CORE-COLLAPSE SUPERNOVAE AND NUCLEAR WEAK-INTERACTION REACTIONS

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What is a Supernova ?

Supernovae are one of the most energetic explosive events in Nature.

Energy output:

- $\sim 10^{53}\,\mathrm{erg/sec}$ ($1\ \mathrm{erg} = 10^{-7}$ J) is released.
- 99% is emitted in neutrinos.
- 1% goes to the kinetic energy of the ejecta and radiation.

Fritz Zwicky
1898-1974

Wilhelm Walter Baade
1893-1960

In 1934 Baade and Zwicky proposed that **supernovae** represents the transition to an ordinary star into a neutron star.

Neutron was discovered in 1932 by James Chadwick.

What is a Supernova ?

- $\sim$ 1 SN/sec in the Universe
- $\sim$ 1 SN/day is discovered
- $\sim$ 1 SN/50-100 years in the Milky Way

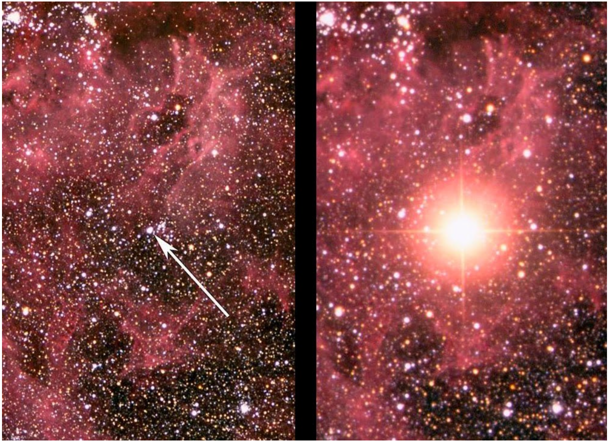
There are two general classes of supernovae:

- ~ 20**% thermonuclear SNe (Type Ia)**
  exploding white dwarfs
- ~ 80**% core-collapse SNe (CCSNe)**
  exploding massive stars ($M > 10M_{\odot}$)

G1.9+0.3
Explosion ~ 140 yrs ago
25000 light-years from Earth

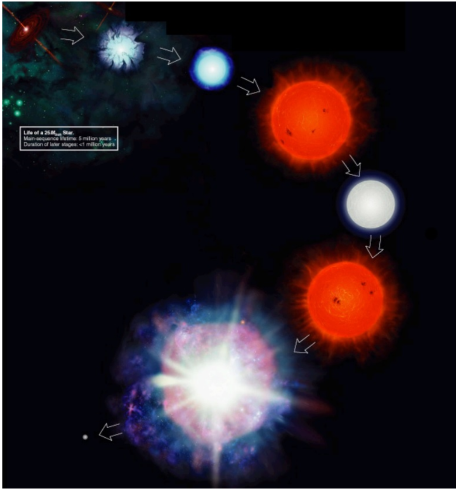
Supernova 1987a

SN1987A in the Large Magellanic Claud ($\sim$ 168000 light years from Earth)



Progenitor – a star of ~ 18 solar masses.

Life Cycle of Massive Stars

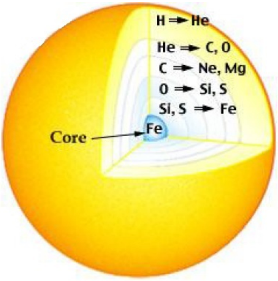


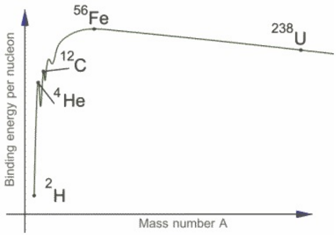
- Protostar
- Main Sequence star: H fuses into He in the core
- Reg Giant: H fuses into He in the shell around He core
- Helium Core Burning: He fuses to C in the core while H fuses to He in then shell
- .....
- Multiple Shell Burning: Many elements fuse is the shells

Heavy-Element Fusion in a 25-$M_\odot$ Star

Fuel	Time	Percentage of Lifetime
H	7,000,000 years	93.3
He	500,000 years	6.7
C	600 years	0.008
O	0.5 years	0.000007
Si	1 day	0.00000004

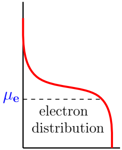
Core-Collapse Supernova

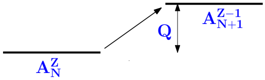




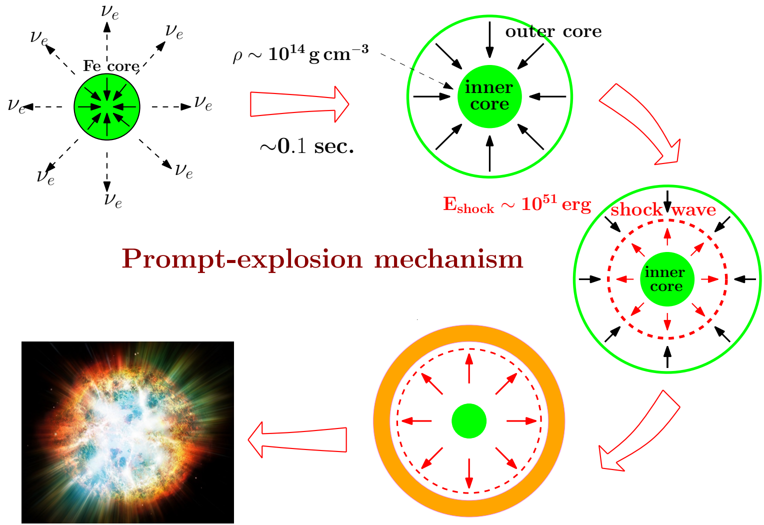
- Massive stars ($M \geq 10 M_\odot$) at the end of their life evolve to an onion like structure;
- At $T \sim 10^9$ K atomic nuclei are completely ionized and dense ($\rho \gtrsim 10^9$ g cm^{-3}) electron-nuclear plasma is formed. For electron gas $kT < E_F$ and $E_F > m_e c^2$;
- Iron core ($R_{core} \approx 10^3 - 10^4$ km) can be stabilized by the pressure of degenerate electron gas as along as $M_{core} < M_{Ch} = 1.44(2Y_e)^2 M_\odot$ ($Y_e \approx 0.45$);
- As the silicon burning proceeds, the iron core approaches M_{Ch} and contracts;
- Increasing density leads to a rise of the electron chemical potential $\mu_e \approx 11.1(\rho_{10} Y_e)^{1/3}$ (ρ_{10} is the density in 10^{10} g cm^{-3});
- When $\mu_e \gtrsim Q$ ($Q = M_f - M_i \approx 2 - 5$ MeV) electron capture on nuclei (neutronization) becomes possible

$$\mathbf{e^- + A_N^Z \rightarrow A_{N+1}^{Z-1} + \nu_e}$$





Core-Collapse Supernova



Prompt-explosion mechanism

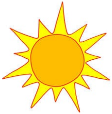
Core-Collapse Supernova

Two reasons for energy loss in a shock-wave:

1. Dissociation nuclei into nucleons (~ 8.8 MeV per nucleon)

$$E_{shock} \geq \frac{0.1 M_{\odot}}{M_{nucleon}} \times 8.8 \text{ MeV} \approx 2 \, \text{foe}$$

1 foe = [ten to the Fifty-One Ergs] 10^{51} erg (1 erg $= 6.24 \times 10^{5}$ MeV)



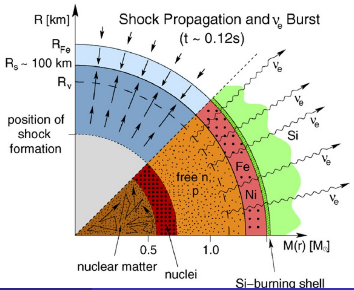
produces ~ 1.2 foe throughout its entire lifetime (10^{10} years)

2. Neutrino emission:

$$e^{-} + p \rightarrow n + \nu_e$$

from H.-Th. Janka et al, Phys. Rep. **442** (2007) 38

Prompt mechanism is unable to trigger supernova explosion!



Core-Collapse Supernova

2 **Neutrino-heating mechanism**

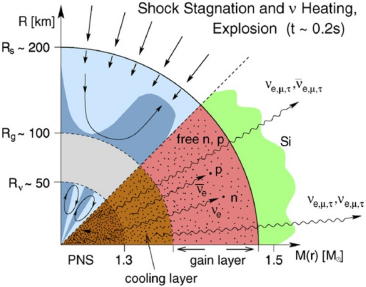
- (Anti)neutrinos carry away energy

$$E_{gr} = G\frac{M_{ns}^2}{R_{ns}} \sim 10^{53}\ \text{erg};$$

- Charged-current captures

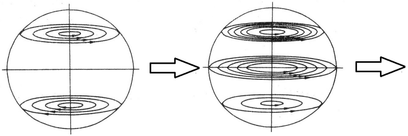
$$\nu_e + n \rightarrow e^- + p$$
$$\overline{\nu}_e + p \rightarrow e^+ + n$$

transfer energy to the shock wave.

