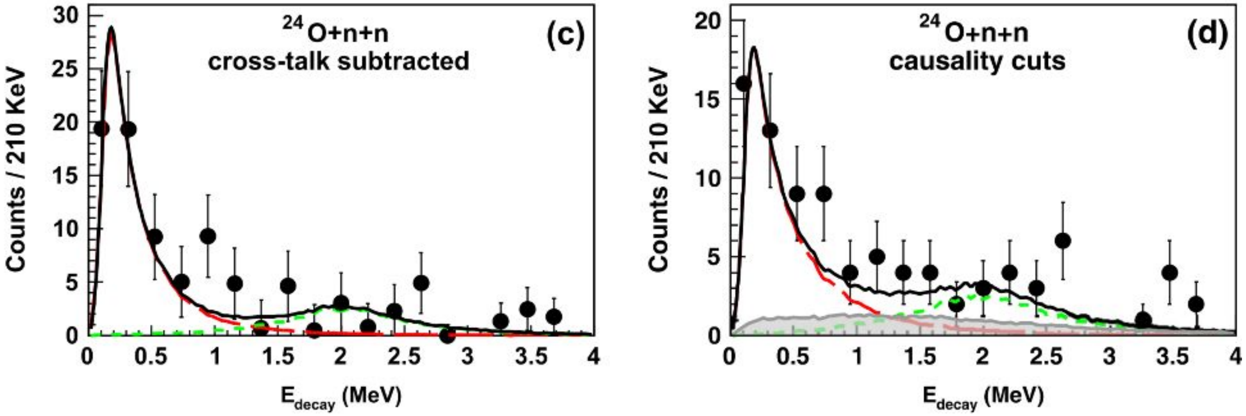
Long-living true four-neutron decay states are most probable.

Nearest candidates for 4n radioactive decay: ^{7}H, ^{18}Be, ^{28}O

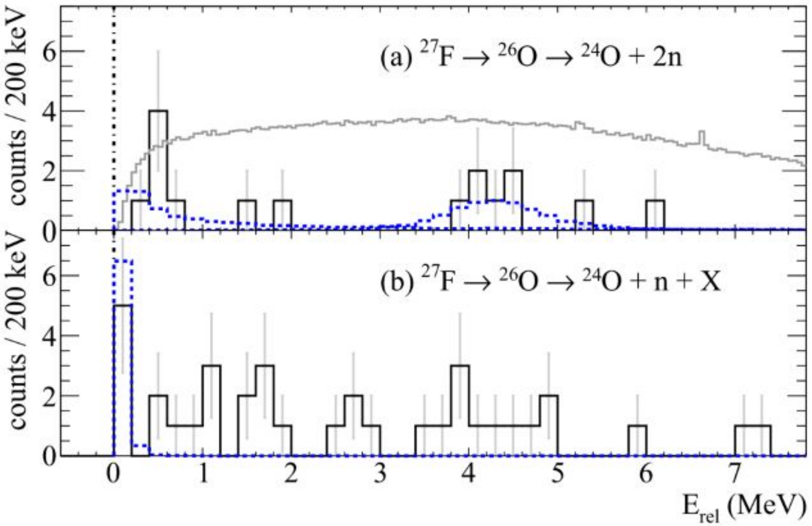
Quest for 2n radiactivity in ^{26}O

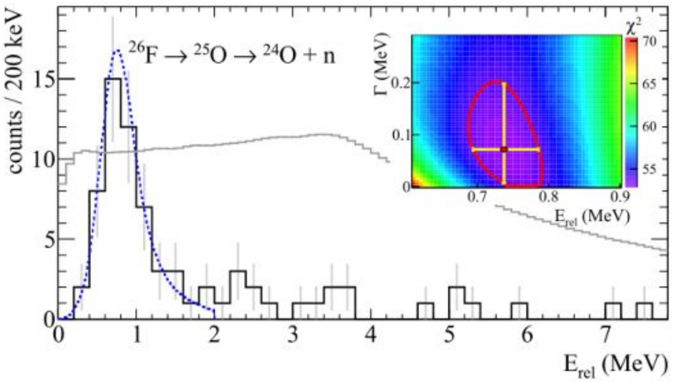
Progress of ^{26}O studies

^{26}O g.s. is somewhere quite low…



MSU: E. Lunderberg et al., PRL 108, 142503 (2012)





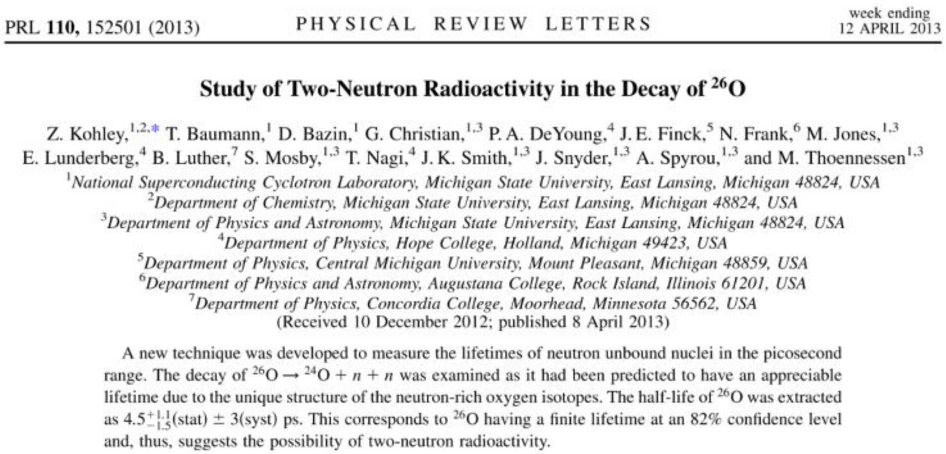
GSI: C. Caesar et al., PRC 88, 034313 (2013)

^{26}O g.s. is really at very low energy $Q_{2n} < 140$ keV

2*n* radioactivity is not impossible!

2n radioactivity in ^{26}O ?

L.V. Grigorenko, I.G. Mukha, M.V. Zhukov, PRL **111** (2013) 042501



PRL **110,** 152501 (2013) PHYSICAL REVIEW LETTERS week ending 12 APRIL 2013

Study of Two-Neutron Radioactivity in the Decay of ^{26}O

Z. Kohley,[1,2,*] T. Baumann,[1] D. Bazin,[1] G. Christian,[1,3] P. A. DeYoung,[4] J. E. Finck,[5] N. Frank,[6] M. Jones,[1,3] E. Lunderberg,[4] B. Luther,[7] S. Mosby,[1,3] T. Nagi,[4] J. K. Smith,[1,3] J. Snyder,[1,3] A. Spyrou,[1,3] and M. Thoennessen[1,3]

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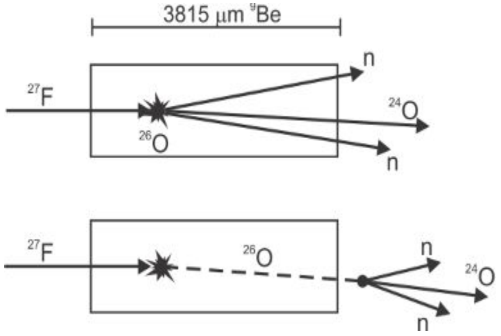
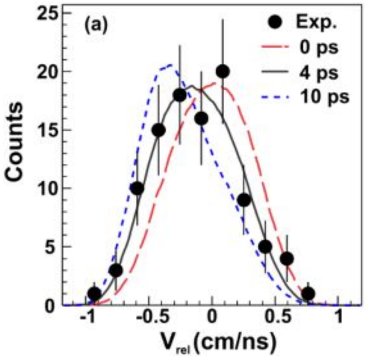
[2]*Department of Chemistry, Michigan State University, East Lansing, Michigan 48824, USA*
[3]*Department of Physics and Astronomy, Michigan State University, East Lansing, Michigan 48824, USA*
[4]*Department of Physics, Hope College, Holland, Michigan 49423, USA*
[5]*Department of Physics, Central Michigan University, Mount Pleasant, Michigan 48859, USA*
[6]*Department of Physics and Astronomy, Augustana College, Rock Island, Illinois 61201, USA*
[7]*Department of Physics, Concordia College, Moorhead, Minnesota 56562, USA*

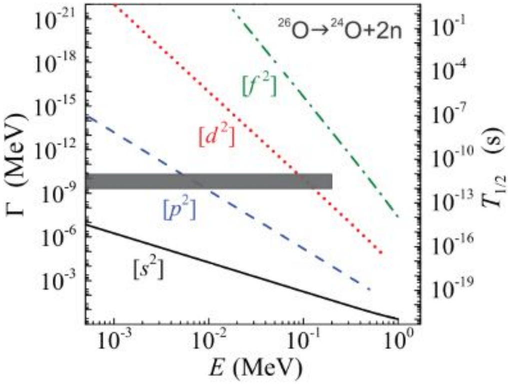
(Received 10 December 2012; published 8 April 2013)

A new technique was developed to measure the lifetimes of neutron unbound nuclei in the picosecond range. The decay of $^{26}O \rightarrow {}^{24}O + n + n$ was examined as it had been predicted to have an appreciable lifetime due to the unique structure of the neutron-rich oxygen isotopes. The half-life of ^{26}O was extracted as $4.5^{+1.1}_{-1.5}$(stat) ± 3(syst) ps. This corresponds to ^{26}O having a finite lifetime at an 82% confidence level and, thus, suggests the possibility of two-neutron radioactivity.

