4D and 6D infinite spin (super)particles and fields

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based on

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International Workshop “Problems of the Modern Mathematical Physics” (PMMΦ)
19-23 February 2024, Dubna, Russia

Our results on describing the infinite (continuous) spin representations will be presented.

In $4D$ space-time infinite spin representations are massless ($P^m P_m = 0$) unitary irreps of the Poincaré group which obey [E.Wigner, V.Bargmann]:

$$W^m W_m = -\mu^2 \neq 0\,, \qquad W_m = \tfrac{1}{2}\,\varepsilon_{mnkl} P^n M^{kl}\,.$$

Each of these representations contains infinite tower of states with all (integer or half-integer) helicities.

The same spectrum of states is used in higher spin theory [M.Vasiliev's team] that considers infinite tower of the helicity states ($W^m W_m = 0$, $W_m = \lambda P_m$).

There are some similarities higher spin theory and infinite spin theory.
But there are fundamental differences:

- in highest spin theory all spins unite with each other through interaction, whereas in infinite spin theory all spins are combined together initially;
- in infinite spin theory there is dimensionfull parameter μ.

Infinite spin representations were researched in the 2000s in many studies (see, e.g., the papers by [L.Brink, A.Khan, P.Ramond, X.Xiong, J.Mund, B.Schroer, J.Yngvason, X.Bekaert, J.Mourad, N.Boulanger, P.Schuster, N.Toro, V.Rivelles, E.Skvortsov, M.Najafizadeh, M.Khabarov, Yu.Zinoviev, K.Alkalaev, M.Grigoriev, A.Chekmenev, A.Bengtsson, R.Metsaev, K.Zhou]).

$4D$ infinite (continuous) spin fields

Since infinite spin representations are infinite-dimensional (in a fixed space-time coordinate), it is necessary

- to consider an infinite tower of space-time fields
- or consider generalized fields that depend on an additional coordinate.

To take into account symmetries, the second choice is preferred.

One common choice is to use commuting vector y^m as additional coordinate (see, for example, [E.Wigner, V.Bargmann] and [P.Schuster, N.Toro, V.Rivelles, X.Bekaert, J.Mourad, E.Skvortsov, M.Najafizadeh, M.Khabarov, Yu.Zinoviev, K.Alkalaev, M.Grigoriev, A.Chekmenev, R.Metsaev, K.Zhou]).

In such formulation, infinite spin field $\Phi(x^m, y^m)$ depends on the position vector x^m and vector coordinate y^m.

To describe irreducible infinite spin representations, the field $\Phi(x^m, y^m)$ obeys the Wigner-Bargmann equations of motion.

This talk presents different formulation of infinite (continuous) spin representations.

4D infinite spin fields with additional spinor coordinate (integer spins)

In our formulation, the additional variable is

commuting Weyl spinor ξ^α, $\bar{\xi}^{\dot\alpha} = (\xi^\alpha)^*$.

The field $\Phi(x;\xi,\bar\xi)$ describing infinite spin representation obeys the following equations of motion (our papers + [Buchbinder, Krykhtin, Takata, 2018])

$$\begin{aligned}
\partial^{\alpha\dot\alpha}\partial_{\alpha\dot\alpha}\,\Phi(x;\xi,\bar\xi) &= 0\,,\\
\left(i\xi^\alpha\partial_{\alpha\dot\alpha}\bar\xi^{\dot\alpha} + \mu\right)\Phi(x;\xi,\bar\xi) &= 0\,,\\
\left(i\frac{\partial}{\partial\xi_\alpha}\partial_{\alpha\dot\alpha}\frac{\partial}{\partial\bar\xi_{\dot\alpha}} - \mu\right)\Phi(x;\xi,\bar\xi) &= 0\,,\\
\left(\xi_\alpha\frac{\partial}{\partial\xi_\alpha} - \bar\xi_{\dot\alpha}\frac{\partial}{\partial\bar\xi_{\dot\alpha}}\right)\Phi(x;\xi,\bar\xi) &= 0\,.
\end{aligned}$$

Due to the first three equations, the field $\Phi(x;\xi,\bar\xi)$ satisfies

$$W^2\Phi(x;\xi,\bar\xi) = -\mu^2\Phi(x;\xi,\bar\xi)\,.$$

and describes irreducible massless infinite spin representation.