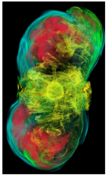


3 **Magnetorotational mechanism** (G. Bisnovatiy-Kogan, 1970)

$$E_{gr} + \frac{\boldsymbol{J}_{core}^2}{M_{core}R_{core}^2} \rightarrow \frac{\boldsymbol{J}_{core}^2}{M_{PNS}R_{PNS}^2}$$

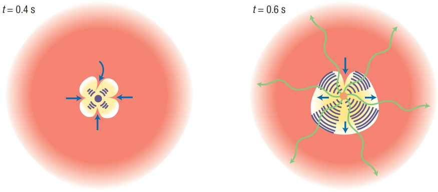


Tarroiadal magnetic field amplification



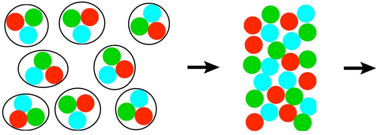
Core-Collapse Supernova

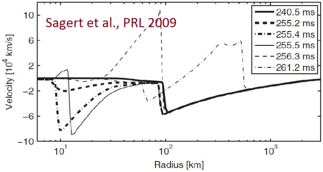
4. **Acoustic mechanism** (A. Burrows, 2006)



adopted from T. Creighton
Nature Physics, 2 (2006) 581

5. **Phase-Transition mechanism** (I. Sagert, 2009, T. Fischer, 2011)



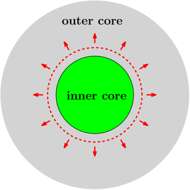


Core-Collapse Supernova

Input physics for core-collapse supernova simulations:

1. cross-sections and rates for nuclear weak-interaction processes under extreme conditions realized in the supernova environment;
2. nuclear equation of state around nuclear matter densities and hight temperatures;
3. full Boltzmann neutrino radiation transport;
4. magnetic field;
5. general relativity;
6. hydrodynamics and so on.

Nuclear weak-interaction processes in supernovae



$$\Delta M = M_{\text{iron core}} - M_{\text{inner core}}$$

- $M_{\text{iron core}} \approx M_{\text{Ch}} \sim Y_e^2$, therefore

$$e^- + (A,Z) \rightarrow (A,Z-1) + \nu_e$$
$$(A,Z) \rightarrow (A,Z+1) + e^- + \overline{\nu}_e$$

  determine the **iron core** mass.

- ν-nucleus reactions become important at $\rho \geq 10^{12}$ g cm^{-3}:

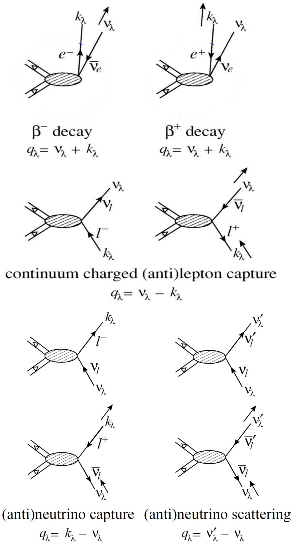
$$\nu + (A,Z) \rightarrow (A,Z) + \nu$$
$$\nu + (A,Z) \rightarrow (A,Z) + \nu'$$
$$\nu_e + (A,Z) \rightarrow (A,Z-1) + e^-$$

  trap neutrinos in the **inner core** and determine its mass:

$$M_{\text{inner core}} \sim Y_L^2, \text{ where } Y_L = Y_e + Y_\nu$$

- Weak-interaction processes affect the equation-of-state: nucleons remain bound in nuclei and do not contribute to pressure; due to nuclear excitations temperature keeps low.

Weak reactions with nuclei in SN matter



adopted from K. Langanke, G. Martínez-Pinedo Rev. Mod. Phys. 75 (819) 2003

$$|\mathcal{M}_{if}|^2 = \frac{1}{2J_i + 1} \sum_{lepton\ spins} \sum_{M_i,\, M_f} |\langle f|H_W|i\rangle|^2$$

Fermi-Dirac distributions of electrons and positrons

$$f_e(E) = \frac{1}{1 + \mathrm{e}^{(E-\mu_e)/kT}}$$

$$\mathbf{2\gamma \leftrightarrow e^+ + e^- \quad \Rightarrow \quad \mu_{e^-} = -\mu_{e^+}}$$

$$\rho Y_e = \frac{1}{\pi^2 \hbar^3 N_A} \int_0^\infty (f_{e^-} - f_{e^+}) p_e^2 dp_e,$$

$\mu_{e^-} \sim (\rho Y_e)^{1/3}$ at hight densities

Capture and decay rates

$$\lambda_{if}^{\text{capt}} \sim |\mathcal{M}_{if}|^2 \int_0^\infty f_e(E) E^2 (E - E_{if})^2 F(Z, E) dE,$$

$$\lambda_{if}^{\beta} \sim |\mathcal{M}_{if}|^2 \int_0^Q (1 - f_e(E)) E^2 (E - E_{if})^2 F(Z, E) dE.$$

Allowed transitions

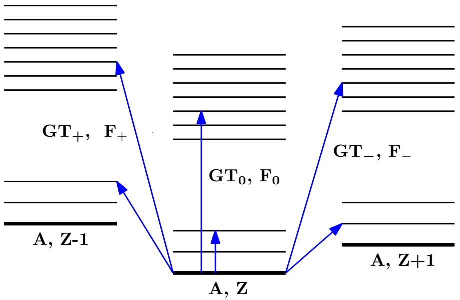
For low-energy weak-interaction processes ($E_{e^{\mp},\nu,\bar{\nu}} \leq 30$ MeV)

$$H_W = G(\mathsf{F}_{\pm,0} + g_A\, \mathsf{GT}_{\pm,0}), \quad \text{where}$$

G – weak interaction constant, g_A – axial coupling constant,

$$\left.\begin{aligned} \mathsf{F}_{\pm,0} &= \sum_{i=1}^{A} \tau_{\pm,0}(i) \quad - \quad \text{Fermi operator} \\ \mathsf{GT}_{\pm,0} &= \sum_{i=1}^{A} \sigma(i)\tau_{\pm,0}(i) \quad - \quad \text{Gamow-Teller operator} \end{aligned}\right\} \text{allowed transitions}$$

$\boxed{\tau_0|\mathbf{n}\rangle = +\tfrac{1}{2}|\mathbf{n}\rangle}$ $\boxed{\tau_0|\mathbf{p}\rangle = -\tfrac{1}{2}|\mathbf{p}\rangle}$ $\boxed{\tau_-|\mathbf{n}\rangle = |\mathbf{p}\rangle}$ $\boxed{\tau_+|\mathbf{p}\rangle = |\mathbf{n}\rangle}$



- charge-neutral transitions

  $$|A, Z\rangle \longrightarrow |A, Z\rangle$$

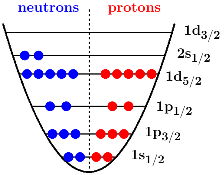
  $\nu, \bar{\nu}$-scattering, $\nu\bar{\nu}$-emission

- charge-changing transitions
  - $|A, Z\rangle \longrightarrow |A, Z+1\rangle$
    β^--decay, e^+-, ν_e-capture;
  - $|A, Z\rangle \longrightarrow |A, Z-1\rangle$
    β^+-decay, e^--, $\bar{\nu}_e$-capture.

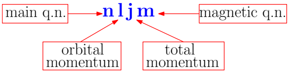
Selection rules

- Fermi transitions: $\pi_i = \pi_f, \ \Delta J = 0, \ \Delta T = 0$
- Gamow-Teller transitions: $\pi_i = \pi_f, \ |\Delta J| = 0, \ 1, \ |\Delta T| = 0, \ 1$

Allowed transitions within the Independent Particle Model

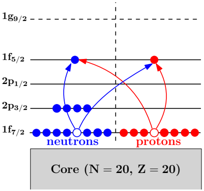


IPM single-particle states



$(nlj)_i \to (nlj)_f$
$n_f = n_i, \ \ l_f = l_i$
Fermi: $j_f = j_i$
GT : $j_f = j_i, \, j_i \pm 1$

Allowed transitions in iron-group nuclei (A=45-65)



$(1f_{7/2,5/2})_i \to (1f_{7/2,5/2})_f$
$(2p_{3/2,1/2})_i \to (2p_{3/2,1/2})_f$

Spin-flip transition
$(\mathbf{1f_{7/2}})_\mathrm{i} \to (\mathbf{1f_{5/2}})_\mathrm{f}$

Strength function

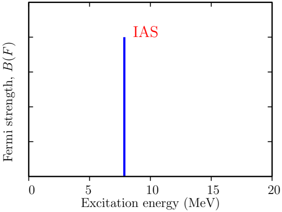
$$S_A(E) = \sum_f B_{if}(A)\delta(E - E_{if}), \qquad E_{if} = E_f - E_i;$$

$$B_{if}(\mathsf{F}_{\pm,0}) = \frac{1}{2J_i + 1}|\langle J_i T_i|\mathsf{F}_{\pm,0}|J_f T_f\rangle|^2, \quad B_{if}(\mathsf{GT}_{\pm,0}) = \frac{1}{2J_i + 1}|\langle J_i T_i|\mathsf{GT}_{\pm,0}|J_f T_f\rangle|^2.$$