$T_{1/2}$ = 4.5 ps:
2*n* radioactivity discovered?



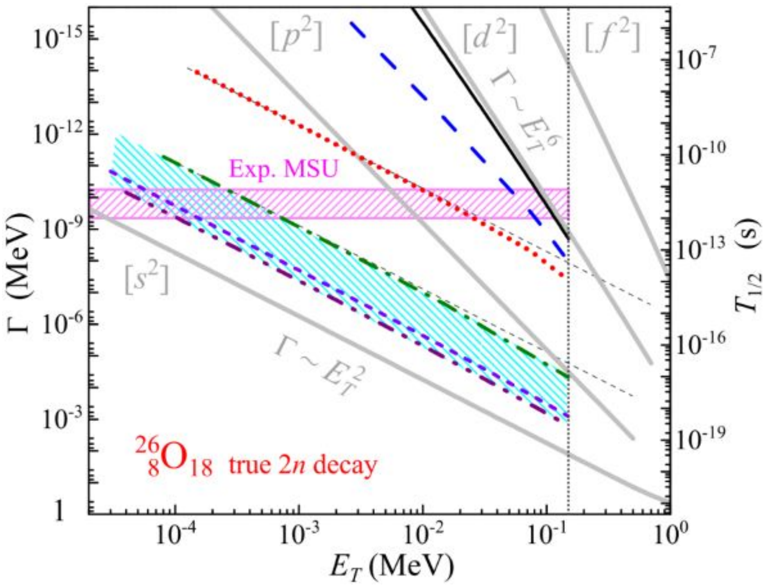




Importance of fine three-body effects

2p radioactivity:
Core recoil – negligible
Paring - factor 200-500

2n radioactivity:
Core recoil – factor 5-10
Paring - factor 2000-10000

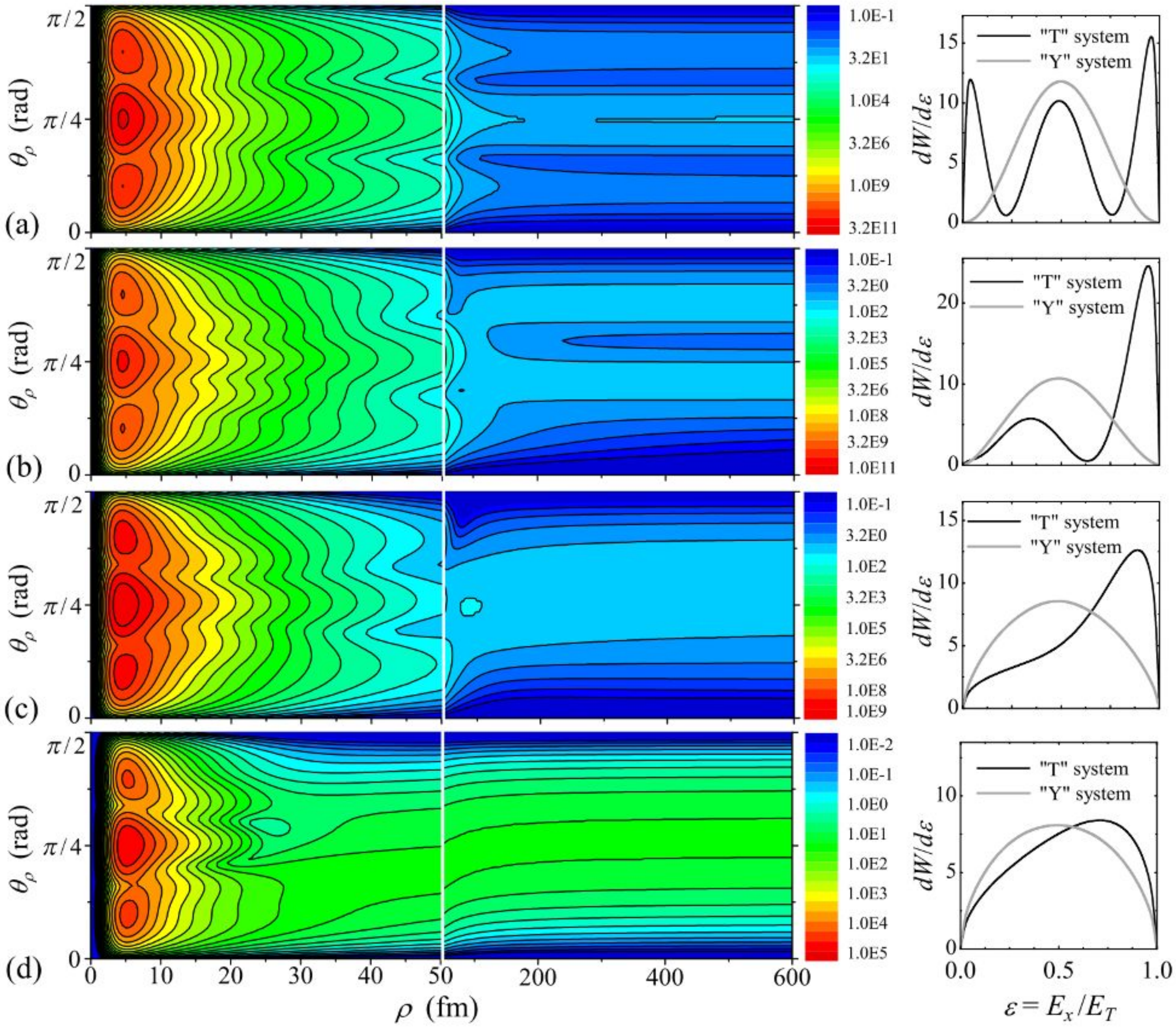


Extreme low-energy decay of ^{26}O should be inferred

From simplistic to precise studies of 2n decay in ^{26}O

L.V. Grigorenko, I.G. Mukha, M.V. Zhukov, PRL **111** (2013) 042501

Subbarrier tunneling to low-l configurations



Independent particle model

Core recoil is important in 2n (not in 2p)

“mini” NN FSI smooth correlations

full NN FSI wash out correlations

2n radioactivity in ^{26}O ?

PRL **116,** 102503 (2016) PHYSICAL REVIEW LETTERS week ending 11 MARCH 2016

Nucleus ^{26}O: A Barely Unbound System beyond the Drip Line

Y. Kondo,[1] T. Nakamura,[1] R. Tanaka,[1] R. Minakata,[1] S. Ogoshi,[1] N. A. Orr,[2] N. L. Achouri,[2] T. Aumann,[3,4] H. Baba,[5] F. Delaunay,[2] P. Doornenbal,[5] N. Fukuda,[5] J. Gibelin,[2] J. W. Hwang,[6] N. Inabe,[5] T. Isobe,[5] D. Kameda,[5] D. Kanno,[1] S. Kim,[6] N. Kobayashi,[1] T. Kobayashi,[7] T. Kubo,[5] S. Leblond,[2] J. Lee,[5] F. M. Marqués,[2] T. Motobayashi,[5] D. Murai,[8] T. Murakami,[9] K. Muto,[7] T. Nakashima,[1] N. Nakatsuka,[9] A. Navin,[10] S. Nishi,[1] H. Otsu,[5] H. Sato,[5] Y. Satou,[6] Y. Shimizu,[5] H. Suzuki,[5] K. Takahashi,[7] H. Takeda,[5] S. Takeuchi,[5] Y. Togano,[4,1] A. G. Tuff,[11] M. Vandebrouck,[12] and K. Yoneda[5]