4D infinite spin fields with additional spinor coordinate (half-integer spins)

One possible way to describe half-integer spins is to consider the field $\Phi_\alpha(x;\xi,\bar{\xi})$ with external spinor index α.

This field describes irreducible infinite half-integer spin representation if it obeys the equations

$$
\begin{aligned}
\partial^{\dot\alpha\alpha}\,\Phi_\alpha(x;\xi,\bar{\xi}) &= 0\,,\\
\left(i\xi^\beta\partial_{\beta\dot\beta}\bar{\xi}^{\dot\beta}+\mu\right)\Phi_\alpha(x;\xi,\bar{\xi}) &= 0\,,\\
\left(i\frac{\partial}{\partial\xi_\beta}\partial_{\beta\dot\beta}\frac{\partial}{\partial\bar{\xi}_{\dot\beta}}-\mu\right)\Phi_\alpha(x;\xi,\bar{\xi}) &= 0\,,\\
\left(\xi_\beta\frac{\partial}{\partial\xi_\beta}-\bar{\xi}_{\dot\beta}\frac{\partial}{\partial\bar{\xi}_{\dot\beta}}\right)\Phi_\alpha(x;\xi,\bar{\xi}) &= 0\,.
\end{aligned}
$$

For irreducible representation we must put also additional constraint

$$
\frac{\partial}{\partial\xi_\alpha}\,\Phi_\alpha(x;\xi,\bar{\xi})=0\,.
$$

Twistor infinite spin fields

The spin composition of the resulting fields is clear in the twistor formulation. The fields in these two formulations are related to each other by

the twistor transform:

$$\Phi(x;\xi,\bar{\xi}) = \int d^4\pi\, e^{\,ip_{\beta\dot{\beta}}x^{\dot{\beta}\beta}}\, \Psi^{(0)}(\pi,\bar{\pi};\xi,\bar{\xi})\,,$$

$$\Phi_\alpha(x;\xi,\bar{\xi}) = \int d^4\pi\, e^{\,ip_{\beta\dot{\beta}}x^{\dot{\beta}\beta}}\, \pi_\alpha\, \Psi^{(-1/2)}(\pi,\bar{\pi};\xi,\bar{\xi})\,,$$

where $\quad p_{\alpha\dot{\alpha}} = \pi_\alpha\bar{\pi}_{\dot{\alpha}} \quad \Rightarrow \quad p^{\dot{\alpha}\alpha}p_{\alpha\dot{\alpha}} = 0$

Commuting spinors π_α and ξ_α determine half the components of two twistors:

$$Z_A = \left(\pi_\alpha, \bar{\omega}^{\dot{\alpha}}\right)\,, \qquad Y_A = \left(\mu^{1/2}\xi_\alpha, \bar{\eta}^{\dot{\alpha}}\right).$$

$\Psi^{(c)}(\pi,\bar{\pi};\xi,\bar{\xi})$ are the twistor fields

(prepotentials for the fields $\Phi(x;\xi,\bar{\xi})$, $\Phi_\alpha(x;\xi,\bar{\xi})$).

c is the $U(1)$ charge of the twistor fields.

Helicity expansions of infinite spin fields

The twistor fields are represented in the form

$$\Psi^{(c)}(\pi,\bar\pi;\xi,\bar\xi) \;=\; \delta\left((\pi\xi)(\bar\xi\bar\pi)-2\mu^2\right)\, e^{-i\mu^{1/2}\left(\dfrac{\xi_1}{\pi_1}+\dfrac{\bar\xi_1}{\bar\pi_1}\right)}\,\hat\Psi^{(c)}(\pi,\bar\pi;\xi,\bar\xi)\,,$$

$$\hat\Psi^{(c)}(\pi,\bar\pi;\xi,\bar\xi) = \psi^{(c)}(\pi,\bar\pi) + \sum_{k=1}^{\infty}(\bar\xi\bar\pi)^k\,\psi^{(c+k)}(\pi,\bar\pi) + \sum_{k=1}^{\infty}(\pi\xi)^k\,\psi^{(c-k)}(\pi,\bar\pi)\,.$$