Fermi transitions

$F_{\pm,0} = \Sigma_{i=1}^{A} \tau_{\pm,0}(i)$

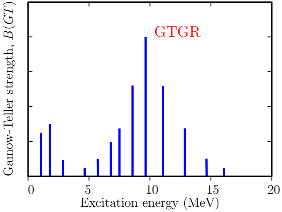
$[F_{\pm,0}, H] = 0$



Gamow-Teller transitions

$GT_{\pm,0} = \Sigma_{i=1}^{A} \sigma(i)\tau_{\pm,0}(i)$

$[GT_{\pm,0}, H] \neq 0$



Fermi transitions

- Transition strength

$$B_{if}(\mathsf{F}_{\pm,0}) = \frac{1}{2J_i+1}|\langle J_i T_i T_{zi}|\mathsf{F}_{\pm,0}|J_f T_f T_{zf}\rangle|^2 = T(T+1) - T_{zi}T_{zf}$$

where $T = T_i = T_f, \quad T_{zf} = T_{zi} \pm 1, 0$,

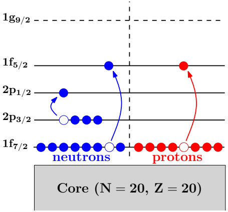
- Ikeda sum rule:

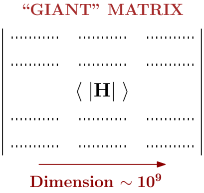
$$\sum_f B_{if}(\mathsf{F}_-) - \sum_f B_{if}(\mathsf{F}_+) = N - Z$$

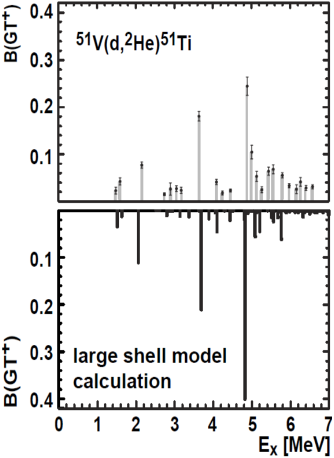
- Transition energy:

$$E_{if}(F_\pm, 0) = \begin{cases} 0, & \text{for} \quad \mathsf{F}_0 \\ E(\mathsf{IAS}) = M_p - M_d + \Delta E_{Coulomb}, & \text{for} \quad \mathsf{F}_\pm \end{cases}$$

Large-Scale Shell Model calculations for $A = 45 - 65$ (K. Langanke et al)







G. Martínez-Pinedo et al. Nucl. Phys. 777 (2006) 395

The Lanchzos iterative algorithm is applied to find low-energy eigenvalues

Nuclear statistical equilibrium

Averaged rates and cross sections: $\langle\lambda\rangle = \sum_i Y_i\lambda_i, \quad \langle\sigma\rangle = \sum_i Y_i\sigma_i$

For $\boldsymbol{T > 10^9}$ **K** and $\rho > 10^9$ g cm^{-3} all electromagnetic and strong reactions

$$(A,Z) + p \rightleftarrows (A+1, Z+1) + \gamma$$
$$(A,Z) + n \rightleftarrows (A+1, Z) + \gamma$$

as well as $(\alpha,\gamma),\ (\alpha,n),\ (\alpha,p),\ (p,n)$ are in equilibrium.

Saha equation ($Y_{A,Z} = Y_{A,Z}(T,\rho,Y_e)$):

$$Y_{A,Z} = \frac{G(A,Z)A^{3/2}}{2^A} Y_p^Z Y_n^N \left(\frac{2\pi\hbar^2}{m_u kT}\right)^{3/2(A-1)} e^{B(A,Z)/kT}$$

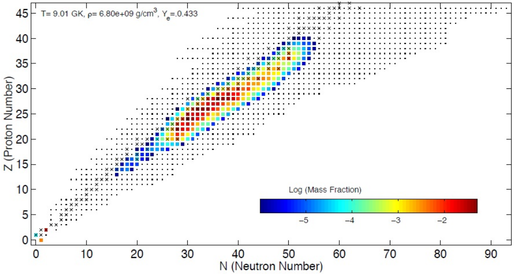
with the constrains ($Y_i = Y(A,Z)$)

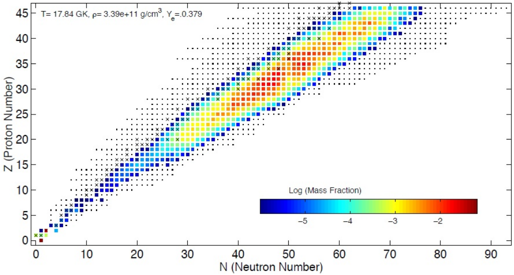
- $\sum_i Y_i A_i = 1$ (baryon number conservation)
- $\sum_i Y_i Z_i = Y_e$ (charge conservation)

Partition function: $G(A,Z) = \sum_i (2J_i+1)e^{-E_i/kT} \approx \dfrac{\pi}{6akT}\exp(akT)$

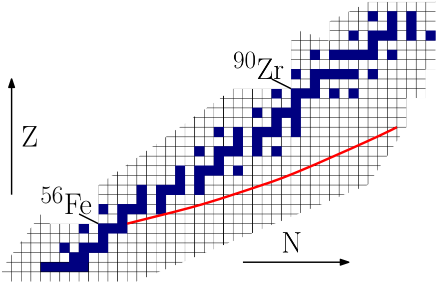
Binding energy: $B(A,Z) = (ZM_p + NM_n) - M(A,Z)$

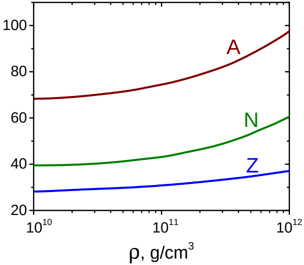
Nuclear statistical equilibrium





LSSM calculations are limited by iron group nuclei ($A = 45 - 65$)





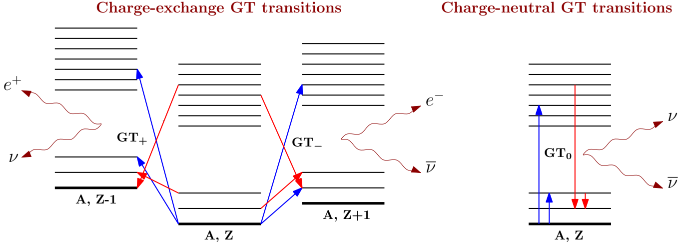
Element trajectory (left figure) and evolution of the mean nucleus as a function of the matter density in the core (right figure).

Finite temperature

The temperature of supernova matter varies from **a few hundreds keV** to **few MeV** ($0.86\ \mathrm{MeV} \approx 10^{10}\ \mathrm{K}$). Therefore, nuclear excited states are thermally populated according to Boltzmann distribution: $g_i(T) \sim \exp(-E_i/kT)$.

Weak reactions with hot nuclei:

- reaction threshold disappears ($Q_{\mathrm{ec}}(^{56}\mathrm{Fe}) \approx 4\,\mathrm{MeV}$);
- stable nuclei undergo β-decay.



$$\sigma(E,T) = \sum_i g_i(T)\sigma_i(E), \quad \lambda(E,T) = \sum_i g_i(T)\lambda_i(E),$$

For iron-group nuclei ($A = 45 - 65$) the mean excitation energy at $T = 1$ MeV is $\langle E \rangle_{\text{Fermi gas}} = AT^2/8 \approx 7 \div 8$ MeV and density of states is $\rho(E) \approx 100\,\mathrm{MeV}^{-1}$.