[1]*Department of Physics, Tokyo Institute of Technology, 2-12-1 O-Okayama, Meguro, Tokyo 152-8551, Japan*
[2]*LPC Caen, ENSICAEN, Université de Caen, CNRS/IN2P3, F-14050 Caen, France*
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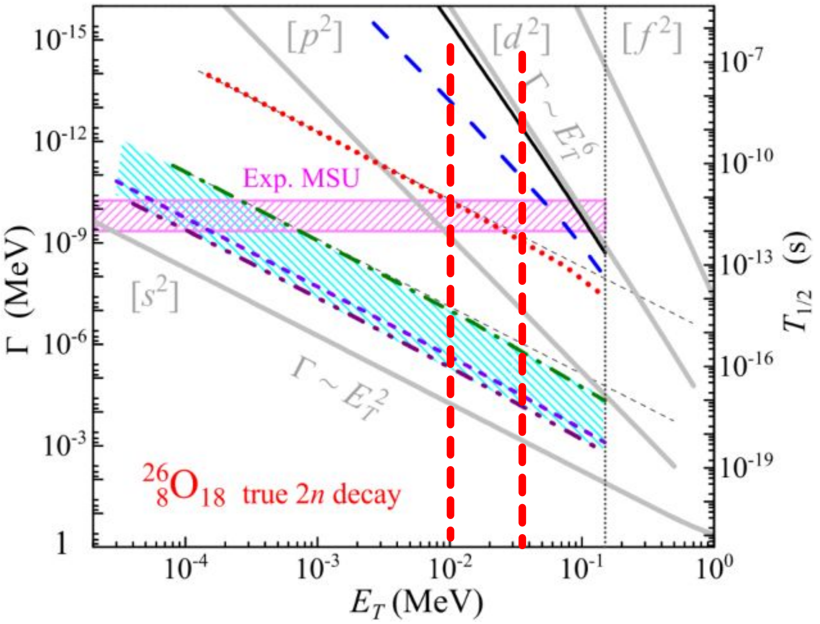
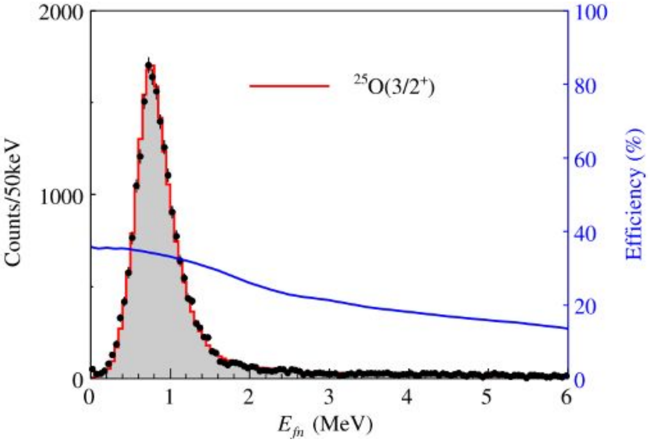
[8]*Department of Physics, Rikkyo University, Toshima, Tokyo 171-8501, Japan*
[9]*Department of Physics, Kyoto University, Kyoto 606-8502, Japan*
[10]*Grand Accélérateur National d'Ions Lourds (GANIL), CEA/DRF-CNRS/IN2P3, Bvd Henri Becquerel, 14076 Caen, France*
[11]*Department of Physics, University of York, Heslington, York YO10 5DD, United Kingdom*
[12]*Institut de Physique Nucléaire, Université Paris-Sud, IN2P3-CNRS, Université de Paris Sud, F-91406 Orsay, France*

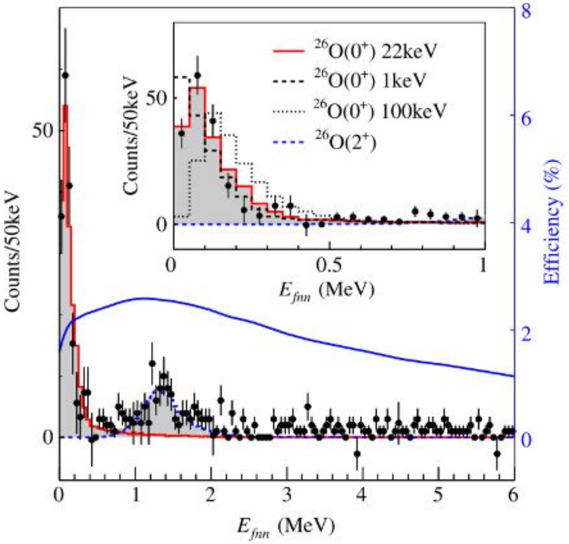
(Received 27 August 2015; published 9 March 2016)

The unbound nucleus ^{26}O has been investigated using invariant-mass spectroscopy following one-proton removal reaction from a ^{27}F beam at 201 MeV/nucleon. The decay products, ^{24}O and two neutrons, were detected in coincidence using the newly commissioned SAMURAI spectrometer at the RIKEN Radioactive Isotope Beam Factory. The ^{26}O ground-state resonance was found to lie only $18 \pm 3(\mathrm{stat}) \pm 4(\mathrm{syst})$ keV above threshold. In addition, a higher lying level, which is most likely the first 2^+ state, was observed for the first time at $1.28^{+0.11}_{-0.08}$ MeV above threshold. Comparison with theoretical predictions suggests that three-nucleon forces, pf-shell intruder configurations, and the continuum are key elements to understanding the structure of the most neutron-rich oxygen isotopes beyond the drip line.



Inconsistency between 3 results





Very precise experiemnent, but conclusion id MC-based

Q_{2n} = 22 keV:
2*n* radioactivity disproved?

Mini-conclusion

Quest for 2n radiactivity in ^{26}O is still opened

One out of three existing important results is wrong:

- Poor lifetime measurements (MSU)

- Overestimated invariant mass precision (RIKEN)

- Theoreticians do not well understand 2n penetration process (not impossible)

Quest for 4n radiactivity in ^{7}H

^{7}H studied in the ^{2}H(^{8}He, ^{3}He)^{7}H reaction

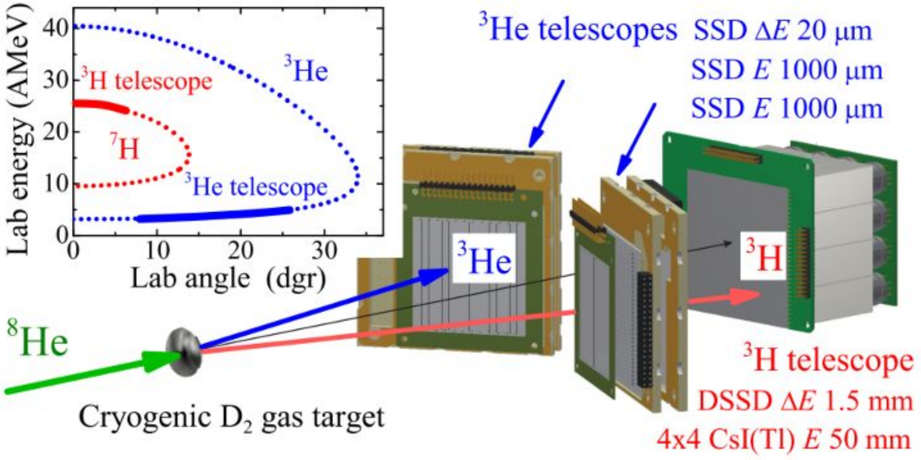
“Flagship“ experiment of ACCULINNA-2 facility

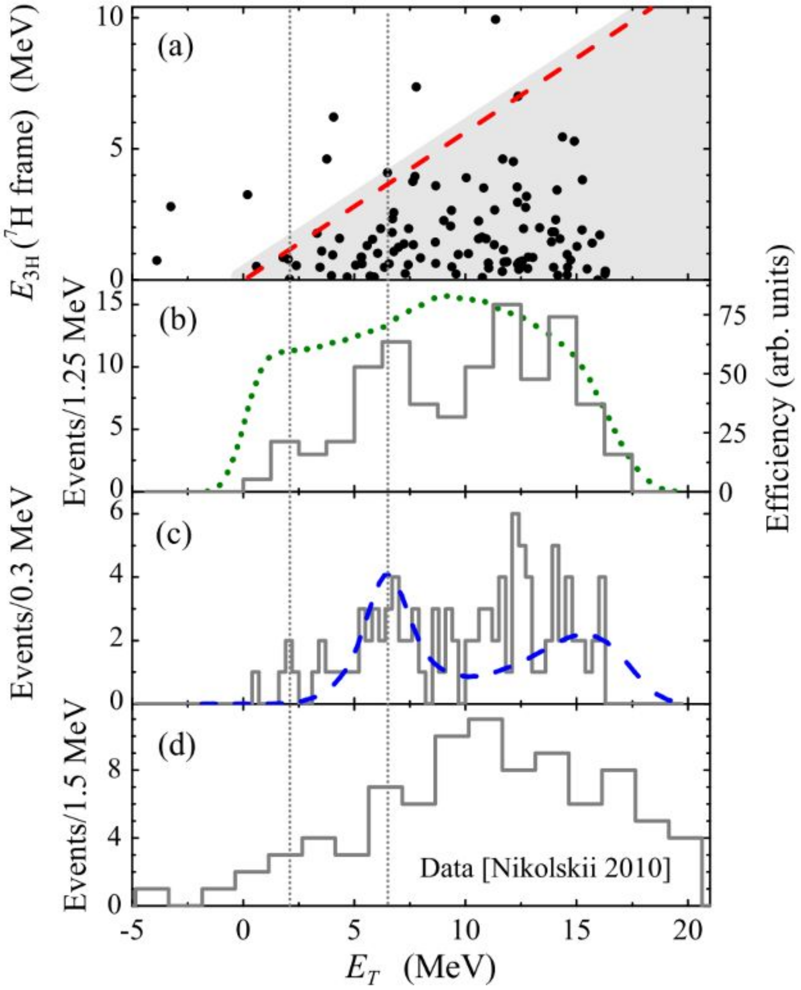
^{8}He beam 26 AMeV, 10^5 pps
2018, two weeks

PHYSICAL REVIEW LETTERS **124,** 022502 (2020)

Evidence for the First Excited State of ^{7}H

A. A. Bezbakh,[1,2] V. Chudoba,[1,2,*] S. A. Krupko,[1,3] S. G. Belogurov,[1,4] D. Biare,[1] A. S. Fomichev,[1,5] E. M. Gazeeva,[1] A. V. Gorshkov,[1] L. V. Grigorenko,[1,4,6] G. Kaminski,[1,7] O. A. Kiselev,[8] D. A. Kostyleva,[8,9] M. Yu. Kozlov,[10] B. Mauyey,[1,11] I. Mukha,[8] I. A. Muzalevskii,[1,2] E. Yu. Nikolskii,[6,1] Yu. L. Parfenova,[1] W. Piatek,[1,7] A. M. Quynh,[1,12] V. N. Schetinin,[10] A. Serikov,[1] S. I. Sidorchuk,[1] P. G. Sharov,[1,2] R. S. Slepnev,[1] S. V. Stepantsov,[1] A. Swiercz,[1,13] P. Szymkiewicz,[1,13] G. M. Ter-Akopian,[1,5] R. Wolski,[1,14] B. Zalewski,[1,7] and M. V. Zhukov[15]





- Excited state at 6.5 MeV
- Indication of ground state at 1.8 MeV
- May be something at 12 MeV

^{7}H studied in the ^{2}H(^{8}He, ^{3}He)^{7}H reaction. Second run.