The coefficient functions $\psi^{(c\pm k)}(\pi,\bar\pi)$ obey the condition

$$\Lambda\cdot\psi^{(c\pm k)}(\pi,\bar\pi) \;=\; -(c\pm k)\,\psi^{(c\pm k)}(\pi,\bar\pi)\,,$$

where $\Lambda = \dfrac{\vec J\vec P}{P_0} = \dfrac{W_0}{P_0}$ is the helicity operator.

Thus, twistor fields $\Psi^{(0)}(\pi,\bar\pi;\xi,\bar\xi)$ and $\Psi^{(-1/2)}(\pi,\bar\pi;\xi,\bar\xi)$ contain in their expansions all integer and half-integer helicities respectively.

Infinite spin supermultiplet

Similar to the Wess-Zumino supermultiplet,
the fields $\Phi(x;\xi,\bar{\xi})$ and $\Phi_\alpha(x;\xi,\bar{\xi})$ with integer and half-integer helicities are unified into one $\mathcal{N}=1$ supermultiplet.

On-shell supersymmetry transformations of the fields Φ and Φ_α are represented in the form

$$\delta\,\Phi \;=\; \varepsilon^\alpha\Phi_\alpha\,, \qquad \delta\,\Phi_\alpha \;=\; 2i\bar{\varepsilon}^{\dot\beta}\partial_{\alpha\dot\beta}\Phi\,.$$

Supersymmetry transformations for the infinite spin twistor fields $\Psi^{(0)}(\pi,\bar\pi;\xi,\bar\xi)$ and $\Psi^{(-1/2)}(\pi,\bar\pi;\xi,\bar\xi)$ are

$$\delta\,\Psi^{(0)} \;=\; \varepsilon^\alpha\pi_\alpha\Psi^{(-1/2)}\,, \qquad \delta\,\Psi^{(-1/2)} \;=\; -2\,\bar\varepsilon^{\dot\alpha}\pi_{\dot\alpha}\Psi^{(0)}\,.$$

This infinite-component supermultiplet stratifies into an infinite number of levels with pairs of the fields $\psi^{(k)}$, $\psi^{(-\frac{1}{2}+k)}$ at fixed $k\in\mathbb{Z}$. Supersymmetry transforms bosonic and fermionic fields into each other inside a given level k. The boosts of the Poincaré group mix the fields with different values of k.

BRST Lagrangian formulation of infinite spin fields

BRST field description of infinite spin fields is presented for both the case of integer spins ([Buchbinder, Krykhtin, Takata, 2018]) and half-integer spins.

Summarizing the results:

- Spinor quantities ξ_α, $\bar{\xi}_{\dot{\alpha}}$, $\partial/\partial\xi_\alpha$, $\partial/\partial\bar{\xi}_{\dot{\alpha}}$ are realized by using creation and annihilation operators in the Fock space.
- BRST charge is constructed by using operators of the first class constraints, which determine equations of motion for infinite spin fields.
- To obtain the off-shell Lagrangian formulation, it is necessary to use triplet of infinite spin fields, both in the case of integer spins and in the case of half-integer spins.

A similar picture takes place in off-shell superfield Lagrangian formulation of infinite spin supermultiplet which will be presented very briefly.

Off-shell superfield Lagrangian formulation of $4D$, $\mathcal{N}=1$ infinite spin theory

We use three (anti)chiral superfield ket-vectors

$$
\begin{aligned}
|\Phi(x_L,\theta)\rangle &= |\phi(x_L)\rangle + \theta^\alpha|\psi_\alpha(x_L)\rangle + \theta^\alpha\theta_\alpha|f(x_L)\rangle\,, \\
|\bar{S}_1(x_R,\bar\theta)\rangle &= |e(x_R)\rangle + \bar\theta_{\dot\alpha}|\bar\psi_1^{\dot\alpha}(x_R)\rangle + \bar\theta_{\dot\alpha}\bar\theta^{\dot\alpha}|\phi_1(x_R)\rangle\,, \\
|S_2(x_L,\theta)\rangle &= |\phi_2(x_L)\rangle + \theta^\alpha|\psi_{2\alpha}(x_L)\rangle + \theta^\alpha\theta_\alpha|g(x_L)\rangle\,,
\end{aligned}
$$

where $x_L^m = x^m + i\,\theta\sigma^m\bar\theta$, $x_R^m = x^m - i\,\theta\sigma^m\bar\theta$.