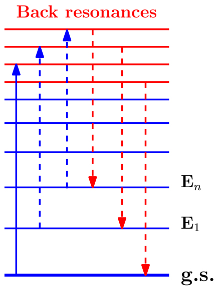
Cross section for $\nu + A \to \nu' + A$:

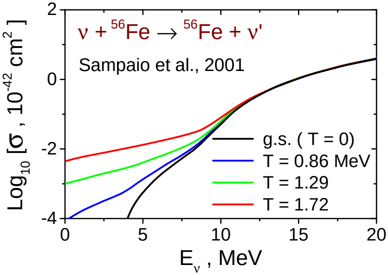
$$\sigma(E_\nu, T) = \sigma_d(E_\nu) + \sigma_{up}(E_\nu, T),$$

$$\sigma_d(E_\nu) \sim \sum_f E_{\nu'}^2 |\langle g.s.|\boldsymbol{\sigma} t_0|f\rangle|^2, \quad (E_{\nu'} = E_\nu - E_f);$$

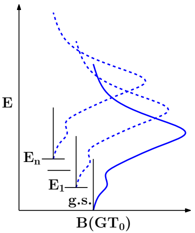
$$\sigma_{up}(E_\nu, T) \sim \sum_{i,f} E_{\nu'}^2 |\langle i|\boldsymbol{\sigma} t_0|f\rangle|^2 \exp\left(-\frac{E_i}{T}\right), \quad (E_i > E_f)$$

$$|\langle i|\boldsymbol{\sigma} t_0|f\rangle|^2 = \frac{2J_f + 1}{2J_i + 1}|\langle f|\boldsymbol{\sigma} t_0|i\rangle|^2;$$

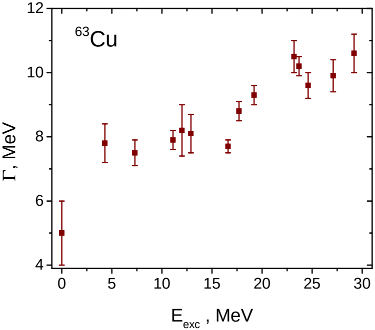




Brink hypothesis

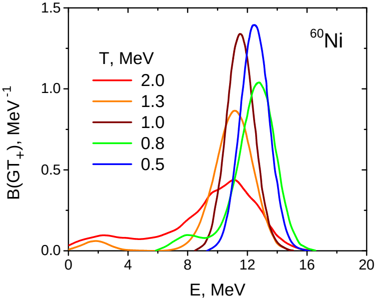


Violation of the Brink Hypothesis



The width of GDR in ^{63}Cu as a function of excitation energy.

M. Kicińska-Habior, et al, Phys. Rev. C **36**, 612 (1987)



Temperature evolution of GT_+ strength distribution for ^{60}Ni (SMMC calculations).

P. B. Radha, D. J. Dean et al, Phys. Rev. C **56** , 3079 (1997)

Cross-sections and rates at $T \neq 0$

Shell-model calculations

1. Thermal averaging: $\sigma(E,T) = \sum_i p_i(T)\sigma_i(E)$, $\lambda(T) = \sum_i p_i(T)\lambda_i$.
2. Cross-section $\sigma_i(E)$ and rates λ_i for thermally excited states;
3. GT strength distributions $S_{GT}(E,E_i)$ for the nuclear ground and excited states;

Shortcomings of shell-model calculations

- application of Brink hypothesis;
- violation of the detailed balance principle

$$\boldsymbol{S(T,-E) \neq S(T,E)\exp(-E/T)};$$

- contribution of low- and negative-energy transitions from nuclear excited states is underestimated;
- shell-model calculations are limited by iron-group nuclei ($A = 45 - 65$)

Statistical approach

1. Temperature dependent GT strength distributions $S_{GT}(E,T)$ in the hot nucleus;
2. Cross-sections $\sigma(E,T)$ and rates $\lambda(T)$ for hot nuclei.

Advantages of the statistical approach

- thermodynamically consistent

$$\boldsymbol{S(T,-E) = S(T,E)\exp(-E/T)};$$

- there is no restrictions on A.

Strength function for transition operator A at $T \neq 0$

Definition

$$S_A(E,T) = \sum_i \frac{e^{-E_i/T}}{Z(T)} S_A(E,i) =$$

$$= \sum_i \frac{e^{-E_i/T}}{Z(T)} \sum_i |\langle f|A|i\rangle|^2 \delta(E - E_f + E_i), \quad Z(T) = \sum_i e^{-E_i/T}.$$

Detailed balance: $\boldsymbol{S_{\mathcal{T}}(-E,T) = S_{\mathcal{T}}(E,T) \exp\left(-\frac{E}{T}\right)}$.

Example: cross-section for inelastic neutrino-scattering on a hot nucleus

$$\boldsymbol{\sigma(E_\nu,T)} = \sum_i \frac{e^{-E_i/T}}{Z(T)} \sigma_i(E_\nu)$$

$$\sim \sum_i \frac{e^{-E_i/T}}{Z(T)} \sum_f (E_\nu - E_f + E_i)^2 |\langle f|GT_0|i\rangle|^2$$

$$= \int dE (E_\nu - E)^2 \sum_{i,f} \langle f|GT_0|i\rangle|^2 \frac{e^{-E_i/T}}{Z(T)} \delta(E - E_f + E_i)$$

$$= \int \boldsymbol{dE(E_\nu - E)^2 S_{GT_0}(E,T)}$$

Strength function at $T \neq 0$

Applying $\delta(E) = \dfrac{1}{2\pi}\displaystyle\int_{-\infty}^{+\infty} dt e^{iEt}$ we find ($A = GT_{\pm,0}$):

$$\boldsymbol{S_A(E,T)} = \sum_{i,f} |\langle f|A|i\rangle|^2 \frac{e^{-E_i/T}}{Z(T)} \delta(E - E_f + E_i)$$
$$= \frac{1}{2\pi}\int dt \sum_{i,f} |\langle f|A|i\rangle|^2 \frac{e^{-E_i/T}}{Z(T)} e^{i(E-E_f+E_i)t}$$
$$= \frac{1}{2\pi}\int dt e^{iEt} \sum_{i,f} \frac{e^{-E_i/T}}{Z(T)} \langle i|e^{iHt}A|f\rangle\langle f|e^{-iHt}A|i\rangle$$
$$\boldsymbol{= \frac{1}{2\pi}\int dt e^{iEt} \langle\langle A(t)A(0)\rangle\rangle}$$

where $\langle\langle A^\dagger(t)A(0)\rangle\rangle$ is the correlation function

$$\langle\langle A^\dagger(t)A(0)\rangle\rangle = \mathrm{Tr}\left\{\hat\rho(T)A^\dagger(t)A(0)\right\}, \quad \hat\rho(T) \sim \mathrm{e}^{-H/kT}, \quad A(t) = e^{iHt}Ae^{-iHt}.$$

Correlation functions can be computed either by applying the Matsubara Green's function technique or by the formalism of superoperators in Liouville space.

Superoperators in Liouville space (A.A.D. and A.I.V. Phys. Part. Nuclei 53 (2022) 885-938)

Liouville space is the space of operators on Hilbert state

$$A \leftrightarrow \|A\rangle\!\rangle, \quad \langle\!\langle A\|B\rangle\!\rangle = \mathrm{Tr}\{A^\dagger B\}, \quad |A| = \langle\!\langle A\|A\rangle\!\rangle^{1/2}$$

Superoperators – operators acting in Liouville space. **Left** and **right** creation and annihilation superoperators:

$$\boldsymbol{a}_k^\dagger \|mn\rangle\!\rangle \leftrightarrow a_k^\dagger\, |m\rangle\langle n|\,, \quad \boldsymbol{a}_k^\dagger \|mn\rangle\!\rangle \leftrightarrow \beta(m,n)\, |m\rangle\langle n|\, a_k\,,$$
$$\boldsymbol{a}_k \|mn\rangle\!\rangle \leftrightarrow a_k\, |m\rangle\langle n|\,, \quad \boldsymbol{a}_k \|mn\rangle\!\rangle \leftrightarrow \alpha(m,n)\, |m\rangle\langle n|\, a_k^\dagger,$$

where $\alpha(m,n)$ and $\beta(m,n)$ are obtained from $\{\boldsymbol{a}_k, \boldsymbol{a}_{k'}\} = 0, \;\; \{\boldsymbol{a}_k, \boldsymbol{a}_{k'}^\dagger\} = \{\boldsymbol{a}_k, \boldsymbol{a}_{k'}^\dagger\} = \delta_{kk'}$.

- statistical average is given by $\langle\!\langle A(t)A(0)\rangle\!\rangle = \langle\!\langle 0(T)\|A(t)A(0)\|0(T)\rangle\!\rangle$, where $\|0(T)\rangle\!\rangle \leftrightarrow \sqrt{\hat\rho(T)}$;
- thermal Hamiltonian $\mathcal{H} = H(\boldsymbol{a}^\dagger, \boldsymbol{a}) - H(\boldsymbol{a}^\dagger, \boldsymbol{a})$ determines spectral properties of a hot nucleus:

$$S_A(E,T) = \sum_k \Big\{ |\langle\!\langle \mathcal{O}_k\|A\|0(T)\rangle\!\rangle|^2 \delta(E-\mathcal{E}_k) + |\langle\!\langle \widetilde{\mathcal{O}}_k\|A\|0(T)\rangle\!\rangle|^2 \delta(E+\mathcal{E}_k) \Big\},$$

  where $\mathcal{H}\|0(T)\rangle\!\rangle = 0$, $\mathcal{H}\|\mathcal{O}_k\rangle\!\rangle = +\mathcal{E}_k\|\mathcal{O}_k\rangle\!\rangle$ and $\mathcal{H}\|\widetilde{\mathcal{O}}_k\rangle\!\rangle = -\mathcal{E}_k\|\widetilde{\mathcal{O}}_k\rangle\!\rangle$;
- the equation-of-motion method at $T \neq 0$

$$\langle 0(T)|[\delta O, [\mathcal{H}, \mathcal{O}_k^\dagger]]|0(T)\rangle = \mathcal{E}_k \langle 0(T)|[\delta O, \mathcal{O}_k^\dagger]|0(T)\rangle,$$

$$\langle \widetilde{O}_k|\, A\, |0(T)\rangle = \mathrm{e}^{-\mathcal{E}_k/2kT} \langle O_k|\, A^\dagger\, |0(T)\rangle^*;$$

- the detailed balance principle

$$S_{A^\dagger}(-E,T) = \mathrm{e}^{-E/kT} S_A(E,T).$$

Calculation of GT$_{\pm}, 0$ strength functions within the TQRPA

Nuclear Hamiltonian : $H = H_{sp} + H_{pair} + H_{ph}$, where H_{sp} and $H_{ph} = \sum_k h_k^\dagger h_k$ are obtained from the Skyrme energy density functional SkM* and H_{pair} is the BCS pairing interaction.