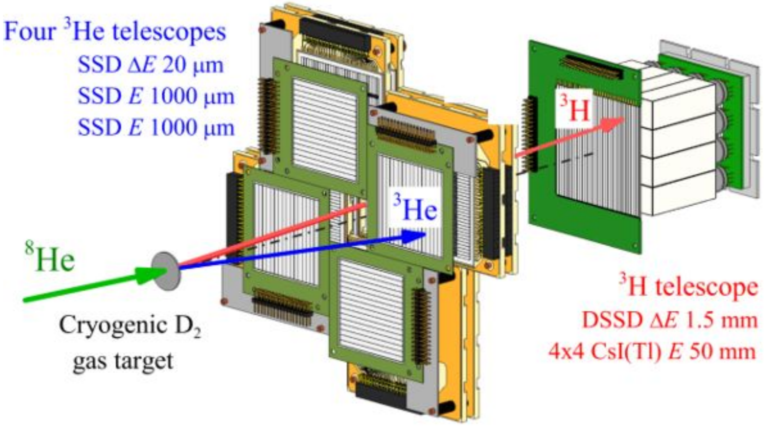
PHYSICAL REVIEW C **103**, 044313 (2021)

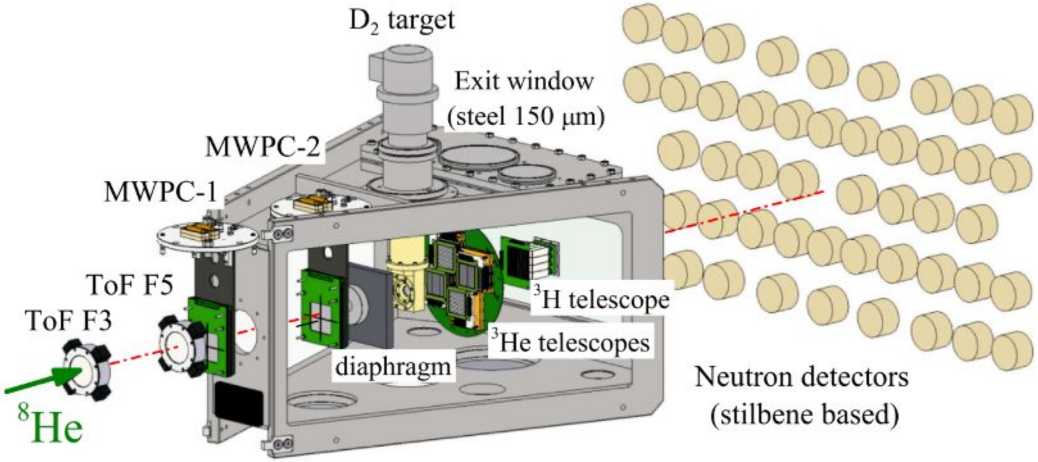
Resonant states in ^{7}H: Experimental studies of the ^{2}H(^{8}He, ^{3}He) reaction

I. A. Muzalevskii,[1,2,*] A. A. Bezbakh,[1,2] E. Yu. Nikolskii,[3,1] V. Chudoba,[1,2] S. A. Krupko,[1] S. G. Belogurov,[1,4] D. Biare,[1] A. S. Fomichev,[1,5] E. M. Gazeeva,[1] A. V. Gorshkov,[1] L. V. Grigorenko,[1,4,3] G. Kaminski,[1,6] O. Kiselev,[7] D. A. Kostyleva,[7,8] M. Yu. Kozlov,[9] B. Mauyey,[1,10] I. Mukha,[7] Yu. L. Parfenova,[1] W. Piatek,[1,6] A. M. Quynh,[1,11] V. N. Schetinin,[9] A. Serikov,[1] S. I. Sidorchuk,[1] P. G. Sharov,[1,2] N. B. Shulgina,[3,12] R. S. Slepnev,[1] S. V. Stepantsov,[1] A. Swiercz,[1,13] P. Szymkiewicz,[1,13] G. M. Ter-Akopian,[1,5] R. Wolski,[1,14] B. Zalewski,[1,6] and M. V. Zhukov[15]

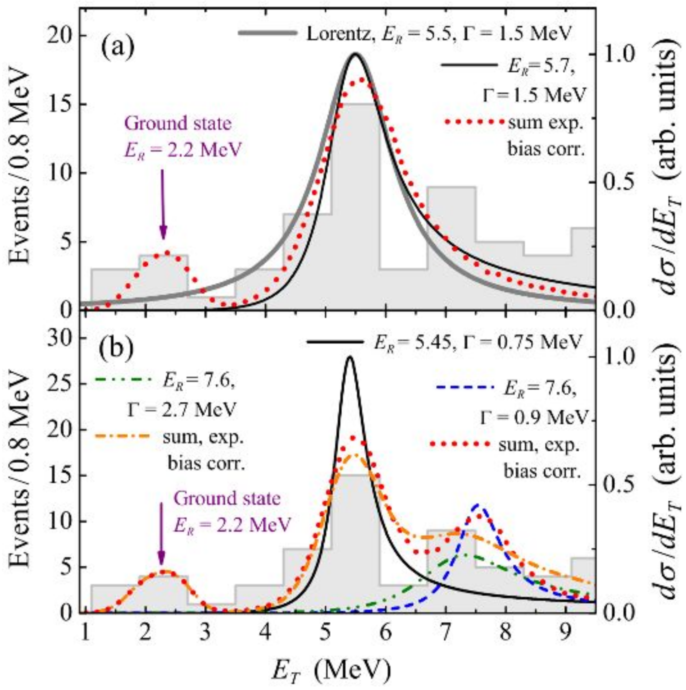
^{8}He beam 26 AMeV, 10^{5} pps
2019, 3 weeks

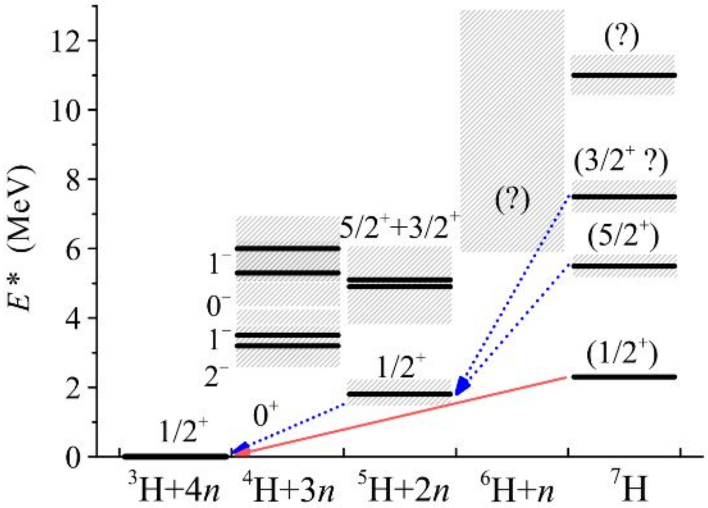
“Comming out party” for the neutron wall