Superfield Lagrangian is written as follows:

$$
\begin{aligned}
\mathcal{L}_{super} = \; & \int d^2\theta d^2\bar\theta \Big(\langle\bar\Phi|\Phi\rangle - \langle S_1|K|\bar{S}_1\rangle - \langle\bar{S}_2|S_2\rangle\Big) \\
& + \int d^2\theta \Big(\langle S_1|(\ell-\mu)|\Phi\rangle - \langle S_1|(\ell^+-\mu)|S_2\rangle\Big) \\
& + \int d^2\bar\theta \Big(\langle\bar\Phi|(\ell^+-\mu)|\bar{S}_1\rangle - \langle\bar{S}_2|(\ell-\mu)|\bar{S}_1\rangle\Big),
\end{aligned}
$$

where ℓ, ℓ^+, K are counterparts of $\xi^\alpha\partial_{\alpha\dot\alpha}\bar\xi^{\dot\alpha}$, $\dfrac{\partial}{\partial\xi_\alpha}\partial_{\alpha\dot\alpha}\dfrac{\partial}{\partial\bar\xi_{\dot\alpha}}$, $\xi_\alpha\dfrac{\partial}{\partial\xi_\alpha}+\bar\xi_{\dot\alpha}\dfrac{\partial}{\partial\bar\xi_{\dot\alpha}}$.

$6D$ infinite spin representations

Poincaré group in $6D$ space has three Casimir operators. They are defined as the squares of tensors

$$P_m\,, \qquad W_{mnk}=\varepsilon_{mnklpr}P^l M^{pr}\,, \qquad \Upsilon_m=\varepsilon_{mnklpr}P^n M^{kl}M^{pr}\,.$$

Casimir operators:

$$C_2=P^mP_m\,, \qquad C_4=\frac{1}{24}\,W^{mnk}W_{mnk}\,, \qquad C_6=\frac{1}{64}\,\Upsilon^m\Upsilon_m\,.$$

On states of irreducible massless unitary representations of infinite spin:

$$C_2=0\,, \qquad C_4=-\mu^2\,, \qquad C_6=-\mu^2 s(s+1)\,.$$

Thus, such representations are characterized by two quantum numbers

$\mu\in\mathbb{R}_{>0}$ and $s\in\mathbb{Z}_{\geq 0}/2$.

$6D$ infinite spin fields

As additional coordinate, we use $6D$ spinor $\xi^{\rm A}_\alpha$,
where $\rm A$ is $SU(2)$ index, α is $SU^*(4) \simeq Spin(1,5)$ index

$6D$ infinite spin fields have external spinor indices:

$$\Phi_{\alpha_1\ldots\alpha_{2s}}(x,\xi) \;=\; \Phi_{(\alpha_1\ldots\alpha_{2s})}(x,\xi)\,.$$

Equations of motion of the $6D$ infinite spin fields:

$$\partial^{\beta\alpha_1}\,\Phi_{\alpha_1\ldots\alpha_{2s}} \;=\; 0\,, \qquad \partial^m\partial_m\,\Phi = 0 \;\text{ at }\; s=0\,,$$

$$\big(i\xi^{\rm A}_\beta\partial^{\beta\gamma}\xi_{\gamma\rm A} + \mu\big)\,\Phi_{\alpha_1\ldots\alpha_{2s}} \;=\; 0\,,$$

$$\left(i\frac{\partial}{\partial\xi^{\rm A}_\beta}\partial_{\beta\gamma}\frac{\partial}{\partial\xi_{\gamma\rm A}} - \mu\right)\Phi_{\alpha_1\ldots\alpha_{2s}} \;=\; 0\,,$$

$$\xi^{\rm A}_\beta(\sigma_a)_{\rm A}{}^{\rm B}\frac{\partial}{\partial\xi^{\rm B}_\beta}\,\Phi_{\alpha_1\ldots\alpha_{2s}} \;=\; 0\,, \qquad a{=}1{,}2{,}3\,.$$