Thermal Hamiltonian : $\mathcal{H} = H(\boldsymbol{a}^\dagger, \boldsymbol{a}) - H(\boldsymbol{a}^\dagger, \boldsymbol{a}) = \mathcal{H}_{sp} + \mathcal{H}_{pair} + \mathcal{H}_{ph}$.

1. Thermal quasiparticles:

$$\mathcal{H}_{sp+pair} \approx \sum_{jm} \varepsilon_j(T)(\beta^\dagger_{jm}\beta_{jm} - \widetilde{\beta}^\dagger_{jm}\widetilde{\beta}_{jm})$$

2. Thermal phonons (TQRPA - Thermal Quasiparticle Random Phase Approximation):

$$\mathcal{H} \approx \sum_{JMi} \omega_{Ji}(T)(Q^\dagger_{JMi}Q_{JMi} - \widetilde{Q}^\dagger_{JMi}\widetilde{Q}_{JMi}) + \mathcal{H}_{qph}, \quad Q_{JMi}\,|0(T)\rangle = \widetilde{Q}_{JMi}\,|0(T)\rangle = 0,$$ where

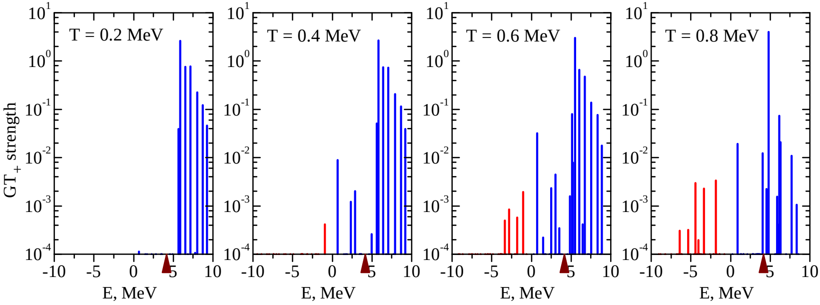
$$Q^\dagger_{JMi} = \sum_{1,2}\psi_{12}\beta^\dagger_1\beta^\dagger_2 + \ldots + \sum_{1,2}\varphi_{12}\beta_1\beta_2 + \ldots$$

3. GT$_{\pm,0}$ strength function within the TQRPA

$$S_{\mathsf{GT}_{\pm,0}}(E,T) = \sum_k \Big\{ |\langle\!\langle Q_{i,1^+}\|\boldsymbol{\sigma} t_{\pm,0}\|0(T)\rangle\!\rangle|^2 \delta(E - \omega_{i,1^+}) + \\ + |\langle\!\langle \widetilde{Q}_{i,1^+}\|\boldsymbol{\sigma} t_{\pm,0}\|0(T)\rangle\!\rangle|^2 \delta(E + \omega_{i,1^+}) \Big\}.$$

4. Calculation of nuclear weak-interaction rates and cross-sections for given supernova conditions ($T,\ \rho,\ Y_e$).

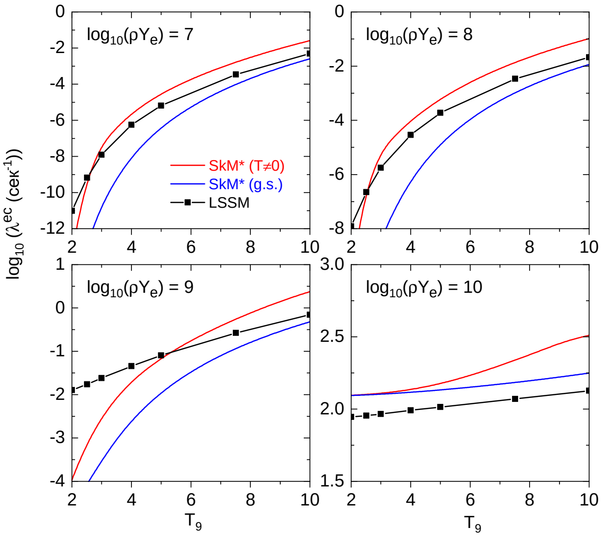
GT$_+$ strength distribution in ^{56}Fe.



$\Delta_p \approx 1.4$ **MeV** $\Rightarrow$ $T_{\rm cr} \approx 0.5\Delta(\approx 0.7$ **MeV**).

The brown arrows indicate the zero-temperature EC threshold:
$Q = M(^{56}\text{Mn}) - M(^{56}\text{Fe}) = 4.2\ \text{MeV}$.

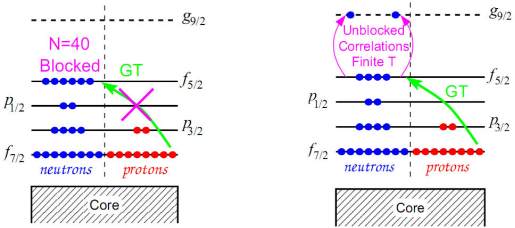
Electron capture rates for ^{56}Fe



$T_9 = 10^9$ K $(0.086$ MeV), ρY_e – electron gas density

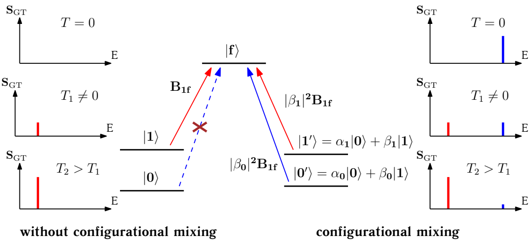
Electron capture on neutron-rich nuclei

Neutron-rich nuclei ($N > 40, \quad Z < 40$)



- Within the **I**ndependent **P**article **M**odel GT_+ transitions are Pauli-blocked;
- EC can proceed only through forbidden transitions or on free protons;
- EC on neutron-rich nuclei was neglected untill 2000s;
- Unblocking mechanisms:
  - configuration mixing $|\Psi_0\rangle = c_0|\mathbf{IPM}\rangle + c_1\alpha_1^+\alpha_2|\mathbf{IPM}\rangle + c_2\alpha_1^+\alpha_2^+\alpha_3\alpha_4|\mathbf{IPM}\rangle + \ldots$;
  - thermally excited states in statistical ensemble $\hat{\rho} \sim \sum_n e^{-E_n/T}|\Psi_n\rangle\langle\Psi_n|$

Unblocking due to thermal effects and configurational mixing



Without configurational mixing:

- there is no GT strength at $T = 0$;
- GT strength increases with T, but its energy remains the same ($E_{1f} = E_f - E_1$).

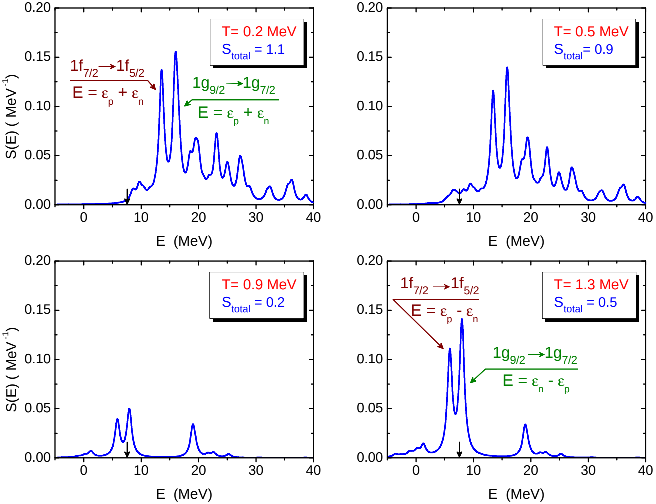
Configurational mixing:

$$S_{GT}(E,T) = \frac{1}{Z(T)} \left\{ |\langle f|\mathrm{GT}|0'\rangle|^2 \delta(E - E_{0'f}) + e^{-E_{1'f}/T} |\langle f|\mathrm{GT}|1'\rangle|^2 \delta(E - E_{1'f}) \right\}$$

$$= \frac{B_{1f}}{1 + e^{-E_{1'}/T}} \left\{ |\beta_0|^2 \delta(E - E_{0'f}) + e^{-E_{1'}/T} |\beta_1|^2 \delta(E - E_{1'f}) \right\}$$

For a weak configurational mixing $E_{1'f} - E_{0'f} \approx E_1 - E_0$.