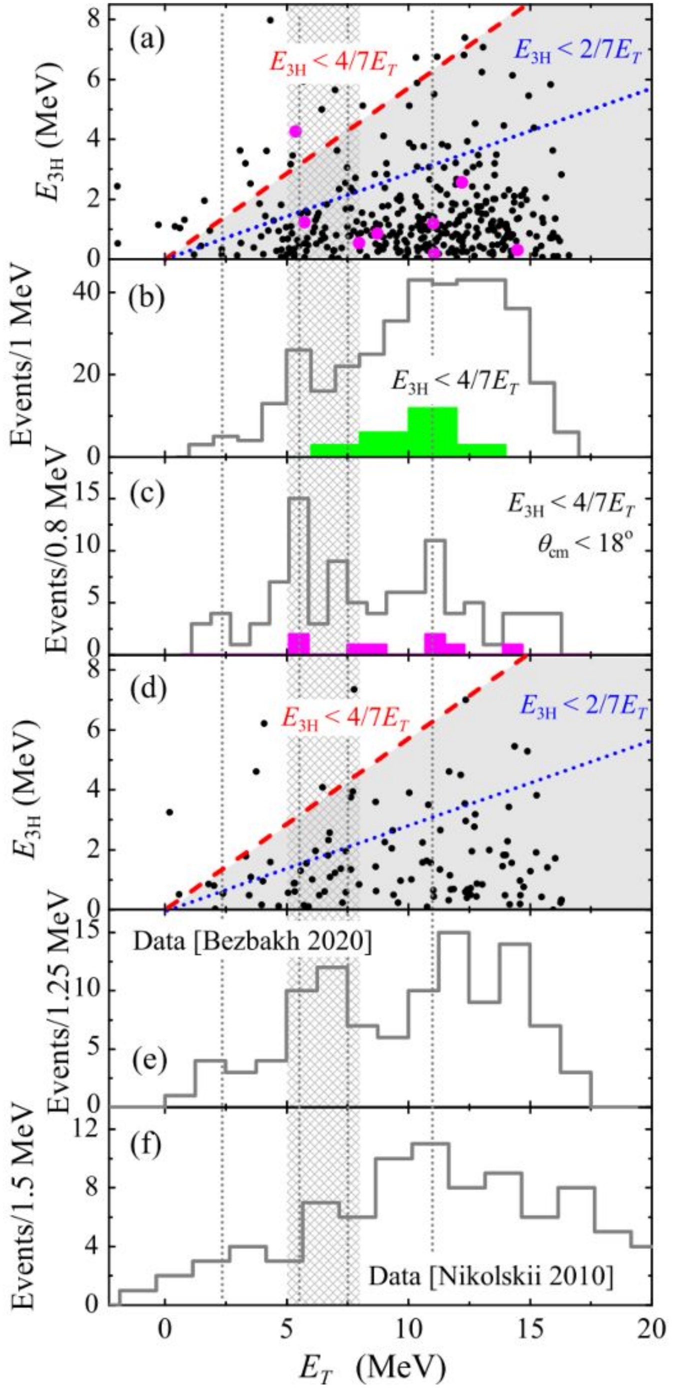




^{7}H data and spectrum



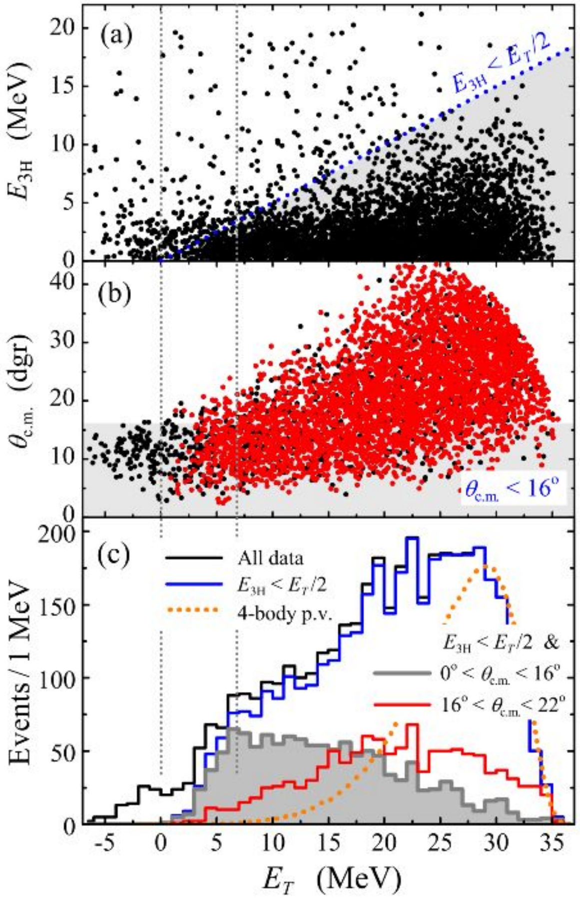




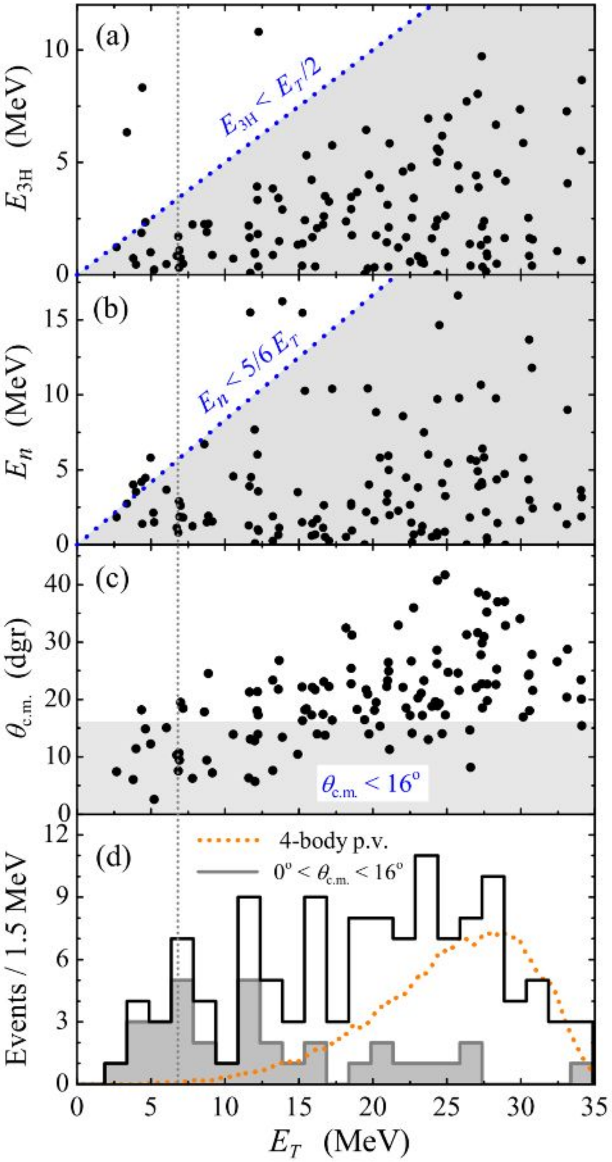
Data on ^{6}H from ^{2}H(^{8}He,^{4}He)^{6}H

Nikolskii *et al.*,
PRC 105, 064605
(2022)

Double coincidences ^{4}He-^{3}H



Triple coincidences ^{4}He-^{3}H-n

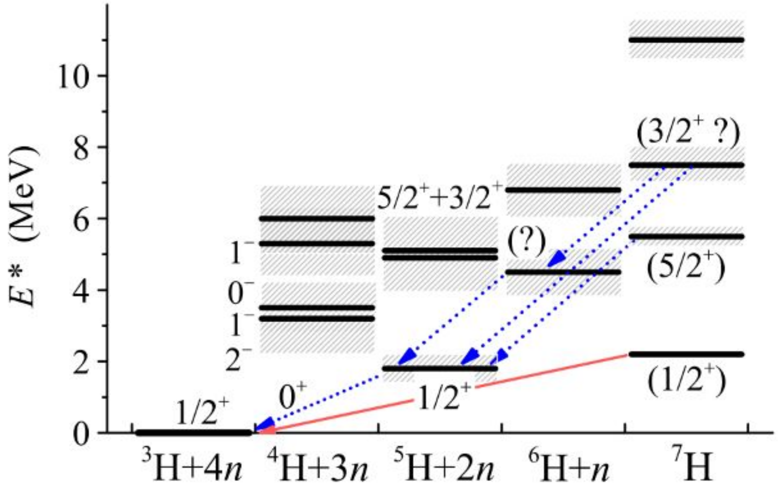


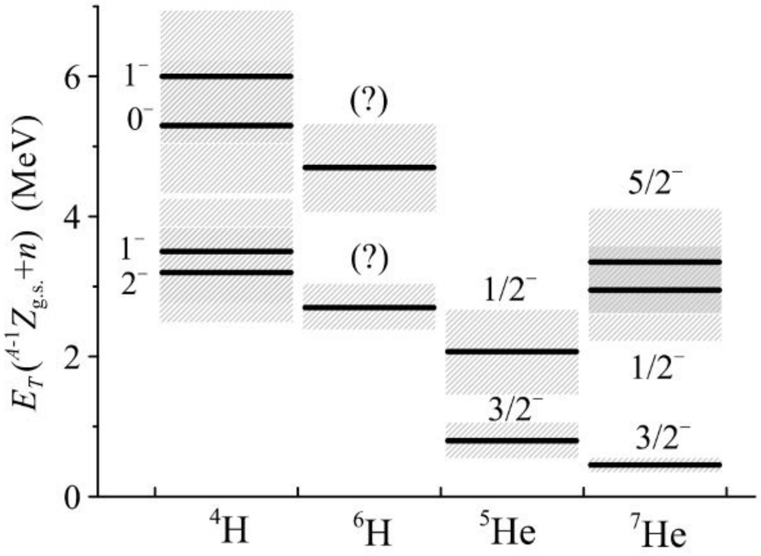
- Setup is not specially suited for this experiment
- Higher cross section and high statistics (factor 10)
- Large backgrounds (accidental alphas)
- Neutron coincidence data

Superheavy hydrogens

Excitation spectra relative to ^{3}H ground state

Analogies in the excitation spectra relative ^{3}H and ^{5}H, ^{4}He and ^{6}He ground states





For the first time the 3-neutron decay was studied with at least some details

The true 5-body decay (true 4n emission) was observed for the first time

Open questions for ^{6}H and ^{7}H

Predictions: the ground state at 7-9 MeV, not at 2 MeV

No good systematic understanding of such systems (4 orbiting neutrons)

Poor convergence of the calculations

Cross section for ^{6}H population

FRESCO: 200 μb/sr at 5-15 deg

Experiment: 190 μb/sr at 5-15 deg

Cross section for ^{7}H population

FRESCO: 5 mb/sr at 5-10 deg

Experiment: 24 μb/sr at 5-10 deg

Mini-conclusion

The properties of ^{6}H and ^{7}H are studied at incredible level of precision

However, there is no understanding of the overall situation

«Decisive» experiments are still due for these nuclides

At ACCULINNA-2 after U-400M upgrade we have a good prerequivisites to «complete the mission»