Twistor formulation of $6D$ infinite spin fields

Integral twistor transform

$$\Phi_{\alpha_1\ldots\alpha_{2s}}(x,\xi) = \int \mu(\pi)\, e^{ip_m x^m}\, \pi_{\alpha_1}^{A_1}\ldots\pi_{\alpha_{2s}}^{A_{2s}}\, \Psi_{A_1\ldots A_{2s}}(\pi,\xi)$$

links space-time fields $\Phi_{\alpha_1\ldots\alpha_{2s}}(x,\lambda)$ and bi-twistor fields $\Psi_{A_1\ldots A_{2s}}(\pi,\xi)$. Here

$$p_m = \pi_\alpha^A(\tilde{\sigma}_m)^{\alpha\beta}\pi_{\beta A} \qquad \Rightarrow \qquad p_m p^m = 0\,.$$

Spinors π_α^A, ξ_α^A define half components of two $6D$ twistors ($\mathrm{Spin}(2,6)$ spinors).

BRST formulation of $6D$ infinite spin fields

- Within the framework of the BRST approach, Lagrangian formulation of infinite spin fields for arbitrary μ and s has been constructed.
- Conditions defining the irreducible representation were reformulated as constraints on the vectors of the Fock space.
- It is necessary to double the number of states due to fields depending on spinor operators with indices of opposite spinor chirality.
- To obtain $\mathrm{SU}(2)$ covariant formulation, the use of $\mathrm{SU}(2)/\mathrm{U}(1)$ harmonics [GIKOS, 1984] is required.

Light-front description of $6D$ infinite spin fields

Coordinate $x^+ = x^0 + x^5$ is interpreted as a "time" evolution parameter.
The role of the Hamiltonian is played by $H = P^-$.

In light-front (not Lorentz covariant) formulation, infinite spin field

$$\Phi^{(2s)}(x^{\pm}, x^{\hat{a}}, u^{\pm}, v^{\pm})$$

has polynomial expansion in $u_i^{\pm}$, $v_{\underline{i}}^{\pm}$ (components of ξ_α^{A} and its momentum)
They are $[\mathrm{SU}(2) \otimes \mathrm{SU}(2)]/\mathrm{U}(1)$ harmonics. $(2s)$ is the harmonic charge.

Action of infinite spin field in this formulation has the form

$$S \; = \; \int d^{\,6}x \, du \, dv \; \bar{\Phi}^{(-2s)} \Box \, \Phi^{(2s)} \, .$$

Derived harmonic light-front approach opens a possibility to construct an interacting theory for $6D$ infinite spin fields: $H \;\rightarrow\; H + H_{\mathrm{int}}$
H_{int} should have zero harmonic charge $\Rightarrow$

$H_{\mathrm{int}} \sim \bar{\Phi}^{(-2s)} \bar{\Phi}^{(-2s)} \Phi^{(2s)} \Phi^{(2s)}$ for self-interaction of charged fields;

$H_{\mathrm{int}} \sim \Big(\Phi_1^{(q_1)} \Phi_2^{(q_2)} \Phi_3^{(q_3)} + c.c. \Big)$, $q_1 + q_2 + q_3 = 0$ for fields with different charges.

Conclusion

In a series of papers, we obtained the following results:

- Field description of $4D$ infinite (continuous) spin representations with additional commuting spinor variables is obtained.
- Corresponding twistor description of such a model has been found.
- Superfield description of $4D$ infinite spin superparticle is obtained.
- Using BRST approach, the Lagrangians described $4D$ infinite spin fields and superfields are obtained.
- Space-time (with additional spinor variable) and twistor descriptions of $6D$ infinite spin representations are obtained.
- Lagrangian descriptions of $6D$ infinite spin fields are found in BRST approach and in light-front formulation.

Thank you very much for your attention !