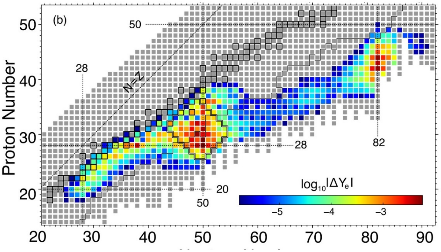
GT_+ strength distribution in ^{76}Ge ($T \neq 0$)



A. Dzhioev et al, Phys. Rev. C **81** (2010) 015804

Electron capture by $N = 50$ neutron-rich nuclei

Ch. Sullivan et al, The Astrophysical Journal 816 (2016)



Top 500 electron-capturing nuclei with the largest absolute change to the electron fraction up to neutrino trapping.

R. Titus et al, PRC 100 (2019)

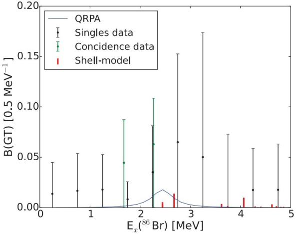


FIG. 5. Gamow-Teller strength distribution extracted from the ^{86}Kr(t, ^{3}He) data and comparison with shell-model and QRPA calculations, as described in the text.

J.C. Zamora et al, PRC100(R), 2019

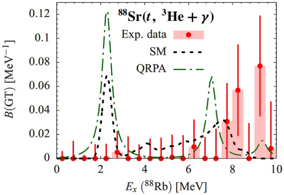
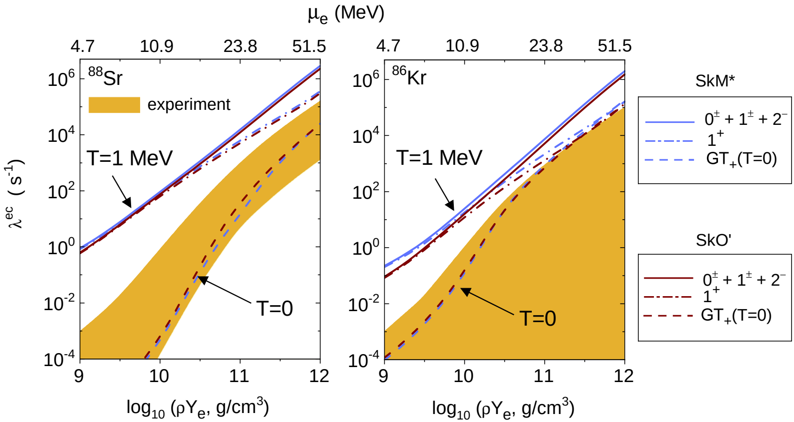


FIG. 2. B(GT) distribution extracted from MDA for $E_x <$ 10 MeV. The error bars denote only the statistical uncertainties.



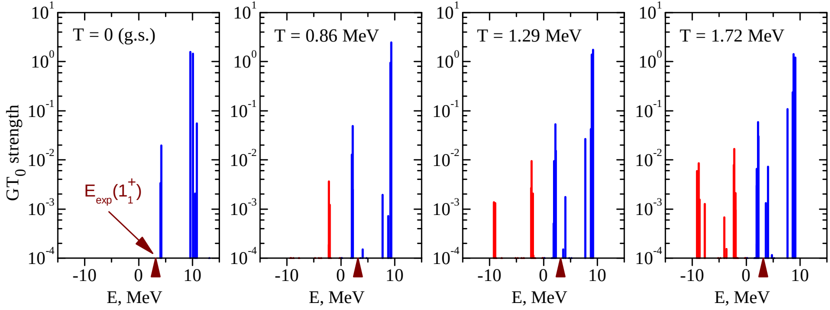
Electron capture rates at $T = 1$ MeV (10^{10} K=0.86 MeV)

A. A. Dzhioev et al, PRC101 (2020) 025805

GT_0 strength distribution in ^{56}Fe

$$\nu + {}^{56}\text{Fe} \to {}^{56}\text{Fe} + \nu'$$

$T = 0.86$ MeV (10^{10} K) corresponds to the condition of a presupernova model for a $15M_{\odot}$ star; $T = 1.29$ MeV (1.5×10^{10} K) - relates to neutrino trapping, $T = 1.72$ MeV (2×10^{10} K) - to neutrino thermalization.



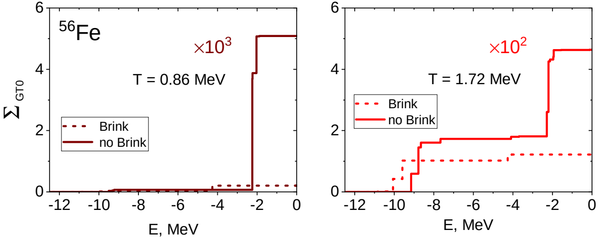
Detailed balance: $S_{\text{GT}_0}(-E, T) = S_{\text{GT}_0}(E, T) \exp\left(-\frac{E}{T}\right)$.

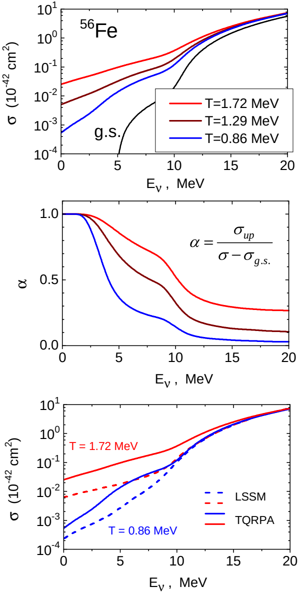
GT_0 strength distribution in ^{56}Fe

Detailed balance

$$\boldsymbol{S_{\mathsf{GT}_0}(-\varepsilon, T) = S_{\mathsf{GT}_0}(\varepsilon, T)\exp\Big(-\frac{\varepsilon}{T}\Big).}$$

Cumulative sum of the GT_0 strength : $\Sigma_{\mathsf{GT}_0}(E,T) = \int\limits_{-\infty}^{E} S_{\mathsf{GT}_0}(\varepsilon, T)d\varepsilon$





$$\sigma(E_\nu,T) = \frac{G_F^2}{\pi(\hbar c)^4}\int_{-\infty}^{E_\nu}(E_\nu - E)^2 S_{\mathsf{GT}_0}(E,T)dE$$
$$= \sigma_d(E_\nu,T) + \sigma_{up}(E_\nu,T),$$

- down-scattering ($E'_\nu < E_\nu$) cross section

$$\sigma_d(E_\nu,T) \sim \int_0^{E_\nu}(E_\nu - E)^2 S_{\mathsf{GT}_0}(E,T)dE;$$

- up-scattering ($E'_\nu > E_\nu$) cross section

$$\sigma_{up}(E_\nu,T) \sim \int_{-\infty}^{0}(E_\nu - E)^2 S_{\mathsf{GT}_0}(E,T)dE.$$

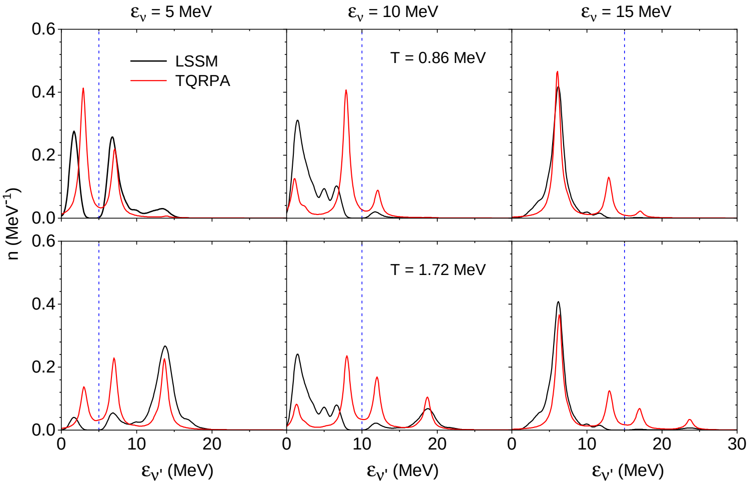
Shell-model calculations

$$\sigma(E_\nu,T) = \sigma_{\mathsf{g.s}}(E_\nu) + \sigma_{up}(E_\nu,T)$$

and $\alpha = 1$.