Quest for 4n «nucleus»

Experiment on ^{4}n

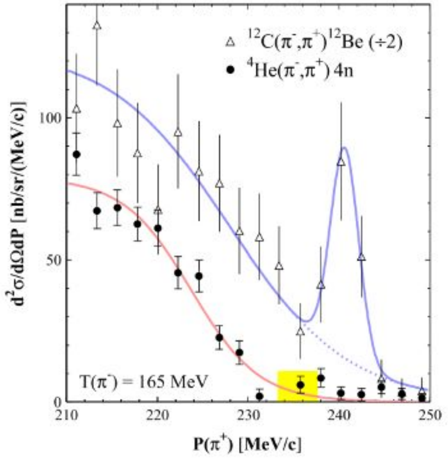


Fig. 3 Experimental results for the reactions ^{4}He(π^-,π^+)4n at $\theta = 0°$ (circles) and ^{12}C(π^-,π^+)^{12}Be at $\theta = 8°$ (triangles, divided by 2). The curves are fits to guide the eye, with a Woods-Saxon distribution only (red) plus an additional Gaussian function (blue). The peak in the ^{12}C channel corresponds to the formation of the ^{12}Be ground and two first excited states, and the range in yellow in the ^{4}He channel to the region expected for a bound tetraneutron. Adapted from Ref. [26]

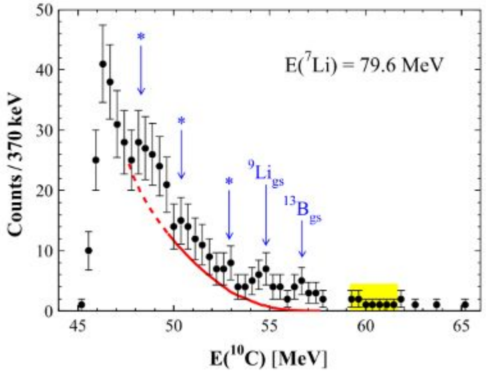


Fig. 5 Energy spectrum of ^{10}C from the ^{7}Li(^{7}Li,^{10}C)4n reaction at $\theta = 7.4°$. Known contaminant reactions are indicated either explicitly or with an asterisk. The red curve corresponds to five-body phase space, and the range in yellow to the region expected for a bound tetraneutron. Adapted from Ref. [44]

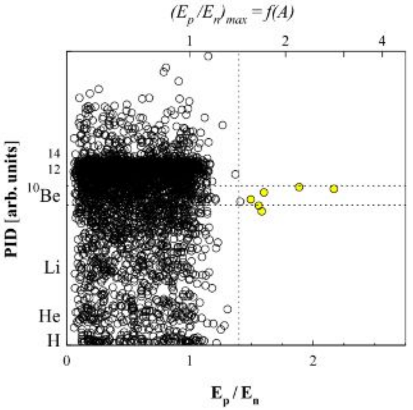


Fig. 6 Scatter plot of the particle identification parameter PID vs the proton recoil in the neutron detector (normalized to the neutron energy) for the reaction (^{14}Be, $X+n$). The dotted lines show the region centered on the ^{10}Be peak and with $E_p/E_n > 1.4$, and the 6 events in yellow are candidates to the formation of a bound tetraneutron. The scale on the upper axis shows the maximum proton recoil as a function of the multineutron mass number. Adapted from Ref. [3]

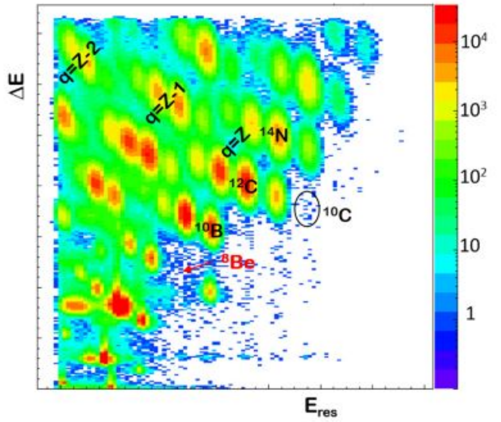


Fig. 1. Identification spectrum, energy loss vs. residual energy, for the run with a central ^{10}C energy of 20.5 MeV selected through the magnetic field. Clusters of ions with two, one and no electrons are denoted. For the completely stripped ions (q=Z) a few N=Z nuclei are labelled as well as the region of the ^{10}C ions.

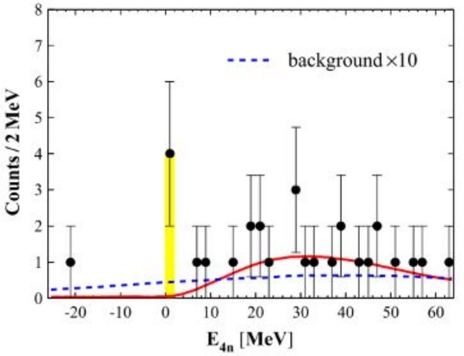
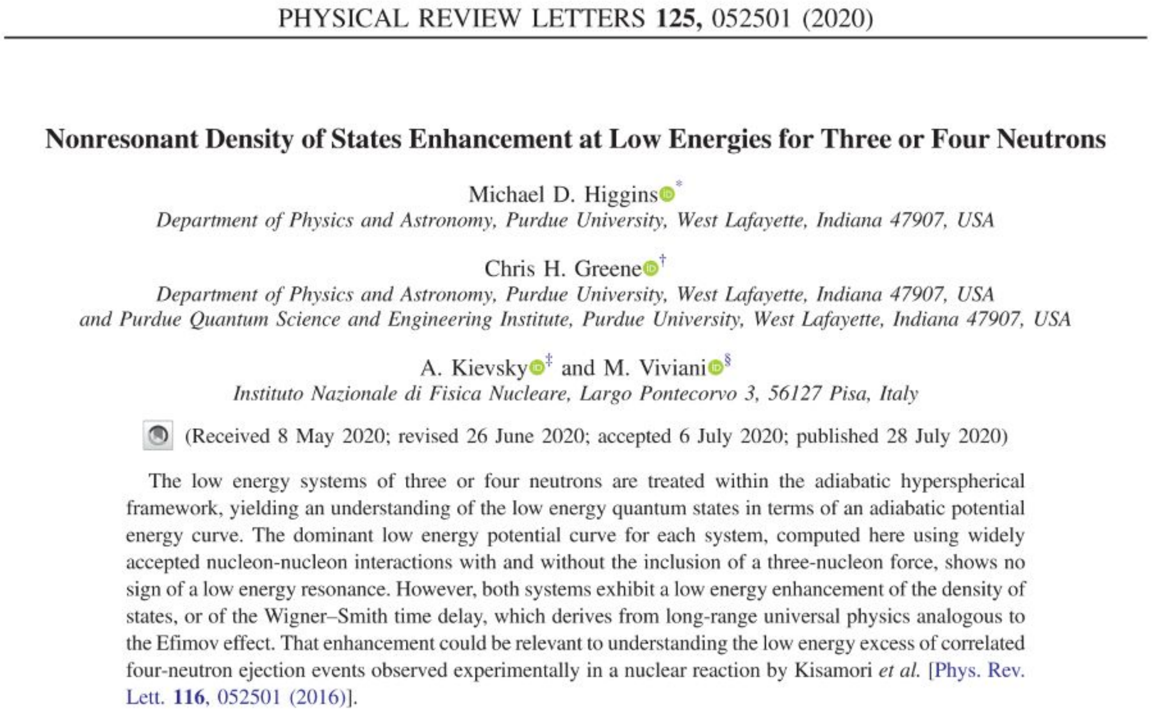


Fig. 7 Missing-mass spectrum of the ^{4}He(^{8}He,^{8}Be)4n reaction. The solid (red) curve represents the sum of the direct decay of correlated 2n pairs plus the estimated background. The dashed (blue) curve represents only the latter, multiplied by a factor of 10 in order to make it visible. The 4 events at threshold are highlighted in yellow. Adapted from Ref. [4]

Recent theory works on ^{4}n



PHYSICAL REVIEW LETTERS **125,** 052501 (2020)

Nonresonant Density of States Enhancement at Low Energies for Three or Four Neutrons

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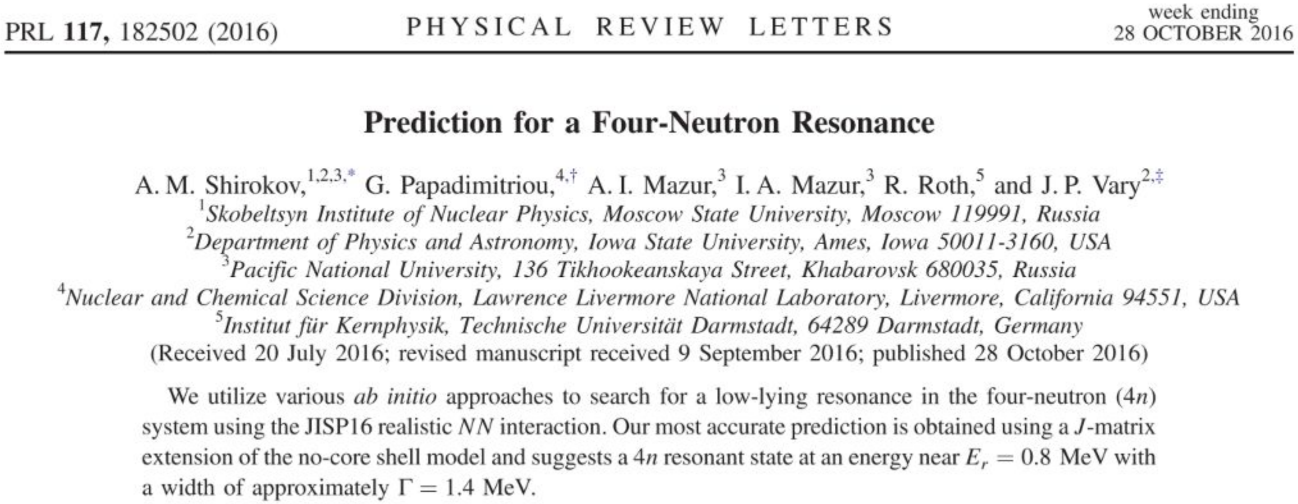
(Received 8 May 2020; revised 26 June 2020; accepted 6 July 2020; published 28 July 2020)

The low energy systems of three or four neutrons are treated within the adiabatic hyperspherical framework, yielding an understanding of the low energy quantum states in terms of an adiabatic potential energy curve. The dominant low energy potential curve for each system, computed here using widely accepted nucleon-nucleon interactions with and without the inclusion of a three-nucleon force, shows no sign of a low energy resonance. However, both systems exhibit a low energy enhancement of the density of states, or of the Wigner–Smith time delay, which derives from long-range universal physics analogous to the Efimov effect. That enhancement could be relevant to understanding the low energy excess of correlated four-neutron ejection events observed experimentally in a nuclear reaction by Kisamori *et al.* [Phys. Rev. Lett. **116**, 052501 (2016)].



PRL **117,** 182502 (2016) PHYSICAL REVIEW LETTERS week ending 28 OCTOBER 2016

Prediction for a Four-Neutron Resonance

A. M. Shirokov,[1,2,3,*] G. Papadimitriou,[4,†] A. I. Mazur,[3] I. A. Mazur,[3] R. Roth,[5] and J. P. Vary[2,‡]
[1]*Skobeltsyn Institute of Nuclear Physics, Moscow State University, Moscow 119991, Russia*
[2]*Department of Physics and Astronomy, Iowa State University, Ames, Iowa 50011-3160, USA*
[3]*Pacific National University, 136 Tikhookeanskaya Street, Khabarovsk 680035, Russia*
[4]*Nuclear and Chemical Science Division, Lawrence Livermore National Laboratory, Livermore, California 94551, USA*
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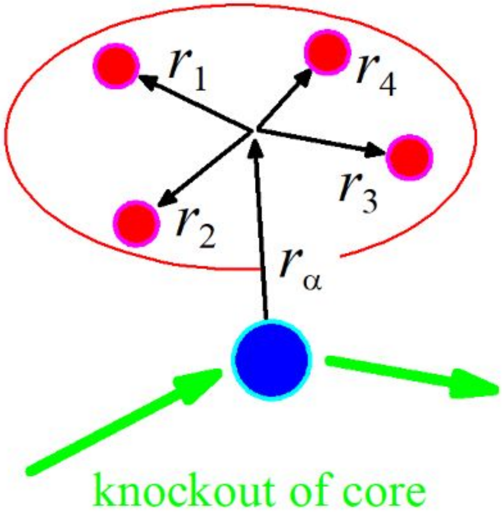
We utilize various *ab initio* approaches to search for a low-lying resonance in the four-neutron ($4n$) system using the JISP16 realistic NN interaction. Our most accurate prediction is obtained using a J-matrix extension of the no-core shell model and suggests a $4n$ resonant state at an energy near $E_r = 0.8$ MeV with a width of approximately $\Gamma = 1.4$ MeV.

How to get real ^{4}n?



Eur. Phys. J. A **19**, 187–201 (2004)
DOI 10.1140/epja/i2003-10124-1

THE EUROPEAN PHYSICAL JOURNAL A

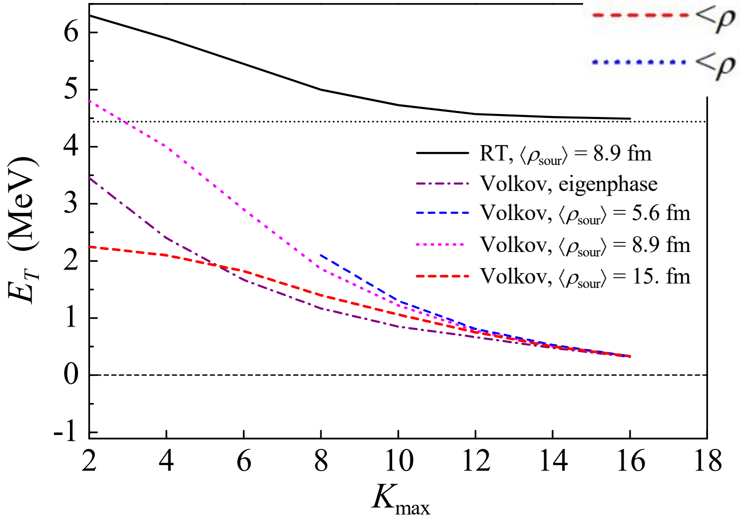
Broad states beyond the neutron drip line

Examples of ^{5}H and ^{4}n

L.V. Grigorenko[1,2,a], N.K. Timofeyuk[3], and M.V. Zhukov[4]

$$(H_4 - E_T)\,\Psi^{(+)} = \Phi$$

$$\Phi = \int d\Omega_q Y_{00}(q) \int d^3 r_\alpha \, e^{iqr_\alpha} \left\langle \alpha \middle| {}^8\mathrm{He} \right\rangle$$

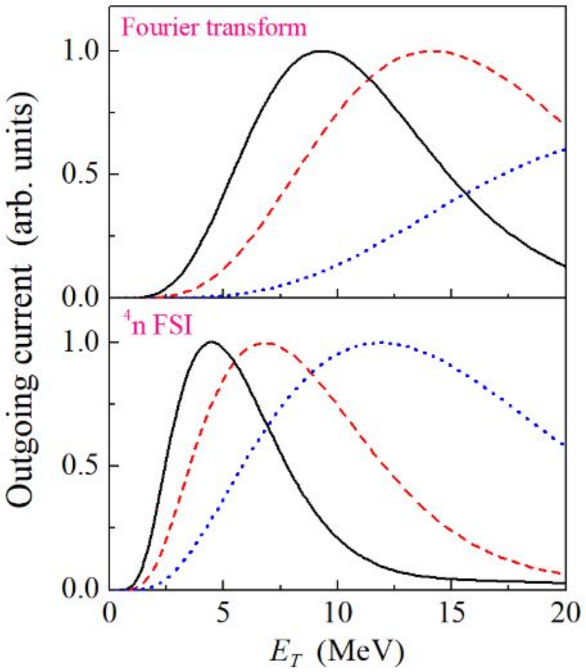


RMS size of the source

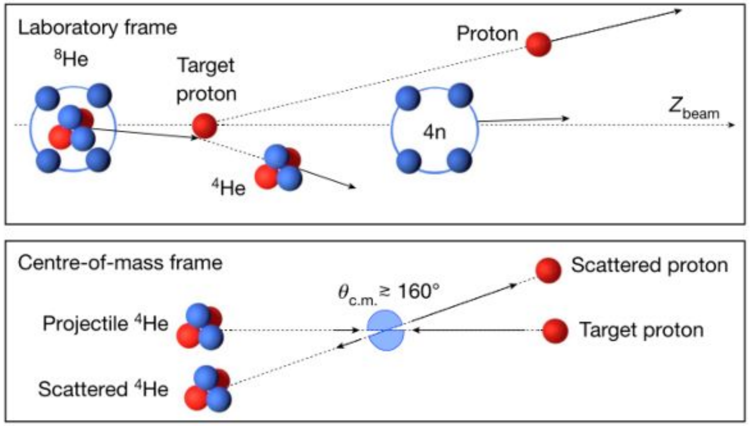
—— $<\rho> = 8.9$ fm

- - - $<\rho> = 7.3$ fm

······ $<\rho> = 5.6$ fm



Experiment ^{4}n



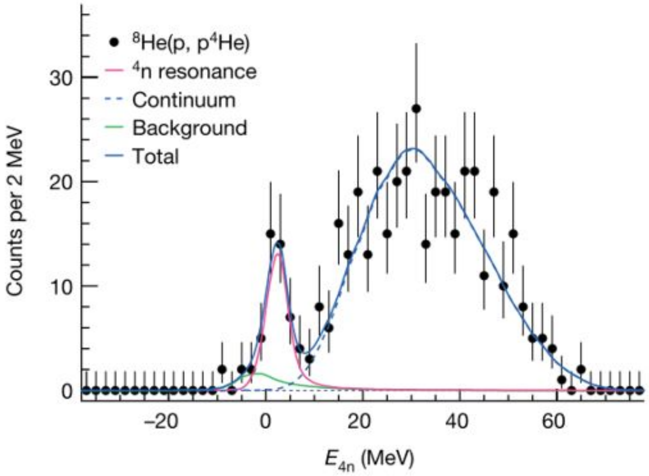
Article

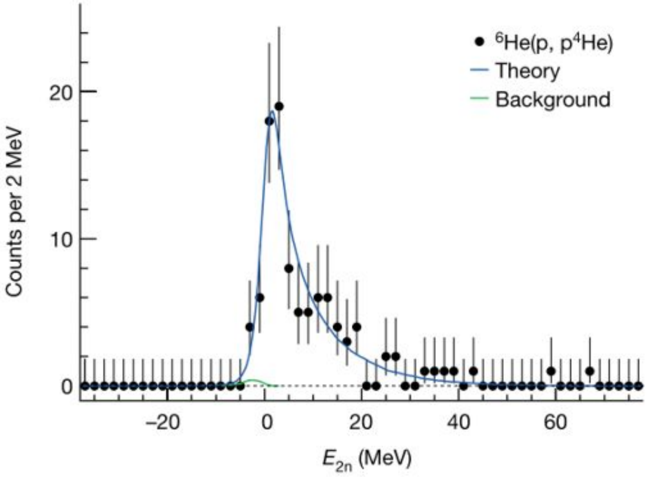
Observation of a correlated free four-neutron system

https://doi.org/10.1038/s41586-022-04827-6
Received: 4 August 2021
Accepted: 28 April 2022