Thermal effects on neutrino spectrum

$\nu + {}^{56}\mathrm{Fe} \rightarrow \nu' + {}^{56}\mathrm{Fe}$



Pre-supernova neutrinos

- SN neutrinos are detectable (SN1987A).
- Neutrino luminosity grows by orders of magnitude in last hours/days before collapse.
- Can we see pre-SN neutrinos too ?
  - alarm for an upcoming SN explosion
  - direct observation of stellar interiors
- Inverse β-decay

$$\bar{\nu}_e + p \to n + e^+$$

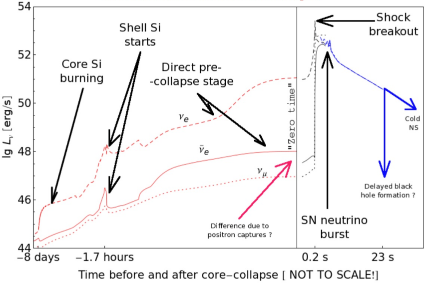
- Properties of $\bar{\nu}_e$ from pre-SN
  - weak-interaction nuclear reactions;
  - oscillations $\bar{\nu}_e \leftrightarrow \bar{\nu}_{\mu,\tau}$

Detector	Mass [kton]	Reactions	Number of Targets	Flux at 1 kpc [$cm^{-2}\,day^{-1}$]	Event rate [day^{-1}]
		$\nu_x + d \to \nu_x + p + n$	6.00 · 10[illegible]	3.8 · 10[illegible]	0.038
		$\bar{\nu}_x + d \to \bar{\nu}_x + p + n$	$6.00 \cdot 10^{31}$	$3.8 \cdot 10^{11}$	0.032
Super-K	32 (H_2O)	$\bar{\nu}_e + p \to e^+ + n$	$2.14 \cdot 10^{33}$	$2.8 \cdot 10^{11}$	41
UNO	440 (H_2O)	$\bar{\nu}_e + p \to e^+ + n$	$2.94 \cdot 10^{34}$	$2.8 \cdot 10^{11}$	560
Hyper-K	540 (H_2O)	$\bar{\nu}_e + p \to e^+ + n$	$3.61 \cdot 10^{34}$	$2.8 \cdot 10^{11}$	687

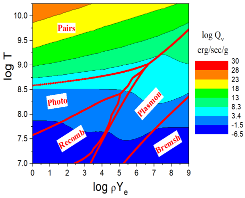
Event rate per day in selected neutrino detectors from silicon burning stage in neutrino-cooled star at distance of 1 kpc.

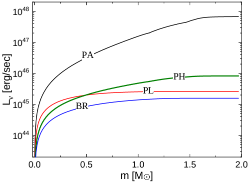
Odrzywolek, Misiaszek, and Kutschera Astropart. Phy. 21, 303 (2004)

Neutrinos BEFORE and AFTER collapse

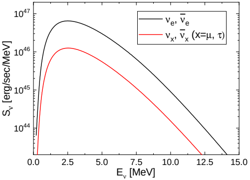


Odrzywolek and Heger Acta Physica Polonica B41(2010) 1611





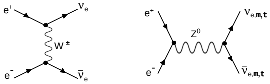
Spectra of ν and $\bar{\nu}$ from the PA process



Thermal processes produce all flavors of neutrinos

- Electron-positron pair annihilation

$$\gamma + \gamma \leftrightarrows e^- + e^+ \to \nu + \bar{\nu},$$
$$P_{\nu\bar{\nu}}/P_{2\gamma} \approx 10^{-19}, \quad Q_\nu \sim T^9/\rho$$



- Plasmon decay

$$\gamma^* \to \nu + \bar{\nu}$$

- Electron-nucleus bremsstrahlung

$$e^- + (Z, A) \to e^- + (Z, A) + \nu + \bar{\nu}$$

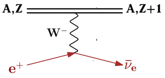
- Photo-neutrino

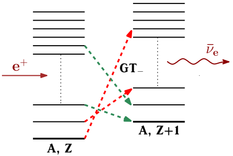
$$e^- + \gamma \to e^- + \nu + \bar{\nu}$$

Emission of $\bar{\nu}$ in nuclear processes

positron capture (PC)

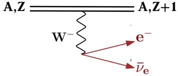
$$(A,Z) + e^{+} \rightarrow (A,Z+1) + \bar{\nu}_e$$

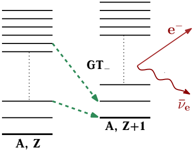




β^{-}-decay

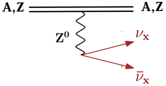
$$(A,Z) \rightarrow (A,Z+1) + e^{-} + \bar{\nu}_e$$

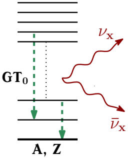




$\nu\bar{\nu}$-pair emission via nuclear de-excitation (ND)

$$(A,Z) \rightarrow (A,Z) + \nu_x + \bar{\nu}_x, \quad x = e,\, \mu,\, \tau$$





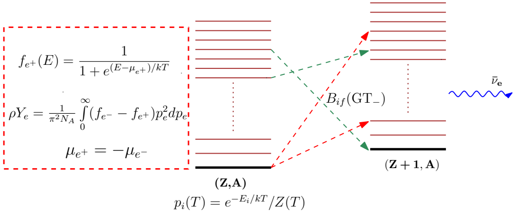
A.A. Dzhioev et al, MNRAS 527 (2024) 7701

- Positron capture (PC) on a hot nucleus ($e^+ + A_N^Z \rightarrow A_{N-1}^{Z+1} + \bar{\nu}_e$)

$$\lambda_{if}^{\mathsf{PC}} = \frac{\ln 2}{K} B_{if}(\mathsf{GT}_-)\Phi_{if}^{\mathsf{PC}} = \int_0^{\infty} \phi_{if}^{\mathsf{PC}}(E_\nu)dE_\nu$$

$$\phi_{if}^{\mathsf{PC}}(E_\nu) = \frac{\ln 2}{K} B_{if}(\mathsf{GT}_-)E_\nu^2 E_e p_e f_{e+}(E_e)F(-Z, E_e)\Theta(E_e - m_e)$$

where $E_e = E_\nu + E_{if}$ and $B_{if}(\mathsf{GT}_-) = |\langle f\|\boldsymbol{\sigma}t_-\|i\rangle|^2/(2J_i + 1)$.



$$\phi^{\mathsf{PC}}(E_\nu) = \sum_{i,f} p_i(T)\phi_{if}^{\mathsf{PC}}(E_\nu) = \frac{\ln 2}{K} E_\nu^2 \int_{-E_\nu+m_e}^{+\infty} S_{\mathsf{GT}_-}(E,T)E_e p_e f_{e\mp}(E_e)F(-Z, E_e)dE$$

where

$$S_{\mathsf{GT}_-}(E,T) = \sum_{i,f} p_i(T)B_{if}(\mathsf{GT}_-)\delta(E - E_{if})$$

$\bar{\nu}$ spectra from nuclear processes

- β^--decay of hot nucleus ($A_N^Z \to A_{N-1}^{Z+1} + e^- + \bar{\nu}_e$)

$$\phi^{\beta}(E_\nu) = \frac{G_{\mathsf{F}}^2 V_{ud}^2}{2\pi^3} E_\nu^2 \int_{-\infty}^{-E_\nu - m_e} S_{\mathsf{GT}\_}(E,T) E_e p_e \big(1 - f_{e^-}(E_e)\big) F(Z+1, E_e) dE$$

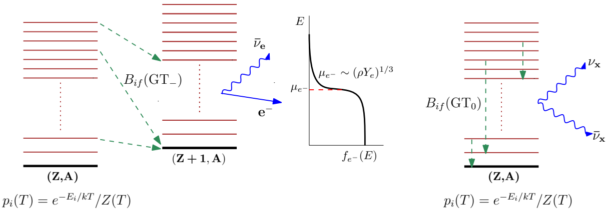
where $E_e = -E_\nu - E$;

- $\nu\bar{\nu}$-pair emission via nuclear de-excitation (ND) ($A_N^Z \to A_N^Z + \nu_x + \bar{\nu}_x,\ x = e,\ \mu,\ \tau$)

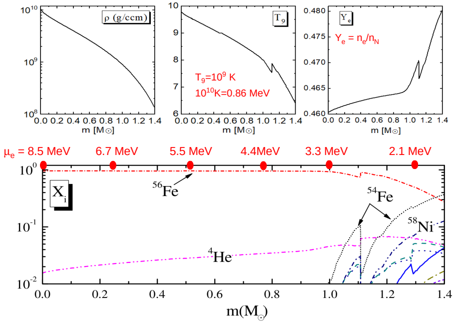
$$\phi^{\mathsf{ND}}(E_\nu) = \frac{G_{\mathsf{F}}^2}{2\pi^3} E_\nu^2 \int_{-\infty}^{-E_\nu} S_{\mathsf{GT}_0}(E,T)(E + E_\nu)^2 dE$$

where

$$S_{\mathsf{GT}_0}(E,T) = \sum_{i,f} p_i(T) B_{if}(\mathsf{GT}_0)\delta(E - E_{if}) \text{ and } B_{if}(\mathsf{GT}_0) = |\langle f \| \boldsymbol{\sigma} t_0 \| i \rangle|^2 / (2J_i + 1)$$



- Realistic pre-supernova conditions via **MESA** (Modules for Experiments in Stellar Astrophysics)
- Pre-supernova model `25_79_0p005_ml` ($M = 25\, M_\odot$)
- The profile that we use corresponds to the onset of the core collapse