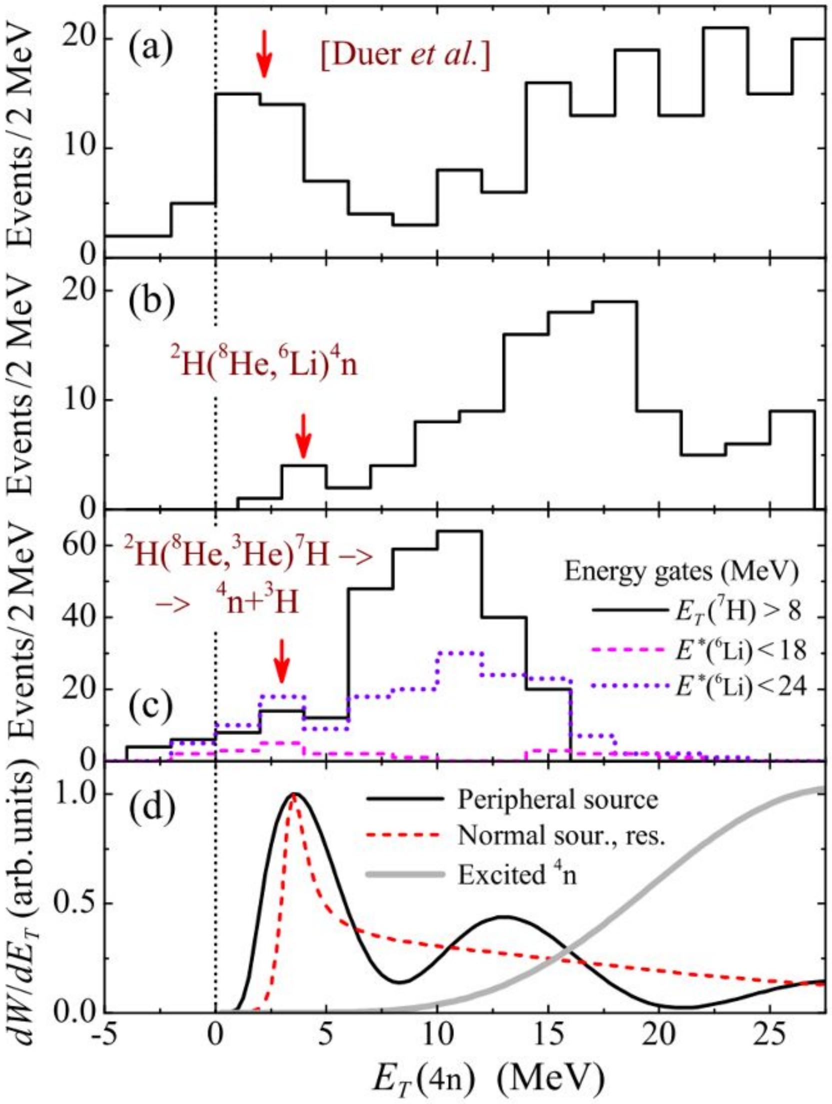
M. Duer[1✉], T. Aumann[1,2,3], R. Gernhäuser[4], V. Panin[2,5], S. Paschalis[1,6], D. M. Rossi[1], N. L. Achouri[7], D. Ahn[5,16], H. Baba[5], C. A. Bertulani[8], M. Böhmer[4], K. Boretzky[2], C. Caesar[1,2,5], N. Chiga[5], A. Corsi[9], D. Cortina-Gil[10], C. A. Douma[11], F. Dufter[4], Z. Elekes[12], J. Feng[13], B. Fernández-Domínguez[10], U. Forsberg[6], N. Fukuda[5], I. Gasparic[1,5,14], Z. Ge[5], J. M. Gheller[9], J. Gibelin[7], A. Gillibert[9], K. I. Hahn[15,16], Z. Halász[12], M. N. Harakeh[11], A. Hirayama[17], M. Holl[1], N. Inabe[5], T. Isobe[5], J. Kahlbow[1], N. Kalantar-Nayestanaki[11], D. Kim[16], S. Kim[1,16], T. Kobayashi[18], Y. Kondo[17], D. Körper[2], P. Koseoglou[1], Y. Kubota[5], I. Kuti[12], P. J. Li[19], C. Lehr[1], S. Lindberg[20], Y. Liu[13], F. M. Marqués[7], S. Masuoka[21], M. Matsumoto[17], J. Mayer[22], K. Miki[1,18], B. Monteagudo[7], T. Nakamura[17], T. Nilsson[20], A. Obertelli[1,9], N. A. Orr[7], H. Otsu[5], S. Y. Park[15,16], M. Parlog[7], P. M. Potlog[23], S. Reichert[4], A. Revel[7,9,24], A. T. Saito[17], M. Sasano[5], H. Scheit[1], F. Schindler[1], S. Shimoura[21], H. Simon[2], L. Stuhl[16,21], H. Suzuki[5], D. Symochko[1], H. Takeda[5], J. Tanaka[1,5], Y. Togano[17], T. Tomai[17], H. T. Törnqvist[1,2], J. Tscheuschner[1], T. Uesaka[5], V. Wagner[1], H. Yamada[17], B. Yang[13], L. Yang[21], Z. H. Yang[5], M. Yasuda[17], K. Yoneda[5], L. Zanetti[1], J. Zenihiro[5,25] & M. V. Zhukov[20]

A long-standing question in nuclear physics is whether chargeless nuclear systems can exist. To our knowledge, only neutron stars represent near-pure neutron systems, where neutrons are squeezed together by the gravitational force to very high densities. The experimental search for isolated multi-neutron systems has been an ongoing quest for several decades[1], with a particular focus on the four-neutron system called the tetraneutron, resulting in only a few indications of its existence so far[2–4], leaving the tetraneutron an elusive nuclear system for six decades. Here we report on the observation of a resonance-like structure near threshold in the four-neutron system that is consistent with a quasi-bound tetraneutron state existing for a very short time. The measured energy and width of this state provide a key benchmark for our understanding of the nuclear force. The use of an experimental approach based on a knockout reaction at large momentum transfer with a radioactive high-energy ^{8}He beam was key.





ACCULINNA-2 data for ^{4}n



Conclusion

We reviewed the status of several extreme neutron-rich and proton-rich systems

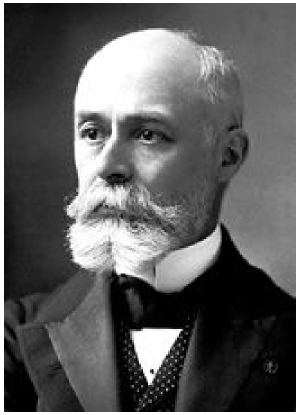
Cases of complicated dynamics: 2p radioactivity, 2n radioactivity, transitional dynamics, 3n, 4n emission, 5-body decay

Recent developments both in theory and in experiment are very important

Nevertheless there are NO rock-solid results in the field

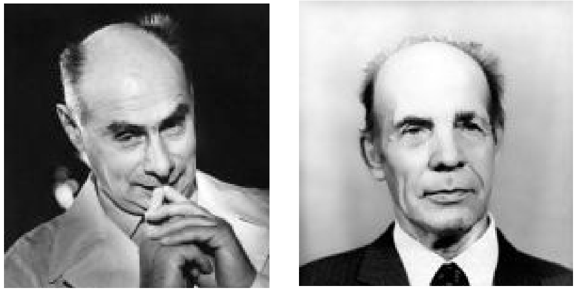
It is often unclear, on which side is the problem – theory or experiment

Radioactivity “hall of fame”



Henri Becquerel: three classes of radioactivity - negative, positive, and electrically neutral

F. Jouliot and I. Curie:
β^+



G.N. Flerov and K.A. Petrzhak
spontaneous fission

V.A. Karnaukhov and
G.M. Ter-Akopian
β-delayed p

S. Hofmann:
p

M. Pfutzner:
2p

?

2n radioactivity

?

4n radioactivity