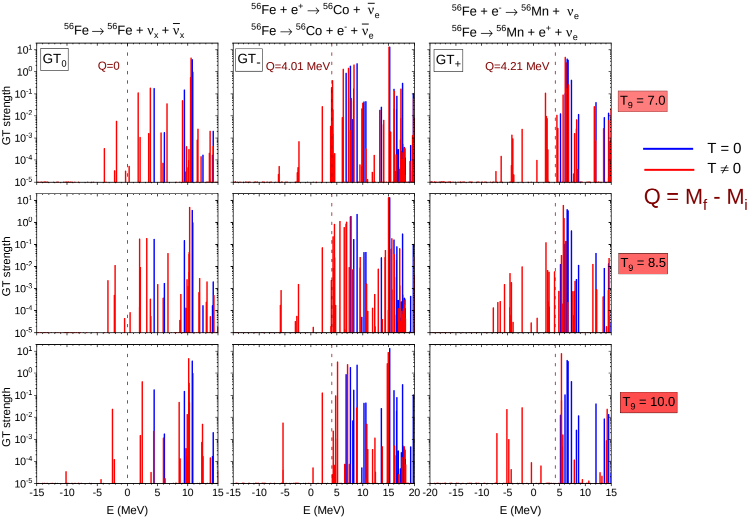
Fermi-Dirac distributions of electrons and positrons in stellar matter

$$\gamma \leftrightarrow e^+ + e^- \quad \Rightarrow \quad \mu_{e^-} = -\mu_{e^+}$$

$$\rho Y_e = \frac{1}{\pi^2 \hbar^3 N_A} \int_0^\infty (f_{e^-} - f_{e^+}) p_e^2 dp_e, \quad f_e(E) = \frac{1}{1 + \mathrm{e}^{(E-\mu_e)/kT}}$$

$\mu_{e^-} \sim (\rho Y_e)^{1/3}$ at hight densities

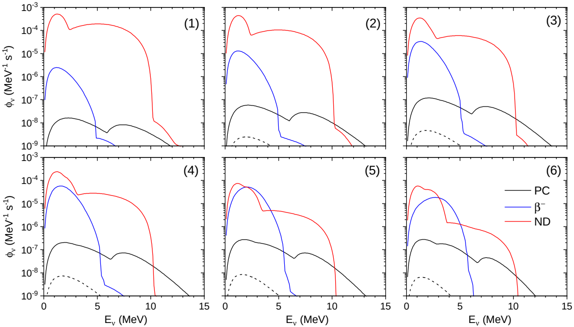
Thermal effects on GT_0 and GT_- strength functions in ^{56}Fe



$S_{\mathrm{GT}_0}(-E,T) = \mathrm{e}^{-E/kT} S_{\mathrm{GT}_0}(E,T)$

$S_{\mathrm{GT}_\mp}(-E,T) = \mathrm{e}^{-(E \mp \Delta_{np})/kT} S_{\mathrm{GT}_\pm}(E,T) \quad (\Delta_{np} \approx 0.83\,\mathrm{MeV})$

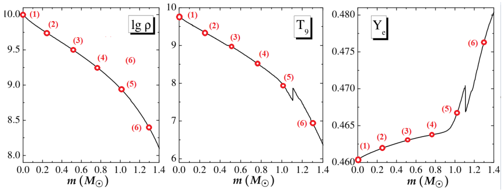
$\bar{\nu}_e$ spectra produced by hot ^{56}Fe



PC: $^{56}Fe + e^+ \to {}^{56}Co + \bar{\nu}_e$

β^- : $^{56}Fe \to {}^{56}Co + e^- + \bar{\nu}_e$

ND: $^{56}Fe \to {}^{56}Fe + \nu_e + \bar{\nu}_e$



$\mu_e \sim (\rho Y_e)^{1/3}$

$\mu_e(1) = 8.45$ MeV

$\mu_e(2) = 6.68$ MeV

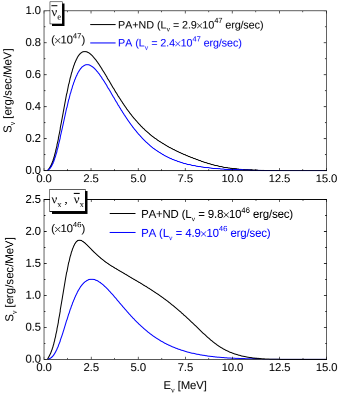
$\mu_e(3) = 5.53$ MeV

$\mu_e(4) = 4.44$ MeV

$\mu_e(5) = 3.34$ MeV

$\mu_e(6) = 2.12$ MeV

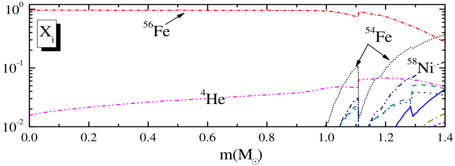
Total $\bar{\nu}$ spectra and luminosities from nuclear and thermal processes



The total $\bar{\nu}$ spectrum from nuclear processes is found by integration over the volume of the star of a weighted sum over all the isotopes present:

$$\Phi_{\bar{\nu}}(E_\nu) = \int dV \sum_i \phi^i_{\bar{\nu}}(E_\nu) n_i,$$

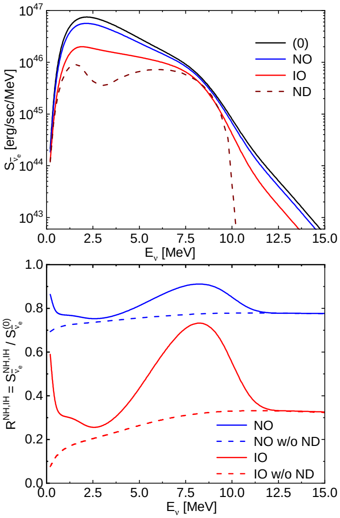
where $n_i = X_i \rho/(m_N A_i)$ is the number density of isotope i.



Antineutrino energy luminosity

$$L_{\bar{\nu}} = \int S_{\bar{\nu}}(E_\nu) dE_\nu,$$

where $S_{\bar{\nu}}(E_\nu) = \Phi_{\bar{\nu}}(E_\nu) E_\nu$ – energy luminosity spectrum.



Flavor neutrinos are a linear combination of mass neutrinos

$$\nu_\alpha = \sum_{i=1,2,3} U_{\alpha i}\nu_i, \quad (\alpha = e,\ \mu,\ \tau).$$

The probabilities of oscillations in a vacuum

$$P(\nu_\alpha \to \nu_\beta) = \delta_{\alpha\beta} - 4\sum_{i<j} \mathrm{Re}[U_{\alpha i}U^*_{\beta i}U^*_{\alpha j}U_{\beta j}] \sin^2 \frac{\Delta m^2_{ij} L}{4E} + 2\sum_{i<j} \mathrm{Im}[U_{\alpha i}U^*_{\beta i}U^*_{\alpha j}U_{\beta j}] \sin^2 \frac{\Delta m^2_{ij} L}{2E}.$$

Mikheev-Smirnov-Wolfenstein effect amplifies oscillations.

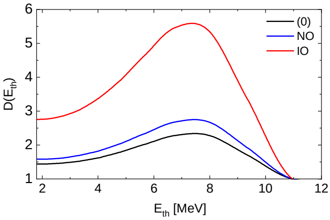
The final $\bar{\nu}_e$ flux reaching the Earth can be written as

$$S_{\bar{\nu}_e} = pS^{(0)}_{\bar{\nu}_e} + (1-p)S^{(0)}_{\bar{\nu}_x}, \quad (x = \mu,\ \tau),$$

where p is the survival probability

- $p \approx 0.68$ for the **normal mass ordering (NO)** ($m_1 < m_2 < m_3$);
- $p \approx 0.02$ for the **inverted mass ordering (IO)** ($m_3 < m_1 < m_2$).

Influence of nuclear processes on $\bar{\nu}_e$ detection



The dominant detection process for $\bar{\nu}_e$ is the inverse β-decay:

$$\bar{\nu}_e + p \rightarrow n + e^+.$$

Then e^+ are registered by Cherenkov and liquid scintillation detectors. The cross section for IBD is

$$\sigma_{\text{IBD}}(E_\nu) \sim p_{e^+} E_{e^+},$$

where $E_{e^+} = E_{\bar{\nu}_e} - (M_n - M_p)$. The minimum energy required to induce IBD is $E^{\text{min}}_{\bar{\nu} i_e} = M_n - M_p + m_e \approx 1.8$ MeV. We assume the detection efficiency 100% above the threshold $E_{\text{th}} \geq E^{\text{min}}_{\bar{\nu}_e}$. Then the number of detected events is expresses as

$$N(E_{\text{th}}) \sim \int_{E_{\text{th}}}^{\infty} \sigma_{\text{IBD}}(E_\nu) \Phi_{\bar{\nu}_e}(E_\nu) dE_\nu.$$

Detection rate enhancement factor due to the ND process:

$$D(E_{\text{th}}) = \frac{N(E_{\text{th}})}{N^*(E_{\text{th}})},$$

where $N^*(E_{\text{th}})$ is computed without the ND contribution.